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**GAUSS™ Programs for the Estimation of State-Space  
Models with ARCH Errors: A User's Guide**

by

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## Abstract

State-space models have long been popular in explaining the evolution of various economic variables. This is mainly because they generally have more economic content than do others in their class of parsimonious models (for example, VARs). Yet, in spite of their advantages, use of these models until recently was limited by the assumption that all the innovations therein had to be conditionally normally distributed. Consequently, one could not model conditionally heteroskedastic series within that framework. The study by Harvey, Ruiz, and Sentana (1992) changed that. These authors showed how ARCH effects could be handled in a state-space framework, whether such innovations were in the measurement equations or in the transition ones. For these purposes, the authors modified the usual Kalman filter and developed an approximate (or quasi-optimal) filter to estimate these models.

An application of the above framework was made recently by Kichian (1999) to estimate Canadian potential output. Because no code was publicly available at that time to perform this task, GAUSS programs were developed at the Bank of Canada. In fact, the code allows for the estimation of a wide variety of state-space models with or without ARCH errors.

This paper explains how to use this Bank code. We show, step-by-step, how to use the programs and give several examples. Also included is additional code for calculating out-of-sample forecast errors on the observable variables in order to assess the goodness of fit of the estimated models.

*JEL classification:* C32, C82, C87, C89

*Bank classification:* *Econometric and statistical methods*

## Résumé

Les modèles espaces d'états servent depuis longtemps à expliquer l'évolution de diverses variables économiques. La raison en est surtout qu'ils renferment généralement plus d'information au sujet de l'économie qu'ils cherchent à décrire que d'autres modèles parcimonieux (les modèles VAR par exemple). Malgré leur attrait, leur utilisation était limitée jusqu'à tout récemment par la contrainte voulant que la distribution des innovations obéisse à une loi normale conditionnelle. Il n'était donc pas possible de modéliser des séries conditionnellement hétéroscedastiques dans un cadre espace d'états. Dans un article publié en 1992, Harvey, Ruiz et Sentana ont levé cette contrainte. Ils ont montré comment le cadre espace d'états permet de tenir compte des effets ARCH, que ceux-ci touchent les équations de mesure ou les équations de transition. Pour arriver à leurs fins, les

auteurs ont modifié le filtre habituel de Kalman et mis au point un filtre approché (ou quasi-optimal) permettant d'estimer ces modèles.

Récemment, Kichian (1999) a appliqué le cadre en question à l'estimation de la production potentielle canadienne. Comme aucun programme adapté à cette tâche ne semblait disponible, les programmes GAUSS nécessaires ont dû être élaborés à la Banque même. Les programmes créés permettent l'estimation d'un large éventail de modèles espaces d'états que ces derniers comportent ou non des erreurs de type ARCH.

L'auteure explique, étape par étape, comment se servir des programmes et donne quelques exemples. Un programme additionnel permet de calculer les erreurs de prévision hors échantillon qui entachent les variables observables et d'évaluer par conséquent la qualité de l'ajustement statistique des modèles estimés.

*Classification JEL: C32, C82, C87, C89*

*Classification de la Banque: Méthodes économétriques et statistiques*



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## 1. Introduction

The purpose of this paper is to explain the use of GAUSS programs developed at the Bank of Canada to estimate a state-space model with autoregressive conditional heteroskedastic (ARCH) errors. The programs are based on the Harvey, Ruiz, and Sentana (1992) paper (hereafter HRS) and are quite flexible. They allow the user to estimate a wide variety of models with or without ARCH errors, whether these errors are in the measurement equations or in the transition equations.

Before HRS, latent variable models existing in the empirical literature always assumed the error terms in the equations to be homoskedastic. But this presented a considerable modelling limitation if one believed that important information was embedded in the conditional volatility of certain variables in the model. Thus, potentially important information was lost in a model that assumed conditionally homoskedastic errors. This was especially true if a series exhibited episodes of low variance followed by episodes of high variance, in which case it is said to have ARCH errors.

In a recent paper, Harvey, Ruiz, and Sentana (1992) showed how ARCH effects could be handled theoretically in a state-space model where the conditional heteroskedasticity was present in either the measurement or the transition equation innovations. For the estimation, the authors proposed using an approximate (or a quasi-optimal) filter, which is a modification of the usual Kalman filter.

In economics, many financial and certain macroeconomic series exhibit thick-tailed empirical distributions indicative of conditional heteroskedasticity, especially at high frequencies. Accordingly, it is easy to see why applying the HRS methodology to latent variable models that include such series could be useful. However so far, to our knowledge, no computer code was publicly available for carrying out such estimations.

A GAUSS code was therefore developed at the Bank of Canada to perform such estimations.<sup>1</sup> The code is general enough to allow the user to estimate state-space models with ARCH effects either in the measurement equation innovations or in the transition ones, or without ARCH effects. It can also accommodate moving-average errors in the observation equations. In addition, code is included for calculating both in-sample and out-of-sample forecast errors on the observable variables.

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1. The impetus to write these programs originally came from the need to estimate an unobserved components model with ARCH expectations for the explicit purposes of estimating Canadian potential output and forecasting inflation (see Kichian [1999]).

