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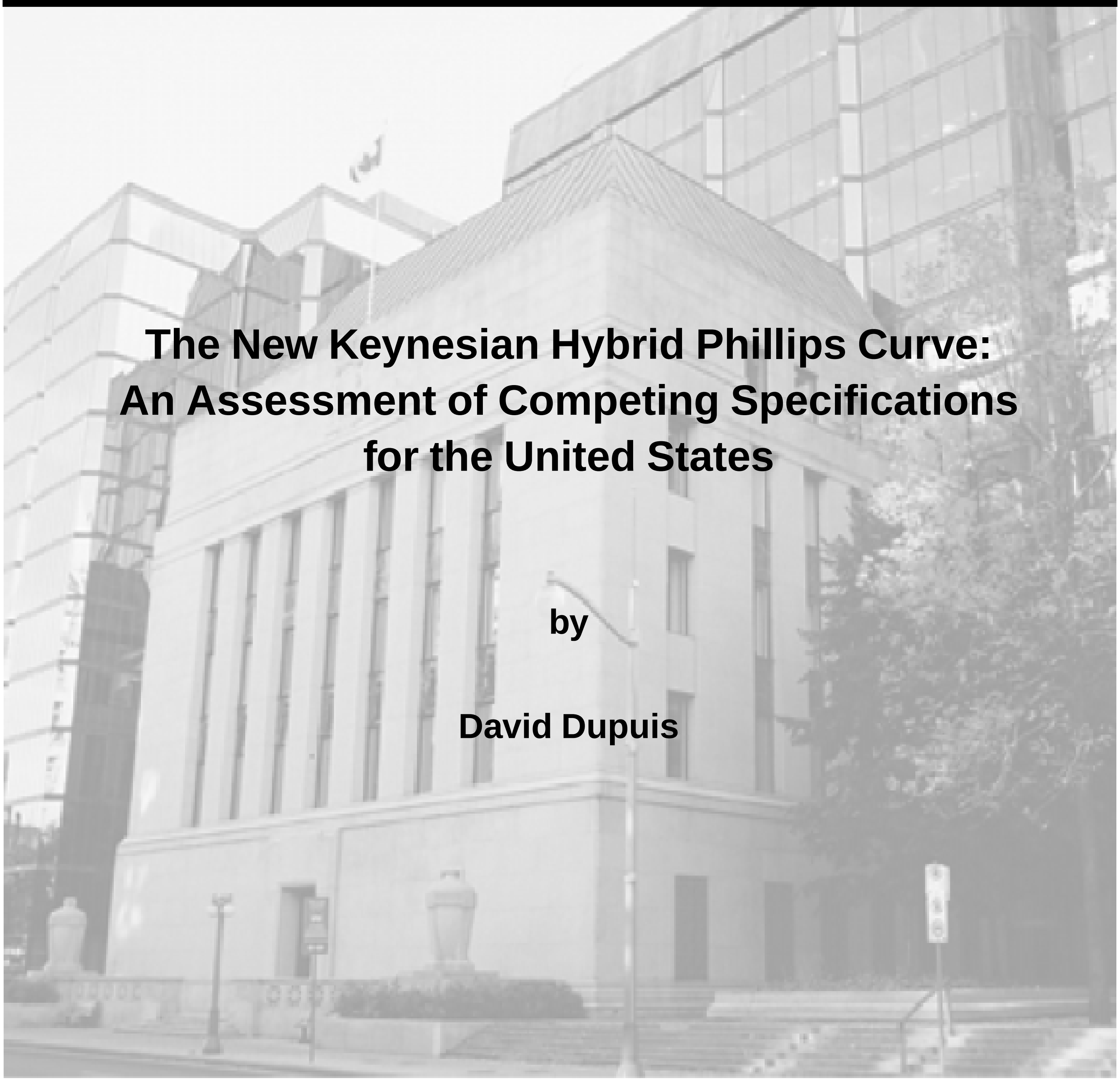

The New Keynesian Hybrid Phillips Curve: An Assessment of Competing Specifications for the United States

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Abstract

Inflation forecasting is fundamental to monetary policy. In practice, however, economists are faced with competing goals: accuracy and theoretical consistency. Recent work by Fuhrer and Moore (1995), Galí and Gertler (1999), Galí, Gertler, and Lopez-Salido (2001), Sbordone (2002), and Kozicki and Tinsley (2002a, b) suggests that the two objectives need not be mutually exclusive in the context of inflation forecasts. The New Keynesian Phillips curve is theoretically appealing, because its purely forward-looking specification is based on a model of optimal pricing behaviour with rational expectations. This specification, however, does not properly capture observed inflation persistence. The author estimates three structural models of U.S. inflation that incorporate price frictions to justify the presence of lags in the forward-looking New Keynesian Phillips curve. The models, based on Galí and Gertler (1999) and Kozicki and Tinsley (2002a, b), are tested on the basis of forecast performances. The results show that the new Keynesian hybrid Phillips curve with the output gap as an explanatory variable performs marginally better than the two alternative specifications.

