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# **Uninsurable Investment Risks**

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**Césaire A. Meh and Vincenzo Quadrini**



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The views expressed in this paper are those of the authors.  
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## Abstract

The authors study a general-equilibrium economy in which agents have the ability to invest in a risky technology. The investment risk cannot be fully insured with optimal contracts, because shocks are private information. The authors show that the presence of these risks may lead to an underaccumulation of capital relative to an economy where idiosyncratic shocks can be fully insured. They also show that, although the availability of state-contingent (optimal) contracts cannot provide full insurance, it brings the aggregate stock of capital close to the complete markets level. Institutional reforms that make the use of these contracts possible have important welfare consequences.

*JEL classification: D31, E21, G0*

*Bank classification: Economic models; Financial institutions; Financial markets*

## Résumé

Dans une perspective d'équilibre général, les auteurs étudient une économie où les agents sont en mesure d'investir dans une technologie risquée, mais où le risque d'investissement ne peut être entièrement couvert par des contrats optimaux, car les chocs y constituent une information privée. Les auteurs montrent que la présence de tels risques peut provoquer une sous-accumulation du capital par rapport à ce que l'on observe dans une économie où il est possible de se prémunir entièrement contre les chocs idiosyncrasiques. Ils montrent également que, bien que l'existence de contrats (optimaux) modulés selon l'état de la nature ne procure pas une assurance totale, elle amène le stock global de capital à s'établir à un niveau voisin de celui que l'on obtient dans une économie dotée de marchés complets. Les réformes institutionnelles qui rendent possible l'emploi de ce type de contrat ont une incidence considérable sur le bien-être.

*Classification JEL : D31, E21, G0*

*Classification de la Banque : Modèles économiques; Institutions financières; Marchés financiers*

# 1 Introduction

A large body of literature that studies saving behaviour in the presence of uninsurable idiosyncratic risks assumes that these risks are not associated with investment. As in Bewley (1986), the most common assumption is that earnings or endowments are subject to shocks that cannot be insured away (e.g., Aiyagari 1994, 1995; Hansen and İmrohorođlu 1992; Huggett 1993, 1996; İmrohorođlu 1989; Ríos-Rull 1994). In this class of models, the inability to fully insure idiosyncratic risk implies that the equilibrium interest rate is lower than in a complete markets economy, whether market incompleteness is taken as given or modelled endogenously. Because the interest rate is equal to the marginal productivity of capital, the presence of uninsurable risks implies that the stock of capital is larger than in the complete markets economy (overaccumulation). Aiyagari (1995) shows that in this case a positive capital income tax is desirable in the long run. Golosov, Kocherlakota, and Tsyvinsky (2003) show that a positive capital income tax can improve the allocation when market incompleteness is endogenous, but the mechanism that justifies the positive tax is different.

Although earnings or labour income uncertainty is an important source of idiosyncratic risk, investment activities are also subject to uninsurable risks. For instance, entrepreneurs invest heavily in their own business<sup>1</sup> and managers of corporations hold a large number of their firm’s shares.<sup>2</sup> Even the return from investing in education is highly uncertain and cannot be insured away. Unlike with earnings or endowment risks, however, an agent can avoid investment risks by choosing safer allocations of savings. Earnings or endowment risks in the class of Bewley’s economies are beyond the control of the agent. The agent can use only the available markets to (incompletely) insure them.

In this paper, we model investment risks in three environments. In the first environment, the “Optimal Contract Economy,” agents can sign optimal state-contingent contracts. These contracts, however, cannot provide full insurance, because there are agency problems in the form of asymmetric information. In the second environment, the “Bond Economy,” agents cannot sign state-contingent contracts. Only non-contingent contracts (borrowing and lending) are available. In the third environment, the “Complete Markets

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<sup>1</sup>See Cagetti and DeNardi (2002), Carroll (2002), Gentry and Hubbard (2000), Hurst and Lusardi (2002), and Quadrini (1999).

<sup>2</sup>See Mikkelson, Partch, and (Shah 1997) and Himmelberg, Hubbard, and Love (2000).



Economy,” there are no agency problems, and therefore full insurance against investment risks is possible.

By comparing these three economies, we show that:

- (i) In the two economies with incomplete markets (the Bond Economy and the Optimal Contract Economy), the equilibrium risk-free interest rate is smaller than in the Complete Markets Economy. For certain specifications of the model, however, the aggregate stock of capital is smaller than in the Complete Markets Economy (i.e., there is underaccumulation).
- (ii) Even with very large agency problems, the availability of optimal contracts brings the aggregate stock of capital and the equilibrium interest rate very close to the corresponding levels in the Complete Markets Economy. Also, the feasibility of optimal contracts increases welfare significantly.

The first result, the underaccumulation of capital, may bring into question our conclusion about the desirability of long-term capital taxes. Because in Aiyagari (1995) the optimality of capital taxes derives from the overaccumulation of capital, if the model does not generate overaccumulation, the rationale for the taxation of capital may also vanish.<sup>3</sup>

The second result highlights the importance of factors that make state-contingent contracts feasible. Among these factors, formal and informal institutions play a central role. State-contingent contracts are not used extensively, because the enforcement system may be highly inefficient and costly. For instance, the resolution of contractual disputes may be extremely long and uncertain. Cross-country studies show that the degree of contract enforcement is correlated with the degree of financial development; Levine (1997) and Dolar and Meh (2002) review the empirical literature. In this study, we interpret the economy with state-contingent contracts as an economy in which financial markets are more developed, in part as a result of the higher efficiency of the institutional enforcement. Our study therefore provides a welfare assessment of institutional reforms (for example, legal systems) that lead to greater contract enforceability.

The model studied in this paper has some similarities with the model studied in Khan and Ravikumar (2001), but there are two important differences. First, their model allows for endogenous growth. Consequently,

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<sup>3</sup>The full investigation of this conjecture is beyond the scope of this paper.

agency problems affect the long-term growth of the economy. In our model, agency problems have only level effects, since there is no endogenous growth. Second, Khan and Ravikumar compare only the Optimal Contracts Economy with the Complete Markets Economy. In our paper, we are primarily interested in comparing the Optimal Contract Economy with the economy in which state-contingent contracts are not available (the Bond Economy), to determine the welfare implications of institutional reforms that make possible the availability of state-contingent contracts.

Our paper shares some similarities with a paper by Angeletos (2003), in which he shows that uninsurable investment risks may induce underaccumulation of capital. In his paper, however, market incompleteness is not endogenous, and it therefore does not identify whether the availability of state-contingent contracts has large welfare implications in the presence of agency problems. Our analysis is more general, because we show that the underaccumulation result requires specific assumptions about the model (see section 4).