In Section 2, we present the HRS model and explain notations. In the following section, we explain the code, indicating which parts to modify in order to set up a particular model. Then in Section 4, we provide two examples that walk the user through the programs. The last section concludes.

## 2. The Harvey, Ruiz, and Sentana model

In this section, we describe the HRS model that is the one used, with only minor modifications, in our programs.

Consider the general model given by the following system of equations:

$$y_t = Z\alpha_t + \beta X_t + \Lambda \varepsilon_t + \varepsilon_t^* \quad t = 1, 2, \dots, nobs \quad (2.1)$$

$$\alpha_t = T\alpha_{t-1} + \delta W_t + \Psi \eta_t + \eta_t^* \quad (2.2)$$

$$\varepsilon_t = \sqrt{h_t} \cdot \bar{\varepsilon}_t \quad \bar{\varepsilon}_t \sim NID(0, 1)$$

$$\eta_t = \sqrt{q_t} \cdot \bar{\eta}_t \quad \bar{\eta}_t \sim NID(0, 1)$$

$$h_t = \alpha_0 + \sum_{i=1}^{q_h} \alpha_i \varepsilon_{t-i}^2 \quad (2.3)$$

$$q_t = \gamma_0 + \sum_{i=1}^p \gamma_i \eta_{t-i}^2$$

Here,  $y_t$  is an  $n \times 1$  vector of observed variables and  $\alpha_t$  is an  $M \times 1$  vector of unobserved state variables.

Equation (1) denotes the set of measurement equations and links the observables to the non-observables through the parameter matrix  $Z$  that is of dimension  $n \times M$ . It also includes a vector of observable exogenous variables  $X_t$  of dimension  $k \times 1$  with  $n \times k$  coefficients  $\beta$ . In addition, the equation contains a disturbance vector  $\varepsilon_t^*$ , of dimension  $n \times 1$ , and which is distributed normally with mean zero and variance  $H_t^*$ .

Equation (2) describes the dynamics of the state vector and is therefore the set of transition equations. The matrix  $T$  is the  $M \times M$  matrix of coefficients on the lagged state variables. In

addition, a vector of exogenous observed variables  $W_t$  of dimension  $s \times 1$  is assumed to affect the state variable vector via the coefficient matrix  $\delta$  of dimension  $n \times s$ . A disturbance vector  $\eta_t^*$  of dimension  $M \times 1$ , which is distributed normally with mean zero and variance  $Q_t^*$ , also enters the equation.

Equations (1) and (2) also each contain a vector of ARCH disturbances. In the case of the measurement equations, these are collected in the  $n \times 1$  vector  $\varepsilon_t$  and are assumed to influence  $y_t$  through the  $n \times n$  coefficient matrix  $\Lambda$ . For the transition equations, the ARCH errors are in the  $M \times 1$  vector  $\eta_t$  and are assumed to influence the state vector via the  $M \times M$  matrix  $\Psi$ . The conditional variances of  $\varepsilon_t$  and  $\eta_t$  are given by  $h_t$  and  $q_t$  respectively and are of lag  $q_h$  and  $p$ .

Finally, it is assumed that the different non-ARCH error terms in the model are independently and identically distributed.

Note that, while the above model is based on HRS, there are nonetheless some differences. First, the parameter matrices  $Z, T, \beta, \delta$  are assumed to be constant while the matrices  $\Lambda$  and  $\Psi$  are set equal to identity. These assumptions are essentially made to reduce the parameter space and should not represent a very serious modelling limitation. Instead, we have generalized the model in another direction by allowing for the presence of exogenous variables in the transition equation. Second, our exogenous variables are allowed to vary across measurement and transition equations, but unlike HRS, are identical within equations of the same category.

## 2.1 The quasi-optimal filter

For the Kalman filter to yield minimum mean-square estimates of the state variables, the distribution of  $y_t$  conditional on information available at time  $t - 1$  is required to be normal. However, since the past values of the  $\varepsilon_t$  and  $\eta_t$  disturbances are not directly observed, this may not necessarily be the case.

To address this issue, HRS suggest treating the distributions of  $\varepsilon_t$  conditional on  $\varepsilon_{t-1}$  and  $\eta_t$  conditional on  $\eta_{t-1}$  as though they were normal and obtaining the first two moments of these errors using elements from the conditional mean and variance terms of the state variables. Evidently, in this case, the obtained estimates are not necessarily minimum mean square and the Kalman filter is then designated as being only quasi-optimal.

To use this modified filter, it is first necessary to re-write equations (2.1) and (2.2) of the model in their augmented form,<sup>2</sup> as in HRS. Thus:

$$y_t = Z^A \alpha_t^A + \beta X_t + \varepsilon_t^* \quad (2.4)$$

$$\alpha_t^A = T^A \alpha_{t-1}^A + \delta^A W_t + G \cdot U_t. \quad (2.5)$$

In the augmented framework, the superscript  $A$  denotes the augmented vector. Here, the ARCH disturbances are re-interpreted as state variables and collected in the vector  $\alpha_t^A$ . The disturbance vector  $U_t$  now includes all the disturbances in the model except for  $\varepsilon_t^*$ .

Let the number of contemporaneous state variables in the system be  $m$ . Note that  $m$  is a subset of  $M$ —the total number of state variables in the system—and it is perfectly possible for  $\alpha_t$  to include lagged state variables as well (as will be shown in the examples below).