JEL classification: E31
Bank classification: Inflation and prices; Economic models

Résumé

La prévision de l'inflation est fondamentale pour la politique monétaire. Dans la pratique, toutefois, les économistes doivent s'efforcer de concilier deux objectifs : l'exactitude et la rigueur théorique. Des travaux récents de Fuhrer et Moore (1995), de Galí et Gertler (1999), de Galí, Gertler et Lopez-Salido (2001), de Sbordone (2002) et de Kozicki et Tinsley (2002a et b) donnent à penser que les deux objectifs ne sont pas forcément mutuellement exclusifs dans le contexte de la prévision de l'inflation. La nouvelle courbe de Phillips keynésienne est séduisante sur le plan théorique, car sa formulation strictement prospective repose sur un modèle de tarification optimale où les anticipations sont rationnelles. Cette spécification ne permet pas cependant de saisir la persistance de l'inflation. L'auteur estime trois modèles structurels de l'inflation aux États-Unis qui intègrent des frictions relatives aux prix afin de tenir compte de la présence de retards dans la formulation prospective de la nouvelle courbe de Phillips keynésienne. Les modèles, qui s'inspirent de ceux de Galí et Gertler (1999) et de Kozicki et Tinsley (2002a et b), sont évalués sur la base de la qualité de leurs prévisions. Les résultats montrent que la nouvelle courbe de Phillips hybride keynésienne où l'écart de production intervient à titre de variable explicative permet de prévoir l'inflation un peu mieux que les deux autres formulations envisagées.

Classification JEL : E31
Classification de la Banque : Inflation et prix; Modèles économiques

1. Introduction

In forecasting, economists are often faced with competing goals: accuracy and theoretical consistency. This is of particular importance in the case of inflation, which is fundamental to monetary policy. Recent work by Fuhrer and Moore (1995), Galí and Gertler (1999), Galí, Gertler, and Lopez-Salido (2001), Sbordone (2002), and Kozicki and Tinsley (2002a, b) suggests that the two objectives need not be mutually exclusive in the context of inflation forecasts. The New Keynesian Phillips curve is theoretically appealing, because its purely forward-looking specification is based on a model of optimal pricing behaviour with rational expectations.

The literature, however, has found that, under such a specification, inflation displays a low level of persistence, which is inconsistent with observed inflation dynamics.[1] Alternative and more general specifications of pricing frictions lead to Phillips curves that contain additional lags and expected leads of inflation. By assuming the presence of adjustment costs associated with price changes, these specifications are more consistent with observed inflation and do not violate the assumption of rational expectations. Such specifications are proposed by Galí and Gertler (1999) and Kozicki and Tinsley (2002a, b).

The goal of this paper is to identify, among three competing model specifications, which formulation of the hybrid Phillips curve provides the best forecasts of U.S. inflation. The three competing specifications are: a marginal cost-based hybrid Phillips curve (HPC^{mc}) proposed by Galí and Gertler (1999), its output-gap counterpart (HPC^{gap}), and a polynomial adjustment-cost (PAC) specification proposed by Kozicki and Tinsley (2002a, b). Section 2 reviews the theory behind the New Keynesian Phillips curve and frictions on price adjustments central to all three approaches. Section 3 presents and estimates both hybrid Phillips curves. Section 4 presents and estimates the PAC specification. Section 5 tests and compares the forecasting properties of each model with other competing specifications. Section 6 offers some conclusions.

2. The New Keynesian Phillips Curve: Basic Derivation

In the basic model, the business sector is assumed to be composed of a continuum of monopolistically competitive firms, indexed by i, each producing a differentiated good, $Y_{i,t}$ at time t, with the production function given by $Y_{i,t} = Z_t L_{i,t}$, where Z_t corresponds to total factor productivity.[2] In this context, households are being paid the nominal wage, W_t, and each firm

1. See, for example, Galí and Gertler (1999).
2. For simplicity, this model does not include capital.

faces the same nominal marginal cost of production. Aggregate price, P_t, and output, Y_t, are represented by:

$$P_t = \left[\int_0^1 P_{i,t}^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}}, \tag{1}$$

$$Y_t = \left[\int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di\right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2}$$

where ε is the constant price elasticity of demand and the Dixit-Stiglitz (1977) elasticity of substitution between differentiated products. In the absence of price frictions, each firm would select, at any given time, the price level, $P^*_{i,t,}$ that would maximize real profits. The first-order condition of this system can be simplified to the familiar markup equation,

$$p_t^* = m + nmc_t, \tag{3}$$

where the aggregate equilibrium log price level, p_t^*, would be given by a fixed markup $[m = \log(\varepsilon/(\varepsilon-1))]$ over nominal marginal costs, nmc_t.[3]