This paper is organized as follows. In section 2 we describe the basic model and characterize the problems solved by agents when state-contingent contracts are available (Optimal Contract Economy) and when they are not available (Bond Economy). Section 3 conducts the quantitative analysis using parameterized versions of the model. Section 4 considers several extensions. Section 5 offers some conclusions.

## 2 The Basic Model

We consider a heterogeneous agent's model where there is a continuum of households that maximize the expected lifetime utility:

$$E \sum_{t=0}^{\infty} \beta^t U(c_t), \quad U(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}, \quad (1)$$

where  $c_t$  is consumption at time  $t$  and  $\beta$  is the intertemporal discount factor. Households are endowed with one unit of time per period, supplied inelastically at the market wage rate,  $w_t$ .

Each household can run a risky technology that returns  $F(k_t, l_{t+1}, z_{t+1})$  in the next period with the inputs of capital,  $k_t$ , and labour,  $l_{t+1}$ . The variable  $z_{t+1}$  is an idiosyncratic independent, identically distributed (i.i.d.)

shock that is unknown when  $k_t$  is chosen, but it is known when  $l_{t+1}$  is chosen. For simplicity, we assume that the shock can take only two values, denoted by  $z_L$  and  $z_H$ , with  $z_L < z_H$ . The probability, denoted by  $p(z)$ , is strictly positive for both realizations of the shock. The function  $F$  is strictly concave in the production inputs and satisfies  $\lim_{k_t \rightarrow 0} EF_k(k_t, l_{t+1}, z_{t+1}) = \lim_{l_t \rightarrow 0} EF_l(k_t, l_{t+1}, z_{t+1}) = \infty$ .

The agent has the ability to divert the retained capital to get a private benefit. Diversion of capital is not observable and generates efficiency losses in the form of a lower probability of the good shock,  $z_H$ . We assume that the probability of the good shock becomes zero in the case of diversion. The private and unobservable return from diversion is additive to consumption. Given  $c_t$ , the agent's consumption, the current utility is  $U(c_t + \alpha k_t)$ , where  $\alpha$  is a utility parameter that is constant in the model. When we later specify the functional form for  $F(k_t, l_{t+1}, z_{t+1})$ , we will impose some restrictions on the parameter  $\alpha$  that guarantee the inefficiency of diversion. Because of the asymmetric information, the agent's problem in the next section will be subject to an incentive-compatibility constraint.

For the analysis that follows, it is convenient to define the gross revenue net of the labour cost:

$$R(w_{t+1}; k_t, l_{t+1}, z_{t+1}) = F(k_t, l_{t+1}, z_{t+1}) - w_{t+1}l_{t+1}. \quad (2)$$

Given the specification of the return from diversion, the optimal input of labour is fully determined by the input of capital, the shock, and the wage rate:  $l_{t+1} = l(k_t, w_{t+1}, z_{t+1})$ . We can eliminate  $l_{t+1}$  as an explicit argument of the gross revenue and write it simply as  $R(w_{t+1}; k_t, z_{t+1})$ .

In addition to the risky investment, there are state-contingent assets that pay  $b(z_{t+1})$  units of output in the next period conditional on the realization of  $z_{t+1}$ . The current value of these assets is  $\delta_t \sum_{z_{t+1}} p(z_{t+1})b(z_{t+1})$ , where  $\delta_t = 1/(1 + r_t)$  is the market discount rate and  $r_t$  is the equilibrium riskless interest rate.

## 2.1 The agent's problem

We use  $a$  to denote the agent's wealth or net worth before consumption. Given the sequence of prices,  $P^t \equiv \{r_j, w_{j+1}\}_{j=t}^{\infty}$ , the optimization problem can be written as follows:

$$V_t(a) = \max_{c,k,b(z_i)} \left\{ U(c) + \beta \sum_i V_{t+1}(a(z_i))p(z_i) \right\}, \quad (3)$$

subject to

$$a = c + k + \delta_t \sum_i p(z_i)b(z_i), \quad (4)$$

$$a(z_i) = w_{t+1} + b(z_i) + R(w_{t+1}; k, z_i), \quad \text{for } i = L, H, \quad (5)$$

$$U(c) + \beta \sum_i V_{t+1}(a(z_i))p(z_i) \geq U(c + \alpha k) + \beta V_{t+1}(a(z_L)), \quad (6)$$

$$a(z_i) \geq \underline{a}_{t+1}. \quad (7)$$

This is the optimization problem for any deterministic sequence of prices, not only for steady states. The time subscript,  $t$ , in the value function is motivated by the non-stationarity of the problem. Note that  $z_i$ , with  $i \in \{L, H\}$ , denotes the next-period realization of the shock, which is unknown when the agent chooses the consumption and investment plan. Equation (4) is the budget constraint. Equation (5) is the law of motion for the next-period net worth before consumption, the variable  $a$ . Equation (6) is the incentive-compatibility constraint and equation (7) imposes limited liability, which is justified by the assumption that the agent can renegotiate any liability that has a net worth smaller than a minimum value,  $\underline{a}_{t+1}$ . The size of this lower bound depends on the assumptions about the penalty that can be imposed on a defaulting agent. Two assumptions that can be made are as follows:

- **No market exclusion:** It can be assumed that there is no market exclusion if the contract is renegotiated and the investor can confiscate only the current net worth of the agent. This can be justified using an argument similar to Kiyotaki and Moore (1997). In this case, the lower bound is  $\underline{a}_{t+1} = 0$ . A variation would assume that labour income cannot be confiscated. In this case, the lower bound is  $\underline{a}_{t+1} = w_{t+1}$ .
- **Exclusion from the investment:** It can be assumed that, as an extreme form of punishment, the agent is precluded from running the risky technology and a fraction,  $\phi$ , of their current and future (labour) income is confiscated in every period. The lifetime utility after repudiation is  $\underline{V}_{t+1} = \sum_{j=0}^{\infty} \beta^j U((1 - \phi)w_{t+1})$ . The lower bound is then determined by the condition  $V_{t+1}(\underline{a}_{t+1}) = \underline{V}_{t+1}$ .

Throughout the paper, we will adopt the first assumption and we impose  $\underline{a}_{t+1} = 0$ .