The dimensions of the augmented matrices are the following:  $Z^A$  is  $n \times d$ ,  $\alpha_t^A$  and  $U_t$  are  $d \times 1$ ,  $\delta^A$  is  $d \times s$ , and  $T^A$  and  $G$  are  $d \times d$ .  $d$  is determined by  $v$ , which is the number of state variable lags in the model; by  $q_h$ , which is the number of the ARCH lags in the measurement equation; and by  $p$ , which is the number of ARCH lags in the transition equation. Therefore,  $d = mv + nq_h + mp$ .

For example, for  $q_h = 2$ ,  $p = 1$  and  $v = 2$ , we have that  $d = 2m + 2n + m$ . Thus, in this case, and assuming  $s = 1$ , the transition equation is the following:

$$\alpha_t^A \equiv \begin{bmatrix} \alpha_t \\ \eta_t \\ \varepsilon_t \\ \varepsilon_{t-1} \end{bmatrix} = \begin{bmatrix} T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \alpha_{t-1} \\ \eta_{t-1} \\ \varepsilon_{t-1} \\ \varepsilon_{t-2} \end{bmatrix} + \begin{bmatrix} \delta \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot W_t + \begin{bmatrix} I_{mv} & \Psi & 0 & 0 \\ 0 & I_M & 0 & 0 \\ 0 & 0 & I_n & 0 \\ 0 & 0 & 0 & I_n \end{bmatrix} \cdot \begin{bmatrix} \eta_t^* \\ \eta_t \\ \varepsilon_t \\ \varepsilon_{t-1} \end{bmatrix} \quad (2.6)$$

---

2. See HRS for more details.

Once the augmented framework has been determined, the quasi-optimal filter can be applied. This is given by

$$\begin{aligned}
\varepsilon_t^\alpha &= T^A \alpha_{t|t-1}^A + \delta^A W_t \\
\varepsilon_t^P &= T^A P_{t|t-1}^A T^{A'} + G \cdot E_{t-1}(U_t U_t') \cdot G' \\
B_t &= Z^A \cdot \varepsilon_t^P \cdot Z^{A'} + H_t^* \\
\varepsilon_t^m &= y_t - Z^A \alpha_{t|t-1}^A - \beta X_t \\
K_t &= \varepsilon_t^P \cdot Z^{A'} \cdot B_t^{-1} \\
\alpha_{t|t}^A &= \varepsilon_t^\alpha + K_t \cdot \varepsilon_t^m \\
P_{t|t}^A &= (I - K_t \cdot Z^A) \cdot \varepsilon_t^P
\end{aligned} \tag{2.7}$$

where  $\alpha_{t|t-1}^A$  is the conditional mean of the augmented state vector,  $P_{t|t-1}^A$  is its conditional covariance matrix,  $\varepsilon_t^\alpha$  and  $\varepsilon_t^P$  are the errors on the estimated conditional mean and variance equations of the state variables respectively,  $B_t$  is the variance-covariance matrix of the observable measurement equation dependent variables,  $\varepsilon_t^m$  is the mean residual term and  $K_t$  is the Kalman gain term. Finally,  $\alpha_{t|t}^A$  and  $P_{t|t}^A$  are the updated conditional estimates of the state vector mean and its variance.

Thus, given starting values for the conditional mean of the state vector  $\alpha_{1|0}^A$  and its conditional variance  $P_{1|0}^A$ , the state residual terms  $\varepsilon_1^\alpha$  and  $\varepsilon_1^P$  can be calculated. The respective terms are then substituted in the measurement equations to obtain the variance-covariance matrix  $B_1$  as well as the mean residual term  $\varepsilon_1^m$ . Now it is possible to calculate the Kalman gain term  $K_1$ . Substituting the last three terms in the state-updating equations, it is possible to obtain  $\alpha_{1|1}^A$  and  $P_{1|1}^A$ . At this point, the log likelihood function for that observation can be calculated. For the time  $t$  observation, this function is given by

$$lf_t = -\frac{n}{2} \log(2\pi) - \frac{\log|B_t|}{2} - \frac{1}{2} (\varepsilon_t^m)' \cdot B_t^{-1} \cdot \varepsilon_t^m. \tag{2.8}$$

In this fashion, the cycle repeats as above until all the observations are processed. The product of the conditional distributions of the  $y_t$ 's constitutes the likelihood function that is numerically maximized with respect to the parameters in the model.

### 3. Explanation of the GAUSS codes

A total of seven files comprise the necessary code. These files are

qo_fe.gau	front-end program
qo_lf.gau	filter and likelihood function
qo_st.gau	returns estimates for mean and diag(variance) of state variables
qo_is.gau	in-sample error for the measurement equation mean
qo_fp.gau	out-of-sample prediction errors
qo_stT.gau	returns estimates of state variables for the last observation
qo.lcg	library of codes

Of these files, the user need be concerned only with the front-end program (but all seven files should be in the same directory). The front-end program is the file where one loads the data, assigns initial values, and specifies a particular model. In addition, those sections that should be user-defined have upper case letter titles. Below are the explanations about this file.

