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The Bank of Canada's New Quarterly Projection Model

Part 1

The Steady-State Model: SSQPM

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CONTENTS

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ABSTRACT

This report is the first documenting the Bank of Canada's new model of the Canadian economy, the Quarterly Projection Model (QPM). QPM is used at the Bank of Canada for both economic projections and policy analysis. Here the authors focus on the model's long-run properties, describing SSQPM, a model of the steady state of QPM that is maintained separately and used to study the determinants of long-run equilibrium in the economy and the permanent effects of economic disturbances or changes of policy.

SSQPM is based on the simple Blanchard-Weil model with overlapping generations. In such a model, household preferences determine the steady-state level of financial wealth, relative to output. The equilibrium is achieved primarily through variation in the level of net foreign assets. This then determines foreign debt service and the capital account of the balance of payments. Given these "asset" considerations, the current account identities provide the required trade balance, and in SSQPM, the real exchange rate adjusts to ensure that this level of trade is achieved.

The authors present the simplest form of such a model and then introduce a series of elaborations and extensions that are judged necessary to support a working projection environment. They then describe the choices made by Bank staff in calibrating the model and the numerical steady state that emerges. Finally, the authors describe the properties of SSQPM, as revealed by its responses to a number of shocks to exogenous variables.

RÉSUMÉ

Le présent rapport est le premier à traiter du nouveau modèle de l'économie canadienne de la Banque du Canada, soit le Modèle trimestriel de prévision (MTP). Le MTP est utilisé à la Banque du Canada aux fins aussi bien d'analyse de politiques que de projections économiques. Dans ce rapport, les auteurs se concentrent sur les propriétés de long terme du modèle et décrivent le MTPRP, un modèle distinct de régime permanent du MTP, qui sert à l'étude des déterminants de l'équilibre de long terme dans l'économie et des effets permanents des chocs économiques ou des modifications des politiques.

Le MTPRP repose sur le modèle simple de Blanchard-Weil à générations imbriquées. Dans un modèle de ce type, les préférences des ménages déterminent le rapport de la richesse financière à la production en régime permanent. L'équilibre est atteint grâce surtout à la variation du niveau des actifs étrangers nets, qui détermine à son tour le service de la dette extérieure et le compte de capital de la balance des paiements. Compte tenu de ces considérations axées sur les actifs, les identités de la balance courante fournissent le niveau requis de la balance commerciale; dans le MTPRP, le taux de change réel s'ajuste de façon à ce que cette dernière atteigne le niveau qui lui est assigné.

Les auteurs du rapport présentent la forme la plus simple d'un modèle de type Blanchard-Weil, puis une série de précisions et d'extensions qu'ils jugent nécessaires pour soutenir l'élaboration des projections. Ils expliquent ensuite les choix effectués par la Banque dans l'étalonnage du modèle et les valeurs numériques du régime permanent qui en résulte. Enfin, ils donnent une description des propriétés du MTPRP, telles qu'elles se dégagent des réactions de celui-ci à plusieurs chocs qui ont été appliqués sur les variables exogènes.

1 INTRODUCTION

Staff at the Bank of Canada have recently begun using a new computer simulation model of the Canadian economy for projections and policy analysis. That model is called MTP / QPM from *Modèle trimestriel de prévision* / Quarterly Projection Model. In discussions in English, we refer to it as simply QPM.

The structure of QPM differs somewhat from that of previous models maintained at the Bank and elsewhere. In any model of an economy, decisions must be taken as to what will be explained in the simultaneous core and what will be left exogenous or explained by recursive structure that does not influence the core variables. Also, decisions must always be taken as to what level of formality is to be used in specifying the longer-term equilibrium that the model may attain and how such considerations should influence the modelling of shorter-term dynamics.

In most models, these elements are blended into a single product and the distinctions are unclear. At the Bank of Canada, we have decided to break the pieces apart and create a system of interrelated models. For example, we maintain a separate model of the steady state, which we call SSQPM, and use it to establish the long-term equilibrium conditions for the main model, QPM. QPM provides the dynamic path followed by the core macro variables of the economy from the initial conditions to the steady state described by SSQPM.

Finally, we maintain a number of satellite models that are used to create a more detailed scenario consistent with the macro scenario generated by QPM, but which we have chosen to keep separate in the sense that no formal feedback is permitted from the satellite models to QPM and the macro solution. The QPM system is therefore very much a top-down macroeconomic model.

This report deals with SSQPM. We describe the underlying theory of the steady state and the numerical values the model attains under the chosen calibration. We also illustrate the response of the numerical steady state to a number of shocks to exogenous variables. We limit discussion in this report to comparative statics;[1] we have nothing to say about the adjustment paths implied by the shocks. That is the role of QPM. However, since QPM is dynamically stable under all the shocks considered here, the properties we report are relevant as descriptions of the properties of equilibria in the larger model.

In many cases, the study of equilibrium effects of permanent shocks is interesting in its own right, and we have found it useful to have the independent SSQPM for this purpose. Moreover, for many questions that arise in practical policy analysis, such as how to respond to temporary shocks, the steady state is not affected and it is convenient to have a dynamic model that exploits this fact. Nevertheless, the more interesting issues of practical policy analysis are usually issues of dynamic paths; in this sense, the analysis we provide here is merely an appetizer to the full meal that will follow in the documentation of the properties of QPM.

Consider the issue of what a model of a steady state must achieve. We focus on three particularly important points. First, full macroeconomic equilibrium requires that all stocks attain a fixed level, relative to output. The structure of the model must respect not just the accounting identities linking stocks and flows, but also behavioural conditions that determine levels for the stocks in a framework that takes into account all intertemporal budget or financing constraints.[2] Second, there must be a market structure with clearly identified prices (or quantities) that are determined to support the notional equilibrium. Moreover, users ought to be free to change the assumptions about exogenous variables or parameters and see the implications in terms of endogenous variables responding according to consistent, rational behavioural rules.[3] In other words, the model must be

1. Strictly speaking, we deal with comparative steady-state dynamics, since the analysis is always about the properties of the steady-state growth path.

2. Note that this implies that the corresponding flows must also settle at fixed proportions of output.

3. Note that we are not discussing dynamic stability here; that is another important issue, but for the dynamic model, not the steady-state model.

able to describe how an equilibrium is determined, and the solution should not depend on arbitrary assumptions about misinformation, persistent errors or myopia on the part of agents, or any other arbitrary rigidities in markets.[4] Third, for a model to be useful for policy analysis it must elaborate a clear structure for the way policy choices influence outcomes in the overall economy. Many of the more interesting aspects of this issue involve dynamics, but the discussion must start with the steady state.

In SSQPM, we use a high level of formality in developing the theory of household decisions. We use an overlapping generations framework developed by Yaari (1965), Blanchard (1985), Buiter (1988), Weil (1989) and Frenkel and Razin (1992). This model has become standard for open economy analysis because it can explain the determination of a country's net foreign asset position and the consequences for the trade balance and (with some elaboration) the real exchange rate, which many of its simple competitors cannot do. We modify the basic version of the model, adding a number of features aimed at reconciling it with the data and adapting it for use in practical policy analysis.

The model is completed with specifications of the behaviour of firms and government, as well as an elaboration of the links to the external sector. The model takes most foreign variables as exogenous. We do, however, abandon the extreme small-open-economy assumptions made in some models and substitute a specification of what we call the "almost small open economy." An important example is that we do not assume that the world price of Canadian exports is determined solely by world conditions. Rather, we specify that to increase the level of exports, all else assumed the same, the world price of Canadian exports must fall.

We specify atomistically competitive firms that maximize profits. The main role of this behaviour in the steady state is to ensure that the real wage is equated to the marginal product of labour and that the level of capital is consistent with zero excess profits and full employment of labour.

4. Such issues can certainly be entertained as part of policy analysis. However, this should be considered and deliberate and not the result of lack of attention.

For a steady state to exist in the usual sense, it is necessary to have stable levels of government spending and debt as a proportion of output. We treat these steady-state ratios as policy choices. The government budget or financing constraint is respected by having taxes on households adjust to the level required to support the choices. In essence, taxes must cover spending and debt service, net of any new debt issue necessary to sustain the ratio of debt to output.

The elaboration of the links to the world economy and, in particular, how the household sector's preferences influence the steady state for net foreign assets are very important. In this model, households determine the steady-state level of financial wealth. With firms determining the level of physical capital and government determining the level of its debt, reconciliation with household preferences comes through variation in the level of net foreign assets, relative to output.[5] This, in turn, pins down the foreign debt-service requirement and the capital account of the balance of payments.[6] With these "asset" considerations, the current account identities determine the required level of the trade balance. In SSQPM, the equilibrium real exchange rate is the key price that adjusts to ensure that this level of trade is achieved. One can think of this last step as setting a price, given a foreign demand curve, such that the appropriate quantity is exported, where "appropriate" has an elaborate definition coming from the optimizing decisions of domestic agents and the policy decisions of government.

Section 2 describes the formal theory that supports SSQPM. We start with a simplified one-good framework, in which we can present the model clearly and provide closed-form solutions for the main variables. This leads to a model we call the simple, overlapping generations model (SOLGM). Then, in Section 3, we describe the modifications introduced to make the model more realistic and suitable for providing the steady state for QPM. The result is the SSQPM model. In SOLGM, for example, there is

5. The model is calibrated to respect the fact that Canadians have typically been net debtors of the rest of the world, but the accounting is done from the "asset" perspective.

6. In a growing economy, a fixed *ratio* of net foreign assets to output implies absolute capital flows in each period, except in the very special case of a zero desired stock.

only one tax, a lump-sum tax on households. For SSQPM the tax system is expanded to allow for indirect taxes and taxes on profits. There are numerous other elaborations, all designed to support reconciliation of a simplified model with the data and use of the model for practical policy analysis.

Section 4 describes the calibration of the model and the numerical steady state it attains. For a description of the determinants of steady state, traditional econometric methods are rarely applicable. We do, however, use the data, wherever possible, in choosing the model's parameters and in setting values for certain key unobservable variables. An example of the latter is the rate of time preference, which we set, along with other things, to reconcile the model's predictions with the stylized facts with respect to Canada's net foreign debt.

In many cases, especially where policy choices are involved, the past may not provide a very good indicator as to sensible choices for future stationary values. A good example of this arises in the choice of the ratio of government debt to output. In such cases, we have relied on the judgment of specialists in constructing a set of assumptions that we think is defensible. An advantage of the rigorous framework provided by SSQPM is that all such assumptions are brought together and made consistent, which allows us to provide a check on the overall macro plausibility of the various individual choices.

In Section 5, we describe a number of shocks to exogenous variables that are designed to illustrate the properties of SSQPM and to enable us to quantify some of the points made analytically in the theoretical discussion. This is accompanied by some pure sensitivity analysis, where we report the impact of particular calibration decisions on the numerical steady state of the model.

We end with a brief summary of this report and its main conclusions along with a sketch of the additional documentation on QPM to come.

2 A SIMPLE MODEL WITH OVERLAPPING GENERATIONS

In this section, we present a simple macro model with overlapping generations. At the heart of this model is a description of the consumption and saving decision and the associated choice of a level of wealth by the household sector. This model was first suggested by Yaari (1965), then popularized by Blanchard (1985) and extended by Buiter (1988) and by Weil (1989), among others. It provides key elements of the steady state of the economy described by QPM. The model is completed with a discussion of the behaviour of firms and government and a description of the role of the links to the external sector in supporting the domestic steady state. The system that emerges constitutes SOLGM (simple, overlapping generations model).

In a closed economy, one could consider models centred around an infinitely lived, representative consumer with a constant rate of time preference. Models in this class are simplistic, but they do satisfy many of the requirements for modern economic analysis. Unfortunately, for a small open economy, like Canada's, such models are seriously incomplete, since they fail to define the country's net foreign asset position. One implication of the small-open-economy assumption is that Canada is a price-taker on financial markets (that is, the equilibrium real interest rate is given exogenously by world markets). In such a world, economic agents can borrow an unlimited amount from foreigners without affecting the interest rate. If Canadians were less patient than consumers in the rest of the world, they would choose to borrow unlimited amounts to indulge their desire for consumption. There is no steady state in such a world.

It is possible to adapt the infinitely lived consumer model to remove this problem. One method is to weaken the small-open-economy assumption by introducing a risk premium such that the more agents borrow, the higher is the interest rate that they face.[7] The net foreign asset position is defined when the real interest rate faced by consumers is equal to their rate

7. This approach was adopted for the Small Annual Model (SAM). See Rose and Selody (1985). See also Black, Macklem and Poloz (1994) and Obstfeld (1980, 1981).

of time preference. Another method is to assume that the rate of time preference increases with wealth so that the higher the level of borrowing, the less a consumer will *want* to borrow; this approach is less appealing because it assumes that the consumer's utility function has a convenient form. There is no reason to believe that wealth should affect a consumer's rate of time preference in any particular way.[8]

An advantage of SOLGM is that it can support a steady state with a determined path for net foreign assets without appealing to the special assumptions described above. In this model, new generations are born with only human wealth, and gradually they accumulate financial assets.[9] Individuals who survive for a long time may end up with large amounts of financial wealth; but aggregate wealth still converges to a steady-state growth path.[10] This approach does not preclude the possibility of incorporating a risk premium that increases with indebtedness, or a rate of time preference that increases with wealth; however, neither addition is *necessary* to determine the net foreign asset position.[11]

Although models like SOLGM have been discussed extensively in the literature, we review its structure here to establish the basis for our extensions in SSQPM. Our exposition, which uses a discrete-time representation, most closely resembles that found in Frenkel and Razin (1992). First, the individual consumer's problem is presented. We then show how aggregation over individuals in a growing economy results in closed-form solutions for per capita consumption and wealth. SOLGM is completed with the addition of a government, competitive firms and a foreign sector.

8. See Mendoza (1991) and references therein, as well as Svensson and Razin (1983).

9. Throughout our discussion we assume that consumers choose to hold a positive amount of financial wealth.

10. Convergence is not absolutely guaranteed. However, the conditions under which a stable steady state does exist are not particularly stringent.

11. Laxton and Tetlow (1992) overlay a varying risk premium on a model similar to the one presented below. See also Macklem, Rose and Tetlow (1994).

2.1 The individual consumer's problem

In our hypothetical economy, individual consumers determine their desired levels of consumption and holdings of financial assets in order to maximize the discounted sum of expected utility over their lifetime, subject to a lifetime budget constraint. They supply labour exogenously and in return are paid a fixed wage income, net of tax, each period. The only uncertainty facing consumers is when they will die. To facilitate aggregation, we assume that the probability of death is constant through time and independent of the consumer's age.

For a consumer born at the end of time period *a*, the problem is to maximize expected lifetime utility for $t > a$:[12]

$$\max_{\{c_{a,t+s}\}_{s=0}^{\infty}} \quad E_t\left(\sum_{s=0}^{\infty} \delta^s u(c_{a,t+s}, \varepsilon_{a,t+s})\right), \tag{1}$$

subject to the budget constraint that, in each period, expenditures, including asset accumulation, must equal income:

$$c_{a,t+s} + fa_{a,t+s} = y^{lab}_{a,t+s} + (1 + r^c_{t+s-1}) fa_{a,t+s-1}, \tag{2}$$

where $c_{a,t+s}$ is the consumption at time $t+s$ of each individual born in period *a*, $fa_{a,t+s}$ is their end-of-period financial asset holdings, which earn r^c_{t+s} in interest, and $y^{lab}_{a,t+s}$ is their after-tax wage income. Future utility, $u(c_{a,t+s}, \varepsilon_{a,t+s})$, is discounted at rate δ.[13] Finally, $\varepsilon_{a,t+s}$ is a stochastic variable indicating whether the consumer is alive or dead.

Assuming that utility is zero when the consumer is dead, the problem can be reformulated as its equivalent under certainty:

12. Note that throughout the model the convention is that births and deaths occur at the end of each time period. One of the implications of this assumption is that the first period of consumption, for a consumer born at time a, is $a + 1$.

13. The rate of time preference or pure discount rate, ρ, is related to the discount factor by $\delta = 1/(1+\rho)$, so for $\rho > 0$, $\delta < 1$.

$$\max_{\{c_{a,t+s}\}_{s=0}^{\infty}} \sum_{s=0}^{\infty} (\delta\gamma)^s u(c_{a,t+s}) , \tag{3}$$

subject to (2). This has the same structure as the problem faced by an infinitely lived consumer, but with a discount factor of $\delta\gamma$ that is modified to include the influence of the probability of survival, γ. The extra discounting introduced shortens the effective planning horizon. This permits the model to have the attributes of an overlapping generations framework while retaining the analytical simplicity of the infinitely lived representative agent model.

A loose end that must be taken care of is "What happens to consumers' financial assets when they die?"[14] One institutional arrangement is a compulsory insurance scheme. Consumers contract with the state to receive a dividend each period they survive, proportional to their financial wealth, and in exchange the state collects their assets when they die. If we assume that there are a large number of individuals, the number of deaths is a constant and known proportion of the existing population, even though individual consumers are uncertain as to when they will die. This means that for every dollar of assets, the state collects, with certainty, $1-\gamma$, on which interest accrues at the market rate. In return, the state transfers $\gamma\pi$, where π is the contracted dividend, to surviving consumers. Assuming that the scheme is run as pure insurance, we have $(1-\gamma)(1+r) = \gamma\pi$.[15] The return to consumers is given by the market interest rate they receive from their investments plus the dividend:

$$1 + r^c_{t+s-1} = (1 + r_{t+s-1}) + \pi = \frac{1 + r_{t+s-1}}{\gamma} . \tag{4}$$

14. We assume that consumers are not allowed to leave bequests (positive or negative). This is not essential, but it does facilitate the analysis. As Weil (1989) has shown, the steady state remains well defined if the birth rate is considered internal to separate "dynasties" rather than applicable to individuals. Bequests among members of a dynasty do not destroy the existence of a steady state.

15. Following Blanchard (1985), the same scheme could be run by private insurance companies, with the assumption that there is perfect competition in the insurance business.

The importance of this point is that the rate for consumers, which they use to discount future income, is above the market rate of interest.

To provide a specific closed-form solution, we need to specify a particular utility function. We assume that the utility function exhibits constant relative risk aversion with an elasticity of intertemporal substitution σ:[16]

$$U_j = \left(\frac{\sigma}{\sigma - 1}\right) C_j^{\frac{(\sigma - 1)}{\sigma}} \quad . \tag{5}$$

With this assumption, the solution of the individual consumer's optimization problem yields the following predictions about behaviour. At each time $t > a$, the individual consumes at a rate proportional to total wealth, $tw_{a,t}$:

$$c_{a,t} = \Omega tw_{a,t} \tag{6}$$

$$\Omega = 1 - \gamma\delta^{\sigma}(1 + r)^{\sigma - 1} , \tag{7}$$

where Ω is the marginal propensity to consume out of wealth. For risk-neutral agents, for whom $\sigma = 1$, this marginal propensity is simply one minus the discount factor – the more agents discount the future, whether because they expect to die or because they simply prefer present to future consumption, the more they consume out of their total wealth each period. In general, however, the propensity to consume out of wealth also depends on the real interest rate and the elasticity of intertemporal substitution.

Total wealth has two components: human wealth, which is defined as the sum of future labour earnings discounted at the interest rate faced by the consumer; and financial wealth, which is the value of their financial assets at the start of the period, plus accrued interest:

$$tw_{a,t} = hw_{a,t} + fw_{a,t} \tag{8}$$

16. The coefficient of relative risk aversion is simply $1/\sigma$.

$$hw_{a,t} = \sum_{s=0}^{\infty} \left(\frac{\gamma}{1+r}\right)^s y_{a,t+s}^{lab} \tag{9}$$

$$fw_{a,t} = \frac{(1+r)}{\gamma} fa_{a,t-1} . \tag{10}$$

At this stage no restriction has been placed on labour income; however, for the solution to be well defined, it is necessary for human wealth, as defined by (9), to be finite. If we introduce technical progress, such that an individual's income grows at a constant rate ζ, irrespective of age, then a necessary and sufficient condition for human wealth to be finite is

$$\frac{\gamma(1+\zeta)}{1+r} < 1 . \tag{11}$$

This says that productivity growth cannot exceed the effective discount rate, which is not a particularly stringent requirement. We return to this issue in subsection 2.3 in discussing the stability of the aggregate system.