The structure of problem (3) is not standard, because the unknown value functions,  $V_j$ , for  $j = t, t+1, \dots$ , enter the constraints of the problem and there are no guarantees that it is concave. We will describe in the next section how we deal with these analytical problems. For the moment, we assume that a solution exists. This solution consists of the sequence of policy functions  $\{c_j(a), k_j(a), b_j(a)(z_i)\}_{j=t}^\infty$ . Given the solution to the agent's problem and the initial distribution of households over asset  $a$ —which we denote by  $M_t(a)$ —the general equilibrium can be defined as follows:

**Definition 1** *Given the initial distribution,  $M_t(a)$ , a general equilibrium is defined by (i) a sequence of prices,  $P^t \equiv \{r_j, w_{j+1}\}_{j=t}^\infty$ ; (ii) a sequence of aggregate demands for labour,  $L(P^t) \equiv \{L_{j+1}(P^t)\}_{j=t}^\infty$ ; (iii) a sequence of aggregate capital,  $K(P^t) \equiv \{K_j(P^t)\}_{j=t}^\infty$ ; and (iv) a sequence of aggregate consumption,  $C(P^t) \equiv \{C_j(P^t)\}_{j=t}^\infty$ . These sequences must satisfy the following requirements: (i) the aggregate demands for labour, capital, and consumption must be the aggregation of individual demands, and they must satisfy  $L_{j+1}(P^t) = 1$  and  $C_j(P^t) + K_j(P^t) = \int aM_j(da)$ ; and (ii) the distribution,  $M_t(a)$ , must evolve according to individual decisions and the stochastic properties of the shock.*

## 2.2 Complete Markets and Bond Economies

We compare the allocation when state-contingent contracts are feasible with the allocations in two alternative environments: when state-contingent contracts are not available (Bond Economy) and when shocks are public information (Complete Markets Economy).

The optimization problems solved in the Complete Markets Economy and in the Bond Economy are special cases of problem (3). In the Complete Markets Economy, the agent's problem is not subject to the incentive-compatibility constraint (6). This allows the agent to self-insure against the investment risk, and the first-order conditions imply that  $ER_k(w_{t+1}; k_t, z_{t+1}) = 1 + r_t$ , where  $R_k$  is the derivative of the gross revenue with respect to  $k$ . Of course, in the steady state it must be that  $1 + r_t = 1/\beta$  for all  $t$ .

The optimization problem solved in the bond economy is also a special case of problem (3). This is obtained by restricting  $b(z_L) = b(z_H) = b$ . In this case, the incentive-compatibility constraint never binds and the optimization problem simplifies to:

$$V_t(a) = \max_{c,k,b} \left\{ U(c) + \beta \sum_i V_{t+1}(a(z_i))p(z_i) \right\}, \quad (8)$$

subject to

$$a = c + k + \delta_t b, \quad (9)$$

$$a(z_i) = w_{t+1} + b + R(w_{t+1}; k, z_i), \quad (10)$$

$$a(z_i) \geq 0. \quad (11)$$

This is a standard concave problem, as formally stated in the following proposition:

**Proposition 1** *For any sequence of prices, there is a unique solution to problem (8) and the function  $V_t(a)$  is strictly increasing, concave, and differentiable at all  $t$ .*

**Proof 1** *It can be verified that the feasible set in problem (8) is convex and the objective function is strictly concave. Therefore, if  $V_{t+1}$  is concave,  $V_t$  is strictly concave. Moving backward, we can establish that  $\lim_{t \rightarrow -\infty} V_t$  is concave. Because the objective of problem (8) is strictly concave, the solution is unique. Standard arguments can be used to prove that the value function is differentiable. QED*

Given proposition (1), the solution to problem (8) can be characterized by the following first-order conditions:

$$U'(c_t) = \beta (1 + r_t) E \left\{ U'(c_{t+1}) \right\} + \lambda_t, \quad (12)$$

$$U'(c_t) = \beta E \left\{ U'(c_{t+1}) \cdot R_k(w_{t+1}; k, z) \right\} + \lambda_t \cdot R_k(w_{t+1}; k, z_L), \quad (13)$$

where  $\lambda_t$  is the Lagrange multiplier associated with the limited liability constraint (11). This is positive if the solution is binding.

The first-order conditions make clear that the expected return from the risky investment is always greater than the return from the risk-free asset;

that is,  $1 + r_t < ER_k(w_{t+1}; k, z)$ . To see this, consider the case in which the solution is not binding. Conditions (12) and (13) imply that:

$$(1 + r_t) \cdot EU'(c_{t+1}) = ER_k(w_{t+1}; k, z) \cdot EU'(c_{t+1}) + \text{Cov}\left(R_k(w_{t+1}; k, z), U'(c_{t+1})\right). \quad (14)$$

Because  $U'(c_{t+1})$  is negatively correlated with  $R_k(w_{t+1}; k, z)$ , the last term on the right-hand side is negative, and, therefore,  $1 + r_t < ER_k(w_{t+1}; k, z)$ .

We can compare this with the case in which  $z_L = z_H = z$  (no shocks). In this case, the covariance term in equation (14) is zero and the marginal returns from the two investments are equal; that is,  $1 + r_t = ER_k(w_{t+1}; k, z)$ . In this case, the environment is similar to that described in Aiyagari (1995), except that  $w_{t+1}$  is deterministic in our framework. Even if  $w_{t+1}$  is stochastic at the individual level, however, the condition  $1 + r_t = ER_k(w_{t+1}; k, z)$  still holds. Because the equilibrium interest rate,  $r_t$ , is smaller than the intertemporal discount rate, the model with only earnings risks generates an overaccumulation of capital.

With investment risks, the result that the interest rate is lower than the intertemporal discount rate still holds. The marginal return on capital, however, is not necessarily smaller than the intertemporal discount rate, and there could be an underaccumulation of capital. This result will be shown numerically in section 3.

### 2.3 Optimal contract economy

One of the complications in solving problem (3) is that the unknown function,  $V_t$ , enters the constraints of the problem. It is convenient to study the dual problem, which minimizes the cost of providing a certain level of utility to the agent.