The following sections are self-evident and will not be explained. These are the sections entitled:

```
/* SPECIFY OUTPUT FILES */
/* LOADING DATA */
/* DATA TRANSFORMATIONS */
/* PRINTING OUT RESULTS */
```

The most important section is the one with the title: `/* MODIFY THIS SECTION AS NEEDED */`.

First, write the model being used in the comment section according to the system of equations given by (2.1) to (2.3). This is the only framework needed for the front-end program so the user need not be concerned with the augmented structure of the model.

Next, designate the names of the individual parameters that will be estimated in the model. Change the names in `_max_ParNames` to reflect the designated names.

Now collect the data for the model in the `swd` matrix where the number of rows is the number of observations `nobs`, while the number of columns = `n+k+s`. The order of entry is as

follows: start with the (observed)  $\mathbf{n}$  dependent variables, next the  $\mathbf{k}$  exogenous variables of the measurement equations, followed by the  $\mathbf{s}$  exogenous variables of the transition equations.

Next, enter the appropriate dimensions of the variables following the definitions of the symbols and dimensions in the program. These definitions are listed below for convenience.

*Definition of symbols (minimum required for program to run)*

$\mathbf{n}$	=	number of contemporaneous dependant variables (1)
$\mathbf{m}$	=	number of contemporaneous state variables only (1)
$\mathbf{k}$	=	number of exogenous variables in each measurement equation (1)
$\mathbf{s}$	=	number of exogenous variables in each transition equation (1)
$\mathbf{l}$	=	$\mathbf{n}$ (1)
$\mathbf{q\_h}$	=	number of lags in ARCH errors in each of $\mathbf{l}$ errors (2)
$\mathbf{r}$	=	$\mathbf{m}$ (1)
$\mathbf{p}$	=	number of lags in ARCH errors in each of $\mathbf{r}$ errors (2)
$\mathbf{v}$	=	1 + number of lags of the state variable with the longest lags (2)

To follow the above, count the number of contemporaneous state variables in the system and designate this as  $\mathbf{m}$ . Now single out the state variable with the longest number of lags and let  $\mathbf{v}$  equal to this number plus one. Also set  $\mathbf{r}$  equal to  $\mathbf{m}$  even if the matrix  $\Psi$  has a dimension of  $M \times M$ .

Notice the number in parentheses at the end of each of the above definitions. This number designates the minimum number required for the program to run. Thus, for instance, the minimum number required for the number of lags in the ARCH errors is two. However, this does not mean that one cannot turn off the ARCH effects, or that it is impossible to set up models with no exogenous variables. The way to accomplish the last two objectives is to set these constants to zero further down in the code when assigning **beta0** and parameter restrictions (this will be explained in more detail in the parameter restrictions section). In the meantime, one must impose **q\_h=p=2** at minimum, and enter a column of ones for  $X_t$  and for  $W_t$  in **swd** (thereby setting **k=s=1**) for the code to run.

Before discussing the initialization of the parameters of the model, that is, **beta0**, we define two new matrices, **TT** and **ZZ**, and explain their dimensions (note that these are not the augmented matrices  $T^A$  and  $Z^A$ ).

Knowing that there are  $m$  contemporaneous state variables in the model being used and knowing  $v$ , define the vector  $\alpha_t^*$  of dimension  $mv \times 1$ . This vector contains a state variable and  $v - 1$  of its lags, followed by another state variable with as many lags, and so on. Now, in order to define **ZZ** and **TT**, write down the system:

$$y_t = \mathbf{ZZ}\alpha_t^* + \beta X_t + \Lambda \varepsilon_t + \varepsilon_t^* \quad (3.1)$$

$$\alpha_t^* = \mathbf{TT}\alpha_{t-1}^* + \delta^* W_t + \Psi^* \eta_t + \eta_t^* \quad (3.2)$$

The  $\mathbf{ZZ}$  matrix is therefore of dimension  $n \times mv$ , with each of its rows containing the parameters that link the respective dependant variable to the present and  $v - 1$  lagged values of the state variables. From the second equation, define the block diagonal matrix  $\mathbf{TT}$  of dimension  $mv \times mv$ , each block of dimension  $v \times v$ . The first row of any block contains the parameters linking a contemporaneous state variable to  $v - 1$  of its lags. Finally, the  $\delta$  and  $\Psi$  matrices are augmented to suit the dimensions of  $\alpha_t^*$  but the user need not be concerned with these.

Having said this, we now move on to the initialization of the parameters **beta0** of the model. Note that any parameters that are constrained should not be initialized. Therefore, do not enter an initial value in **beta0** for those parameters that are constrained.

As mentioned in the description of the model, the matrix of coefficients  $\beta$  for the exogenous variables  $X_t$  is of dimension  $n \times k$ . Enter initial values for  $\beta$  by successive rows of  $1 \times k$ . Similarly, enter the  $M \times s$  coefficients in the matrix  $\delta$  row by row. These are followed by the initial values first for  $H_t^*$ , then for  $Q_t^*$ .