In this framework, each firm faces a constraint on the frequency of price adjustment. This constraint reflects sticky prices and Taylor-type staggered price contracts, which makes aggregation cumbersome. Calvo (1983) provides a popular way to simplify this problem, where each firm is subjected to a geometric distribution of price-adjustment delays.[4] Under this specification, the probability that a firm is allowed to adjust its price in any period t is $(1-\theta)$. This probability is time-independent, which implies that the mean lag of adjustment is $1/(1-\theta)$, and that the probability of a price reset after $i-1$ periods of price stability is $(1-\theta)\theta^{i-1}$.

From the Calvo contract, it is possible to show that the aggregate price level, p_t, is a combination of the lagged price level, p_{t-1}, and the optimal reset price, p_t^*, such that[5]:

$$p_t = \theta p_{t-1} + (1-\theta)p_t^*, \tag{4}$$

3. See Kozicki and Tinsley (2002a) for the complete derivation.
4. An alternative, provided by Rotemberg (1982), assumes a quadratic cost of price adjustment.
5. For a complete and explicit derivation, see, among others, Goodfriend and King (1997), King and Wolman (1996), and Woodford (1996), as reported by Galí and Gertler (1999).

where the variables are expressed in deviation from the steady-state inflation rate. Then, for a firm that resets its price at time t to maximize expected discounted profits, the optimal reset price, given the Calvo contract, can be expressed as[6]:

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{nmc_{t+k}\}, \tag{5}$$

where β is a discount factor.

It follows that firms allowed to reset prices at time t will take account of the expected future path of nominal marginal cost (expressed in per cent deviation from steady state), in view of the possibility that the new reset price might be subject to future adjustment constraints. In the case of perfect price flexibility ($\theta = 0$), firms merely adjust prices proportionately to movements in current marginal costs. As the degree of price rigidity, θ, increases, prices are expected to remain fixed for an extended period of time, and the firm will place more weight on expected marginal costs in setting current prices.

Hence, a firm's real marginal cost, given cost minimization and Cobb-Douglas technology, will equal the real wage divided by the marginal product of labour. Therefore, the real marginal cost in $t + k$ for a firm that has optimally set prices in t is given by,

$$MC_{t,t+k} = \frac{(W_{t+k}/P_{t+k})}{(1-\alpha)(Y_{t,t+k}/N_{t,t+k})}, \tag{6}$$

where $Y_{t,t+k}$ and $N_{t,t+k}$ represent output and employment for a firm that has set prices in t at the optimal value, P_t^*. Individual firm marginal cost, however, is not observable in the absence of firm level data. It is therefore helpful to define the observable average marginal cost as:

$$MC_t \equiv \frac{(W_t/P_t)}{(1-\alpha)(Y_t/N_t)}. \tag{7}$$

Following Woodford (1996), Galí, Gertler, and Lopez-Salido (2001), and Sbordone (2002), I assume that the Cobb-Douglas production technology framework and isoelastic demand curve obtains the following log-linear relationship between $MC_{t,t+k}$ and MC_t:

6. Since all variables are expressed in deviation from steady state, the constant markup (m) drops out of equation (5).

$$\hat{mc}_{t,\,t+k} = \hat{mc}_{t+k} - \frac{\varepsilon\alpha}{1-\alpha}(p_t^* - p_{t+k}), \tag{8}$$

where $\hat{mc}_{t,\,t+k}$ and $\hat{mc}_{t+k}$ are the log deviations of $MC_{t,\,t+k}$ and MC_t from their respective steady-state values. It follows that, in the limiting case, where technology is linear (i.e., $\alpha = 0$), all firms will face the same marginal cost.

Combining equations (4), (5), and (8), the Calvo formulation leads to the New Keynesian Phillips curve, which relates to the real marginal cost and takes the following form:

$$\pi_t = \kappa E_t\{\pi_{t+1}\} + \lambda\hat{mc}_t, \tag{9}$$

where the coefficient

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta[1+\alpha(\varepsilon-1)]} \tag{10}$$

is a function of the frequency of price adjustment, θ, a discount factor, β, the degree of curvature of the production function, α, and the elasticity of demand, ε.