To complete this part of the discussion, we must define how an individual's wealth accumulates. This is simply a matter of enforcing the budget constraint and the condition that, at birth, the consumer has no financial assets and hence no financial wealth:

$$tw_{a,a+1} = hw_{a,a+1} \tag{12}$$

$$fw_{a,t} = \frac{1+r}{\gamma}(fw_{a,t-1} + y_{a,t-1}^{lab} - c_{a,t-1}), \tag{13}$$

where (13) is the consumer's budget constraint, (2), rewritten in terms of financial wealth. Using (8) and the subsequent development above, we can derive the following summary condition describing the evolution of the individual's total wealth:

$$tw_{a,t} = \frac{(1+r)}{\gamma}(1-\Omega)tw_{a,t-1} . \tag{14}$$

2.2 Aggregate consumer behaviour

Aggregation across individuals completes the overlapping generations framework and determines the aggregate level of financial wealth. An essential additional complication at this point is to introduce a birth rate. A death rate allows us to specify the individual problem with a higher rate of discounting, with the advantages noted above. A birth rate is then logically necessary to prevent the economy from disappearing, although its real role is to allow us to describe a growing economy. A practical model of a growing economy requires a growing population as well as rising productivity.

Let the constant birth rate be β.[17] Recall that γ is the probability of survival. Assuming a large number of consumers, the proportion remaining in the population at the end of each period can be taken as constant and equal to γ. Together, these stipulations imply that the population growth rate, n, of the economy is constant and given by

$$1 + n = \gamma(1 + \beta) . \qquad (15)$$

With no loss in generality, we assume that the population at time zero is one. For what follows, we also normalize the level of productivity to one at time zero.

When we aggregate across individual consumers, it is convenient to write the aggregate variables in a particular way – deflated to remove the influence of growth. In all that follows, we denote the aggregate quantity of a variable X, expressed in per capita terms and deflated by the productivity index, by x_t.[18] That is,

17. "Birth" should be interpreted as including net immigration.

18. For readers wishing to replicate the derivation of the aggregate equations, the following information will be useful. The size of the generation born in time a, at time $t > a$, is $\beta(1+n)^a\gamma^{t-a}$ – the probability of being born, β, times the size of the generation prior to birth, $(1+n)^a$, times the probability of surviving to time t, γ^{t-a}. The quantity of X assigned to this generation is $\beta\gamma^{t-a}(1+n)^a x_{a,t}$. Aggregating over generations and adjusting for labour-augmenting productivity growth ζ, and population growth n:

$$x_t = \frac{1}{(1+n)^t(1+\zeta)^t}\sum_{a=-\infty}^{t-1}\beta\gamma^{t-a}(1+n)^a x_{a,t} = \frac{1}{(1+\zeta)^t}\sum_{a=1}^{\infty}\beta\left(\frac{\gamma}{1+n}\right)^a x_{t-a,t} \quad .$$

$$x_t = X_t / ((1+n)^t (1+\zeta)^t) \quad . \tag{16}$$

The levels of the variables at any point in time can be computed by inverting (16).

The resulting model of aggregate behaviour is written as follows:

$$c_t = \Omega t w_t \tag{17}$$

$$\Omega = 1 - \gamma \delta^{\sigma} (1+r)^{\sigma - 1} \tag{18}$$

$$tw_t = hw_t + fw_t \tag{19}$$

$$hw_t = \frac{1+r}{1+r-\gamma(1+\zeta)} y^{lab} \tag{20}$$

$$fw_t = \frac{1+r}{(1+n)(1+\zeta)} fa_{t-1} \tag{21}$$

$$fw_t = \frac{1+r}{(1+n)(1+\zeta)} (fw_{t-1} + y^{lab} - c_{t-1}) \, . \tag{22}$$

In addition, the dynamic accumulation of total wealth is given by

$$tw_t = \frac{(1+r)(1-\Omega)}{(1+n)(1+\zeta)} tw_{t-1} + \left(\frac{\beta}{1+\beta}\right) hw_t \quad . \tag{23}$$

While the aggregate system resembles closely the model of individual behaviour, there are important differences. Note, in particular, the difference embodied in (22) and carried through to (23). For the individual, human wealth is the flow of future labour income discounted at the interest rate faced by the consumer. This holds at the aggregate level too, as shown by the presence of the consumer's rate, $(1+r)/\gamma$, in (20). An individual's financial wealth is evaluated the same way, as shown in (13).[19] In

19. The financial wealth equations are written as cumulation conditions, but they can be interpreted as valuation equations in the sense that the future stream of income from the assets is being discounted at the rate shown in the cumulation equation.

aggregate, however, financial wealth is evaluated (and pays) at the market rate, as shown by the fact that (22) does not have any adjustment for γ. In effect, the uncertainty about death is netted out in the aggregate. This introduces a phenomenon that is often referred to as "overdiscounting" in the aggregate model. Some aspects of the aggregate economy, including the marginal propensity to consume out of wealth, (18), reflect the probability of death, which raises the individual discount rate above the market rate. This has some interesting implications for policy, to which we return in discussing the model's properties.

Another difference, illustrated in (23) as compared with (14), is that the newly born augment the total wealth in the economy (with their human wealth); the higher the birth rate, the more they augment it.

2.3 A note on stability

There are two conditions for this model to have a steady state. As noted above, condition (11) must hold for human wealth to be well defined at the level of the individual consumer and hence in aggregate (the aggregate condition is the same in this case). In addition, from (23) it is apparent that for total wealth to attain a steady state, the following must hold:

$$\frac{(1+r)(1-\Omega)}{(1+n)(1+\zeta)} < 1 \quad . \tag{24}$$

The denominator of (24) is the (gross) growth rate of the economy (the combined effects of population and productivity growth). The numerator is a bit more complicated. The term $(1–\Omega)$, which can be identified from (18), is the marginal propensity to save (from wealth). Thus for every unit of wealth there will be $(1–\Omega)$ of saving that will yield return r and grow to the value in the numerator of (24). If interest rates are so high that this compounding effect exceeds the growth rate of the economy, wealth would grow so much each period, just from the flow yield, that total wealth would expand without limit and never attain a steady state.

2.4 Adding government, firms and foreigners to the model

To complete SOLGM we must add a government and a foreign sector as well as the supply side of the domestic economy. The latter includes the introduction of producing capital, which is presumed to be owned by domestic consumers as part of financial wealth. The rest of financial wealth then consists of government debt and net claims on foreigners. In a nutshell: the ratio of government debt to output is treated as a policy choice; the capital stock is determined by profit-maximizing competitive firms; the household sector's determination of the equilibrium total financial wealth then determines simultaneously the level of net foreign assets. In SOLGM, we assume that the three assets all have the same rate of return, the market rate r, which is treated as exogenously determined in world markets.[20]

In interpreting the following, the reader should recall that aggregate variables are always written in standardized units that eliminate growth; that is, they are written in per capita terms and deflated by the gross rate of technical progress.

By definition, household financial assets are claims to physical capital, *k*, government bonds, *gb*, or foreign bonds, *fb*:[21]

$$fa_t = fb_t + gb_t + k_t. \tag{25}$$

Note that the sign convention defines assets from the perspective of the consumer – for example, while gb is a liability of the government, its value and sign in (25) are positive. Similarly, we record foreign bonds as an asset; the fact that there is net debt will show up as a negative value for *fb*.

Output, *y*, must be consumed, invested in capital, absorbed by government, or exported:

$$y_t = c_t + i_t + g_t + netx_t. \tag{26}$$

20. Relaxing this simplification is an important part of the elaboration in SSQPM.

21. Note that there is no "money" in SOLGM.

2.4.1 Government

The government is a relatively simple entity in this model. It buys some of the output and finances that spending by borrowing from consumers or taxing them. There are two important points regarding the interaction of the government and households concerning the way government collects revenues and spends. First, the tax is effectively lump-sum. That is, since consumers are assumed to supply labour inelastically, there are no distortions introduced by taxing income. Second, government expenditures do not influence the marginal decisions of consumers. Since we have not shown any utility coming from the activities of government, one could say that the model treats such activities as if they were pure waste. However, nothing would change in terms of the real equilibrium if we introduced an additively separable contribution to welfare from the activities of government. This is an important point that would require attention if one wanted to use the model for welfare analysis in experiments involving the size of government.

The government's budget constraint is

$$g_t = \tau^w w_t + gb_t - \frac{(1+r)}{(1+n)(1+\zeta)} gb_{t-1} , \tag{27}$$

where w_t is wage income, taxed at rate τ^w, and gb_t is the end-of-period stock of government debt. The imposition of (27) is the essential step in permitting the model to be complete with respect to stock-flow accounting in the government sector. However, imposing (27) is not sufficient to ensure the existence of a steady state. General equilibrium considerations would lead us to expect the real rate of interest to exceed the growth rate. If this is so, (27) has no steady state for debt, in general, meaning for arbitrary choices for the primary deficit, $g_t - \tau^w w_t$. Rather, we must appeal to behaviour to ensure the steady state. There are two choices that must be made to complete the model – the level of debt and something that guarantees the appropriate level of the deficit, given that choice for debt. We characterize fiscal policy for the steady-state analysis as consisting of a choice of the ratio of debt to output and a choice of the size of government in the sense of the share of output taken by government. The tax rate is then set to ensure the debt ratio, based on (27). Although we could reverse the roles of taxes and spending and still have a well-defined steady state, we find it clearer to characterize the choice of government as being what it will spend, leaving the consequences for tax rates technically residual.[22]

2.4.2 Firms and aggregate supply

In each period, firms employ labour and rent the existing capital from consumers in return for wages and rental payments. The production function is assumed to be Cobb-Douglas with constant returns to scale and elasticity of output with respect to capital α:

$$y_t = \left(\frac{k_{t-1}}{(1+\zeta)(1+n)}\right)^{\alpha} , \tag{28}$$

where y_t is output and k_{t-1} is the available stock of capital, both measured in our standardized units. Assuming free entry, profit maximizing behaviour provides the following familiar marginal conditions:

$$w_t = (1-\alpha)y_t \tag{29}$$

$$\frac{\alpha y_t(1+\zeta)(1+n)}{k_{t-1}} = r + \phi , \tag{30}$$

where ϕ is the depreciation rate of capital. Equation (29) requires the real wage to be set to the marginal product of labour. Equation (30) is the equivalent marginal condition for capital, where the user cost of capital is the market interest rate plus the depreciation rate, $r + \phi$. Equation (30) determines the optimal ratio of capital to output. Given the supply of labour (one in these units), equations (28) and (30) simultaneously determine the level of capital and the level of output. The standard result for income distribution, that the share of output that goes to capital is α, emerges from the solution, as shown from the labour share in (29).

22. This view also accommodates more easily the consequences of observed changes in the level of debt, where it is natural to think of current developments with respect to debt having implications for tax rates in the future.

The supply side of the model also provides an important part of the flow demand, namely, the investment spending necessary to maintain the optimal level of capital. The identity linking stocks and flows provides the appropriate condition:

$$i_t = k_t - \frac{(1-\phi)}{(1+\zeta)(1+n)} k_{t-1} , \tag{31}$$

where i_t is gross investment.

Finally, since we chose to define human wealth based on after-tax income, we must add the formal reconciliation:

$$y_t^{lab} = (1-\tau^w) w_t . \tag{32}$$

2.4.3 Links to the world economy

Given the desired level of financial wealth that emerges from the optimization by consumers, as well as the level of debt chosen by the government and the level of capital that emerges from the optimization by firms, the stock of net foreign assets can be computed residually from definition (25). It is important to understand that there is nothing behaviourally residual in this calculation. Consumers determine the level of total financial wealth, and this simultaneously determines the level of net foreign assets, given the levels of the other stocks.

As is the case for all stocks, the maintenance of the stock equilibrium imposes a condition on the flow accounts. The implied cumulation of net foreign assets determines the current account, ca_t, through the balance of payments identity as shown in (33):

$$ca_t = fb_t - \frac{1}{(1+n)(1+\zeta)} fb_{t-1} . \tag{33}$$

In turn, the current account identity – ca_t is defined to be net exports of the domestic good plus income from net foreign assets – determines the level of net exports, $netx_t$, as shown in (34):

$$netx_t = ca_t - \frac{r}{(1+n)(1+\zeta)} fb_{t-1} . \tag{34}$$

Net exports in SOLGM are determined to support the net foreign asset position, which, in turn, comes from optimizing decisions of domestic consumers and firms, and the level of debt chosen by the government. World demand plays no role in the usual sense. What is determined by world markets is the real interest rate.

2.5 Summary

We now have all the pieces of a complete solution for the steady state of SOLGM. For the flows, government spending is a policy choice and investment and output come from the analysis of supply and the optimizing decisions of firms. Reconciliation comes in the simultaneous determination of a level of wealth, a level of net foreign assets (and hence net exports) and consumption, with the latter emerging as satisfying both the optimization conditions of households and the market conditions of full employment and a zero-excess-profit distribution of income.

SOLGM serves as the point of departure for the more elaborate model, SSQPM, which is described in the next section. SOLGM is also used in the simulation analysis to provide a starting point for the interpretation of the properties of SSQPM.

3 SSQPM: THE STEADY-STATE STRUCTURE OF QPM

In this section, we discuss the extensions of SOLGM that we have introduced to provide a steady-state structure suitable for use in support of a working projection model. For SSQPM, SOLGM is enriched to include indirect taxes and taxes on profits, relative prices, a decomposition of net exports and a real exchange rate. The small-open-economy assumption is also relaxed, allowing Canada to affect its terms of trade. Finally, a number of "risk" premiums are introduced to enable us to escape the assumption that all assets have the same rate of return.

The complicated nature of some of these elaborations makes it impossible to model all their effects and interactions within an optimization framework like that presented in Section 2. SOLGM is a one-good model. It would be relatively straightforward to make the imported good conceptually distinct, but this would not provide enough flexibility to explain, for example, all the movements in relative prices among the expenditure components of the national accounts.

In building a model of an economy, researchers always face difficult choices concerning how much complexity to accept in the name of realism. The choice made for the new model at the Bank of Canada was to limit the core theory and analytical structure of the model to the simple one-good paradigm. This reflects a judgment that any serious extension to a multi-good framework carries with it too high a cost in terms of complexity and model maintenance and an unfeasible computational burden, given the level of rigour now demanded in serious policy analysis.

Our goal is then to exploit the logical rigour and power of the analytical framework of the simplest model, but without accepting its simplicity literally. What this means is that we must add structure that is formally incompatible with the core theory in order to preserve its integrity, as defined by its ability to cope with the complexity of real-world data. This is not an easy task, and without the benefit of formal theory we must rely on our judgment as to how to do it in a reasonable way.

We provide the reader with two types of evidence to assess our success in this regard. In Section 4, we describe how we calibrate the model to respect certain aspects of the data and a number of judgments about the nature of the steady state we wish to portray. In Section 5, a detailed analysis of the impact of our elaborations on SOLGM is provided through comparisons of the responses to shocks of various models, beginning with SOLGM and building to SSQPM as the elaborations are added. This information should give the reader a good understanding of SSQPM as well as a basis for judging how well we have succeeded in adapting the simple model for practical use.

3.1 Exports and imports

SOLGM explains the determination of net exports. For practical purposes, the net exports must, at a minimum, be divided into exports and imports. The framework of the previous section cannot easily address this problem; after all, there is only one good and no logical basis for gross flows in both directions. In reality, trade occurs for a number of reasons, not the least being that different countries produce different goods.[23]

Our solution is to model the share of each component of spending that is imported (including imports for reexport).[24] Total imports are then defined by adding the components:

$$m = c^{mshar}c + i^{mshar}i + g^{mshar}g + x^{mshar}x, \tag{35}$$

where m denotes imports, x denotes exports, and z^{mshar} denotes the share of goods in expenditure category z that is imported. Equation (35) and the trade balance identity,

$$netx = x - m, \tag{36}$$

23. One way to tackle the problem is to introduce more goods, including tradable and non-tradable goods, and to model each country separately. See Macklem (1993), for an example of how the model of the previous section can be extended in this manner. However, we are not prepared to entertain for QPM the extra complexity this approach entails.

24. It is not possible to get direct measures of all these shares. See subsection 4.7 for information on how we have proceeded.

simultaneously determine the levels of exports and imports, using the level of net exports from the formal model.

This approach to modelling trade is obviously a simple one, but it seems adequate for our purposes. Our solution takes the fact of two-way trade as given and ties the explanation of the level of imports to the expenditure components that are already determined in the model. At this point in the discussion, the import shares have been treated as exogenous, but in the discussion that follows they are made to vary with the exchange rate and have further implications for prices in the economy.

3.2 Relative prices and indirect taxes

In the one-good framework of SOLGM, relative prices play no role. The same good is sold at the same price to consumers, firms, the government and foreigners. In reality, there are many goods, and important changes in relative prices have occurred at the level, for example, of the expenditure deflators in the national accounts. The downward trend in the investment deflator associated with falling computer prices is one example. To apply the model to real world data, some mechanism for introducing relative prices is necessary.

Indirect taxes must also be added to any model intended for practical policy analysis. They represent an important source of government revenue in Canada, as in most industrial economies, and the issue of the appropriate mix of taxes remains at the centre of Canadian policy debates. It is natural to consider indirect taxes as part of the discussion of relative prices, because changes in relative indirect tax rates constitute an important part of the explanation of overall changes in relative market prices.

In SSQPM, there are two distinct prices for each component of expenditure – the market price, which includes any indirect tax, and the price at factor cost. For the analysis of steady states, we are not concerned with absolute price levels; all prices are measured relative to the equivalent price of aggregate output. To the extent that observed relative prices reflect differences in rates of indirect tax, which could be interpreted as different taxes on the same good in different uses, the logic of SOLGM is undisturbed; indeed, the model can easily be extended to incorporate such taxes with no loss of rigour. However, one cannot explain all variation in the data on relative prices by simply taking account of differing tax rates. To allow SSQPM to reflect real-world data, we must allow an element of unexplained exogenous structure in relative prices, over and above what can be explained by taxes.

The relative market price of a good z is denoted by p^z; the addition of a tilde, $\tilde{p}^z$, denotes a relative price at factor cost. Similarly, the rate of indirect tax on good z is denoted by τ^z. The three are related by

$$p^z = \frac{1+\tau^z}{1+\tau^y}\tilde{p}^z, \tag{37}$$

where τ^y is the aggregate indirect tax rate defined by

$$\tau^y y = \tau^c \tilde{p}^c c + \tau^i \tilde{p}^i i + \tau^g \tilde{p}^g g + \tau^x \tilde{p}^x x. \tag{38}$$

With this definition of τ^y we have $p = (1+\tau^y)pfc$, where p is aggregate market price (the GDP deflator) and pfc is the related price at factor cost.[25]

The introduction of relative prices and indirect taxes affects the consumer's budget constraint (2). We now have

$$(1+\tau^c)\tilde{p}^c c_{a,t+s} + p^{fa} fa_{a,t+s} = y^{lab}_{a,t+s} + \frac{(1+r_{t+s-1})}{\gamma} p^{fa} fa_{a,t+s-1}. \tag{39}$$

The solution to the consumer's problem is exactly the same, with $c_{a,t}$ being replaced with $(1+\tau^c)\tilde{p}^c c_{a,t+s}$ and $fa_{a,t+s}$ by $p^{fa} fa_{a,t+s}$, where p^{fa} is the price in terms of output that consumers must pay for a unit of financial

25. Note that this approach is slightly different from that used in the national accounts. See Smith (1991), where an additional adjustment for the rate of indirect taxation in a base period is introduced, so that both market and factor cost price indexes are one in the base period. This is simply an issue of notation; it has no real consequences in the model.

assets (defined more precisely below). Similar transformations to the aggregate solution are also necessary.