Next, enter the first rows of each of the  $v \times v$  blocks of the  $\mathbf{TT}$  matrix successively. This is followed by the initialization of the  $\mathbf{ZZ}$  matrix where parameters are entered, again row by row. Next come the  $\Lambda$  and  $\Psi$  parameters.<sup>3</sup> If either one of these matrices is zero, indicating no ARCH errors in that category of equations, enter nothing for the respective parameters in the initialization vector. Similarly, if either of these matrices is identity, enter nothing for its parameters.

Finally, we come to the initialization of the ARCH parameters. Parameters for each of the measurement equations are entered first, row by row, followed by those of the transition equations.

Following this is the section entitled `/* ASSIGNING PRIORS FOR STATE VARIABLES */`. For an idea about which priors to use in a particular model, we refer the reader to Harvey (1989).

---

3. In the code, the names corresponding to these two matrices are **lambda** and **epsi** and their dimensions are **n** and **m** respectively.



The section where parameter constraints are imposed is the one with the heading:  
 /\* PROCEDURES -- MODIFY FROM beta TO archp ONLY, restrictions should be same across  
 all procs \*/. Impose restrictions according to the model being used and to the parameter dimen-  
 sions. These should be the same for all five procedures in the code.

In the next section, procedures are called in turn. Once terms, pterms, and the priors are assigned, the filter and maximum likelihood are applied. The output consists of the estimated parameters and their standard errors. Then procedure *temp2* returns the “one-sided” estimated state vector in **esalpha** as well as the diagonal of its variance-covariance matrix in **esp**. Procedure *temp4* returns the measurement errors  $\varepsilon_t^m$ , i.e., in-sample prediction errors. Finally, *temp5* returns the last value of the estimated state vector and its variance-covariance matrix. This is used in procedure *temp3* below in order to calculate out-of-sample prediction errors.

## 4. Examples

To demonstrate the flexibility and the limitations of our code, some simple examples are provided in this section. We start with a basic model for the estimation of potential output with no ARCH or moving-average error terms. The next example adds moving-average terms to the observable equations of the model. Finally, we show how to add ARCH errors to the basic framework.

Before continuing, the reader is cautioned that care should be taken, as in any multivariate model, to ensure that the model used is well identified. If, during the estimation, one of the parameters goes to zero, this is a sign that there are such problems in the model.

### 4.1 The basic model

Consider the following model:

$$\begin{aligned}
 y_t &= y_t^P + g_t \\
 y_t^P &= \mu + y_{t-1}^P + \varepsilon_t^{y*} \\
 g_t &= \phi_1 g_{t-1} + \eta_t^{g*} \\
 \pi_t &= \pi_{t-1} + \beta_0 g_t + \varepsilon_t^{\pi*}
 \end{aligned}
 \tag{4.1}$$

where  $y_t$  is the log of real GDP,  $y_t^P$  is the log of potential output, and  $g_t$  is the output gap. Potential is assumed to follow a random walk with drift while the gap is a stationary AR(1) process. A

Phillips curve then links inflation to the gap via the parameter  $\beta_0$ . Finally, we assume all error terms to be uncorrelated and to be normally and independently distributed.

Let us first simplify the model by substituting  $y_t^P$  for its expression in the identity and by recalling that  $y_{t-1}^P = y_{t-1} - g_{t-1}$ . The system becomes

$$\begin{aligned}\Delta y_t &= \mu + g_t - g_{t-1} + \varepsilon_t^{y*} \\ \Delta \pi_t &= \beta_0 g_t + \varepsilon_t^{\pi*} \\ g_t &= \phi_1 g_{t-1} + \eta_t^{g*}\end{aligned}\quad (4.2)$$

In this system, the observed dependent variables are  $\Delta y_t$  and  $\Delta \pi_t$ . Thus, set  $\mathbf{n}=2$ . The unobserved contemporaneous variable is  $g_t$  so that  $\mathbf{m}=1$  while  $\mathbf{M}=2$  since the lag of the gap also enters one of the measurement equations.

In the first measurement equation, there is one exogenous variable, i.e., the constant  $\mu$ . Thus, we set  $\mathbf{k}=1$ . There are no exogenous variables in the transition equation but we must set  $\mathbf{s}=1$  and enter a vector of constants in  $\mathbf{swd}$  for this last variable because of the minimum requirement. There are no ARCH effects in the above model, yet we should still set  $\mathbf{q}_h=\mathbf{p}=2$ . Further down, we will see how to turn off the ARCH effects. We also have that  $\mathbf{l}=2$ ,  $\mathbf{r}=1$  and, since only one lag of the only contemporaneous state variable is present in the measurement equations,  $\mathbf{v}=\mathbf{1}+\mathbf{1}=2$ .