We use  $v_t$  to denote the lifetime utility of the agent and  $A_t(v_t)$  to denote the cost for the intermediary. This is defined as:

$$A_t(v) = \min_{c,k,v(z_i)} \left\{ c + k + \delta_t \sum_i \left[ -w_{t+1} - R(w_{t+1}; k, z_i) + A_{t+1}(v(z_i)) \right] p(z_i) \right\}, \quad (15)$$

subject to

$$v = U(c) + \beta \sum_i v(z_i) p(z_i), \quad (16)$$

$$U(c) + \beta \sum_i v(z_i) p(z_i) \geq U(c + \alpha k) + \beta v(z_L), \quad (17)$$

$$v(z_i) \geq \underline{v}_{t+1}, \quad \text{for } i = L, H. \quad (18)$$

Equation (16) is the promise-keeping constraint, equation (17) is the incentive-compatibility constraint, and equation (18) imposes limited liability. The lower bound,  $\underline{v}_{t+1}$ , is the equivalent of  $\underline{a}_{t+1}$  imposed in the original problem.

This can be interpreted as the problem solved by a financial intermediary that enters into a long-term contractual relation with an agent. If we can show that the long-term contract is equivalent to a sequence of short-term contracts, we can claim that the solution of the dual problem is equivalent to the solution of the original problem.

There are two main difficulties with the dual problem. The first derives from the fact that the constraint set is not convex. Consequently, we cannot prove that the problem is concave and use first-order conditions to characterize the solution. Therefore, in solving the problem, we use a direct optimization technique, described in the appendix.

The second difficulty is to show that the optimal long-term contract is free from renegotiation and can be implemented with a sequence of short-term contracts. As Fudenberg, Holmstrom, and Milgrom (1990) show, if the utility frontier is downward sloping, the long-term contract is free from renegotiation and can be implemented with a sequence of short-term contracts. In our model, the utility frontier is represented by the negative form of the function  $A_t(v)$ . Therefore, it is enough to show that  $-A_t(v)$  is not increasing for all  $v < \underline{v}_t$ . In section 3, we will show this result numerically for the parameterizations of the model considered in this paper.

Once we have (numerically) established that the solution of the dual problem (15) is equivalent to the solution of the original problem (3), we can easily



see the correspondence between the two problems. The cost value,  $A_t(v)$ , is equal to the net worth,  $a$ , in the original problem. Likewise, the agent's value,  $V_t(a)$ , in the original problem corresponds to the agent's promised utility,  $v$ , in the dual problem. Therefore,  $a = A_t(v)$  and  $v = V_t(a)$ . In addition, the lower bound,  $\underline{v}_{t+1}$ , is such that  $A(\underline{v}_{t+1}) = 0$ . This guarantees that the limited liability constraint,  $a(z_i) \geq 0$ , is satisfied in the original problem.

### 3 Numerical Analysis

This section shows numerically the macroeconomic and welfare implications of market incompleteness. Although it does not match specific observations, it provides important information about the potential magnitude of these implications.

**Parameterization:** We assign the following parameter values. The period in the model is one year and the intertemporal discount rate is  $\beta = 0.95$ . The risk-aversion parameter is  $\sigma = 1.5$ .

We assume that the shock affects the efficiency units of capital. If the investment at time  $t$  is  $k_t$ , the efficiency units of capital at the beginning of the next period (before choosing labour) are  $\tilde{k}_{t+1} = z_{t+1}k_t$ . The total resources returned by the risky technology are:

$$F(k_t, l_{t+1}, z_{t+1}) = \tilde{k}_{t+1} + (\tilde{k}_{t+1}^\epsilon l_{t+1}^{1-\epsilon})^\theta.$$

The first component is non-depreciated capital and the second is the output produced. After setting  $z_L = 0.5$  and  $z_H = 1.0$ , the probability of the low shock is chosen to have an expected depreciation rate of 8 per cent; that is,  $p(z_L) \cdot z_L + (1 - p(z_L)) \cdot z_H = 0.92$ . This implies that, with 16 per cent probability, capital depreciates by 50 per cent, and with 84 per cent probability there is no depreciation. A sensitivity analysis is conducted by changing the value of  $z_L$  (keeping the average depreciation rate constant). The return-to-scale parameter is set to  $\theta = 0.95$  and the parameter  $\epsilon = 0.35$ . This implies a labour income share of 60 per cent. We also set  $\alpha = 0.2$ . This value guarantees that diversion is always inefficient. We conduct a sensitivity analysis with respect to this parameter. Table 1 reports the full set of parameter values for the baseline economy.

Table 1: Parameter Values for the Baseline Economy

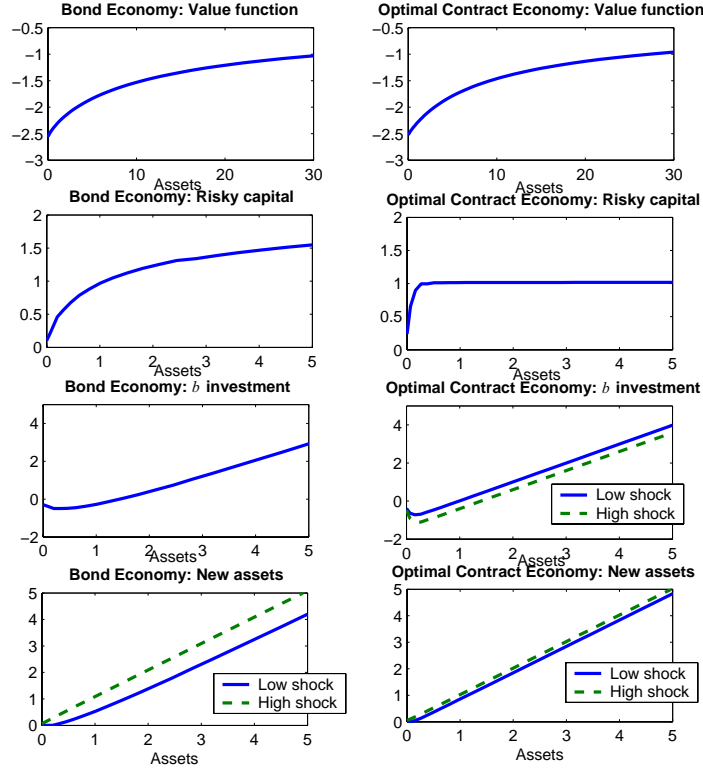
Discount rate	$\beta$	0.95
Risk aversion	$\sigma$	1.50
	$\theta$	0.95
	$\epsilon$	0.35
Technology $zk + [(zk)^\epsilon l^{1-\epsilon}]^\theta$	$z_L$	0.50
	$z_H$	1.00
	$p_L$	0.16
Diversion parameter	$\alpha$	0.20

**Steady-state properties:** Figure 1 plots several variables for an individual household in the steady-state equilibrium, for the Bond Economy (left panels) and for the Optimal Contract Economy (right panels). The top panels plot the household's value as a function of assets; that is, the function  $V(a)$ . In the case of optimal contracts, this function is the inverse of the function  $A(v)$  derived from solving the dual problem. Because  $V(a)$  is monotonically increasing, the function  $A(v)$  is also increasing, which implies that the utility frontier  $-A(v)$  is downward sloping. As Fudenberg, Holmstrom, and Milgrom (1990) show, this guarantees that the long-term contract is free from renegotiation and can be implemented as a sequence of short-term contracts. Therefore, the solution of problem (15) is equivalent to the solution of the original problem (3).