The government's budget constraint and the first-order conditions from the firm's maximization also need to be adjusted. Equation (27) becomes[26]

$$(1+\tau^g)\tilde{p}^g g_t = \tau^w w_t + \tau^y y + gb_t - \frac{(1+r)}{(1+n)(1+\zeta)} gb_{t-1}, \tag{40}$$

and equation (30) becomes

$$\frac{\alpha y_t (1+\zeta)(1+n)}{k_{t-1}} = (1+\tau^i)\tilde{p}^i (r+\phi). \tag{41}$$

Analogous to conditions (25) and (26), the combined budget constraints in the extended environment imply

$$y_t = p^c c_t + p^i i_t + p^g g_t + p^{netx} netx_t \tag{42}$$

and

$$p^{fa} fa_t = \tilde{p}^{netx} fb_t + gb_t + (1+\tau^i)\tilde{p}^i k_t, \tag{43}$$

where

$$p^{netx} = (p^x x - p^m m)/(x-m) \,. \tag{44}$$

These market clearing conditions are used to determine two relative prices: the price of goods for consumption, p^c, and the "price" of financial assets, p^{fa}. This latter price is not an asset valuation in the usual sense. Rather, it reflects the necessary conversion of cumulative flows with different relative prices into output units. Thus, for example, to own a unit of capital, a

26. We do not include a price for government bonds, assuming that in the steady state all debt has a coupon rate equal to the equilibrium interest rate and bonds have par value.

consumer has to pay the relative price of capital goods, and this must be properly reflected in the budget constraint and real asset accounts.

Having inserted relative prices and indirect taxes into the accounting framework, we must now consider how to model them. Of particular interest is the effect of the exchange rate on prices. It seems reasonable, given that part of each expenditure category is imported, to model the relative price of each category as being influenced by both domestic and foreign conditions. This idea is captured by combining import prices and domestic costs in a price index for each component, with the weight on the foreign price being the proportion of the component that is imported.

Thus, for example, the relative price of investment goods is modelled as a geometric average of the relative price of imported investment goods and domestic marginal cost, augmented by the effective indirect tax rate for investment goods, with the weight being the share of investment goods that is imported:

$$p^{i} = \bar{p}^{i}(1+\tau^{i})(p^{i^{m}})^{i^{mshar}}(p^{mc})^{1-i^{mshar}} . \qquad (45)$$

The notation $\bar{p}^{z}$ is used to denote an exogenous price base associated with variable z. It is these variables that we use, in calibration, to introduce any exogenous relative price effects.[27]

The relative price of imported investment goods is, in turn, given by a geometric average of the real exchange rate (the “foreign price”) and marginal cost:

$$\tilde{p}^{i^{m}} = \bar{p}^{i^{m}}(pfx)^{x1}(p^{mc})^{1-x1} . \qquad (46)$$

In this case, the weight, $x1$, reflects the extent to which the law of one price is assumed to hold in the market for imported investment goods – a weight

27. When the model is applied to history, such variables are used to capture trends in the data that cannot be explained within the model’s accounts. For simulations into the future, all such variables are set to converge on particular values chosen as part of the calibration of the steady-state solution.

of one implies that changes in the real exchange rate, pfx, will affect the relative price of investment goods one for one, whereas a weight less than one implies that the domestic price (cost) conditions affect the price of imports. This is the first form of relaxation of the strict small-open-economy assumptions in SSQPM – although Canada does not influence the world price of the imported component, the price in domestic markets is allowed to deviate from the pure purchasing power parity (PPP).

The relative price of goods imported for consumption is modelled in a parallel fashion:

$$\tilde{p}^{c^m} = \bar{p}^{c^m}(pfx)^{x2}(p^{mc})^{1-x2}, \tag{47}$$

but for the prices of the other two components of imports we simply tie their endogenous movements to those of the price of imports for investment, given by (46):

$$\tilde{p}^{g^m} = \bar{p}^{g^m}\tilde{p}^{i^m} \tag{48}$$

$$\tilde{p}^{x^m} = \bar{p}^{x^m}\tilde{p}^{i^m}. \tag{49}$$

The overall import price is then defined as an appropriately weighted average of the component prices:

$$^{m}m = p^{c^m}c^{mshar}c + p^{i^m}i^{mshar}i + p^{g^m}g^{mshar}g + p^{x^m}x^{mshar}. \tag{50}$$

For the price of exports we add one additional feature. The basic structure is similar to the one reported in (45) for investment goods, in that we combine domestic cost and other influences in a geometric average:

$$p^x = \bar{p}^x(p^{cx}/(1+\tau^y))^{x4}(p^{mc})^{1-x4}. \tag{51}$$

However, in this case, p^{cx} is not a single foreign price, but rather a combination of world commodity prices and general world prices, both multiplied by the real exchange rate. This structure is added to allow us to introduce the effects of shocks to world commodity prices into Canada's terms of trade. We will elaborate the equation for export prices further in subsection 3.4, where we add a direct influence of the relative size of the Canadian economy to export prices in further relaxing the strict small-open-economy assumptions.

Finally, we must define the implicit marginal cost (essentially, the price at factor cost of domestic absorption of domestic production) that reconciles the relative price system:

$$1 = ((1 + \tau^{y}) p^{mc})^{x5} (p^{x})^{1 - x5} \quad . \tag{52}$$

The p^{mc} differs from the price at factor cost, in the national accounts sense, because of the wedge introduced by the fact that, in the data, the export price differs from the output price (for reasons other than indirect taxes). Thus, there is an implicit rent that must be accounted for in a one-good model of domestic production.

3.3 Other taxes

To complete our stylized picture of the Canadian tax system, we consider taxes on interest income, dividends and profits. Taxes on investment income received by consumers are accounted for in the tax on labour income. The one tax covers all forms of personal income taxation.

The introduction of a tax on profits means that the cost of capital is divided by $(1 - \tau^{k})$, where τ^{k} is the tax rate, and equation (30) becomes:

$$\frac{\alpha y_t (1 + \zeta)(1 + n)(1 - \tau^{k})}{k_{t-1}} = (1 + \tau^{i}) \tilde{p}^{i} (r + \phi) \quad . \tag{53}$$

The government receives additional revenue, $\tau^{k} \alpha y$, from this tax.[28]

28. Note that we have ignored any difference between marginal and average tax rates in this version of SSQPM.

3.4 The almost-small-open-economy assumption

In many models of open economies, it is assumed that the economy in question takes the price of its exports as given. Furthermore, it is often assumed that the economy can borrow as much capital as it wants without affecting the price of capital. When both of these assumptions hold, the economy is described as a small open economy. In SSQPM, we relax the first assumption so that an increase in the size of the Canadian economy relative to the rest of the world will, all else being held equal, decrease the export price; equation (51) becomes

$$p^x = \bar{p}^x (p^{cx}/(1+\tau^y))^{x4} (p^{mc})^{1-x4} \left(\frac{y^*}{y}\right)^{x7} . \tag{54}$$

The idea is that the Canadian economy as a whole has some effect on the price of exports, even though individual firms act in a competitive manner. Such aggregate market power may arise from the fact that Canada is a large exporter of some goods – wheat, lumber and natural gas, for example. If supply of these goods increases, the price falls, since the foreign demand curve for these products is not perfectly elastic. While this phenomenon is judged to be important enough to be included in the Canadian model, the effect is assumed to be too small to influence the general level of prices in the rest of the world. Moreover, Canada is assumed to have no influence on the world level of the prices of imported goods.

3.5 The exchange rate

The real exchange rate is perhaps the most important relative price in an open economy; it serves a key role in the establishment of external equilibrium and the reconciliation of flow equilibrium conditions with steady-state stock levels. However, as is the case with all relative prices, the real exchange rate has no meaning in a competitive one-good paradigm, where absolute PPP must hold.[29] This does not accord with history and is not satisfactory for a realistic policy model.

For SSQPM, we have altered the model such that PPP does not need to hold. Real variables can affect the real exchange rate and vice versa. We specify that an increase in the steady-state share of exports requires a depreciation in the exchange rate, as does a rise in domestic potential output relative to world potential output (since Canada must lower its price to sell more to foreigners in both cases, continuing the almost-small-open-economy metaphor):

$$pfx = \overline{pfx}\frac{x}{y}\left(\frac{y}{y^*}\right)^{x6}. \tag{55}$$

As well as affecting relative prices, exchange rate movements influence the shares of imports and hence the pattern of trade. These effects are captured in equations of the form

$$z^{mshar} = \frac{\bar{z}^{mshar}}{pfx}, \tag{56}$$

where z is a consumption, investment, government or export good.

3.6 The cost of capital and risk premiums

Mehra and Prescott (1985) document what has now become known as the equity premium puzzle. Simply put, the return on equity is vastly at odds with the return on bonds over likely parameter ranges for such things as the risk aversion of investors, the probability of bankruptcy, and taxes. In models with a supply side based on a Cobb-Douglas production function, like SOLGM, with real interest rates of about 5 per cent and depreciation of about 7 per cent per annum, the capital-output ratio suggested by the model is about 3.0, compared with less than 1.5 observed in the recent data.[30] Given the above depreciation rate, the real interest rate would have

29. Macklem (1993) provides an example of a multigood model where the real exchange rate is defined as the relative price of the tradable good.

to be around 13 per cent in steady state, for the model's predicted capital-output ratio to approach observed levels.

Although many authors have suggested ways to reconcile the equity premium puzzle, it seems reasonable to say that none has gained widespread acceptance. Furthermore, such attempts at reconciliation tend to be specific to the model or its calibration and are usually quite burdensome to implement. Consequently, for SSQPM a simpler approach was chosen.

One such method, used by Rose and Selody (1985) for the SAM model, for example, is to introduce an exogenous risk premium into the firm's problem. This has the effect of raising the cost of capital. A similar strategy was adopted by Macklem (1990). However, Macklem treats the risk premium as a pure cost – net output is reduced by the amount of the premium,[31] whereas Rose and Selody treat the household sector as the residual owner of capital with both a responsibility for the cost of investment and a claim to the net proceeds, including any rents.[32]

We take an approach similar to Rose and Selody in that we do not treat the premium as a pure output cost, but we assume that the risk premium is paid, in a lump-sum manner, to consumers and does not enter into their marginal calculations.

To implement this, we must put the risk premium, φ^{κ}, into the cost of capital and modify equation (53) as follows:

$$\frac{\alpha y_t(1+\zeta)(1+n)(1-\tau^k)}{k_{t-1}} = (1+\tau^i)\tilde{p}^i(r+\phi+\varphi^k) \quad . \qquad (57)$$

This results in a lump-sum redistribution to consumers:

30. In this discussion, capital has been limited to private production capital. However, even if we add housing and government capital, an "equity" premium is still required.

31. This has the unfortunate consequence of upsetting the national accounts identity.

32. Rose and Selody account for exogenous direct foreign ownership of capital. It is in this sense that the domestic household sector is a residual owner.

$$risk_t = \frac{\varphi^k(1+\tau^i)\tilde{p}^i k_{t-1}}{(1+\zeta)(1+n)}. \tag{58}$$

Since the observed returns on government debt and net foreign assets also differ from whatever rate we might pick to represent exogenous world interest rates, we must also add risk premiums for these assets. The appropriate values are inferred from identities like (34) and (27). The interest rate required to balance these equations is typically higher than the rate on the default asset – the foreign bond. These differences are accounted for in a similar way – the interest rate for each asset is increased by a risk premium, and a corresponding lump-sum transfer is assumed to be made to consumers (or from consumers in the case of net debt).[33]

The transfers must show up somewhere in the wealth accounting. One answer would be to introduce asset prices that would reflect the net effect of the transfers and to modify appropriately the accounting for real asset accumulation. Another answer would be to add a new component of wealth, the asset value of transfers. We have chosen to put the transfers into the income base for the computation of human wealth.[34] It would therefore be more accurate to think of it as human and transfer wealth, the value of everything not considered a regular flow payment from financial assets. We will continue to use the simple term "human wealth." Note that the net effect of the transfer wealth on consumption will be the same, wherever we put it, as long as consumers discount it the same way.

3.7 Seigniorage and inflation

The introduction of money into a model with optimizing agents requires a view on why money is held – why agents hold money, which has zero nominal return, when they could hold other assets with positive return.[35]

33. In coding and calibrating the model, we measure these differences relative to the consumer's rate, the central rate in the theory, and not the foreign rate, but the point is the same. See Section 4 for details.

34. This implies an obvious modification to the consumer's budget constraint (39), and the definition of human wealth (20).

In SSQPM, we nevertheless follow tradition and ignore the issue. We simply assume that consumers must hold money, which results in an inflation tax accruing to the government from seigniorage. Furthermore, we assume that all consumers hold the same amount of money so that the inflation tax is not distortionary.[36] With these assumptions, only minor changes need to be made to the model presented so far. A seigniorage term must be included in the aggregate consumer budget constraint, equation (39), in the definition of human wealth, equation (23), and in the government budget constraint, equation (27).

3.8 Concluding remarks on SSQPM

This concludes our discussion of the structure of SSQPM. The complete model is documented in the appendixes. In Appendix 1, we provide definitions of the mnemonics used for all variables and parameters. In Appendix 2, we list the equations of SSQPM.

We turn next to the calibration of the model and its resulting numerical steady state.

35. The literature abounds with methods for introducing money and costs of inflation into models with optimizing agents. For example, Sidrauski (1967) includes money in the consumer's utility function and Clower (1967) requires that consumers hold money in order to purchase consumption goods – the so-called cash-in-advance constraint. Other techniques can be found in Black, Macklem and Poloz (1994) and the references therein.

36. An alternative approach would be to treat money like government debt or any other asset, that is, to specify that consumers accumulate money balances over their lives. Then, since an increase in seigniorage would lead to a decline in the direct tax rate, inflation would be beneficial for the same reason that a decline in the level of government spending is beneficial – see subsection 5.2.

4 CALIBRATION AND THE STEADY STATE

In the elaboration of SSQPM, a number of parameters and exogenous variables have been introduced. The values chosen for these parameters and the measures of the variables, which in some cases are unobservable and must be specified by indirect means or judgment, determine the numerical steady state of the model and have an important influence on its properties. This section discusses what values were chosen and the final steady state that the model obtains, given these choices. After a few general comments about the calibration exercise, we proceed by defining the variables in an order similar to their appearance in the text. Full details can be found in Tables 1 and 2 (pp. 77-78).

Calibration is a difficult task. In some cases, we do not have data for an important concept in the model. In other cases, the data are at odds with the simplified theory and structure of the model. For example, many series that must be stationary if a steady state is to exist in the usual sense have important trends over recent history. In such cases, and in the choices of policy variables, the past may provide little guidance as to what is a reasonable specification for the future. The practical model builder must rely on a mixture of historical observations and judgment and take care to understand how such judgments may matter.

Steady states for variables for which we have data (and for which we can make a case for stationarity) are typically chosen in line with long-run averages in the data. However, when there is a distinct break in a series, or when a series has a historical trend that must disappear if there is to be a steady state, judgment must be applied as to when the system will become stationary. One example, introduced in subsection 3.2, arises from the recent downward trend in the relative price of investment goods associated with declining computer prices. For the model to have a steady state, in the usual sense, judgment must be used to impose an end to this trend and to select a final value for the relative price.

For this discussion, transition issues are not important; we are merely pinning down an end point for use in the larger dynamic model.

However, this provides a good opportunity for us to stress that the discussion here should not be taken to imply that the numerical steady state that emerges is pertinent to the immediate future. For example, the dynamic model retains an important downward trend in the relative price of investment goods well into the next century.

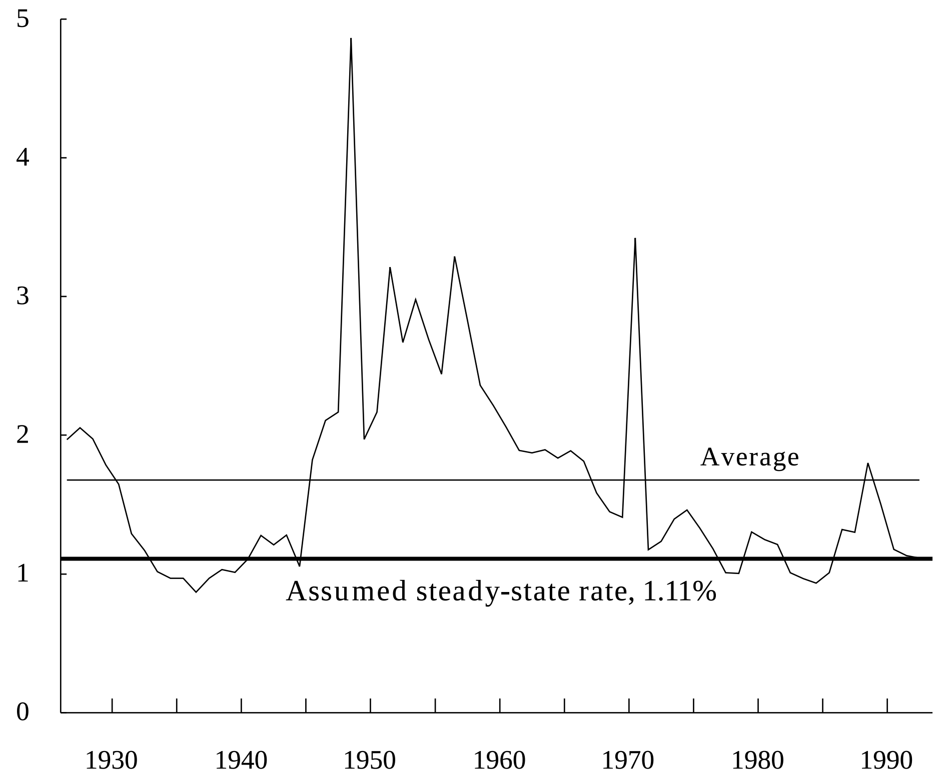
Variables that are unobservable are often set, as part of the calibration of the model, so that the numerical steady state is sensible, based on some other objective criterion or informed judgment. For example, the rate of time preference is chosen, in part, to make the model's predicted net foreign asset position accord with judgment, tempered by the historical data, as to what constitutes a realistic steady state in the case of Canada. In decisions about steady-state asset or debt ratios, the main users of QPM (the staff of the Bank of Canada) played a particularly important role in choosing the details of the calibration of the steady state. This process also extended to other important issues, such as the choice of steady-state values for world real interest rates, the relative cost of capital and associated issues, such as the implied composition of capital and, hence, the depreciation rate.

4.1 Population dynamics

With respect to the population (and labour force), there are three values that must be determined: the population growth rate, the probability of survival and the birth rate. The three are related as shown in equation (15), so only two values can be chosen independently. In this presentation, we focus on the population growth rate and the probability of survival and infer the birth rate from the identity, but the three values should be viewed as determined simultaneously.

Even calibrating the population growth rate is far from straightforward. Dealing with the transition of the postwar population bubble into the labour force and on into normal retirement creates special problems, over and above the usual difficulties with family-level demographics. Moreover, for Canada, an important policy choice, the rate of immigration

Figure 1
Population growth rate
(per cent, per annum)



permitted, plays a central role. We have done no special work on these issues. We set the annual population growth rate at 1.11 per cent, which is below its average from 1926 to 1993 (see Figure 1) but in line with recent experience and a demographic projection taken from Statistics Canada.[37]

Choosing the probability of survival is a more difficult proposition. First and foremost, it is worth emphasizing that the population dynamics behind the model, especially the assumption of a constant probability of survival, are not (and are not intended to be) realistic. The model does not have a realistic life cycle, with a period of childhood dependency, a period of adult working life and a period of retirement. New arrivals go straight from the womb to the production line, where they stay until their number

37. See Statistics Canada (1991). The figure 1.11 per cent is the projected average growth rate for 1990–2011.

comes up and the grim reaper plucks them away to immortality. Consequently, it is not appropriate to use the measured aggregate Canadian mortality rate in setting the model's probability of survival. Indeed, to do so would result in individuals having unacceptably low discount rates.

To understand this, recall that the effective discount factor under certainty equivalence is the individual's pure discount factor, based on time preference, times the probability of survival. Consequently, the individual's effective discount factor will be higher (discount *rate* lower) the higher is the probability of survival. If we were to use the actual Canadian mortality rate, which is currently about 0.7 per cent, we would not add much extra discounting, relative to the model with agents who live forever. As stressed in Section 2, an advantage of the SOLGM framework is that it determines a level of net foreign assets. However, to reflect the data on net foreign assets (as well as certain other features of consumption in the dynamic model), we need a higher effective discount rate. The survival rate in this model serves the role of adding the flexibility to use a simple model and at the same time to respect some important features of the data. With these points in mind, the probability of surviving another year is set to 0.96.