Thus the state-space framework is written

$$\begin{aligned}\begin{bmatrix} \Delta \pi_t \\ \Delta y_t \end{bmatrix} &= \begin{bmatrix} \beta_0 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} g_t \\ g_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \mu \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{\pi*} \\ \varepsilon_t^{y*} \end{bmatrix} \\ \begin{bmatrix} g_t \\ g_{t-1} \end{bmatrix} &= \begin{bmatrix} \phi_1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} g_{t-1} \\ g_{t-2} \end{bmatrix} + \begin{bmatrix} \eta_t^{g*} \\ 0 \end{bmatrix}\end{aligned}\quad (4.3)$$

Therefore, in the code, we have that

```
_max_ParNames={"mu", "var(y)", "var(pi)", "var(g)", "phi1", "beta0"};
swd = (100*(lrgdp-lrgdp_11))~((ircpi-ircpi_11)/4)~ones(nobs,1)~ones(nobs,1);
n=2; m=1; k=1; s=1; l=2; q_h=2; r=1; p=2; v=2;
```

For this example, the matrices  $\alpha_t^*$ ,  $ZZ$ , and  $TT$  coincide with  $\alpha_t$ ,  $Z$ , and  $T$ . Thus we have that

$$\alpha_t^* = \begin{bmatrix} g_t \\ g_{t-1} \end{bmatrix} \quad ZZ = \begin{bmatrix} \beta_0 & 0 \\ 1 & -1 \end{bmatrix} \quad TT = \begin{bmatrix} \phi_1 & 0 \\ 1 & 0 \end{bmatrix}. \quad (4.4)$$

The initializations of parameter values are carried out next. Thus,

```

beta0 =
    @ beta1: 1xk @
0.1~   @ beta2: 1xk @           initializes the constant  $\mu$ 
    @ delta1 1xs @
    @ delta2 1xs @
0.3~0.5~ @ H*: 1xn @           initializes the  $\varepsilon_t^{\pi^*}$  and  $\varepsilon_t^{y^*}$ 
0.3~   @ Q*: 1xn @           initializes the  $\eta_t^{g^*}$ 
0.6~   @ TT1: 1xv @           initializes the  $\phi_1$ 
    @ TT2: 1xv@
0.5    @ ZZ1 1xmv@           initializes the  $\beta_0$ 
    @ ZZ2 1xmv@

```

Notice that, since there are no exogenous variables in the inflation equation and no ARCH effects present in either the measurement or the transition equations, there are no initializations for these.

Finally, the imposed constraints are:

**beta** = 0.0|th[1,1]; @ free parameter for the constant  $\mu$  @

**delta** = zeros(1,1); @ zero exog. variables imposed in transition equations @

**Hstar** = th[2:3,1]; @ free parameters for the  $\varepsilon_t^{\pi^*}$  and  $\varepsilon_t^{y^*}$  @

**Qstar** = th[4,1]; @ free parameter for  $\eta_t^{g^*}$  @

**TT** = th[5,1]|0.0; @ free parameter for  $\phi_1$  @

**ZZ** = th[6,1]|0.0 |1.0|-1.0 ; @ free parameter for  $\beta_0$  and constraints for the gap terms @

**lambda** = 1.0|1.0; @ impose 1.0 for each of the dependent variables@

**epsi** = 1.0; @ impose 1.0 for each of the contemp. state variables @

**archq** = zeros(3,1) |zeros(3,1); @ impose no ARCH effects in the measurement eqns. @

**archp** = zeros(3,1); @ impose no ARCH effects in the state eqns. @

At this point, the model can be run through the filter and the likelihood procedures. If the user also wants to calculate the mean-square errors of forecasts, then one should remember to reserve some data at the end of the sample for this purpose and set the forecast sample duration in the section entitled, /\* CALCULATING OUT-OF-SAMPLE FORECAST ERRORS \*/.

## 4.2 The Gerlach and Smets model:<sup>4</sup> Base model with moving-average errors

Now, supposing we want to refine the model by allowing one of the measurement errors to have a moving-average process with three lags. Similarly, we would like to impose an AR(2) process for the gap instead of the AR(1). We also want to let the gap term enter contemporaneously as well as with a lag in the inflation equation. Finally, we prefer to make the constant in the output equation time varying (a unit root process, to be more precise). To complete the changes to the basic model, we add a constant and exogenous variables to the inflation equation.

---

4. See Kichian (1999).

In this case, the model becomes

$$\begin{aligned}
 y_t &= y_t^P + g_t \\
 y_t^P &= \mu_t + y_{t-1}^P + \varepsilon_t^{y*} \\
 \mu_t &= \mu_{t-1} + \eta_t^{\mu*} \\
 g_t &= \phi_1 g_{t-1} + \phi_2 g_{t-2} + \eta_t^{g*} \\
 \Delta\pi_t &= c + \beta_0 g_t + \beta_1 g_{t-1} + \gamma(L)\omega_t + \theta(L)\varepsilon_t^{\pi*}
 \end{aligned} \tag{4.5}$$

Once again, assuming that  $y_{t-1}^P = y_{t-1} - g_{t-1}$ , the model can be re-written in its more compact form as

$$\begin{aligned}
 \Delta y_t &= \mu_t + g_t - g_{t-1} + \varepsilon_t^{y*} \\
 \mu_t &= \mu_{t-1} + \eta_t^{\mu*} \\
 g_t &= \phi_1 g_{t-1} + \phi_2 g_{t-2} + \eta_t^{g*} \\
 \Delta\pi_t &= c + \beta_0 g_t + \beta_1 g_{t-1} + \gamma(L)\omega_t + \theta(L)\varepsilon_t^{\pi*}
 \end{aligned} \tag{4.6}$$

To integrate the MA(3) term in the system, simply interpret  $\varepsilon_t^{\pi*}$  as a state variable. Do the same for the time-varying constant  $\mu_t$ . Thus, in this new system, the observed dependent variables are still the same but now  $\mathbf{m}=3$ . Since the moving-average term is a state variable and of lag 3, we must set  $\mathbf{v}=1+3=4$ . We also point out the presence of exogenous variables  $\omega_t$  and their first lags in this system. (Note that, by contrast, there are two lags of each variable “dpoil” and “de” in the code. They are not included here in order to save space.)