The other panels plot the investment in the risky technology,  $k$ , the investment in the state-contingent asset,  $b(z)$ , and the next-period wealth,  $a(z)$ . In the Bond Economy, there are no state-contingent assets and  $b$  represents the investment in the riskless asset, or bond. In both, the Bond and the Optimal Contract Economies, the next-period wealth depends on the realization of the shock. Note that state-contingent contracts reduce significantly the volatility of assets, and, therefore, the risk of investing in the risky technology (see the last two panels of Figure 1), which explains why the availability of these contracts can have substantial macroeconomic and welfare consequences.

Table 2 reports the steady-state interest rate, aggregate capital, and concentration of wealth as measured by the Gini index. In the Complete Markets Economy, the interest rate is equal to the intertemporal discount rate, and the stock of capital (normalized to 1) satisfies  $ER_k(w; k, z) = 1/\beta$ . In the

Figure 1: Value Function and Policy Rules in the Bond Economy and in the Optimal Contract Economy



two versions of market incompleteness, the interest rate is smaller than the intertemporal discount rate. This is not surprising, given the results obtained by Huggett (1993) and Aiyagari (1994). What differs here is that the aggregate stock of capital is smaller than in the Complete Markets Economy. In other words, market incompleteness may lead to underaccumulation of capital. This is the direct consequence of the fact that the accumulation of real capital is risky and agents require a premium.

Table 2 also shows that the availability of state-contingent contracts brings the steady-state level of capital very close to the complete markets level. Note that the availability of state-contingent contracts reduces the inequality in the distribution of wealth, but only slightly. The Gini index for wealth is small relative to the data, because shocks are i.i.d. and there is no other source of heterogeneity. If we assume that only a subgroup of agents

Table 2: Steady-State Interest Rate, Capital Stock, and Wealth Inequality for Different Degrees of Market Completeness

	Interest rate	Aggregate capital	Gini index
Bond Economy	4.22	0.911	43.8
Optimal Contract Economy	5.21	0.995	42.4
Complete Markets Economy	5.26	1.000	–

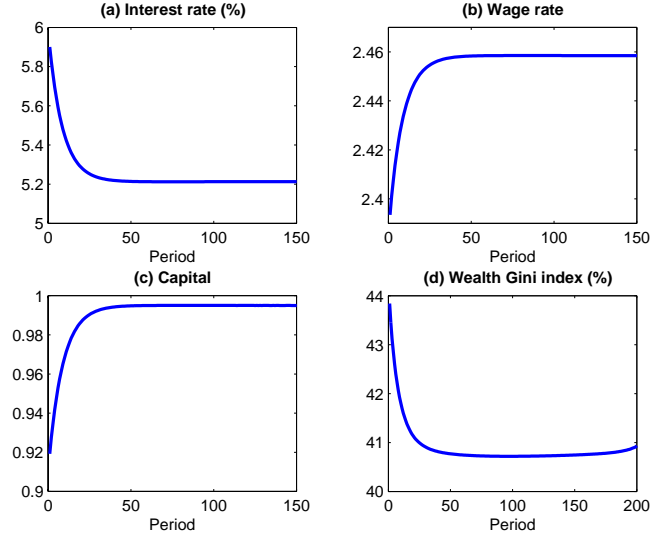
have access to the risky technology—as we do in section 4.1—the model generates a much higher concentration of wealth. In the Complete Markets Economy, the distribution of wealth is not determined. In other words, any distribution of wealth is a steady-state equilibrium if in aggregate there is the same (steady-state) level of capital; see Chatterjee (1994) for a proof of this result.

**Institutional reforms and welfare:** The steady-state comparisons described above show that market incompleteness may have substantial macroeconomic consequences in the absence of state-contingent contracts. Assuming the existence of institutions that make the use of state-contingent contracts feasible, what are the welfare consequences of introducing such institutions?

Figure 2 plots the transition dynamics for the interest rate, the wage rate, the aggregate stock of capital, and the Gini index. After the introduction of state-contingent contracts, the interest rate increases sharply and converges gradually to the new steady-state level: the state-contingent contracts increase the demand for capital immediately, while the supply responds only gradually through capital accumulation. As panel (c) shows, the aggregate stock of capital converges to a higher level, but only gradually. As capital increases, the demand for labour increases and, to clear the labour market, the wage rate must rise (panel (b)). The increase in the wage rate reduces profits and, therefore, the propensity to invest in the risky technology. This effect does not totally offset the higher incentive created by the better insurance possibilities that state-contingent contracts provide. Panel (d) shows that the introduction of state-contingent contracts reduces the concentration of wealth as measured by the Gini index.

The welfare consequences are calculated as the aggregate additional con-

Figure 2: Transition to the Steady-State with State-Contingent Contracts



sumption (appropriately distributed among agents) required to make all agents indifferent between remaining with the existing institutions (and being unable to use state-contingent contracts) and undertaking a transition to the new steady-state equilibrium after the introduction of the new institutions (and having access to state-contingent contracts).

Let  $V^{Bond}(a) = E \sum_{t=0}^{\infty} \beta^t U(c_t^{Bond})$  be the expected lifetime utility of an agent who has a net worth of  $a$  and lives in the steady state of the Bond Economy. The distribution of agents over  $a$  is denoted by  $M(a)$ . Moreover, define by  $V^{OptCon}(a) = E \sum_{t=0}^{\infty} \beta^t U(c_t^{OptCon})$  the expected lifetime utility of an agent who has a net worth of  $a$  after the introduction of state-contingent contracts (and, therefore, after undertaking the transition to the new steady state). The consumption gain from transition for an agent who has a net worth of  $a$  is denoted by  $g(a)$ . This is determined by the following condition:

$$V^{OptCon}(a) = E \sum_{t=0}^{\infty} \beta^t U(c_t^{Bond} \cdot (1 + g(a))) = (1 + g(a))^{1-\sigma} \cdot V^{Bond}(a).$$

In other words, the consumption gain is determined by equalizing the lifetime utility reached in the transition with the lifetime utility obtained by increasing the consumption in the Bond Economy by  $c_t^{Bond} g(a)$  for all  $t$ .