3. The New Hybrid Phillips Curve

3.1 The model

Galí and Gertler (1999) have established that, in the New Keynesian Phillips curve, inflation displays a low level of persistence that is inconsistent with observed inflation dynamics. Hence, following Galí and Gertler (1999) and Galí, Gertler, and Lopez-Salido (2001), a departure is made from the basic Calvo model to allow for sticky-price adjustment. Specifically, the Calvo contract is modified to allow two types of firms to coexist in the model. A subsample of firms, $1-\omega$, will have forward-looking price-setting behaviour “à la Calvo,” while the remaining firms will set their prices using a backward-looking rule of thumb based on the recent history of aggregate price inflation. The aggregate price level is then given by:

$$p_t = \theta p_{t-1} + (1-\theta)\bar{p}_t^*, \tag{11}$$

where $\bar{p}_t^*$ is an index of prices set in period *t*, based on the forward- and backward-looking price-setters’ behaviour such that,

$$\bar{p}_t^* = \omega p_t^b + (1-\omega) p_t^f , \tag{12}$$

where p_t^b is the price set by the backward-looking rule of thumb, p_t^f is the price set by forward-looking firms, and ω is the degree of "backward-lookingness." For the purpose of the hybrid Phillips curve specification, forward-looking firms behave exactly as in the basic Calvo framework described earlier. Consequently, their behaviour can be expressed as in equation (5), such that,

$$p_t^f = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\{nmc_{t+k}\} . \tag{13}$$

For backward-looking firms, we adopt Galí and Gertler's assumptions and posit that these firms follow a rule of thumb based on recent aggregate pricing behaviour, which can be stated as follows[7]:

$$p_t^b = \bar{p}_{t-1}^* + \pi_{t-1} . \tag{14}$$

Intuitively, since forward-looking firms set prices as a markup over marginal costs, and because they must lock in prices for (perhaps) more than one period, a firm's pricing decision is based on the expected future behaviour of marginal costs. Correspondingly, backward-looking price-setters will fix their prices according to the equilibrium price in the previous period, corrected for recent inflation.[8]

Combining equations (9) and (11) through (14), the reduced-form empirical formulation of the hybrid Phillips curve is given by:

$$\pi_t = \lambda mc_t + \gamma_f E_t\{\pi_{t+1}\} + \gamma_b \pi_{t-1} , \tag{15}$$

where $\lambda = \dfrac{(1-\omega)(1-\theta)(1-\beta\theta)(1-\alpha)}{\phi[1+\alpha(\varepsilon-1)]}$,

$\gamma_f = \beta\theta\phi^{-1}$,

$\gamma_b = \omega\phi^{-1}$,

with $\phi = \theta + \omega[1-\theta(1-\beta)]$.

7. Galí and Gertler assume (i) no persistent deviations between the rule and optimal behaviour; (ii) the price in period t given by the rule depends only on information dated t-1 or earlier; and (iii) firms are unable to discern whether any individual competitor is backward or forward looking.
8. This process is akin to the indexation of a backward-looking rule of thumb. Christiano, Eichenbaum, and Evans (1997) use this specification to justify, on the basis of theory, the presence of inflation lags in the New Keynesian Phillips curve.

As determined earlier, the structural parameter β corresponds to a discount factor, θ is the degree of price stickiness, and ω is the degree of "backwardness" in price-setting.

Though firms set prices as a markup over marginal costs, Galí and Gertler (1999) posit that, in the standard sticky-price model without variable capital, there exists an approximate relationship between marginal cost and the output gap, $mc_t = \eta gap_t$, where η is the output elasticity of marginal cost (see the appendix for the formal derivation).[9]

Hence, equation (10) can also be restated as:

$$\pi_t = \lambda\eta gap_t + \gamma_f E_t\{\pi_{t+1}\} + \gamma_b \pi_{t-1}. \tag{16}$$

3.2 Estimation of the new hybrid Phillips curve

The new hybrid Phillips curves (HPC^{mc} equation (15) and HPC^{gap} equation (16)) are estimated via non-linear instrumental variables (generalized method of moments, GMM) on a quarterly basis over the period 1972Q2 to 2003Q2.[10] The model's restrictions allow for the identification of only three structural parameters. As in Galí, Gertler, and Lopez-Salido (2001), I choose to estimate the discount factor, β, the degree of price stickiness, θ, and the degree of "backwardness" in price-setting, ω, around plausible values for the degree of curvature of the production function, α, and the elasticity of demand, ε.

Measures of α and ε are based on the average markup over marginal cost, *m*, and the average of the labour income share, *S*. From these assumptions, it follows that $\alpha = 1 - (S_t / m_t)$ and $\varepsilon = m/(m-1)$. The value for the U.S. average labour share, 2/3, is taken from Cooley and Prescott (1995). The average markup is set equal to 1.4, although values between 1.1 and 1.4 are tested without much impact on the results.