In this case, the state-space representation is given by

$$\begin{bmatrix} \Delta\pi_t \\ \Delta y_t \end{bmatrix} = \begin{bmatrix} 1 & \theta_1 & \theta_2 & \theta_3 & \beta_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_{t-1}^\pi \\ \varepsilon_{t-2}^\pi \\ \varepsilon_{t-3}^\pi \\ g_t \\ g_{t-1} \\ g_{t-2} \\ \mu_t \\ \mu_{t-1} \end{bmatrix} + \begin{bmatrix} c & \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \omega_{1t} \\ \omega_{1t-1} \\ \omega_{2t} \\ \omega_{2t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \varepsilon_t^{y*} \end{bmatrix} \quad (4.7)$$

$$\begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_{t-1}^\pi \\ \varepsilon_{t-2}^\pi \\ \varepsilon_{t-3}^\pi \\ g_t \\ g_{t-1} \\ g_{t-2} \\ \mu_t \\ \mu_{t-1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_1 & \phi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{t-1}^\pi \\ \varepsilon_{t-2}^\pi \\ \varepsilon_{t-3}^\pi \\ \varepsilon_{t-4}^\pi \\ g_{t-1} \\ g_{t-2} \\ g_{t-3} \\ \mu_{t-1} \\ \mu_{t-2} \end{bmatrix} + \begin{bmatrix} \eta_t^{\pi*} \\ 0 \\ 0 \\ 0 \\ \eta_t^{g*} \\ 0 \\ 0 \\ \eta_t^{\mu*} \\ 0 \end{bmatrix} \quad (4.8)$$

while the (3.1) equivalent of the  $\alpha_t^*$  state-space representation looks like this:

$$\begin{aligned}
 \begin{bmatrix} \Delta\pi_t \\ \Delta y_t \end{bmatrix} &= \begin{bmatrix} 1 & \theta_1 & \theta_2 & \theta_3 & \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_t^\pi \\ \varepsilon_{t-1}^\pi \\ \varepsilon_{t-2}^\pi \\ \varepsilon_{t-3}^\pi \\ g_t \\ g_{t-1} \\ g_{t-2} \\ g_{t-3} \\ \mu_t \\ \mu_{t-1} \\ \mu_{t-2} \\ \mu_{t-3} \end{bmatrix} + \begin{bmatrix} c & \gamma_1 & \gamma_2 & \gamma_3 & \gamma_4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \omega_{1t} \\ \omega_{1t-1} \\ \omega_{2t} \\ \omega_{2t-1} \end{bmatrix} + \begin{bmatrix} c \\ 0 \end{bmatrix} \begin{bmatrix} 1 \end{bmatrix} \\
 &+ \begin{bmatrix} 0 \\ \varepsilon_t^{y*} \end{bmatrix}
 \end{aligned} \tag{4.9}$$

so that, in the above,  $ZZ = \begin{bmatrix} 1 & \theta_1 & \theta_2 & \theta_3 & \beta_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$  and,

$$TT = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \varphi_1 & \varphi_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}. \tag{4.10}$$

The corresponding writing in the code (with two lags for each of the exogenous variables instead of one lag shown in the model here) is as follows:

```
_max_ParNames= "const","dpoil","dpoil-11","dpoil-12","de","de-11","de-12","h2star",
                "q1star","q2star","q3star","phi1","phi2","del1","del2","del3","gap",
                "gap-11"};

swd=           ((ircpi-ircpi_11)/4)~(100*(lrgdp-lrgdp_11))~ones(nobs,1)
                ~drpoil~drpoil_11~drpoil_12~de~de_11~de_12~ones(nobs,1);

n=2; m=3; k=7; s=1; l=2; q_h=2; r=3; p=2; v=4;
```

The initialization of parameter values are carried out next. Thus,

```
beta0 =
-0.008~0.83~0.73~-0.39~2.32~3.28~-4.15~ @ beta1: 1xk
                                     @beta2: 1xk @
                                     @ delta1 1xs @
                                     @ delta2 1xs @
                                     @ delta3 1xs @
0.7~ @ H*: 1xn @
0.45~0.46~0.06~ @ Q*: 1xm @
                                     @ T1:1xv (1st row of 1st(vxv) diag block in TT) @
1.52~-0.59~ @ T2 : 1xv (1st row of 2nd(vxv) diag block in TT) @
                                     @ T3:1xv (1st row of 3rd(vxv) diag block in TT) @
-0.57~-0.09~-0.02~0.16~-0.14; @ Z1:1xmv @
                                     @ Z2:1xmv @
                                     @ lmda:nx1 @
                                     @ psi:mxr @
                                     @ alph1: qqx1 @
                                     @ alph2: qqx1 @
                                     @ alph3: qqx1 @
                                     @ gamm1:ppx1 @
                                     @ gamm2:ppx1 @
                                     @ gamm3:ppx1 @
```

Once again, since there are no ARCH effects present in either the measurement or the transition equations, there are no initializations for these.