The aggregate consumption gains are given by:

$$\text{Gains} = \frac{\int_a c^{Bond}(1 + g(a))M(da)}{\int_a c^{Bond}M(da)} - 1.$$

For the baseline parameterization, the average gains are 2.32 per cent of aggregate consumption.

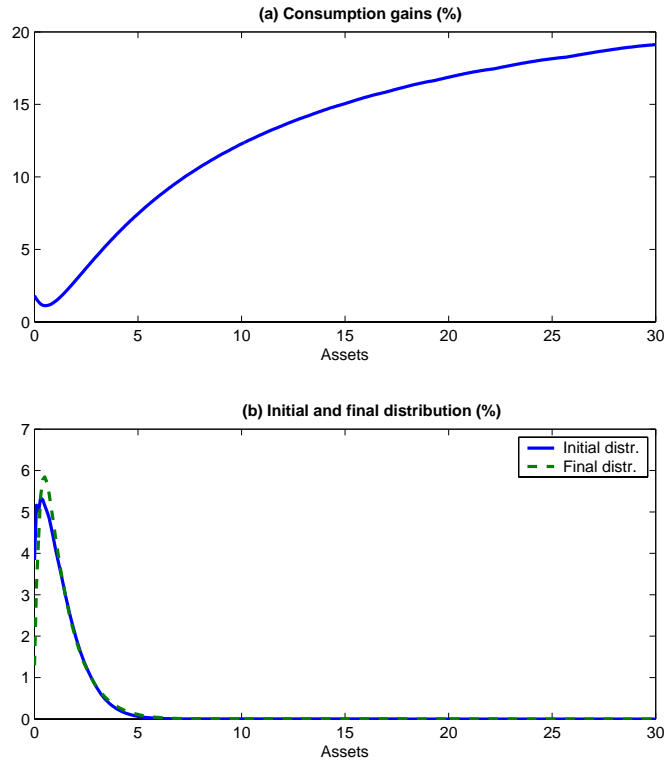
Although the average gains are positive, they are not uniformly distributed across agents. The top panel of Figure 3 plots the welfare gains as a function of the initial wealth. The gains are larger for (initially) wealthier agents. For example, an agent who has average wealth would gain less than 2 per cent, and an agent who has 10 times the average wealth would gain 12 per cent. The bottom panel plots the initial and final distribution of agents over assets, showing the relative importance of poorer agents (who do not gain much from the transition) and wealthier agents (who are the largest beneficiaries).

The distribution of the welfare gains can be explained as follows. After the introduction of state-contingent contracts, the aggregate demand of capital increases. Because the supply responds slowly, the interest rate increases (panel (a) of Figure 2). The increase in the interest rate is beneficial for the holders of wealth; that is, the richest agents. For the poorer agents, the increase in the interest rate represents an increase in the cost of financing, because they are net borrowers. We may have expected the relaxation of financial constraints to be more beneficial for poorer agents, because they have tighter constraints. This would have been the case if the interest rate had remained constant. Because of general-equilibrium effects, however, the interest rate does increase, and this benefits those who receive interest payments: the rich.

**Sensitivity analysis:** We conduct a sensitivity analysis regarding the utility parameter for diversion,  $\alpha$ , the concavity of the production function,  $\theta$ , and the volatility of the shock,  $z_H - z_L$ . Table 3 reports key statistics for the steady-state equilibrium and the welfare gains from the transition.

First, we observe that the higher utility from diversion does not significantly affect our results. A similar conclusion seems to hold for the curvature of the production function. The Gini index for the Bond Economy is smaller, but the difference is not large. The volatility of the shock seems to play an

Figure 3: Distribution of Welfare Gains Following the Introduction of State-Contingent Contracts



important role. The increase in volatility has significant macroeconomic consequences when state-contingent contracts are not available. For example, the aggregate stock of capital drops by 8 per cent when the low realization of the shock changes from 0.5 to 0.25. The drop in the risk-free interest rate is also large. The availability of state-contingent contracts, however, still brings the aggregate stock of capital very close to the complete markets level. As a result, the introduction of state-contingent contracts leads to much larger welfare gains, almost 5 per cent.

Table 3: Sensitivity Analysis: Steady-State Values and Welfare Gains from Transition

	Interest rate	Aggregate capital	Gini index	Welfare gains
<b>Baseline, <math>\alpha = 0.2, \theta = 0.95, z_L = 0.5</math></b>				
Bond Economy	4.22	0.911	43.8	–
Optimal Contract Economy	5.21	0.995	42.4	2.32
Complete Markets Economy	5.26	1.000	–	–
<b>Higher utility from diversion, <math>\alpha = 0.3</math></b>				
Bond Economy	4.22	0.911	43.8	–
Optimal Contract Economy	5.18	0.992	41.1	2.19
Complete Markets Economy	5.26	1.000	–	–
<b>Higher curvature of production, <math>\theta = 0.915</math></b>				
Bond Economy	4.19	0.911	38.8	–
Optimal Contract Economy	5.21	0.994	41.7	2.25
Complete Markets Economy	5.26	1.000	–	–
<b>Higher volatility of shocks, <math>z_L = 0.25</math></b>				
Bond Economy	2.96	0.832	48.3	–
Optimal Contract Economy	5.23	0.997	42.1	4.73
Complete Markets Economy	5.26	1.000	–	–

## 4 Extensions of the Model

The model studied in the previous sections is very stylized. For example, we have assumed that all agents in the economy have access to the risky investment. It seems more reasonable to assume that only a subgroup of households have access to this investment. We have also assumed that agents do not face any earnings risks. Another assumption is that the labour supply is fixed, although in an actual economy it may respond to wages. Moreover, we have assumed that the input of labour is chosen after the observation of the shock. This section extends the previous model by considering alternative assumptions.



## 4.1 Only a subgroup of the population have access to the risky technology

One possible interpretation of the risky investment is that it captures the risk associated with entrepreneurial activities. Therefore, we can assume that the households that have access to this type of investment are the ones engaged in entrepreneurial activities and/or high managerial positions. If we adopt this interpretation, then about 10 per cent of households are in the position of investing in the risky technology (see Quadrini 1999). We will refer to these households as “entrepreneurs” and to the others as “workers.”