Hence, for simplicity of estimation, marginal costs are adjusted for α and ε following Sbordone (2002). I measure inflation as the per cent change in the core personal consumption expenditure (core PCE) deflator; nevertheless, the results are robust to using the full PCE index.[11] Marginal

9. Traditionally, empirical work on the Phillips curve underlines the relevance of the output gap as an indicator of economic activity, as opposed to marginal cost. Nevertheless, in the sticky-price framework with fixed capital, there exists an approximate relationship between the two variables. Making use of the fact that firms will demand labour at the real wage, which equates the marginal product of labour, and that, at this real wage, households will produce a corresponding level of output, one can write, in deviation from the steady-state value, $mc_t = \eta gap_t$. Galí and Gertler do not estimate the new hybrid Phillips curve with the output gap as an explanatory variable.
10. The sample is constrained by the methodology surrounding the estimation of our output gap. For a review of the methodology, see Gosselin and Lalonde (2002)
11. The use of the PCE index in place of the consumer price index is justified by the belief that it appears to be central to monetary policy-making at the U.S. Federal Reserve.

costs are the logarithm of the labour share of income in the non-farm business sector in per cent deviation from steady state. The output gap is estimated via an eclectic approach proposed by Rennison (2003) and estimated by Gosselin and Lalonde (2002). It combines the Hodrick-Prescott filter with a Blanchard-Quah structural vector autoregression (SVAR), and provides an accurate end-of-sample estimate compared with competing methods.[12] The instrument set includes four lags of inflation, the labour share of income, wage inflation, and eight lags of the output gap.

The orthogonality conditions are normalized given the specification provided by Galí and Gertler (1999).[13] Table 1 reports the results of the estimation for both models. Overall, the estimates are consistent with the literature. The β's are estimated to be close to unity (0.976 and 0.994 in the marginal cost and output-gap specifications, respectively), consistent with the literature. The degree of price stickiness, θ, is estimated to be about 0.462 for the marginal-cost specification and 0.628 for the output-gap model, which implies that prices are fixed an average of 1.9 and 2.7 quarters (as given by $1/(1-\theta)$) in the marginal-cost and output-gap specifications, respectively. It has been documented that it takes, on average, 4 quarters for U.S. producer prices to adjust to shocks.[14] Consistent with the sticky-price theory, firms do not adjust their prices instantaneously, because it is often costly for them to do so. Examples of such costs may include the deterrent effect of competition and the reluctance of firms to antagonize customers.[15] The parameter θ usually assumes a value close to 0.75 (since adjustment lags = $1/(1-\theta)$) in the literature.

The degree of "backwardness" in price setting, ω, is estimated to be 0.354 and 0.541 in the marginal-cost and output-gap model, respectively, which suggests that roughly 35 per cent and 54 per cent of firms are using backward-looking price-setting rules in their models. In the marginal-cost specification, this implies estimates of the reduced-form coefficient of 0.435 for the lagged component of inflation (γ_b) and of 0.555 for the lead component (γ_f); in the output-gap specification, it implies estimates of 0.463 for the lagged component of inflation and of 0.535 for the lead component. It appears that forward-looking behaviour is important in both models.[16] In

12. Figures 1 to 4 present the data.
13. It is widely known that, in small samples, non-linear estimates obtained through GMM are sometimes sensitive to the way the orthogonality conditions are normalized.
14. Carlton (1986) documents that the average lag in adjusting U.S. producer prices is about a year, while Sbordone (2002) and Galí and Gertler (1999) find the average to be around 3 to 4 quarters. Lower price stickiness found in my results may suggest that, in the 1999–2003 period of high productivity, declining production costs were passed down to customers as lower prices in the face of increased competition.
15. These factors are identified by Blinder et al. (1998).
16. We will see later that forward-looking behaviour becomes more important as the sample is shortened to represent only the latest period.

addition, the slope of the coefficient on the marginal-cost variable ($\lambda = 0.235$) is positive and significant, as expected, as is the corresponding coefficient in the output-gap model ($\lambda\eta = 0.054$).

Table 1: Structural Estimates of the Hybrid Phillips Curves[a]

	Marginal cost	Output gap
θ	0.462 (0.05)	0.628 (0.07)
β	0.976 (0.03)	0.994 (0.04)
λ	0.235 (0.08)	0.054 (0.02)
ω	0.354 (0.05)	0.541 (0.07)
γ_b	0.435 (0.03)	0.463 (0.06)
γ_f	0.555 (0.04)	0.535 (0.05)
D	1.86 (0.09)	2.69 (0.24)
J-statistic[b]	9.67 (0.72)	11.21 (0.59)

a. The normalized equation (13) is being estimated. The standard error is in parentheses. *D* is the estimated duration of price stickiness.
b. The *p*-value is in parentheses.