Finally, the imposed constraints are:

```

beta = th[1:7,1]|zeros(7,1);
delta = zeros(3,1);
Hstar = 0.0|th[8,1];
Qstar = th[9,1]|th[10,1]|th[11,1];
TT = zeros(4,1) |th[12:13,1]|0.0|0.0 |1.0|zeros(3,1);
ZZ = 1.0|th[14:16,1] |th[17:18,1]|zeros(2,1) |zeros(4,1)
|zeros(4,1) |1.0|(-1.0)|0.0|0.0 |1.0|zeros(3,1);
lambda = 1.0|1.0;
epsi = 1.0|1.0|1.0;
archq = zeros(3,1) |zeros(3,1);
archp = zeros(3,1) |zeros(3,1)|zeros(3,1);

```

### 4.3 The basic model with ARCH errors

In this section, we explain how ARCH errors can be integrated in one of the transition equations. For this purpose, we present another version of the basic model found in example 4.1:

$$\begin{aligned}
 \Delta y_t &= \mu + g_t - g_{t-1} + \varepsilon_t^{y*} \\
 \pi_t &= \pi_t^e + \beta_0 g_t + \beta_1 g_{t-1} + \varepsilon_t^{\pi*} \\
 g_t &= \phi_1 g_{t-1} + \phi_2 g_{t-2} + \eta_t^{g*} \\
 \pi_t^e &= \theta_1 \pi_{t-1}^e + \theta_2 \pi_{t-2}^e + \eta_t^e \quad . \\
 \eta_t^e &\sim N(0, q_{2t}) \\
 q_{2t} &= \gamma_0 + \sum_{i=1}^P \gamma_i (\eta_{t-i}^e)^2
 \end{aligned} \tag{4.11}$$

In this model, there are two latent variables,  $g_t$  and  $\pi_t^e$ . The first is the output gap while the second is the inflation expectations term. We assume that current expectations depend on past own values and that the error term follows an ARCH(3) process. For the gap variable, we assume a stationary AR(2) process with normal i.i.d.errors. The state-space framework looks like this:

$$\begin{aligned}
\begin{bmatrix} \Delta y_t \\ \pi_t \end{bmatrix} &= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ \beta_0 & \beta_1 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} g_t \\ g_{t-1} \\ g_{t-2} \\ \pi_t^e \\ \pi_{t-1}^e \\ \pi_{t-2}^e \end{bmatrix} + \begin{bmatrix} \mu \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{y*} \\ \varepsilon_t^{\pi*} \end{bmatrix} \\
\begin{bmatrix} g_t \\ g_{t-1} \\ g_{t-2} \\ \pi_t^e \\ \pi_{t-1}^e \\ \pi_{t-2}^e \end{bmatrix} &= \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta_1 & \theta_2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} g_{t-1} \\ g_{t-2} \\ g_{t-3} \\ \pi_{t-1}^e \\ \pi_{t-2}^e \\ \pi_{t-3}^e \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ 0 \\ 0 \\ \eta_t^e \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \eta_t^{g*} \\ 0 \\ 0 \\ \mathbf{0} \\ 0 \\ 0 \end{bmatrix}. \tag{4.12} \\
q_{2t} &= \gamma_0 + \sum_{i=1}^P \gamma_i (\eta_{t-i}^e)^2
\end{aligned}$$

Notice that the error term corresponding to the expected inflation term is set equal to zero in the  $\eta_t^*$  matrix. Instead, it is set to follow an ARCH(3) in the parameter restriction section as follows:

```

_max_ParNames={"mu", "var(y)", "var(pi)", "var(g)", "phi1", "phi2", "theta1", "theta2",
               "beta0", "beta1", "gamma10", "gamma11", "gamma12", "gamma13"};

```

```

swd = (100*(lrgdp-lrgdp_11))~(ircpi/4)~ones(nobs,1)~ones(nobs,1);

```

```

n=2; m=2; k=1; s=1; l=2; q_h=2; r=2; p=3; v=3;

```

and the restrictions imposed on the parameters are given by:

```

beta = th[1,1]|0.0;
delta = zeros(2,1);
Hstar = th[2:3,1];
Qstar = th[4,1]|0.0;
TT = th[5:6,1]|0.0 |th[7:8,1]|0.0;
ZZ = 1.0|-1.0|zeros(4,1) |th[9:10,1]|0.0|1.0|zeros(2,1);
lambda = 1.0|1.0;
epsi = 1.0|1.0;

```

---

```
archq = zeros(3,1) |zeros(3,1);  
archp = zeros(4,1) |th[11:14,1]; @ free parameters for ARCH(3) in 2nd state  
variable @
```

## 5. Conclusions

In this paper, we show, step-by-step and with examples, how to use the GAUSS codes developed at the Bank to estimate state-space models with and without ARCH errors. The ARCH errors can be present in either the measurement or the transition equations of the model.

With the code for the one-sided estimates in place, the next step is to extend the programs to account for the possibility of obtaining smoothed estimates. In addition, it would be useful to include various diagnostic tests to assess the model's goodness of fit. Finally, another possible extension to the existing code is allowing for parameter variability over time. These issues are left for future versions.

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