In this economy, entrepreneurs solve the same problem we studied earlier. Workers solve a simpler problem: because they do not face a risk, the consumption path can be easily determined using the Euler equation,  $U'(c_t) \leq \beta(1 + r_t)U'(c_{t+1})$ , the budget constraint,  $a_t = c_t + \delta_t b_t$ , and the law of motion for wealth,  $a_{t+1} = w_{t+1} + b_t$ . The Euler equation is satisfied with the inequality sign if  $a_{t+1} = 0$ ; that is, if the borrowing limit is binding. In the steady state, the interest rate is lower than the intertemporal discount rate and the liability constraint will be binding:  $a_t = 0$  for all  $t$ . The level of consumption is then equal to  $c_t = \delta w$ , where  $\delta$  and  $w$  are constant in a steady state.

Table 4: Steady-State Values and Welfare Gains from Transition when 10 Per Cent of the Population Have Access to the Risky Investment

	Interest rate	Aggregate capital	Gini index	Welfare gains
Bond Economy	1.84	0.873	95.1	–
Optimal Contract Economy	5.24	0.993	94.9	5.81
Complete Markets Economy	5.26	1.000	–	–

The basic results do not change by assuming that only a subgroup of the population have access to the risky investment (see Table 4). In particular, the aggregate stock of capital is still smaller than in the Complete Markets Economy. Furthermore, the availability of optimal contracts brings the aggregate stock of capital close to the complete markets level. The most notable change is the increase in the Gini index, which occurs because only a small subgroup of agents (the entrepreneurs) save. Although the model is

stylized, this shows how entrepreneurial activities can generate a much larger concentration of wealth. Also significant is the increase in the welfare gains from the introduction of state-contingent contracts. These larger gains come from the increase in the wage rate. Because 90 per cent of the population are workers who have a low level of consumption, the increase in the wage rate, and therefore consumption, has an important impact on their utilities.

## 4.2 Agents also face earnings risks

Would the result change if agents also faced idiosyncratic risks to earnings, as in the Bewley (1986) economy? To answer this question, we assume that agents have different earnings abilities, which we denote by  $\varepsilon$ . Individual labour income is then the product of the earnings ability with the wage rate:  $\varepsilon w$ . Earnings abilities follow a two-state Markov process with a symmetric transition probability,  $\Gamma(\varepsilon'/\varepsilon)$ .

To keep the problem simple, we assume that earnings abilities are observable. This implies that, with optimal contracts, the earnings risk is insurable. Therefore, the problem solved in the Optimal Contract Economy is the same problem solved before. In the Bond Economy, the optimization problem is also similar, except that we take expectations with respect to the earnings ability,  $\varepsilon$ .

In Table 5, we report the results for the economy with earnings risks where the process for earnings abilities has been calibrated by assuming an autocorrelation of 0.5 and a standard deviation of 0.33. These are the baseline numbers used in Aiyagari (1994).

Even with earnings risks, the aggregate stock of capital is smaller than in the Complete Markets Economy. We observe, however, that the difference between the two levels of capital is somewhat reduced. This occurs because the presence of uninsurable earnings risks brings an extra incentive to save, which reduces the equilibrium interest rate. The lower interest rate facilitates more investment in the risky technology.

## 4.3 Elastic labour supply

In this section, we show how the results would change if labour was elastic. We consider the extreme case in which labour is perfectly elastic. There are two ways to incorporate this in the model. One possibility is to assume that the utility function is of the form  $U(c - \varphi \cdot l)$ . Alternatively, we could

Table 5: Steady-State Values and Welfare Gains from Transition when Agents Face Earnings Risks

	Interest rate	Aggregate capital	Gini index	Welfare gains
All agents have access to risky investment				
Bond Economy	3.09	0.972	44.5	–
Optimal Contract Economy	5.21	0.995	42.4	5.97
Complete Markets Economy	5.26	1.000	–	–
Only 10% have access to risky investment				
Bond Economy	0.01	0.931	88.6	–
Optimal Contract Economy	5.24	0.993	94.9	9.33
Complete Markets Economy	5.26	1.000	–	–

assume that wages are not set competitively and the wage rate is above the market clearing rate with involuntary unemployment. For the calculation of the welfare gains, we use the first assumption.

Table 6 reports steady-state values for the economy with elastic labour. As the table shows, market incompleteness has a much larger impact on the macroeconomy when labour is elastic. In particular, the aggregate stock of capital is substantially smaller (with and without state-contingent contracts) than in the Complete Markets Economy. This is because, with inelastic supply, the fall in the demand for labour induces a fall in the equilibrium wage rate, which in turn increases the return from the risky investment (that is, the expected profit rate increases). This reduces the fall in the demand for risky capital and, in equilibrium, the capital stock is higher. When the supply is perfectly elastic, the lower demand for labour does not lead to lower wages. Consequently, the fall in investment is bigger.

Figure 4 plots the transition path for several variables when labour is elastic and when it is not elastic. The plots are constructed using the baseline economy in which all agents have access to the risky investment. The cases in which only a fraction of agents invest in this technology are qualitatively similar (which we omit for reasons of space).

Of course, the assumption that labour is perfectly elastic is an abstraction. With a more reasonable assumption, in which the elasticity of labour is positive but not infinity, the effects of market incompleteness on the accu-

Table 6: Steady-State Values and Welfare Gains from Transition when Labour is Elastic and All Agents Have Access to the Risky Technology

	Interest rate	Aggregate capital	Gini index	Welfare gains
Bond Economy	4.57	0.650	42.5	–
Optimal Contract Economy	5.21	0.970	42.4	1.37
Complete Markets Economy	5.26	1.000	–	–

mulation of capital are smaller. The point we would like to make, however, is that the elasticity of labour tends to increase the underaccumulation of capital when markets are incomplete.