3.3 Robustness analysis

Following Galí and Gertler, I consider two robustness tests. The first allows additional lags of inflation to enter the model. The second examines subsample stability. Three lags of inflation are added to the specification of the hybrid Phillips curve to test whether the dominance of forward-looking behaviour in the baseline model could reflect inappropriate lag specifications. Table 2 reports the results. The parameter ψ denotes the sum of the coefficients on the three additional lags of inflation. The estimate for ψ is not significantly different from zero in the marginal-cost specification, and the presence of these lags does not significantly alter the results. In the output-gap model, the sum of coefficients on the additional lags rises to 0.129 with a standard error of

0.05, while the value of the β coefficient drops. In this instance, backward-looking behaviour becomes predominant, even though forward-looking behaviour remains important.

Table 2: Robustness Analysis: Extra Inflation Lags[a]

	Marginal cost	Output gap
θ	0.472 (0.05)	0.672 (0.06)
β	0.946 (0.08)	0.627 (0.10)
λ	0.234 (0.08)	0.100 (0.05)
ω	0.345 (0.07)	0.462 (0.08)
ψ	0.012 (0.03)	0.129 (0.05)
γ_b	0.427 (0.06)	0.454 (0.05)
γ_f	0.553 (0.06)	0.414 (0.01)
D	1.89 (0.10)	3.05 (0.19)
J-statistic[b]	9.73 (0.72)	10.33 (0.67)

a. The standard error is in parentheses.
b. The *p*-value is in parentheses.

For robustness tests of subsample stability,[17] Table 3 reports estimates over the intervals 1972Q2 to 1993Q1 and 1979Q3 to 2003Q2. The second interval is associated with the post-Volker period. Results do not appear to vary widely, although the importance of forward-looking behaviour appears to be greater in the post-Volker period, characterized by low and stable inflation. This is evident from the degree of “backwardness” in price-setting, which declines from a high of 46 per cent in the 1972Q2 to 1993Q1 period to a low of 19 per cent in the post-Volker period (both for the output-gap model). The full-sample estimates suggest that, at most, 54 per cent of firms were using backward-looking price-setting rules.

17. More thorough parametric stability tests are shown in Figures 5 and 6.

In addition, marginal costs and the output gap have a significant impact on short-run inflation dynamics of approximately the same magnitude as full-sample models. Duration of price stickiness is estimated to be in the range of 1.9 to 3.1 quarters, not much different from the full-sample estimated range of 1.9 to 2.7 quarters.

Table 3: Robustness Analysis: Subsample Stability[a]

	Marginal cost		Output gap	
Parameters	1972Q2–1993Q1	1979Q3–2003Q2 (post-Volker)	1972Q2–1993Q1	1979Q3–2003Q2 (post-Volker)
θ	0.395 (0.04)	0.520 (0.06)	0.698 (0.06)	0.696 (0.03)
β	1.006 (0.04)	0.959 (0.03)	0.954 (0.05)	0.959 (0.02)
λ	0.332 (0.10)	0.242 (0.11)	0.047 (0.02)	0.093 (0.02)
ω	0.335 (0.04)	0.240 (0.07)	0.456 (0.06)	0.191 (0.05)
γ_b	0.459 (0.03)	0.318 (0.05)	0.405 (0.05)	0.216 (0.05)
γ_f	0.543 (0.04)	0.660 (0.05)	0.580 (0.04)	0.757 (0.03)
D	1.65 (0.07)	2.08 (0.12)	3.31 (0.20)	3.29 (0.11)
J-statistic[b]	8.04 (0.84)	8.51 (0.81)	8.31 (0.82)	6.45 (0.93)

a. The standard error is in parentheses.
b. The p-value is in parentheses.

Estimates of both hybrid Phillips curves are consistent with what has been seen in the literature. As for the PAC model that follows, I tried—without much success—to update the specification of both hybrid Phillips curves with the addition of import prices (or the exchange rate) to account for the possibility of pass-through to U.S. prices.

4. The Polynomial Adjustment-Cost Approach

4.1 The model

An alternative approach to modelling inflation, in the hybrid neo-Keynesian context, is provided by the PAC model as developed by Tinsley (1993) and Kozicki and Tinsley (2002a, b). In the PAC framework, a firm must take into account the prospect of delays in future price adjustment, just as in the basic Calvo model. Consequently, the firm selects a reset price that is best characterized by the weighted average of expected equilibrium prices over future periods, where the weights are the discounted survival probabilities of the current reset price, as demonstrated by Kozicki and Tinsley (2002a). Hence, the log reset price of firms allowed to adjust their prices in period t is

$$\begin{aligned} p_{r,t} &= E_t\left\{(1-\beta\theta)\sum_{i=0}^{\infty}(\beta\theta)^i p^*_{t+i}\right\}, \\ &= E_t\left\{\frac{(1-\beta\theta)}{(1-\beta\theta F)}p^*_t\right\}, \end{aligned} \tag{17}$$

where F is the usual lead operator.