Another perspective on our choice of the survival rate can be had from the calculation of expected lifetime within the model. The expected lifetime is related to the survival rate (under the model's characterization of the population) by the formula $\gamma/(1-\gamma)$. With our calibration of $\gamma = 0.96$, agents in the model act as if they have an expected remaining life of 24 years. Again, "life" here must be interpreted as working life. While this may appear rather low for an average working life, the same model would say that just under 13 per cent of the population would work for 50 years and almost 2 per cent for 100 years.

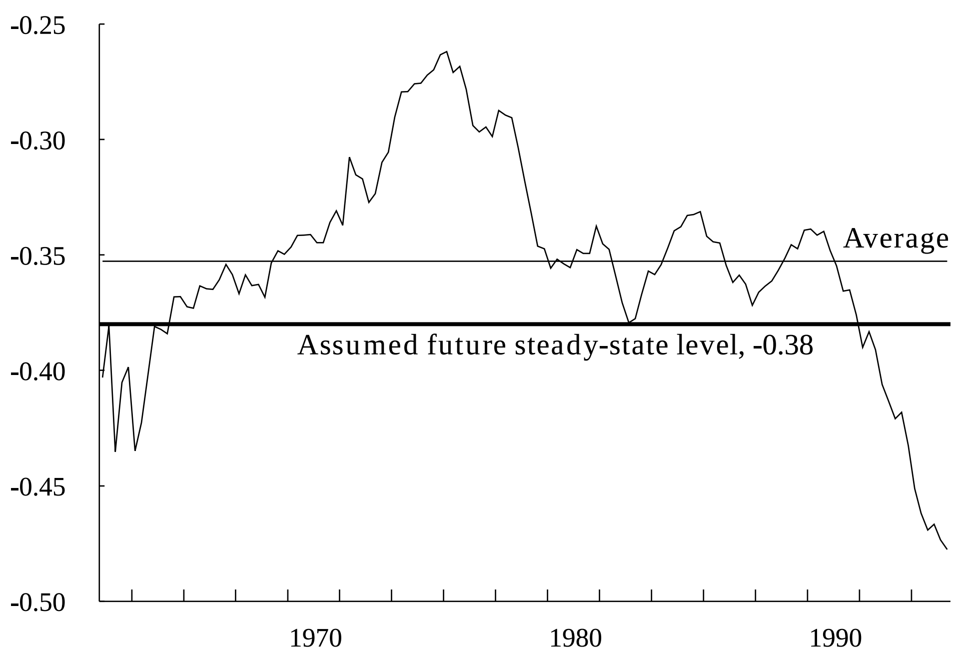
The "birth" rate that emerges is 5.3 per cent per annum. Recall that births refer here to gross entry into the labour force. This includes net immigration. Nevertheless, the counterpart of the higher death rate is a birth rate that is above the figure one would obtain from the statistics.

4.2 The utility function

Two parameters, in addition to the survival probability, are required to specify the utility function. These are the elasticity of intertemporal substitution, σ, and the discount factor, δ. The former is chosen to be 0.5. This is at the low end of the range of values described in Mehra and Prescott's (1985) survey of previous literature, but well above values suggested in some more recent literature.[38] We are satisfied that a value of 0.5 is a reasonable choice; it implies a substantial degree of risk aversion, but less than suggested by the more extreme estimates that exist.

The choice of the discount rate then proximately determines the level of financial wealth, and given the levels for government debt and the capital stock from other considerations in the model, this proximately determines the level of net foreign assets. The ratio of Canada's nominal net foreign assets to GDP over the past 30 years is shown in Figure 2. The series is always negative (Canada is a net debtor), with the average over

Figure 2
Ratio of net foreign assets to GDP



38. See, for example, Carroll and Weil (1993) and their references.

this period being about -35 per cent. Recent numbers, however, are much lower. There is no consensus estimate of where this ratio will settle down. The judgment of Bank of Canada staff was that we should calibrate to a nominal net foreign debt to GDP ratio of 0.38. Given our choice for the survival rate, this implies a pure discount rate (from time preference) of just over 3 per cent (the discount factor $\delta = 0.969$). The combined effective discount factor is about 0.93, with an effective rate of discounting of the future of just over 7 per cent per annum.[39]

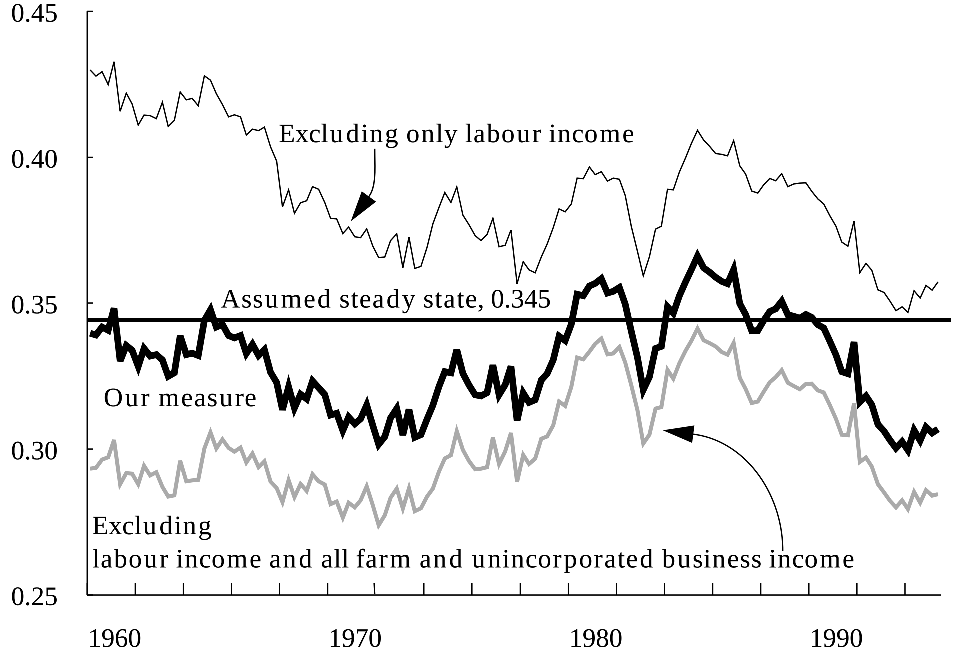
4.3 The technology and the cost of capital

There are two issues to be dealt with in specifying the remaining aspects of supply in the model. The first involves the production technology, the second the cost of capital. The choices are linked by the equilibrium condition, equation (57).

We calibrate the technology by setting capital's share of income to 0.345, based on the long-run average of one measure of this share. There is no clear definition of what is labour income and what is capital income in the national accounts. For example, farm income and unincorporated business income clearly contain elements of both from the perspective of the model. Figure 3 shows several measures of capital's share of income. We prefer the measure obtained if we allocate farm and unincorporated business income to labour and capital using the ratio of wage income to corporate profits. A consideration in choosing this measure was the degree to which it validates the assumptions of the model. We have a Cobb-Douglas technology, which implies that factor shares should be anchored to fixed values. It is therefore natural to choose a measure consistent with the model's maintained hypotheses or predictions. The cost of this is that one must look elsewhere for evidence to support the model in the sense of validating a prediction, but if one is going to use a model for policy analysis, that model may as well be tuned as closely as is practicable to the facts as revealed in published statistics.

39. The issue of the world real interest rate is deferred to subsection 4.8.

Figure 3
Capital share of income



Specifying the cost of capital involves a number of parameters. Steady-state values for the relative price at factor cost of investment goods, the rate of indirect tax on investment goods, the rate of capital income tax, the real interest rate, the depreciation rate and the risk premium on capital all must be set. Except for the real interest rate and risk premium, these variables are observable historically, although it is by no means obvious what relevance the measures have for the future. For example, the relative price of investment goods, far from exhibiting stationary behaviour, has moved from above 1.3 in the early 1980s to below 0.8 in the early 1990s – a change of almost 40 per cent (Figure 4). Similarly, depreciation rates have been steadily increasing since the mid-1960s (Figure 5). These facts both reflect the falling cost of computers and other related automated equipment and the consequent rise in the share of such things in investment. Knowing this does not tell us what the future has in store, however. Judgment from specialists was used in defining a consistent scenario, including choices for the tax and depreciation rates (see Tables 1 and 2 and Figure 5).

Figure 4
Relative price of investment goods at factor cost

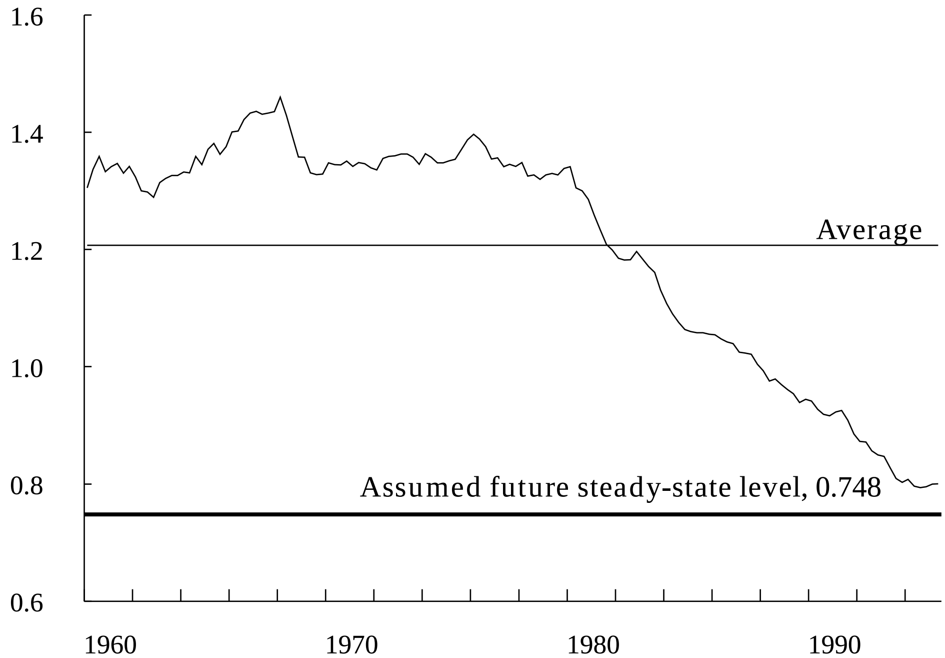
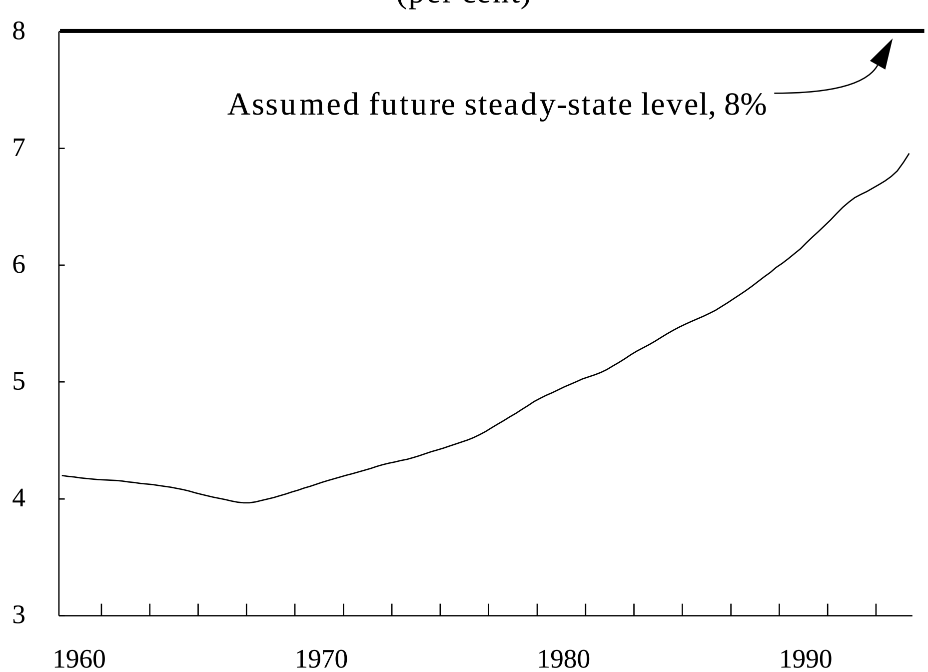


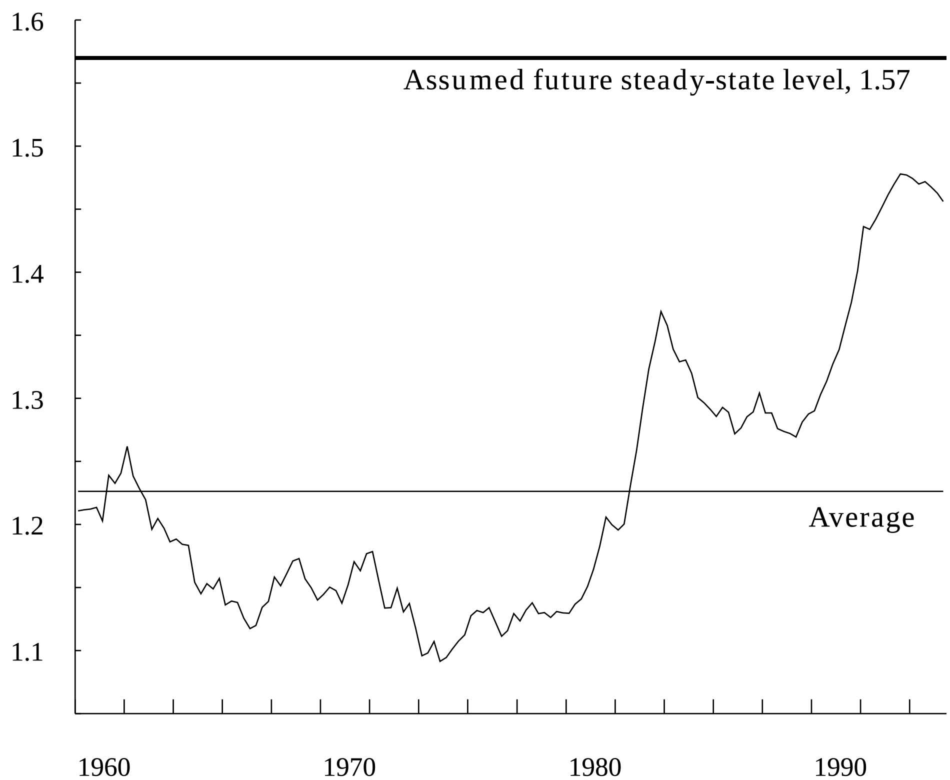
Figure 5
Rate of capital depreciation

(per cent)



Finally, there remains the unobservable risk premium on capital. We set it so that the steady-state value of the capital-output ratio is about 1.6, somewhat above recent history (Figure 6), reflecting a judgment that the downward trend in the relative cost of capital will continue for some time.

Figure 6
Ratio of capital stock to GDP

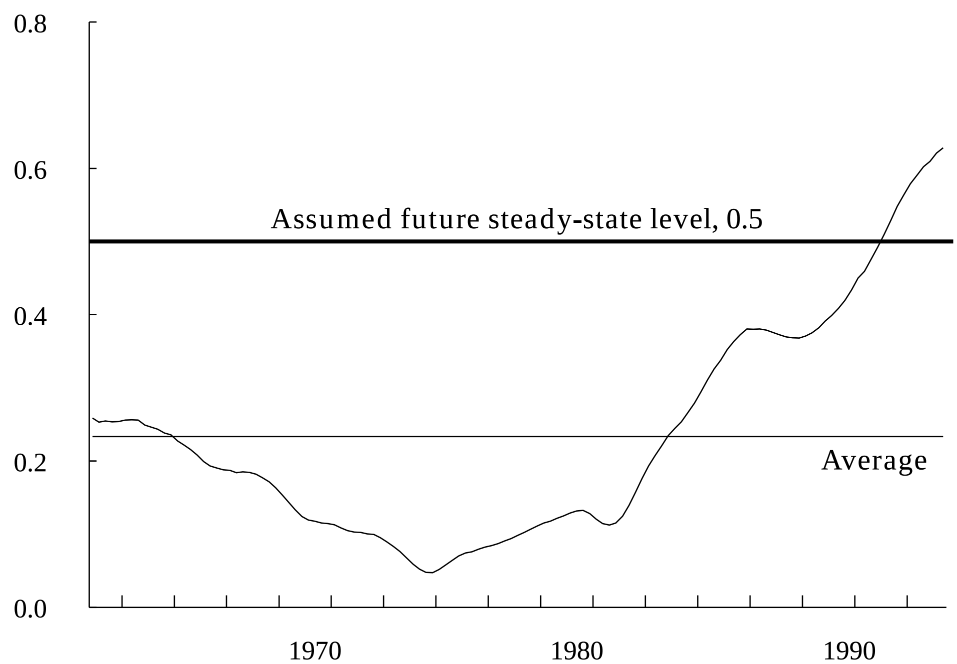


4.4 Fiscal assumptions

Arguably, the fiscal section of SSQPM is the hardest to calibrate. This is because the main variables are essentially free to be chosen by government. Such choices are not normally considered subject to the usual marginal optimizing calculus of private agents. It is therefore necessary to make judgments about these variables, and history may be of little relevance in indicating the future. Of particular importance to the overall steady state are the values for the ratios of government debt and spending to output, the rate of indirect taxation on consumption goods and the rate of tax on corporate profits. The other variables, particularly the other indirect tax rates, are less important.

In this case, staff judgment, based partly on announced fiscal plans, was that the steady-state debt-output ratio should be set at 0.5, higher than the historical average but lower than its current value (see Figure 7).

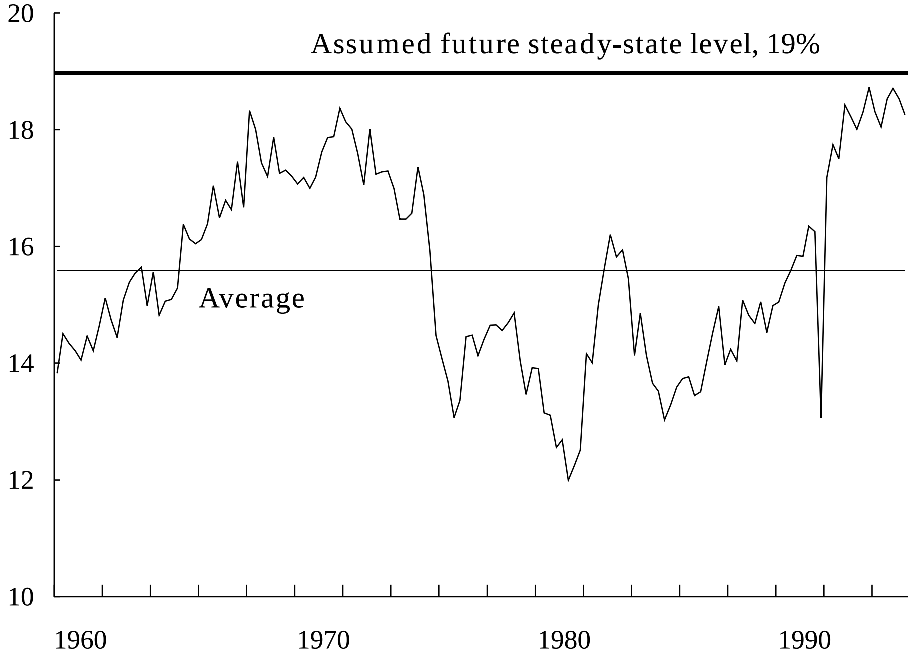
Figure 7
Ratio of government debt to GDP



The indirect tax rate on consumption goods is set at 19 per cent, higher than it has ever been historically but in keeping with the trend towards consumption taxation in recent years (Figure 8). This calibration was completed before the recent reduction in tobacco taxes and before the discussion of a national sales tax began to point to the possibility of some reversal of the recent trend. This is an issue that may have to be revisited in the year ahead.

The ratio of government expenditures to GDP is set at 0.20, lower than in the past (Figure 9). This reflects a judgment that as part of the

Figure 8
Rate of indirect tax on consumer expenditures
(per cent)



process of stabilizing the growth in government debt, some shrinkage in the relative size of the government sector will occur.