#### 4.4 The input of labour is chosen in advance

To this point, we have assumed that the input of labour is decided after shock is experienced. Suppose that both capital and labour have to be decided on one period in advance. To simplify the problem, we make some small changes in the specification of the technology. The total resources returned by the risky technology are as follows:

$$F(k_t, l_t, z_{t+1}) = (1 - d)k_t + z_{t+1}(k_t^\epsilon l_t^{1-\epsilon})^\theta.$$

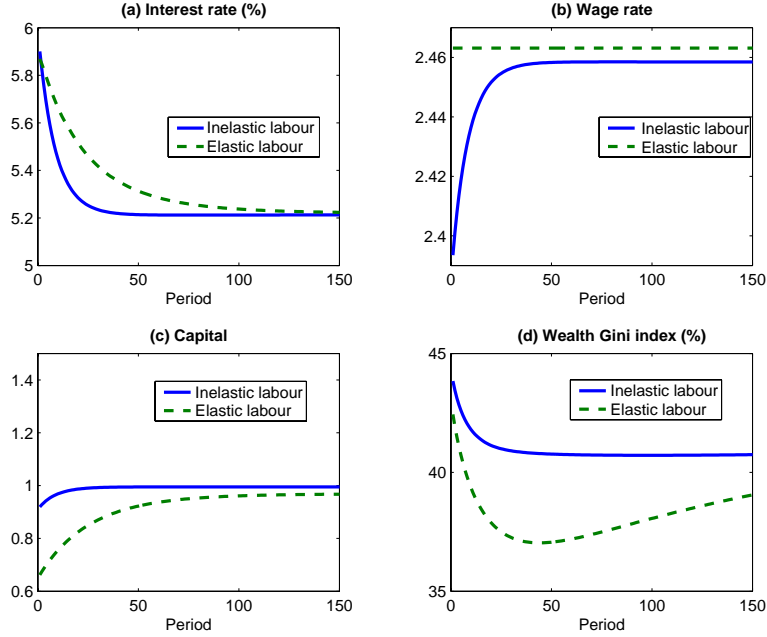
The only relevant change to the technology is that the shock affects output alone, and therefore the depreciation of capital is not stochastic. We also modify the benefit from diversion as follows:

$$U(c_t + \alpha y_{t+1}),$$

where  $y_{t+1} = E z_{t+1} (k_t^\epsilon l_t^{1-\epsilon})^\theta$ . With these changes, the capital-output ratio chosen by the firm depends only on the wage and interest rates, and not on the agent's asset position. This facilitates the computation of the agent's problem (15).

Table 7 reports the steady-state results for the following parameter values:  $\alpha = 1.0$ ,  $z_L = 0$ , and  $p(z_L) = 0.5$ . The most important result is that the aggregate stock of capital is higher than the complete markets level when markets are incomplete (both in the Bond Economy and in the Optimal

Figure 4: Transition to the Optimal Contract Economy for Different Degrees of Labour Elasticity



Contracts Economy). Also note that the overaccumulation of capital is quite large in the bond economy. The introduction of state-contingent contracts brings it very close to the complete markets level.

The overaccumulation of capital can be explained as follows: Because labour is chosen before the shock is experienced, the employment choice is also risky. In other words, if the agent employs more labour, the return from the risky investment is more volatile. This reduces the demand for labour which, in turn, reduces the wage rate. Because of the lower wage rate, the expected profit per unit of capital is higher. This provides an incentive to invest more in the risky technology. As a result, market incompleteness generates overaccumulation of capital, as in the model that has only earnings risks. Note, however, that with only earnings risks the wage rate is higher than in the Complete Markets Economy. Nonetheless, when markets are incomplete, the wage rate is still lower than in the Complete Markets Economy.

Table 7: Steady-State Values When Labour is Chosen in Advance and All Agents Have Access to the Risky Technology

	Interest rate	Aggregate capital	Gini index
Bond Economy	1.70	1.201	63.5
Optimal Contract Economy	4.81	1.010	52.3
Complete Markets Economy	5.26	1.000	–

## 5 Conclusion

We have studied an economy in which agents have investment opportunities in a risky technology. Our consideration of uninsurable investment risks may overturn the assumption that uninsurable risks induce agents to overaccumulate capital. We have shown that, with investment risks, the equilibrium stock of capital may be smaller than in the Complete Markets Economy. This may also change some earlier results that emphasize the benefits of long-run capital taxes. We have also shown, however, that the underaccumulation of capital depends on the assumption that labour is chosen in advance and there is no risk in the employment choice. When labour is chosen in advance, more capital can overaccumulate in a model that has investment risks than in a simpler model that has only earnings risks.

We have also compared economies with different degrees of market incompleteness, focusing on economies in which state-contingent contracts are available but not able to provide full insurance because of information asymmetries. Even if agency problems are quite severe, in the sense that agents can obtain large gains by diverting resources, the use of state-contingent contracts can lead to an aggregate stock of capital that is very close to the one with complete markets, and substantially higher than the capital that would prevail when state-contingent contracts are not available. We have also shown that institutional reforms that make it feasible to use optimal contracts can have important welfare consequences. The next step is to understand which types of institutional environments facilitate or make possible the use of these contracts. That is left for future research.

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## Appendix: Computation of the Equilibrium

**Steady state for the Bond Economy:** We start by guessing the steady-state interest and wage rates. Given the prices, we solve problem (8) on a grid of points for the asset holdings,  $a$ , using value function iteration. After guessing the next-period values of  $V(a)$  at each grid point, we approximate this function with a quadratic polynomial. Given the next-period value function, problem (8) is solved at each grid point using a maximizing routine that does not require a smooth value function. We use the Fortran routine BCPOL.

Once the iteration on the value function has converged, we use the agents' policy rules to find the invariant distribution of agents over  $a$ . Starting from an initial distribution, we iterate until convergence. After aggregating using the invariant distribution, we verify the clearing conditions in the capital and labour markets, update the prices, and restart the procedure until all markets (labour and capital) clear.

**Steady state for the Optimal Contract Economy:** The numerical procedure is similar to the procedure used to solve for the steady state of the Bond Economy based on value function iteration. Because we solve for the dual problem (15), the agent's problem is solved at each grid point of  $v$ . In forming the grid for  $v$ , however, we do not know the lower bound  $\underline{v}$ . Therefore, when we guess the prices  $r$  and  $w$ , we also guess the value of  $\underline{v}$ , which is the first point of the grid. After solving for the individual problem on all grid points, we verify whether  $A(\underline{v}) = 0$ . If it does not, we update the guess for  $\underline{v}$  until this condition is satisfied.

**Transition equilibrium:** To compute the transition from the steady state of the Bond Economy to the steady state of the Optimal Contracts Economy, we first guess the sequences of prices,  $r$  and  $w$ , and lower bounds,  $\underline{v}$ , for a certain number of periods. The number of periods is sufficiently long for the economy to get close to the new steady-state equilibrium. Given the guessed sequences, we solve the agent's problem backward at each grid point, starting from the final transition period. In the final period, the economy is supposed to have converged to the new steady state; therefore, we already know the solution. Once we have solved for all transition periods, we start from the initial period and compute the market clearing conditions and the condition  $A_t(\underline{v}_t) = 0$ . We then update the guessed sequences and continue until all the equilibrium conditions are satisfied in all transition periods.



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