Accordingly, the current aggregate logged price is a geometric average of the current reset price of adjusting firms and the past prices of firms not yet able to adjust their prices,

$$\begin{aligned} p_t &= (1-\theta)[p_{r,t}+\theta p_{r,t-1}+\theta^2 p_{r,t-2}+\ldots], \\ &= \frac{(1-\theta)}{(1-\theta L)}p_{r,t}, \end{aligned} \tag{18}$$

where L is the usual lag operator.

As in Kozicki and Tinsley (2002a), by combining (17) and (18), the dynamic behaviour of the aggregate price can be defined by the linear difference equation,

$$E_t\{(1-\theta L)(1-\beta\theta F)p_t\} = (1-\theta)(1-\beta\theta)p^*_t . \tag{19}$$

Adding $\theta(1+\beta)p_t$ to both sides of equation (19) yields the following:

$$\pi_t = E_t\beta\pi_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta}(p^*_t - p_t) . \tag{20}$$

The sole difference between the New Keynesian Phillips curve in equation (9) and equation (20) is that the price gap in the latter, $p_t^* - p_t$, is represented by marginal cost (or the output gap) in the former.[18]

Hence, following Tinsley (1993), the optimal intertemporal planning can be captured by assuming that firms choose their relative price to minimize per cent deviations from the desired optimal price path subject to frictions on price adjustment. The planning problem therefore can be stated as:

$$\min_{p} E_t\left\{\sum_{i=0}^{\infty} \beta^i [\kappa_0 (p_{t+i} - p^*_{t+i})^2 + \kappa_1 (\Delta p_{t+i})^2 + \kappa_2 (\Delta^2 p_{t+i})^2 + \kappa_3 (\Delta^3 p_{t+i})^2 + \ldots]\right\}, \tag{21}$$

where $E_t\{.\}$ is a cost forecast based on information available at the beginning of period *t*, and β is a discount factor. The term $(p_{t+i} - p^*_{t+i})^2$ is the cost associated with the spread between the actual and desired price level at time *t+i*, and κ_0 is the unit cost associated with this spread. The remainder of this cost function represents an (m-1)-order frictional polynomial that captures frictions on price adjustment. This planning problem is best described by the PAC approach developed by Tinsley (1993), and inflation can therefore be described as[19]:

$$\pi_t = -a_0 (P_{t-1} - P^*_{t-1}) + \sum_{j=1}^{m-1} a_j \pi_{t-j} + E_{t-1}\left(\sum_{i=0}^{\infty} f_i \pi_{t-i+1}\right). \tag{22}$$

Hence, inflation at time *t* is subject to three distinctive elements: the spread at time *t*-1 between the actual and desired price level, past inflation, and a weighted forecast of expected inflation. In this case, the weights, f_i, are a function of the discount rate (β) and of the cost parameters (κ_i).

4.2 Estimates of the PAC model

4.2.1 Cointegration test and desired path

The theoretical framework implies that there must exist a cointegration relationship between the desired path variables. The desired price path, measured by the logarithm of core consumption deflator (core PCE), is a function of the logarithm of labour compensation (lw) and the logarithm of energy prices ($lener$), given by:

18. In the spirit of Galí and Gertler (1999), Kozicki and Tinsley (2002a) also argue that marginal costs and output gaps are closely related.
19. This is a variant of what Kozicki and Tinsley (2002b) propose.

$$p_t^* = \alpha_0 + \alpha_1 lener_t + \alpha_2(lw_t - \rho_t), \tag{23}$$

where ρ_t is the trend labour productivity in the non-farm business sector as estimated by Gosselin and Lalonde (2002). This specification is much in line with what has been estimated in the FRB/US macroeconomic model.[20] To test for cointegration among the desired path variables, I adopt Johansen's procedure.[21] The Hannan-Quinn and Shwartz criteria determine that the model should include four lags. Results show that the null hypothesis of no cointegration is rejected by the data (Table 4).

Table 4: Johansen Cointegration Test[a]

Cointegrating space	*PGp*	Critical value[b]
Prices, labour compensation, and energy prices	44.04	32.00

a. The null hypothesis is rejected if the computed value is greater than the critical one.
b. Threshold of 10 per cent.