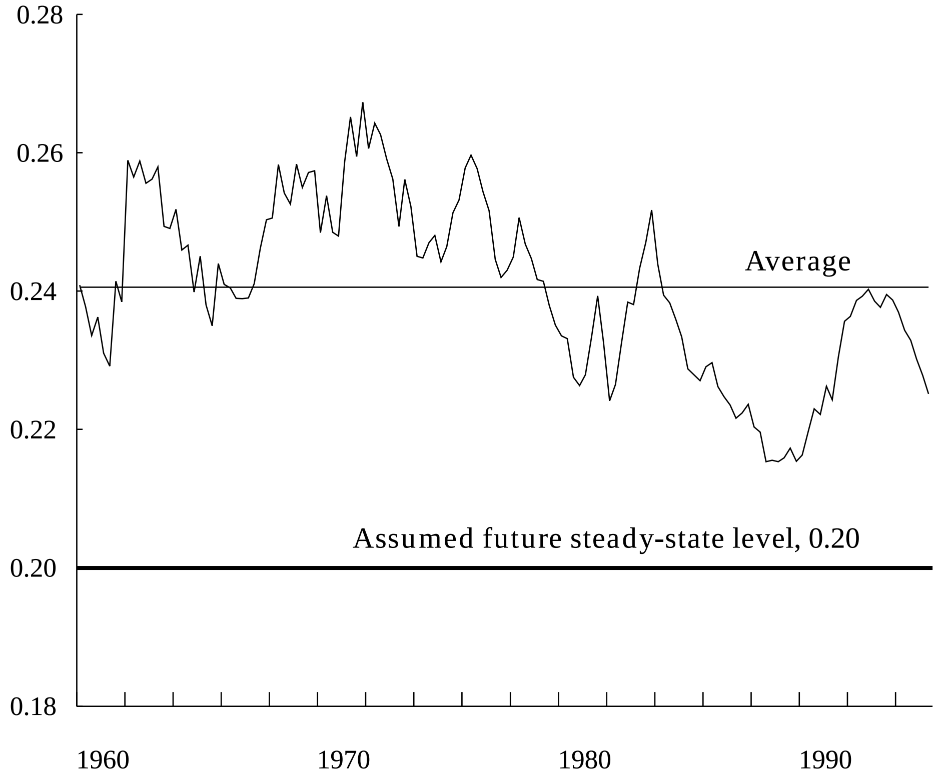
The tax rate on corporate profits is set at 5 per cent. This is in line with recent evidence on average effective tax rates, but is well below the statutory marginal rates.[40]

Finally, recall that, given the calibration of the rest of the government revenue components, the tax on labour income is determined to respect the government's intertemporal financing constraint and the chosen steady-state debt ratio.[41] The effective net tax rate (this includes the netting out of transfers in the data) turns out to be just over 12 per cent.

40. For discussion of effective tax rates see James (1993), Fillion (1992), and Black, Macklem and Poloz (1994).

41. There are a number of other indirect tax rates that must be set, but they are of much less importance. We invite the interested reader to consult Table 2.

Figure 9
Ratio of government spending on goods and services to GDP



4.5 Relative prices and associated variables

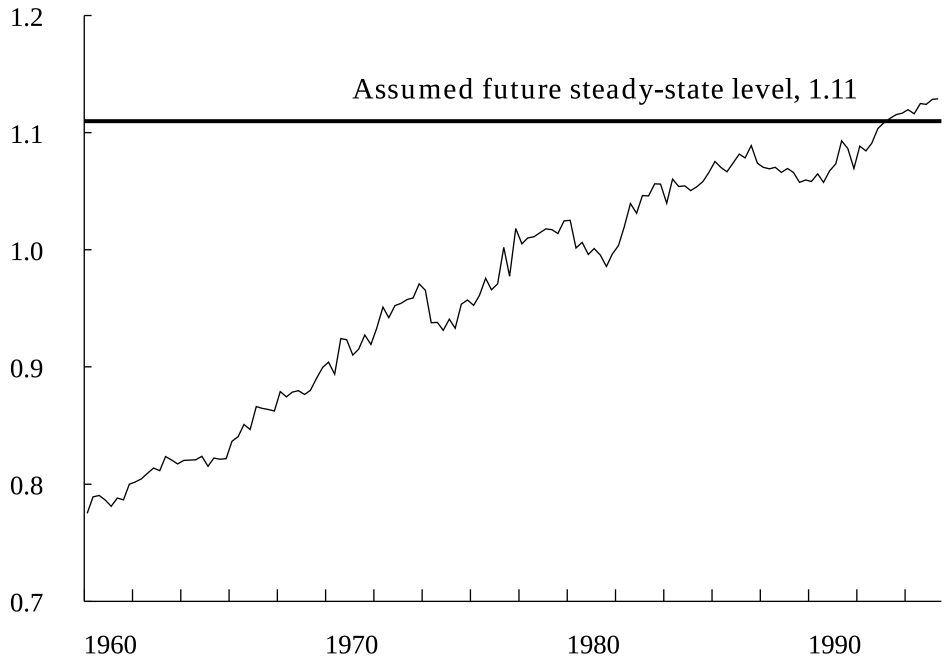
Relative prices have both an exogenous and an endogenous component in SSQPM. The endogenous part is the marginal response to changes in other endogenous variables that we have included in the model. However, there the variables of the form $\bar{p}^z$ must still be chosen to set the levels of these variables for the control solution.[42]

Relative prices for investment goods, government goods, export goods and all the imported components are required. The relative price of

42. Relative prices are not the only variables for which this is the case. Tables 1 and 2 provide additional information.

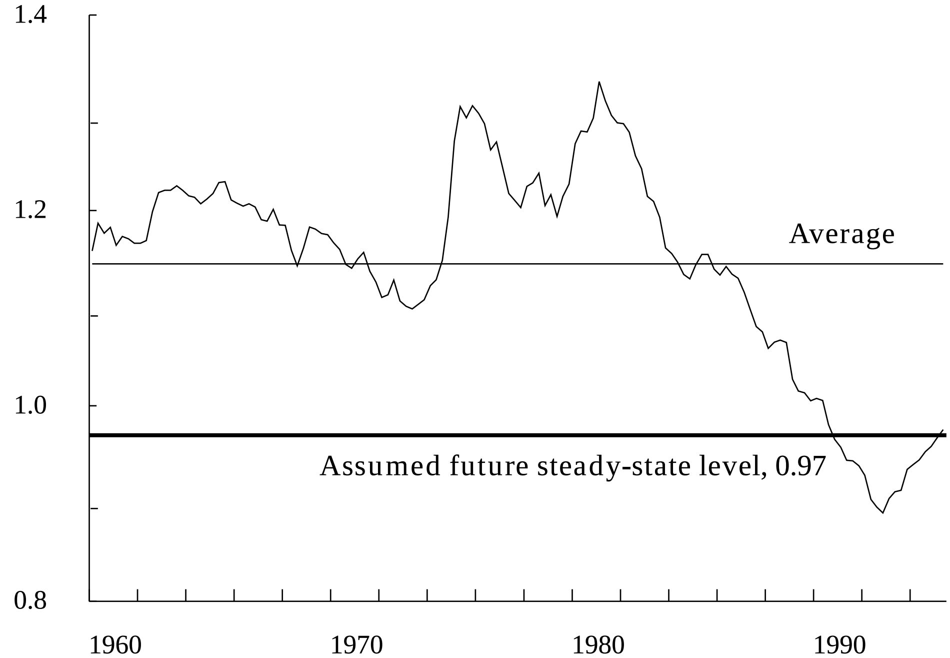
investment goods has already been discussed in subsection 4.3. On advice from the specialists, the relative prices of government and export goods are set at 1.11 and 0.97. In both cases, these choices are in line with current values (Figures 10 and 11).

Figure 10
Relative price of government goods and services at factor cost



Choosing the steady-state prices for the import components is a harder task, since appropriate data are not readily available. As we note in the discussion of import shares (subsection 4.7), estimates can be inferred from input-output tables; however, the infrequent and delayed publication of these data is a problem. In the case of the shares, this method remains the best available solution, but for prices, which are likely to vary more than shares, it may not be. This issue is still under active consideration. For the moment, the relative prices of the import components are set to their domestic counterparts. This seems a reasonable compromise until further analysis has been completed.

Figure 11
Relative price of export goods and services, at factor cost

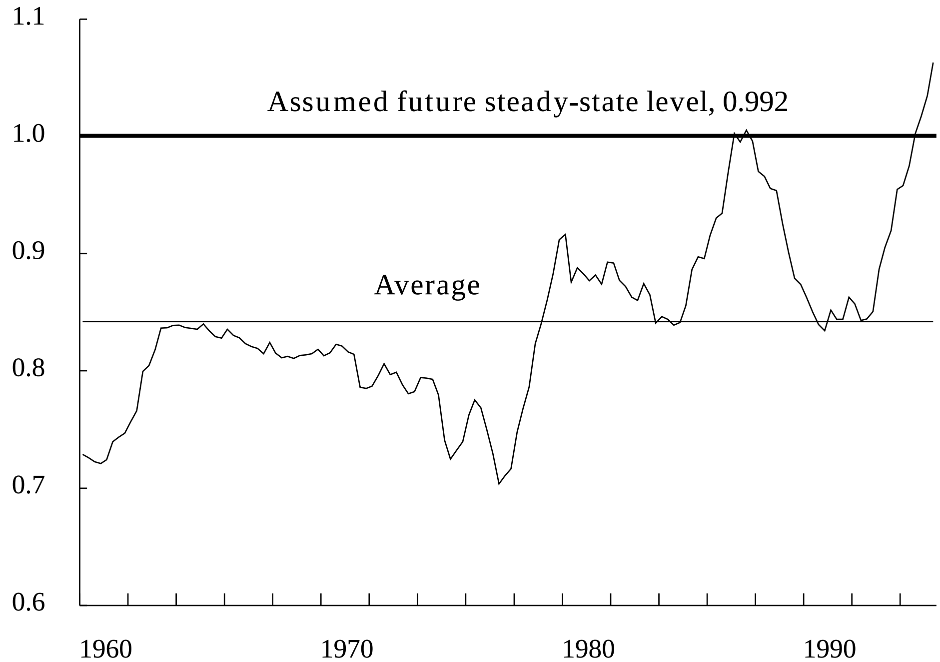


There are a number of other things pertaining to relative prices that must be chosen, such as the exponents *x*1 through *x*7, equations (46) through (55). As reported in Section 3, the equations where these parameters appear are based on the steady-state implications of various reduced-form specifications in the dynamic model. These specifications are calibrated based, in part, on estimation results, so that the dynamic model has satisfactory properties. For this reason, the formal discussion of these parameters is deferred to a future paper dealing explicitly with the dynamic model. The values chosen are nevertheless included in Table 2 (p. 78).

4.6 The real exchange rate

The final relative price is the real exchange rate, which we calibrate with respect to data for the trade-weighted G-6 figures. Again, based on staff judgment, the steady-state exchange rate is set at 0.992, which means that some of the recent depreciation is interpreted as an overshoot of the underlying real equilibrium (Figure 12).

Figure 12
Real exchange rate



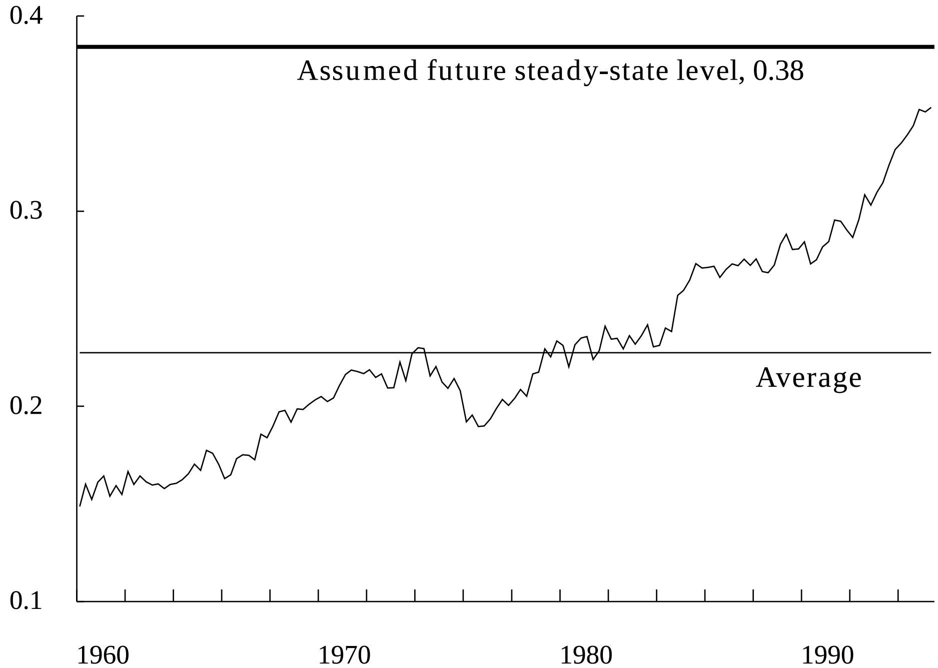
4.7 Import shares and the ratio of exports to GDP

For the control calibration, we pick levels for the import shares of consumption, investment and government goods and for the ratio of exports to GDP. The share of goods imported and reexported is then defined implicitly by the import identity. It is important to note that these variables are endogenous in simulations of SSQPM around the control.

The calibration of import shares is hampered by the fact that Statistics Canada provides data on imports only by commodity, and not by expenditure class. It is possible, for example, to find out how many trucks are imported, but not what portion is for use in production as opposed to personal consumption. It is necessary, therefore, to apportion the commodity groupings to the expenditure categories by ad hoc methods.

Given that the calibration of the import shares, especially for exports, is a little "soft," more reliable data, namely export data, are used to calibrate the import shares. The steady-state value for exports as a share of output is set at 0.38, a value that anticipates some further growth in the relative volume of trade (Figure 13).

Figure 13
Ratio of exports to GDP



4.8 The real interest rate and the risk premiums

In SSQPM, there are three types of income-generating assets held by consumers: government bonds, net foreign assets and capital.[43] Historically, these assets have provided different real rates of return – around 4 per cent for the first two and over 18 per cent for capital (including depreciation). It is this large difference between the rates that makes the calibration difficult. The basic approach is to define the "average" market rate, r, and use

43. Consumers also hold money, which is presumed to have a zero nominal return. Inflation generates a seigniorage transfer to the government. For this calibration, the rate of inflation in the steady state is assumed to be 1 per cent.

the risk premiums to express differences from it.[44] We set r, which enters the model through the rate used by consumers to discount future income in evaluating human wealth, at 8 per cent.[45] This is not quite an average of historical returns, but it is not far off recent values.

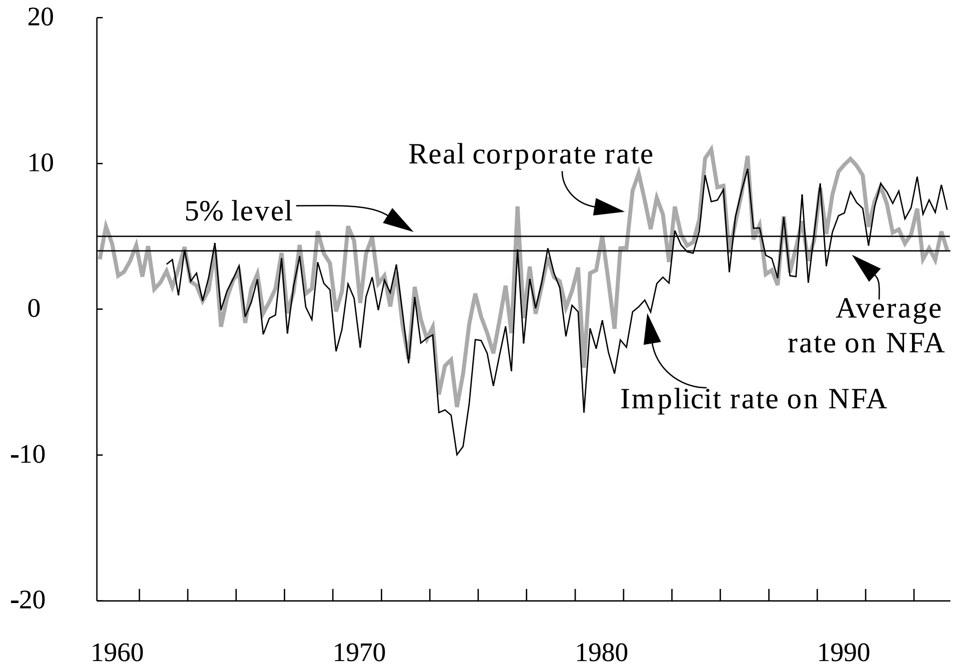
Risk premiums are then used to account for historical differences between the average rates observed for particular assets and the benchmark rate. Interest payments can be expressed as a rate of interest times the appropriate stock. Available data on interest payments and stocks allow an implicit interest rate to be calculated, but such rates often do not resemble comparable market rates. This difference can be attributed to errors of measurement (for example, flows that are inconsistent with the stock), overly simple asset accounts (for example, the lack of a term structure for government debt in the model), as well as something more closely resembling risk assessments in the real world. This last point refers to the fact that some debt – government debt, for example – is typically rated as less risky than average corporate debt.

The imputed interest rates for net foreign assets and government debt together with the observed rate on short-term corporate paper are illustrated in Figures 14 and 15. In the steady state they are both set to 5 per cent, which is more in line with recent observations and above the historical averages. It is a matter of some importance and dispute as to where to set equilibrium real interest rates. We have chosen to treat the more recent data as indicative of the values that must be accepted for the future. This may be overly pessimistic, but given the importance of debt issues in the overall macro scenario, we think that it is dangerous to base analysis on the assumption that real rates will soon return to average historical levels.

44. It is possible, of course, to define the risk premiums relative to some other base like, for example, the foreign real interest rate. We felt, however, that the central role of the average real interest rate in the consumer's problem made it a more natural choice of base.

45. Recall that consumers actually use a higher rate of discount, owing to the effect of the probability of death. The effective discount rate for the human wealth calculation is over 12 per cent.

Figure 14
Implicit real interest rate on net foreign assets
(per cent)



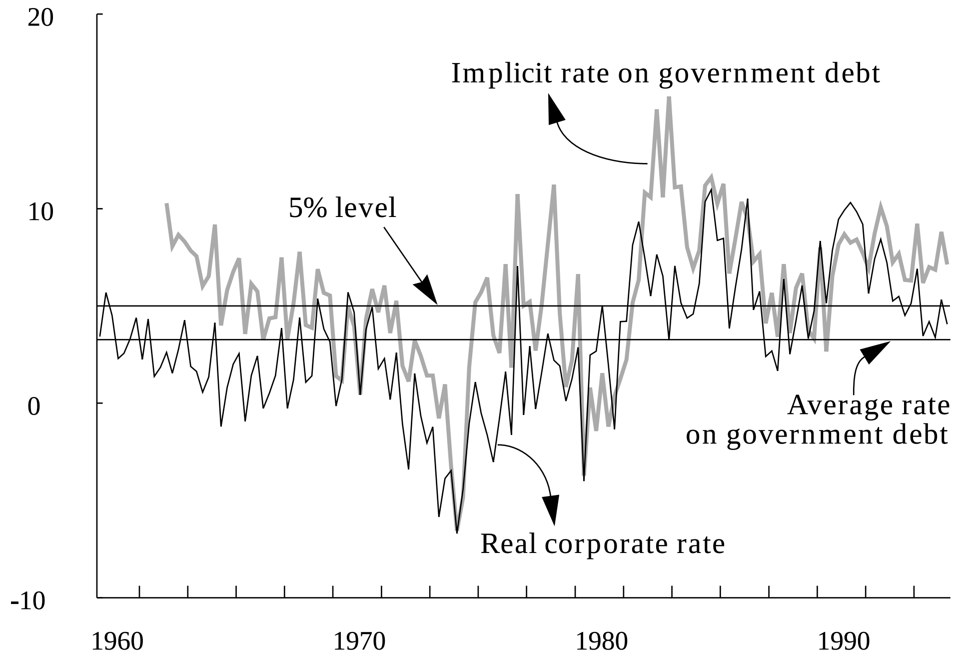
The calibration of this sector, and particularly how one would proceed in endogenizing risk premiums such that they increase with debt levels, is an area of active research at the Bank of Canada. For the moment, however, no such *marginal* effects of debt levels are included in the model or the analysis of its properties.[46]

4.9 The numerical steady state

Table 1 contains the solution of the model for the main macro variables under these calibration decisions. To interpret these results it is important to remember that all quantities are expressed in base period units, that is, deflated by population and by the level of labour productivity. In a growing economy there is no *stationary* state, and it is necessary to choose some

46. See Macklem, Rose and Tetlow (1994) for further discussion of these issues and an example of a study using this model with special added structure to include the effects of endogenous risk premiums.

Figure 15
Implicit real interest rate on government debt
(per cent)



normalization to show the underlying levels. On the left side, the table shows a number of standard decompositions: the expenditure components of output; sources and disposition of household (personal) income; the aggregate sources and disposition of gross saving; and the household wealth accounts. On the right side, we show the various relative prices, the import components of the expenditures, the main tax rates and some miscellaneous other values.

We think we have succeeded in our goal. That is, we are able to produce a control solution that seems reasonable and contains all the judgments of the specialists, while maintaining overall macro plausibility in that none of the results is so odd as to cast the overall solution into doubt.

5 ILLUSTRATIVE SHOCKS

5.1 An increase in the government debt-output ratio

Here we consider what happens in SSQPM when the government increases the steady-state debt-to-GDP ratio by 10 percentage points and changes the income tax rate in a manner compatible with its budget constraint. It is assumed that the money stock and size of government spending (on goods and services) relative to GDP are constant. For government debt to rise to the hypothesized new level, taxes must be reduced initially to create a cumulative deficit of the appropriate amount; but taxes must be higher in the steady state to service the higher level of government debt.[47]

In a typical traditional Keynesian model, this type of question cannot be addressed properly because intertemporal solvency constraints are not enforced. If pushed to answer the question, such models would show taxes going down in the short run, but there would be no mechanism to recognize the long-term implications of the higher level of debt. Moreover, with their focus on current flows, such models would normally show strong short-run positive effects on consumption from the higher disposable income.

At the other extreme, there is the so-called Ricardian-equivalence proposition – that there is no real effect from the method of financing the activities of government. Consumers recognize that the tax break is temporary and save the proceeds to pay the higher taxes they expect in the future. Consumption is not affected by the level of government debt.

The overlapping generations model of the type considered here lies in the middle ground between the Keynesian and Ricardian extremes. All intertemporal budget constraints are respected, but there are real effects of increasing the level of debt – consumption increases initially, but is lower in the steady state.

47. See Macklem, Rose and Tetlow (1994) for examples of simulations of QPM that include the dynamic path under such a shock as well as these steady-state effects.

The results for this experiment are reported in Table 3 (p. 80). Our analysis begins with the simplest model, SOLGM, in the first column, and proceeds, moving right across the table, as we add the various complications described in Section 3, towards SSQPM.

In SOLGM, an important steady-state effect of raising the level of government debt is to decrease consumption. The reason, simply put, is that consumers have less income due to the higher income tax rate.[48] Initially, however, owing to the decrease in taxes and the discounting of the future increase in taxes, consumers perceive themselves to be wealthier following the policy change.[49] This triggers an increase in consumption, financed by borrowing. Over time, consumers who survive (and new consumers) face the tax increase, and the economy moves to a steady state with lower financial wealth and less consumption.

Since consumers must hold the higher level of government debt and the stock of capital is unchanged, net foreign assets must decline by more than the rise in government debt to bring about the lower equilibrium level of financial wealth. This, in turn, necessitates an increase in the trade surplus to service the higher foreign debt and net exports rise. The essential consequence of higher government debt is that consumption falls to make room for the higher exports necessary to support the higher foreign debt service. Consumers alive when the debt is increased get the benefit of higher initial consumption, but future generations pay the price in terms of reduced consumption.

In SSQPM, the qualitative results are similar, but the magnitudes are a bit different. An important effect comes from the depreciation of the exchange rate associated with the move to a higher trade surplus. This increases the cost of capital and lowers the equilibrium capital stock and, hence, output. The depreciation also puts more of the adjustment of net

48. In this sense, the shock can be thought of as illustrating a pure income effect.

49. This is the key to understanding how the transition works. Consumers discount the future at a higher rate than the market because they expect some of the future tax burden to be passed on to the newly born and because they expect to escape still more of that burden when they die.

exports into imports. Exports are still above the control level, but substantially less so than in SOLGM and the intermediate models. Although the introduction of the risk premiums lowers the marginal effect of all of this on steady-state consumption, the same basic result emerges – higher government debt levels imply permanently lower consumption.

5.2 An increase in the relative level of government spending

In this shock, we focus on the effect of changing the size of government, as measured by the ratio of government spending to total output. We increase the ratio of government spending to output by 1 percentage point, which raises the level of government spending by 5 per cent before accounting for any induced changes of output. The results are shown in Table 4 (p. 82).

Given that we hold the government debt ratio and all other tax rates fixed in this shock, the rise in government spending is financed by an increase in taxes on household income. Because this tax does not affect labour supply, there are no permanent output effects in the small open economy (first 3 columns). With higher taxes there is lower human wealth and a lower desired level of wealth overall. Net foreign assets decline as households borrow more from foreigners in an attempt to limit the reduction in consumption.

When we add the exchange rate response in SSQPM, there is a small depreciation, which raises the cost of capital and lowers the level of capital and output (and hence the absolute size of the change in government spending itself). As in the debt shock, more of the external adjustment comes through imports. Consumption must still take the brunt of the impact, but it is down less than in SOLGM.

5.3 A shift from direct to indirect taxation

This shock illustrates the effects in SSQPM of a 10 percentage point increase in the indirect tax rate on consumption goods (from a base of just under 19 per cent in the control) coupled with a decline in the tax rate on household income. This is a very large shock by normal standards. The results are found in Table 5 (p. 84).

The reduction in direct taxes raises disposable income and leads to an increase in human wealth, which, in itself, would act to increase steady-state consumption expenditures.[50] However, with an increase in the level of indirect taxes on consumption goods, it may not be obvious why this higher level of expenditures translates into a higher level of real consumption.[51]

The key to understanding this is to ask who bears the burden of the change in tax policy. Recall that individuals in SSQPM prefer to tilt their consumption toward the future – they save more when they are young and consume more when they are old. Consumption taxes reinforce this by allowing the individual to transfer the payment of the taxes to when they are old and wealthy. Thus, with the change in the tax regime in this shock there is a transfer of wealth from the old to the young. The young receive a larger transfer, in the form of the decrease in direct taxes, than is necessary to pay their portion of the tax, so their real consumption and savings increase. The older, wealthier consumers suffer a loss, because they saved out of relatively highly taxed income and now must pay again when they consume.

The above describes the mechanics, but there is an interesting perspective on policy that emerges. How, one might well ask, could a tax on consumption end up increasing consumption when the alternative tax is effectively lump-sum and hence non-distortionary. The answer is to be found in the phenomenon of "overdiscounting" in this model. The introduction of the possibility of death leads individual consumers to discount the future at a rate higher than the market rate. From a social planner's perspective, this is distortionary, because it induces too little saving and

50. As there is no marginal change in behaviour, owing to the non-distortionary nature of the income tax, all the wealth measures increase by the same proportion.

51. Recall that $(1+\tau^c)\tilde{p}^c c_t = \Omega t w_t$. Thus, higher indirect taxes on consumption goods will lower consumption, all else equal.

overconsumption. Taxing consumption then moves the economy towards the social optimum by compensating for individual overdiscounting. The results shown in Table 5 reveal that this is a quantitatively important point in SSQPM.

5.4 A decrease in the rate of time preference

In this shock, we lower the rate of time preference by 0.1 percentage points. Such a change may seem small, but it is not in terms of its effects. The reason is that changes in rates used for discounting the entire future to obtain a present value have a highly leveraged effect on the answers obtained.

In an overlapping generations model, the level of impatience, as measured by the rate of time preference, relative to the interest rate faced by consumers, is a key determinant in the savings-consumption decision. If the rate of time preference is equal to the effective interest rate, then consumption will be the same in all periods and there will be no individual saving. If the rate of time preference is less than the interest rate, individual consumption increases over time – there is a tilt towards future consumption because the financial incentive to save outweighs the impatience. The consumer will choose to save and accumulate financial wealth. This is the case assumed for the calibration of SSQPM. Conversely, if the rate of time preference is greater than the interest rate, consumption decreases over time – the individual consumer will choose to borrow to finance current spending and pay off the debt by consuming less in the future.

Reducing the rate of time preference increases the weight given to the future in the individual optimization. The result is a higher optimal level of wealth and a lower marginal propensity to consume. The first three columns of Table 6 (p. 86) illustrate this for an environment where there is no effect on the cost of capital. The higher level of wealth is achieved primarily through an increase in net foreign assets.[52] The resulting decline in

52. The decline in human wealth when we add transfers reflects our accounting decision to put the transfers into what we call human wealth. With lower foreign debt, the transfer, which in this case was positive, is reduced.

the required trade balance frees up resources to support a higher equilibrium level of consumption. This provides a good illustration of the paradox of thrift at work in the intertemporal framework. Despite the lower marginal propensity to consume, the higher rate of saving results in higher wealth and a higher overall level of consumption in the steady state.

When we allow for the almost-small-open-economy effects in SSQPM, there are some important changes. With higher wealth and a reduced need to export, there is an appreciation of the exchange rate, which lowers the cost of imported capital and raises the level of capital and output. The resulting need for an increase in net foreign assets is smaller (some of the increase in wealth comes in the form of capital and government debt), muting the decline in exports. A substantial rise in imports, from both the higher investment and the pure price effects on import shares, also contributes to limiting the fall in exports.

5.5 An increase in the world rate of interest

The shock is an increase of 0.1 percentage points in the world real interest rate. This shock affects all domestic rates, including the cost of capital.

Given the close connection between the rate of time preference and real interest rates, it would be surprising to find substantial differences between the effects of an increase in the world real interest rate and an increase in the rate of time preference. Table 7 (p. 88) shows that, indeed, there are clear similarities. As with the time preference shock, an increase in the world real interest rate tends to encourage more consumption in the future at the expense of the present, owing to higher discounting in the agent's human wealth calculation. At the same time, however, there is the added effect that higher market interest rates imply a higher cost of capital, which triggers a lower optimal capital stock and, consequently, lower output and labour income at all points in time. This reduction in consumption possibilities is not a feature of the shock to the rate of time preference.

The first three columns of Table 7 describe the results when all relative prices are held fixed. As with the time preference shock, steady-state consumption rises with the increase in the world real interest rate, but all other components of real expenditure fall, as does aggregate output. The decline in human wealth is not strong enough, however, at least in the first two columns, to offset the large positive effect on the desired level of financial wealth of the higher return to saving. As consumers hold less capital and, given the decline in output, less government debt, the small net increase in total wealth comes in the form of a relatively large increase in net foreign assets. Looking at consumption, we see that the wealth effect is, in this case, reinforced strongly by a rise in the marginal propensity to consume.[53] Risk-averse agents tend to take proportionately more consumption when rates rise and the return to saving is higher.

The last column of Table 7 shows the results for SSQPM when the relative price effects are added. The real exchange rate appreciates in response to the lower trade surplus implied by the lower foreign debt service requirement. This results in a lower price of investment goods, but not of sufficient magnitude to overturn the direct effect of the higher interest rate. The biggest changes come in the composition of trade. Imports increase and the decline in exports is much muted.

It is interesting to contrast the results from SSQPM with those of familiar open-economy models, such as the Mundell-Fleming (MF) model. One important difference between the predictions of the two models is that while SSQPM produces an appreciation of the exchange rate in response to an increase in the foreign real interest rate, the MF model predicts a depreciation. The MF model is an income-expenditure model (that is, stocks play no role) and has no formal supply structure. This means that the sole role of the exchange rate is to return aggregate demand to its former level – higher real interest rates must be offset by a lower dollar. SSQPM, on the other hand, includes the supply-side effect of higher interest rates on the cost of capital and adds the critical intertemporal aspects of consumption

53. The sign reversal in the effect on total wealth when we add the risk premiums comes from the effect of the shock on transfers. Note that this effect is not large enough to offset the effect of the rise in the marginal propensity to consume. Consumption remains above control.

behaviour that provide the formal channel for interest rates to influence consumption in the first place.

It is important to stress that we are discussing the long-run properties of QPM here. In fact, in dynamic simulations, QPM also predicts a depreciation of the dollar in the short run, which is then reversed as the solution goes to the new steady state.

5.6 A decline in foreign productivity

We conclude our discussion of properties with a shock to foreign productivity, holding domestic productivity fixed. Specifically, we lower foreign labour productivity by 1 per cent. This shock focusses on the marginal effects of the almost-small-open-economy assumptions in SSQPM. As illustrated in the first three columns of Table 8 (p. 90), the shock has no effect on domestic variables until we add the extra relative price changes associated with the relaxation of the small-open-economy assumptions.

A fall in world productivity reduces the level of output and demand for the domestic good in the world economy. This leads to a decline in Canadian export prices and to a depreciation in the real exchange rate. These relative price changes mute the reduction in export volumes.

The depreciation of the real exchange rate causes the price of investment goods, 50 per cent of which are imported in our control calibration, to rise. As a result, the optimal levels of capital, output, labour income (and hence consumption) are all lower. The correspondingly lower net foreign asset position is supported, in the face of lower exports, by a proportionately greater decline in imports.

5.7 Sensitivity analysis

Table 9 (p. 92) reports a limited sensitivity analysis. We report the effect on the per cent shock-control results for output to a number of changes in calibration, with a focus on the import shares. The qualitative results are unaffected.

6 CONCLUDING REMARKS

In this report, we have reviewed the structure and properties of SSQPM, a model that describes the steady state of an almost small open economy and that has been calibrated to reflect the Canadian data and a series of judgments about the future course of some important exogenous variables.

The purpose of SSQPM is to define the underlying long-term equilibrium for a complete dynamic model of the Canadian economy, called QPM. These models and a series of satellite models together provide the system now being used by the staff of the Bank of Canada as their core tool for both economic projections and policy analysis.

Bridging the gap between the rigour required for forward-looking policy models and the flexibility required to support practical work in a policy institution has been a long-standing source of tension. Most applied models have focussed on explaining the shorter-term dynamics and have not worried about the underlying specification of the steady state. This may be reasonable, to a point, if the focus is kept on very short-term issues and transitory shocks. However, models are constantly being asked to answer questions that are medium- to long-term in nature, and there is increasing reluctance to accept as reasonable any analysis that is based on dynamically unstable simulations or simulations that push important general equilibrium issues into the undocumented future.

We have provided examples, the clearest being perhaps the case of debt accumulation, where it is simply invalid to simulate the future with arbitrary assumptions about things like sectoral deficits, because such systems can be dynamically unstable and fail to converge to a stock equilibrium. What is required is a true behavioural model of stock equilibrium, which then must have an influence on the specification of shorter-term dynamics. Nothing less is acceptable if one is to entertain seriously the most pressing fiscal policy questions of the day, because otherwise the answers are, in our view, uninterpretable and potentially seriously misleading.

We hasten to add that while we believe that the neoclassical, market-clearing paradigm is useful and empirically defensible, this is not the only view that could be taken. However, while accepting as open for debate the details of the model of equilibrium (or the absence of equilibrium or the multiplicity of equilibria), we would argue that for a model to be taken seriously for macro policy analysis in the 1990s, it must deal with these issues explicitly.

An important practical question is how to simulate a model with forward-looking behaviour in an economy with a well-determined steady state. Solution of a steady-state model like SSQPM is trivial. However, it is far from trivial to solve a full dynamic model like QPM. In the past, computer technology and solution algorithms were not available, or not available at reasonable cost, to permit applied macroeconomists to try to bridge the gap between short-term forecasting and policy analysis. We would argue that computational limitations are no longer an impediment to implementing a coherent view of what determines the macro general equilibrium and the influence of these forces on the economy over the short to medium term.[54] We have also done development work on this aspect of using macroeconomic models. A separate report on a new solution procedure we have implemented is forthcoming.[55]

The difficulty raised by demanding a high level of rigour in the description of the longer-term general equilibrium is that the problem rapidly becomes intractable if there is no real *macro*economics. If it is necessary to model micro-sectoral detail to understand the macro economy, then the task of the macroeconomist is truly daunting. We believe that it *is* possible to gain reliable qualitative and quantitative insight about the economy and macro policy issues from a highly simplified macrotheoretic framework.

What we have tried to do with SSQPM is to take the simple one-good paradigm, which is analytically easy to understand and manipulate

54. QPM is simulated regularly over 100 quarters and we have no technical difficulty in doubling that.

55. See Armstrong, Black, Laxton and Rose (1994).

but which is extremely simplistic, and make it work as a description of a real economy. We think that we have succeeded in introducing sufficient flexibility to achieve that goal, since we are able to calibrate the model to produce a steady state that respects many of the essential features of the Canadian data and the judgment of informed users.

The report contains a number of shocks to illustrate the properties of SSQPM. Being able to do this is another advantage of the highly structured framework of the model. We think that the properties revealed are reasonable as a point of departure and bolster our claim that a working policy framework can be built around a simple core model.

We recognize that there are many important macro policy issues that arise from shocks that cannot be entertained directly in such a simple model or from questions about economic structure that are not encompassed by the model's formal theoretical framework. Attempts to make a simple model more flexible can go only so far. Perhaps the most important issue of this type for Canadian analysis is how to deal with changes in the terms of trade. We have put a number of representations of terms-of-trade effects into the model. Nevertheless, we would not claim to be satisfied with our ability to analyse such shocks.

Our plan is to maintain a number of specialized models, adding the extra analytical complexity required to deal with identified key issues, such as the effects of shocks to the terms of trade, but eliminating all the complexity of another sort that is necessary in the core projection model. In Macklem (1993), for example, a formal model with three domestic goods is described. This model has very similar analytical underpinnings in terms of the theory of consumer behaviour but contains the necessary elaboration on the supply side to represent relative prices for tradable, non-tradable and resource goods. It can be used to provide insight as to the equilibrium effects of terms-of-trade shocks. But it does not have the broad attention to short-term dynamic properties necessary for QPM.

We are working on developing procedures to take information from such models for use in QPM through judgmental adjustments when required. We have not yet gained enough experience to judge how well this will work or how often we will be forced to resort to such supplementary analysis. However, we are encouraged by the results of using QPM in production mode since the autumn of 1993: we seem to have wide scope to deal with the practical complexities of preparing a regular quarterly projection, without needing formal supplementary analysis on a regular basis.

Describing the determinants of a steady state is only the start of what is required for policy analysis. Moreover, we have offered very little here in terms of quantitative evidence on how important these things are over the horizon of normal economic projections. To continue the documentation of the QPM system, we need to turn to issues of the specification of dynamics in the larger model. That will be the subject of a future report.

APPENDIX 1: THE MNEMONICS OF SSQPM

Note: all stocks and flows are defined as real values deflated by population and by the level of productivity.

A1.1 The household sector: consumption, income and wealth

c	Consumption
Ω	Marginal propensity to consume out of wealth
δ	Discount factor
β	Birth rate
γ	Probability of survival
n	Population growth rate
ζ	Growth rate of labour-augmenting technical progress
σ	Elasticity of intertemporal substitution
w	Real wage
y^{lab}	Labour income
tw	Total wealth
hw	Human wealth
fw	Financial wealth
fa	Financial assets

A1.2 The government sector

g	Government expenditures on goods and services

gb	Government debt
τ^w	Tax rate on labour income
τ^k	Tax rate on profits
τ^y	Aggregate indirect tax rate
τ^c	Indirect tax rate on consumption goods
τ^i	Indirect tax rate on investment goods
τ^g	Indirect tax rate on government goods
$seign$	Seigniorage

A1.3 Foreign sector

$netx$	Net exports
fb	Foreign assets of households
ca	Current account balance
x	Exports
m	Imports
c^{mshar}	Portion of consumption that is imported
g^{mshar}	Portion of government spending that is imported
i^{mshar}	Portion of investment goods that is imported
x^{mshar}	Portion of export goods that is imported
pfx	Real exchange rate (in terms of Canadian goods)

A1.4 Firms

i	Investment
k	Stock of capital
ϕ	Rate of physical depreciation
α	Exponent on labour input in the production function (labour share of income)

A1.5 Interest rates and "risk" premiums

r	Real interest rate on financial assets
φ^{k}	Risk premium on capital assets
φ^{gb}	Risk premium on government assets
φ^{fb}	Risk premium on net foreign assets
$risk$	Quantity of consumption goods transferred to consumers in compensation for risk

A1.6 Prices

p^{mc}	Marginal cost to GDP deflator measured at market prices
p^{cx}	Index of foreign prices, converted into Canadian dollars, relative to the GDP deflator at market prices
p^{fa}	Price of financial assets relative to the GDP deflator
$\tilde{p}^{c}$	Price of consumption goods at factor cost relative to the GDP deflator at factor cost

$\tilde{p}^{i}$	Price of investment goods at factor cost relative to the GDP deflator at factor cost
$\tilde{p}^{g}$	Price of government goods at factor cost relative to the GDP deflator at factor cost
$\tilde{p}^{x}$	Price of export goods at factor cost relative to the GDP deflator at factor cost
$\tilde{p}^{m}$	Price of import goods relative to the GDP deflator at factor cost
$\tilde{p}^{netx}$	Price of net export goods at factor cost relative to the GDP deflator at factor cost
p^{c}	Market price of consumption goods relative to the GDP deflator at market prices
p^{i}	Market price of investment goods relative to the GDP deflator at market prices
p^{g}	Market price of government goods relative to the GDP deflator at market prices
p^{x}	Market price of export goods relative to the GDP deflator at market prices
p^{m}	Price of import goods relative to the GDP deflator at market prices
p^{netx}	Market price of net export goods relative to the GDP deflator at market prices
p^{m}	Market price of import goods relative to the GDP deflator at market prices

p^{cm}	Market price of imported consumption goods relative to the GDP deflator at market prices
p^{im}	Market price of imported investment goods relative to the GDP deflator at market prices
p^{gm}	Market price of imported government goods relative to the GDP deflator at market prices
p^{xm}	Market price of imported export goods relative to the GDP deflator at market prices

A1.7 Miscellaneous

$\bar{p}^{i}$	Exogenous term in the equation for the relative price of investment goods (constant in a steady state)
$\bar{p}^{g}$	Exogenous term in the equation for the relative price of government goods (constant in a steady state)
$\bar{p}^{x}$	Exogenous term in the equation for the relative price of export goods (constant in a steady state)
$\bar{p}^{c^m}$	Exogenous term in the equation for the relative price of imported consumption goods (constant in a steady state)
$\bar{p}^{i^m}$	Exogenous term in the equation for the relative price of imported investment goods (constant in a steady state)
$\bar{p}^{g^m}$	Exogenous term in the equation for the relative price of imported government goods (constant in a steady state)

$\bar{c}^{mshar}$	Exogenous term in the equation for imported consumption goods (constant in a steady state)
$\bar{i}^{mshar}$	Exogenous term in the equation for imported investment goods (constant in a steady state)
$\bar{g}^{mshar}$	Exogenous term in the equation for imported government goods (constant in a steady state)
$\bar{x}^{mshar}$	Exogenous term in the equation for goods imported and reexported (constant in a steady state)
$x1$	Weight in the equation for the price of imported investment goods
$x2$	Weight in the equation for the price of imported consumption goods
$x4$	Weight in the equation for the price of export goods
$x5$	Weight in the equation defining marginal cost
$x6$	Weight in the equation for the real exchange rate
$x7$	Weight in the equation for the price of export goods

APPENDIX 2: THE EQUATIONS OF SSQPM

A2.1 Household behaviour: consumption and wealth

$$(1+\tau^c)\tilde{p}^c c = \Omega tw \tag{A1}$$

$$\Omega = 1 - \gamma\delta^{\sigma}(1+r)^{\sigma-1} \tag{A2}$$

$$tw = hw + fw \tag{A3}$$

$$hw = \frac{1+r}{1+r-\gamma(1+\zeta)}(y^{lab} + risk - seign) \tag{A4}$$

$$fw = \frac{1+r}{(1+n)(1+\zeta)-(1+r)}(y^{lab} + risk - seign - (1+\tau^c)\tilde{p}^c c) \tag{A5}$$

A2.2 Breakdown of financial wealth

$$fw = \frac{1+r}{(1+n)(1+\zeta)} p^{fa} fa \tag{A6}$$

$$fa = fb + gb + k \tag{A7}$$

$$p^{fa} fa = \tilde{p}^{netx} fb + gb + (1+\tau^i)\tilde{p}^i k \tag{A8}$$

A2.3 Government's budget constraint

$$(1+\tau^g)\tilde{p}^g g = \tau^w w + \tau^y y + \tau^k \alpha y + seign + \left(1 - \frac{(1+r+\phi^{gb})}{(1+n)(1+\zeta)}\right) gb \tag{A9}$$

A2.4 Population (and labour force) growth

$$1+n = \gamma(1+\beta) \tag{A10}$$

A2.5 Market clearing condition

$$y = c + i + g + netx \tag{A11}$$

A2.6 Reconciliation of income flows: net transfers to households from “risk” premiums

$$risk = \frac{\varphi^k(1+\tau^i)\tilde{p}^i k + \varphi^{gb} gb + \varphi^{fb}\tilde{p}^{netx} fb}{(1+\zeta)(1+n)} \qquad \text{(A12)}$$

A2.7 The supply side, firm behaviour and labour income

$$y = \left(\frac{k}{(1+\zeta)(1+n)}\right)^{\alpha} \qquad \text{(A13)}$$

$$w = (1-\alpha)y \qquad \text{(A14)}$$

$$\frac{\alpha y(1+\zeta)(1+n)(1-\tau^k)}{k} = (1+\tau^i)\tilde{p}^i(r+\phi+\varphi^k) \qquad \text{(A15)}$$

$$i = \left(1-\frac{(1-\phi)}{(1+\zeta)(1+n)}\right)k \qquad \text{(A16)}$$

$$y^{lab} = (1-\tau^w)w \qquad \text{(A17)}$$

A2.8 Foreign trade

$$netx = \left(1-\frac{1+r+\varphi^{fb}}{(1+n)(1+\zeta)}\right)fb \qquad \text{(A18)}$$

$$p^m m = p^{cm}c^{mshar}c + p^{im}i^{mshar}i + p^{gm}g^{mshar}g + p^{xm}x^{mshar}x \qquad \text{(A19)}$$

$$m = c^{mshar}c + i^{mshar}i + g^{mshar}g + x^{mshar}x \qquad \text{(A20)}$$

$$netx = x - m \qquad \text{(A21)}$$

$$\tilde{p}^{netx}netx = \tilde{p}^x x - \tilde{p}^m m \qquad \text{(A22)}$$

$$p^{netx}netx = p^x x - p^m m \qquad \text{(A23)}$$

$$c^{mshar} = \frac{\bar{c}^{mshar}}{pfx} \qquad \text{(A24)}$$

$$i^{mshar} = \frac{\bar{i}^{mshar}}{pfx} \tag{A25}$$

$$g^{mshar} = \frac{\bar{g}^{mshar}}{pfx} \tag{A26}$$

$$x^{mshar} = \frac{\bar{x}^{mshar}}{pfx} \tag{A27}$$

A2.9 Indirect taxes

$$\tau^y y = \tau^c \tilde{p}^c c + \tau^i \tilde{p}^i i + \tau^g \tilde{p}^g g \tag{A28}$$

A2.10 Prices

$$p^i = \bar{p}^i (1 + \tau^i)(p^{i^m})^{i^{mshar}} (p^{mc})^{1 - i^{mshar}} \tag{A29}$$

$$\tilde{p}^{i^m} = \bar{p}^{i^m} (pfx)^{x1} (p^{mc})^{1 - x1} \tag{A30}$$

$$p^g = \bar{p}^g (1 + \tau^g)/(1 + \tau^y) \tag{A31}$$

$$\tilde{p}^{c^m} = \bar{p}^{c^m} (pfx)^{x2} (p^{mc})^{1 - x2} \tag{A32}$$

$$\tilde{p}^{g^m} = \bar{p}^{g^m} \tilde{p}^{i^m} \tag{A33}$$

$$\tilde{p}^{x^m} = \bar{p}^{x^m} \tilde{p}^{i^m} \tag{A34}$$

$$1 = ((1 + \tau^y) p^{mc})^{x5} (p^x)^{1 - x5} \tag{A35}$$

$$p^x = \bar{p}^x (p^{cx}/(1 + \tau^y))^{x4} (p^{mc})^{1 - x4} \left(\frac{y^*}{y}\right)^{x7} \tag{A36}$$

$$pfx = \overline{pfx} \frac{x}{y} \left(\frac{y}{y^*}\right)^{x6} \tag{A37}$$

$$p^c = \frac{1+\tau^c}{1+\tau^y}\tilde{p}^c \tag{A38}$$

$$p^i = \frac{1+\tau^i}{1+\tau^y}\tilde{p}^i \tag{A39}$$

$$p^g = \frac{1+\tau^g}{1+\tau^y}\tilde{p}^g \tag{A40}$$

$$p^x = \frac{1}{1+\tau^y}\tilde{p}^x \tag{A41}$$

$$p^m = \frac{1}{1+\tau^y}\tilde{p}^m \tag{A42}$$

$$p^{cm} = \frac{1}{1+\tau^y}\tilde{p}^{cm} \tag{A43}$$

$$p^{im} = \frac{1}{1+\tau^y}\tilde{p}^{im} \tag{A44}$$

$$p^{gm} = \frac{1}{1+\tau^y}\tilde{p}^{gm} \tag{A45}$$

$$p^{xm} = \frac{1}{1+\tau^y}\tilde{p}^{xm} \tag{A46}$$

TABLES

Table 1
The numerical steady state of SSQPM

Real output and expenditures		**Real market prices**	
Output	1.2447	*Consumption*	1.0789
• *Consumption*	0.7812	*Investment*	0.7091
• *Investment*	0.1981	*Government*	1.0036
• *Government*	0.2489	*Exports*	0.8514
• *Exports*	0.4782	*Imports*	0.8567
• *Imports*	0.4618	**Real prices at factor cost**	
Personal income and expenditures		*Consumption**	1.0331
After-tax labour income	0.7103	*Investment**	0.7480
Financial income	0.2942	*Government**	1.1100
• *Capital*	0.2859	*Exports**	0.9700
• *Government bonds*	0.0346	*Imports*	0.9760
• *Net foreign assets*	-0.0263	**Other prices**	
Asset accumulation	0.0423	*Price of foreign exchange**	0.9920
Consumption expenditure	0.9602	**Import shares**	
Sources and disposition of saving		*Consumption**	0.2200
Gross saving	0.1600	*Investment**	0.5000
• *Personal saving*	0.1642	*Government**	0.2100
• *Government surplus*	-0.0173	*Exports*	0.2898
• *Current account surplus*	0.0132	**Taxes**	
Gross investment	0.1600	*Income tax rate*	0.1301
Capital depreciation	0.1219	*Firm tax rate***	0.0500
Net investment expenditure	0.0381	*Aggregate indirect tax rate*	0.1392
Household wealth accounts		**Miscellaneous**	
Total wealth	10.5972	*Net exports to GDP*	0.0132
Human	8.7722	*Exports to GDP**	0.3842
Financial	1.8250	*NFA to GDP (nominal)**	-0.3800
• *Capital*	1.6457	*Growth rate of GDP*	0.0250
• *Government bonds*	0.7470	*Birth rate*	0.0532
• *Net foreign assets*	-0.5677		

Note: All variables are endogenous unless otherwise noted. Levels are expressed in per capita terms and deflated by the level of labour productivity. Prices are expressed relative to the price of output. On the left-hand side of the table, the accounting follows the conventions of Section 3. That is, with the exception of the decomposition of real output, variables are measured in units of output multiplied by the relative price at factor cost and one plus the rate of indirect tax.

* Exogenous for calibrations, endogenous for simulations
** Exogenous

Table 2

The numerical steady state of SSQPM: additional variables and parameters

Variable	Definition	Value
SOLGM Parameters		
r	Consumer's real interest rate	0.0800
ζ	Growth of labour productivity	0.0138
δ	Discount factor**	0.9691
σ	Intertemporal elasticity of substitution	0.5000
α	Capital's share of income	0.3449
ϕ	Capital depreciation rate	0.0800
γ	Probability of survival	0.9600
β	Population growth	0.0111
gb / y	Government assets to GDP (nominal)	0.5000
g / y	Government share of GDP	0.2000
Other government parameters		
$seign / y$	Seigniorage to GDP	0.0008
	Rate of indirect tax on	
τ^c	• consumption goods	0.1898
τ^i	• investment goods	0.0800
τ^g	• government goods	0.0300
Risk premiums relative to the consumer's market rate		
k^{risk}	Risk premium on capital	0.1076
gb^{risk}	Risk premium on government bonds	-0.0300
fb^{risk}	Risk premium on net foreign assets	-0.0300
Foreign variables		
y	Size of the foreign sector	12.4468
p^{cx}	Price of exportable commodities*	1.0231

(continued next page)

Table 2

(continued)

Variable	Definition	Value
Prices		
$p^{c^{mshar}}$	Price of imported consumption goods*	0.9297
$p^{i^{mshar}}$	Price of imported investment goods*	0.7000
$p^{g^{mshar}}$	Price of imported government goods*	1.0000
$p^{x^{mshar}}$	Price of reexport goods (endogenous)	1.2217
$\tilde{p}^{netx^{mshar}}$	Relative price of net exports*	0.8000
Coefficients		
x^1	PPP coefficient for investment	0.8000
x^2	PPP coefficient for consumption	0.9000
x^4	PPP coefficient for exports	0.7000
x^5	Coefficient for marginal cost	0.9400
x^6	Relative size coefficient for volumes	0.6000
x^7	Relative size coefficient for prices	0.0833

Note: All variables are exogenous unless otherwise noted. Levels are expressed in per capita terms and deflated by the level of (labour) productivity.

* Exogenous for calibrations, endogenous for simulations

** Endogenous for calibrations, exogenous for simulations

Table 3

Effects of increasing the ratio of government debt to output

	SOLGM	Plus other taxes[a]	Plus risk premiums	SSQPM[b]
Real output and expenditures				
Output	-0.0000	0.0000	0.0000	-0.1080
• *Consumption*	-1.2523	-1.3152	-0.4362	-0.5897
• *Investment*	0.0000	0.0000	0.0000	-0.3128
• *Government*	-0.0000	0.0000	0.0000	-0.1080
• *Exports*	1.8177	2.0317	0.8787	0.3220
• *Imports*	0.0153	0.0572	0.1209	-0.5655
Personal income and expenditures				
After-tax labour income	-0.8036	-1.0455	-0.3559	-0.4897
Financial income	-1.2523	-1.3152	-0.1253	-0.2372
• *Capital*	0.0000	0.0000	0.0000	-0.1080
• *Government*	20.0000	19.8471	19.9417	19.8121
• *Net foreign*	-33.9930	-32.9502	-27.6375	-27.5490
Asset accumulation	-1.2523	-1.3152	-0.4362	-0.5579
Consumption expenditure	-1.2523	-1.3152	-0.4362	-0.5579
Sources and disposition of saving				
Gross saving	-0.0000	0.0000	0.0000	-0.1080
• *Personal saving*	-0.3106	-0.3286	-0.1124	-0.2239
• *Government surplus*	-0.3658	-0.3839	-0.2971	-0.3429
• *Current account surplus*	0.4725	0.4844	0.3130	0.3624
Gross investment expenditure	0.0000	0.0000	0.0000	-0.1080
• *Capital depreciation*	-0.0000	0.0000	0.0000	-0.1080
• *Net investment expenditure*	0.0000	-0.0000	0.0000	-0.1080
Household wealth accounts				
Total wealth	-1.2523	-1.3152	-0.4362	-0.5579
Human	-1.2523	-1.3152	-0.4362	-0.5579
Financial	-1.2523	-1.3152	-0.4362	-0.5579
• *Capital*	-0.0000	0.0000	0.0000	-0.1080
• *Government*	20.0000	19.8471	19.9417	19.8121
• *Net foreign*	-33.9930	-32.9502	-27.6375	-27.5490

(continued next page)

Table 3

(continued)

	SOLGM	Plus other taxes[a]	Plus risk premiums	SSQPM[b]
Real market prices (diff.)				
Consumption	0.0000	0.1342	0.0506	0.0870
Investment	0.0000	0.1218	0.0459	0.1803
Government	0.0000	0.1162	0.0438	0.0488
Exports	0.0000	0.1128	0.0425	0.2619
Imports	0.0000	0.1128	0.0425	0.3534
Real prices at factor cost (diff.)				
Consumption	0.0000	0.0000	-0.0000	0.0330
Investment	0.0000	0.0000	0.0000	0.1537
Government	0.0000	0.0000	0.0000	0.0000
Exports	0.0000	0.0000	0.0000	0.2510
Imports	0.0000	0.0000	0.0000	0.3549
Import shares				
Consumption	0.0000	0.0000	0.0000	-0.0801
Investment	0.0000	0.0000	0.0000	-0.1820
Government	0.0000	0.0000	0.0000	-0.0765
Exports	0.0000	0.0000	0.0000	-0.1055
Taxes (diff.)				
Income tax	0.8187	1.1395	0.5085	0.5071
Firm tax rate	0.0000	0.0000	0.0000	0.0000
Aggregate indirect tax rate	0.0000	-0.1442	-0.0555	-0.0554
• *Indirect tax on C*	0.0000	0.0000	0.0000	0.0000
• *Indirect tax on I*	0.0000	0.0000	0.0000	0.0000
• *Indirect tax on G*	0.0000	0.0000	0.0000	0.0000
Miscellaneous				
Price of foreign exchange	0.0000	0.0000	0.0000	0.3625
Net exports to GDP (diff.)	0.6928	0.7599	0.2924	0.3353
NFA to GDP (nominal, diff.)	-12.9173	-12.5855	-10.5258	-10.5446

Note: All results are reported as per cent shock minus control except those labelled "diff.," which are absolute differences multiplied by 100.

a. These include indirect taxes, a tax on profits and seigniorage.
b. These include all relative prices, the real exchange rate and import shares.

Table 4

Effects of increasing the relative level of government spending

	SOLGM	Plus other taxes[a]	Plus risk premiums	SSQPM[b]
Real output and expenditures				
Output	-0.0000	0.0000	0.0000	-0.0310
• *Consumption*	-2.3349	-2.1715	-1.5680	-1.7600
• *Investment*	0.0000	0.0000	0.0000	-0.0899
• *Government*	5.0000	5.0000	5.0000	4.9674
• *Exports*	0.7318	0.6467	0.1151	0.0751
• *Imports*	-0.0291	-0.0173	-0.0185	-0.1768
Personal income and expenditures				
After-tax labour income	-1.4984	-1.7262	-1.2791	-1.6763
Financial income	-2.3349	-2.1715	-0.4504	-0.5485
• *Capital*	0.0000	0.0000	0.0000	-0.0310
• *Government*	-0.0000	-0.1839	-0.1483	-0.1964
• *Net foreign*	-14.3140	-11.0443	-4.8312	-5.5442
Asset accumulation	-2.3349	-2.1715	-1.5680	-1.8331
Consumption expenditure	-2.3349	-2.1715	-1.5680	-1.8331
Sources and disposition of saving				
Gross saving	-0.0000	0.0000	-0.0000	-0.0310
• *Personal saving*	-0.5791	-0.5425	-0.4039	-0.4951
• *Government surplus*	0.0000	0.0036	0.0022	0.0034
• *Current account surplus*	0.1990	0.1624	0.0547	0.0729
Gross investment expenditure	0.0000	0.0000	0.0000	-0.0310
• *Capital depreciation*	-0.0000	0.0000	0.0000	-0.0310
• *Net investment expenditure*	0.0000	-0.0000	0.0000	-0.0310
Household wealth accounts				
Total wealth	-2.3349	-2.1715	-1.5680	-1.8331
Human	-2.3349	-2.1715	-1.5680	-1.8331
Financial	-2.3349	-2.1715	-1.5680	-1.8331
• *Capital*	-0.0000	0.0000	0.0000	-0.0310
• *Government*	-0.0000	-0.1839	-0.1483	-0.1964
• *Net foreign*	-14.3140	-11.0443	-4.8312	-5.5442

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Table 4

(continued)

	SOLGM	Plus other taxes[a]	Plus risk premiums	SSQPM[b]
Real market prices (diff.)				
Consumption	0.0000	0.1937	0.1546	0.0984
Investment	0.0000	0.1759	0.1404	0.1594
Government	0.0000	0.1677	0.1339	0.1663
Exports	0.0000	0.1628	0.1300	0.1919
Imports	0.0000	0.1628	0.1300	0.2743
Real prices at factor cost (diff.)				
Consumption	0.0000	0.0000	0.0000	-0.0769
Investment	0.0000	0.0000	0.0000	0.0441
Government	0.0000	0.0000	0.0000	0.0000
Exports	0.0000	0.0000	0.0000	0.0577
Imports	0.0000	0.0000	0.0000	0.1505
Import shares				
Consumption	0.0000	0.0000	0.0000	-0.0192
Investment	0.0000	0.0000	0.0000	-0.0437
Government	0.0000	0.0000	0.0000	-0.0184
Exports	0.0000	0.0000	0.0000	-0.0253
Taxes (diff.)				
Income tax	1.5265	1.8815	1.8278	2.0295
Firm tax rate	0.0000	0.0000	0.0000	0.0000
Aggregate indirect tax rate	0.0000	-0.2081	-0.1695	-0.1885
• *Indirect tax on C*	0.0000	0.0000	0.0000	0.0000
• *Indirect tax on I*	0.0000	0.0000	0.0000	0.0000
• *Indirect tax on G*	0.0000	0.0000	0.0000	0.0000
Miscellaneous				
Price of foreign exchange	0.0000	0.0000	0.0000	0.0868
Net exports to GDP (diff.)	0.2917	0.2547	0.0511	0.0949
NFA to GDP (nominal, diff.)	-5.4393	-4.2746	-1.8950	-2.1857

Note: All results are reported as per cent shock minus control except those labelled "diff.," which are absolute differences multiplied by 100.

a. These include indirect taxes, a tax on profits and seigniorage.
b. These include all relative prices, the real exchange rate and import shares.

Table 5

Effects of a switch from direct to indirect taxation

	SOLGM	Plus other taxes[a]	Plus risk premiums	SSQPM[b]
Real output and expenditures				
Output	-0.0000	0.0000	0.0000	0.3753
• *Consumption*	2.6388	1.8846	0.3199	1.2316
• *Investment*	0.0000	0.0000	0.0000	1.0920
• *Government*	-0.0000	0.0000	0.0000	0.3753
• *Exports*	-3.8302	-2.9113	-0.6443	-0.4902
• *Imports*	-0.0322	-0.0819	-0.0886	1.2351
Personal income and expenditures				
After-tax labour income	8.2800	8.3054	7.1394	8.3464
Financial income	12.9026	10.4480	2.5137	2.9028
• *Capital*	0.0000	0.0000	0.0000	0.3753
• *Government*	5.6783	5.3849	5.9206	6.1437
• *Net foreign*	71.6275	47.2163	20.2643	20.3277
Asset accumulation	12.9026	10.4480	8.7516	9.1769
Consumption expenditure	12.9026	10.4480	8.7516	9.1769
Sources and disposition of saving				
Gross saving	-0.0000	0.0000	-0.0000	0.3753
• *Personal saving*	3.1998	2.6102	2.2544	2.6421
• *Government surplus*	-0.1039	-0.1042	-0.0882	-0.1063
• *Current account surplus*	-0.9956	-0.6942	-0.2295	-0.2674
Gross investment expenditure	0.0000	0.0000	0.0000	0.3753
• *Capital depreciation*	-0.0000	0.0000	0.0000	0.3753
• *Net investment expenditure*	0.0000	-0.0000	0.0000	0.3753
Household wealth accounts				
Total wealth	12.9026	10.4480	8.7516	9.1769
Human	12.9026	10.4480	8.7516	9.1769
Financial	12.9026	10.4480	8.7516	9.1769
• *Capital*	-0.0000	0.0000	0.0000	0.3753
• *Government*	5.6783	5.3849	5.9206	6.1437
• *Net foreign*	71.6275	47.2163	20.2643	20.3277

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Table 5

(continued)

	SOLGM	Plus other taxes[a]	Plus risk premiums	SSQPM[b]
Real market prices (diff.)				
Consumption	4.0895	3.0130	2.4419	2.1443
Investment	-5.3732	-4.8768	-5.2828	-4.3291
Government	-5.3732	-4.6511	-5.0382	-5.4539
Exports	-5.3732	-4.5156	-4.8915	-4.9214
Imports	-5.3732	-4.5156	-4.8915	-5.8738
Real prices at factor cost (diff.)				
Consumption	-0.0000	0.0000	-0.0000	-0.5301
Investment	0.0000	0.0000	0.0000	-0.5303
Government	0.0000	0.0000	0.0000	0.0000
Exports	0.0000	0.0000	0.0000	-0.3545
Imports	0.0000	-0.0000	0.0000	-1.4671
Import shares				
Consumption	0.0000	0.0000	0.0000	0.1415
Investment	0.0000	0.0000	0.0000	0.3217
Government	0.0000	0.0000	0.0000	0.1351
Exports	0.0000	0.0000	0.0000	0.1865
Taxes (diff.)				
Income tax	-8.4353	-9.0522	-10.2021	-9.8722
Firm tax rate	0.0000	0.0000	0.0000	0.0000
Aggregate indirect tax rate	5.6783	6.0935	6.7658	6.5470
• *Indirect tax on C*	10.0000	10.0000	10.0000	10.0000
• *Indirect tax on I*	0.0000	0.0000	0.0000	0.0000
• *Indirect tax on G*	0.0000	0.0000	0.0000	0.0000
Miscellaneous				
Price of foreign exchange	0.0000	0.0000	0.0000	-0.6341
Net exports to GDP (diff.)	-1.4598	-1.0889	-0.2144	-0.6491
NFA to GDP (nominal, diff.)	27.7978	18.9671	9.3941	9.4769

Note: All results are reported as per cent shock minus control except those labelled "diff.," which are absolute differences multiplied by 100.

a. These include indirect taxes, a tax on profits and seigniorage.
b. These include all relative prices, the real exchange rate and import shares.

Table 6
Effects of a decrease in the rate of time preference

	SOLGM	Plus other taxes[a]	Plus risk premiums	SSQPM[b]
Real output and expenditures				
Output	-0.0000	0.0000	0.0000	0.0834
• *Consumption*	0.9617	1.0278	0.3416	0.4537
• *Investment*	0.0000	0.0000	0.0000	0.2419
• *Government*	-0.0000	0.0000	0.0000	0.0834
• *Exports*	-1.3960	-1.5876	-0.6881	-0.2460
• *Imports*	-0.0117	-0.0447	-0.0947	0.4368
Personal income and expenditures				
After-tax labour income	-0.0000	0.1536	0.0459	0.1113
Financial income	4.2584	4.9794	1.9438	1.9784
• *Capital*	0.0000	0.0000	0.0000	0.0834
• *Government*	-0.0000	0.0996	0.0380	0.1205
• *Net foreign*	26.1057	25.7485	21.6442	21.0809
Asset accumulation	4.2584	4.9794	6.7675	6.6825
Consumption expenditure	0.9617	1.0278	0.3416	0.4277
Sources and disposition of saving				
Gross saving	-0.0000	0.0000	-0.0000	0.0834
• *Personal saving*	1.0561	1.2440	1.7433	1.7829
• *Government surplus*	0.0000	-0.0019	-0.0006	-0.0021
• *Current account surplus*	-0.3629	-0.3786	-0.2451	-0.2773
Gross investment expenditure	0.0000	0.0000	0.0000	0.0834
• *Capital depreciation*	-0.0000	0.0000	0.0000	0.0834
• *Net investment expenditure*	0.0000	-0.0000	0.0000	0.0834
Household wealth accounts				
Total wealth	1.5402	1.5854	0.8625	0.9503
Human	-0.0000	0.1932	-0.3220	-0.2423
Financial	4.2584	4.9794	6.7675	6.6825
• *Capital*	-0.0000	0.0000	0.0000	0.0834
• *Government*	-0.0000	0.0996	0.0380	0.1205
• *Net foreign*	26.1057	25.7485	21.6442	21.0809

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Table 6

(continued)

	SOLGM	Plus other taxes[a]	Plus risk premiums	SSQPM[b]
Real market prices (diff.)				
Consumption	-0.0000	-0.1046	-0.0396	-0.0680
Investment	0.0000	-0.0950	-0.0359	-0.1384
Government	0.0000	-0.0906	-0.0343	-0.0373
Exports	0.0000	-0.0879	-0.0333	-0.2001
Imports	0.0000	-0.0879	-0.0333	-0.2698
Real prices at factor cost (diff.)				
Consumption	-0.0000	-0.0000	-0.0000	-0.0267
Investment	0.0000	0.0000	0.0000	-0.1183
Government	0.0000	0.0000	0.0000	0.0000
Exports	0.0000	0.0000	0.0000	-0.1920
Imports	0.0000	-0.0000	0.0000	-0.2712
Import shares				
Consumption	0.0000	0.0000	0.0000	0.0616
Investment	0.0000	0.0000	0.0000	0.1400
Government	0.0000	0.0000	0.0000	0.0588
Exports	0.0000	0.0000	0.0000	0.0812
Taxes (diff.)				
Income tax	0.0000	-0.1674	-0.0655	-0.0638
Firm tax rate	0.0000	0.0000	0.0000	0.0000
Aggregate indirect tax rate	0.0000	0.1127	0.0435	0.0423
• *Indirect tax on C*	0.0000	0.0000	0.0000	0.0000
• *Indirect tax on I*	0.0000	0.0000	0.0000	0.0000
• *Indirect tax on G*	0.0000	0.0000	0.0000	0.0000
Miscellaneous				
Price of foreign exchange	0.0000	0.0000	0.0000	-0.2770
Net exports to GDP (diff.)	-0.5321	-0.5938	-0.2290	-0.2574
NFA to GDP (nominal, diff.)	9.9202	9.8125	8.2361	8.0469

Note: All results are reported as per cent shock minus control except those labelled "diff.," which are absolute differences multiplied by 100.

a. These include indirect taxes, a tax on profits and seigniorage.
b. These include all relative prices, the real exchange rate and import shares.

Table 7

Effects of an increase in the foreign real interest rate

	SOLGM	Plus other taxes[a]	Plus risk premiums	SSQPM[b]
Real output and expenditures				
Output	-0.3275	-0.3275	-0.1962	-0.1238
• *Consumption*	0.6386	0.6561	0.0844	0.1918
• *Investment*	-0.9466	-0.9466	-0.5677	-0.3586
• *Government*	-0.3275	-0.3275	-0.1962	-0.1238
• *Exports*	-1.4940	-1.6356	-0.6762	-0.2918
• *Imports*	-0.4756	-0.4868	-0.3047	0.1355
Personal income and expenditures				
After-tax labour income	-0.2830	-0.2039	-0.1506	-0.1187
Financial income	3.7561	4.3905	1.5636	1.5914
• *Capital*	0.2916	0.2916	-0.0378	0.0347
• *Government*	0.9184	1.0061	1.8286	1.9016
• *Net foreign*	20.1224	20.0720	15.4540	14.9344
Asset accumulation	2.4751	3.1018	4.7573	4.6783
Consumption expenditure	0.6386	0.6561	0.0844	0.1582
Sources and disposition of saving				
Gross saving	-0.9466	-0.9466	-0.5677	-0.4956
• *Personal saving*	-0.0980	0.0648	0.8040	0.8369
• *Government surplus*	0.0060	0.0047	0.0025	0.0017
• *Current account surplus*	-0.2934	-0.3096	-0.1938	-0.2184
Gross investment expenditure	-0.9466	-0.9466	-0.5677	-0.4956
• *Capital depreciation*	-0.9466	-0.9466	-0.5677	-0.4956
• *Net investment expenditure*	-0.9466	-0.9466	-0.5677	-0.4956
Household wealth accounts				
Total wealth	0.1167	0.1567	-0.3768	-0.3047
Human	-1.2734	-1.0904	-1.4261	-1.3616
Financial	2.5700	3.1972	4.8543	4.7752
• *Capital*	-0.8549	-0.8549	-0.4756	-0.4035
• *Government*	-0.2352	-0.1485	-0.0756	-0.0040
• *Net foreign*	21.0355	20.9857	17.0350	16.5251

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Table 7

(continued)

	SOLGM	Plus other taxes[a]	Plus risk premiums	SSQPM[b]
Real market prices (diff.)				
Consumption	-0.0000	-0.0913	-0.0293	-0.0657
Investment	0.0000	-0.0828	-0.0266	-0.1169
Government	0.0000	-0.0790	-0.0254	-0.0275
Exports	0.0000	-0.0767	-0.0247	-0.1560
Imports	0.0000	-0.0767	-0.0247	-0.2026
Real prices at factor cost (diff.)				
Consumption	-0.0000	-0.0000	-0.0000	-0.0347
Investment	0.0000	0.0000	0.0000	-0.1028
Government	0.0000	0.0000	0.0000	0.0000
Exports	0.0000	0.0000	0.0000	-0.1512
Imports	0.0000	0.0000	0.0000	-0.2041
Import shares				
Consumption	0.0000	0.0000	0.0000	0.0534
Investment	0.0000	0.0000	0.0000	0.1215
Government	0.0000	0.0000	0.0000	0.0510
Exports	0.0000	0.0000	0.0000	0.0704
Taxes (diff.)				
Income tax	0.0745	-0.0617	0.0365	0.0378
Firm tax rate	0.0000	0.0000	0.0000	0.0000
Aggregate indirect tax rate	0.0000	0.0983	0.0322	0.0312
• *Indirect tax on C*	0.0000	0.0000	0.0000	0.0000
• *Indirect tax on I*	0.0000	0.0000	0.0000	0.0000
• *Indirect tax on G*	0.0000	0.0000	0.0000	0.0000
Miscellaneous				
Price of foreign exchange	0.0000	0.0000	0.0000	-0.2404
Net exports to GDP (diff.)	-0.3956	-0.4465	-0.1442	-0.1609
NFA to GDP (nominal, diff.)	7.9227	7.9299	6.4494	6.2783

Note: All results are reported as per cent shock minus control except those labelled "diff.," which are absolute differences multiplied by 100.

a. These include indirect taxes, a tax on profits and seigniorage.
b. These include all relative prices, the real exchange rate and import shares.

Table 8
Effects of a decline in foreign productivity

	SOLGM	Plus other taxes[a]	Plus risk premiums	SSQPM[b]
Real output and expenditures				
Output	-0.0000	0.0000	0.0000	-0.0525
• *Consumption*	0.0000	0.0000	-0.0000	-0.0648
• *Investment*	0.0000	0.0000	0.0000	-0.1521
• *Government*	-0.0000	0.0000	0.0000	-0.0525
• *Exports*	-0.0000	-0.0000	0.0000	-0.2399
• *Imports*	0.0000	0.0000	0.0000	-0.3101
Personal income and expenditures				
After-tax labour income	-0.0000	0.0000	-0.0000	-0.0373
Financial income	0.0000	0.0000	-0.0000	-0.0525
• *Capital*	0.0000	0.0000	0.0000	-0.0525
• *Government*	-0.0000	0.0000	-0.0000	-0.0525
• *Net foreign*	0.0000	0.0000	-0.0000	0.0525
Asset accumulation	0.0000	0.0000	-0.0000	-0.0525
Consumption expenditure	0.0000	0.0000	-0.0000	-0.0525
Sources and disposition of saving				
Gross saving	-0.0000	0.0000	-0.0000	-0.0525
• *Personal saving*	-0.0000	0.0000	-0.0000	-0.0525
• *Government surplus*	0.0000	-0.0000	0.0000	0.0009
• *Current account surplus*	-0.0000	-0.0000	0.0000	-0.0007
Gross investment expenditure	0.0000	0.0000	0.0000	-0.0525
• *Capital depreciation*	-0.0000	0.0000	0.0000	-0.0525
• *Net investment expenditure*	0.0000	-0.0000	0.0000	-0.0525
Household wealth accounts				
Total wealth	0.0000	0.0000	-0.0000	-0.0525
Human	0.0000	0.0000	-0.0000	-0.0525
Financial	0.0000	0.0000	-0.0000	-0.0525
• *Capital*	-0.0000	0.0000	0.0000	-0.0525
• *Government*	-0.0000	0.0000	-0.0000	-0.0525
• *Net foreign*	0.0000	0.0000	-0.0000	0.0525

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Table 8

(continued)

	SOLGM	Plus other taxes[a]	Plus risk premiums	SSQPM[b]
Real market prices (diff.)				
Consumption	0.0000	0.0000	-0.0000	0.0133
Investment	0.0000	-0.0000	0.0000	0.0707
Government	0.0000	-0.0000	0.0000	0.0000
Exports	0.0000	-0.0000	0.0000	0.0607
Imports	0.0000	-0.0000	0.0000	0.1185
Real prices at factor cost (diff.)				
Consumption	0.0000	0.0000	-0.0000	0.0127
Investment	0.0000	0.0000	0.0000	0.0746
Government	0.0000	0.0000	0.0000	0.0000
Exports	0.0000	0.0000	0.0000	0.0691
Imports	0.0000	0.0000	-0.0000	0.1350
Import shares				
Consumption	0.0000	0.0000	0.0000	-0.0387
Investment	0.0000	0.0000	0.0000	-0.0878
Government	0.0000	0.0000	0.0000	-0.0369
Exports	0.0000	0.0000	0.0000	-0.0509
Taxes (diff.)				
Income tax	0.0000	-0.0000	0.0000	0.0000
Firm tax rate	0.0000	0.0000	0.0000	0.0000
Aggregate indirect tax rate	0.0000	0.0000	-0.0000	-0.0000
• *Indirect tax on C*	0.0000	0.0000	0.0000	0.0000
• *Indirect tax on I*	0.0000	0.0000	0.0000	0.0000
• *Indirect tax on G*	0.0000	0.0000	0.0000	0.0000
Miscellaneous				
Price of foreign exchange	0.0000	0.0000	0.0000	0.1746
Net exports to GDP (diff.)	-0.0000	-0.0000	0.0000	0.0236
NFA to GDP (nominal, diff.)	0.0000	0.0000	-0.0000	-0.0000

Note: All results are reported as per cent shock minus control except those labelled "diff.," which are absolute differences multiplied by 100.

a. These include indirect taxes, a tax on profits and seigniorage.
b. These include all relative prices, the real exchange rate and import shares.

Table 9

Sensitivity analysis: effect on per cent shock minus control for output of variation in selected assumptions

	Govt. debt shock	Govt. size shock	Indirect tax shock	Rate of time preference shock	Foreign real interest rate shock	Relative productivity shock
Base case	-0.1080	-0.0310	0.3753	0.0834	-0.1238	-0.0525
c^{mshar} **(base case = 0.22)**						
• *Lower = 0.10*	-0.1399	-0.0982	0.4562	0.1085	-0.1066	-0.0455
• *Higher = 0.30*	-0.0903	0.0061	0.3331	0.0695	-0.1333	-0.0563
i^{mshar} **(base case = 0.50)**						
• *Lower = 0.40*	-0.0890	-0.0246	0.2763	0.0687	-0.1387	-0.0419
• *Higher = 0.60*	-0.1245	-0.0366	0.4645	0.0960	-0.1098	-0.0625
g^{mshar} **(base case = 0.21)**						
• *Lower = 0.10*	-0.1124	0.0206	0.3846	0.0868	-0.1221	-0.0505
• *Higher = 0.30*	-0.1046	-0.0698	0.3682	0.0807	-0.1252	-0.0540
Combination c^{mshar}, g^{mshar}						
• *0.26, 0.10*	-0.1027	0.0384	0.3612	0.0792	-0.1273	-0.0526
γ **(base case = 0.96)**						
• *Lower = 0.95*	-0.1080	-0.0310	0.3753	0.0648	-0.1396	-0.0525
• *Higher = 0.97*	-0.1080	-0.0310	0.3753	0.1126	-0.0988	-0.0525

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