Given the simultaneity problem, the desired price path (23) is estimated via a non-linear instrumental variables (GMM) estimator on a quarterly basis over the period 1972Q2 to 2003Q2. The GMM is instrumented using four lags of each variable included in the desired price-path equation (23).[22] Estimation results are reported in Table 5.[23,24]

20. Equations in the FRB/US model are described in Brayton and Tinsley (1996) and Brayton et al. (1997).
21. Unit-root testing reveals that all variables are integrated of order 1.
22. Results are not sensitive to the number of lags.
23. Figures 7 and 8 show the evolution of the observed and desired price path.
24. The coefficients on labour compensation (adjusted for productivity) and energy prices can be interpreted as the cointegrating vector of the system. A common, though incorrect, inclination is to give an interpretation to the cointegrating vector coefficients. However, one cannot assume that the coefficients in the cointegrating vector represent partial derivatives. Wickens (1996) shows that reduced-form cointegrating vectors should not be interpreted without further structural assumptions. Intuitively, given the endogeneity that characterizes the set of variables, a shock to each variable induces movements in the others.

To account for the highly unusual period of stock market returns that stretched from 1999Q1 to 2003Q2, I also estimate the model over the period 1972Q2 to 1998Q4.[25] This period did not affect the estimates of the desired price path. To provide a more thorough parametric stability test, I perform rolling estimations of the desired path coefficients over the period 1993 to 2003. The parameters are highly stable, as confirmed in Figure 9.

Table 5: Desired Price Path

	1972Q4–2003Q1[a]	**1972Q4–1998Q4**
Constant	**-3.93 (0.00)**	**-3.88 (0.00)**
Labour compensation (adjusted for productivity)	**1.26 (0.00)**	**1.25 (0.00)**
Energy prices	**-0.11 (0.00)**	**-0.11 (0.00)**

a. The *p*-value is in parentheses.

4.2.2 Dynamic equation

Prior to estimating the dynamic equation for the inflation PAC, we need to forecast the desired price path. We use a “satellite” VAR to do so. This small model encompasses four lags of each of the variables included in the desired price path: prices, labour compensation (adjusted for productivity), and energy prices.

The order of the PAC must also be predetermined. Preliminary results suggest that the inflation PAC is of order m = 4 in equation (22).[26] This result is in line with the dynamic inflation equation in the FRB/US model. With m = 4, it appears to be costly for firms to adjust the first four price moments.[27]

In addition to the usual explanatory variables, I include a moving average of four lags of the output gap and one lag of inflation in import prices in the PAC specification (equation (24)).

25. This is in the spirit of Gosselin and Lalonde (2002).
26. This empirical choice is dictated by the absence of residual autocorrelation and by the level of significance in the maximum lag.
27. The first three moments can be identified as the level, the growth rate, and the acceleration of the price variable.

Import prices expressed in domestic currency can offer a proxy for pass-through effects on the U.S. economy.[28] To ensure model convergence, I constrain the sum of the coefficients on the lags of the dependent variable and the expectation term to 1:

$$\pi_t = -a_0(P_{t-1} - P^*_{t-1}) + \sum_{j=1}^{m-1} a_j \pi_{t-j} + E_{t-1}\left(\sum_{i=0}^{\infty} f_i \pi_{t-i+1}\right) + \lambda_1 \sum_{j=1,4} gap_{t-j} + \lambda_2 lpm_t \ . \quad (24)$$

To circumvent the potential simultaneity problem, I estimate the dynamic equation of the PAC specification through GMM. I instrument the model using four lags of inflation, labour compensation, and energy prices. Table 6 shows the results.

The model's R^2 is fairly high at 91 per cent and the residuals are white noise, as confirmed by the Ljung-Box Q test (significance level of 0.39). The coefficient on the error-correction term between the observed and desired price path is in the order of -0.02, compared with -0.07 in FRB/US. As expected, given the observed persistence in inflation, adjustment costs appear to be relatively important. The price level converges to its desired path at a rate of 2 per cent per quarter (8 per cent annually). The sum of the coefficients associated with the lagged dependent variable is 0.70. Correspondingly, the sum of the forward-looking weights is equal to 0.30. This is not surprising, given the observed persistence of the inflation process.[29] As expected, the coefficient on import prices (0.02) is positive for the full sample, denoting a small but significant pass-through effect.

For the hybrid Phillips curve models, subsample stability was considered. I tried to gauge the parameter stability over the intervals 1972Q2 to 1993Q1 and 1979Q3 to 2003Q2. The parameters appear to be stable. The coefficient associated with import prices, however, appears to be sensitive to the sample selection, and quickly falls out of the equation when a more recent subsample is considered, which suggests a negligible pass-through for the U.S. economy in the latter part of the sample.[30] Figure 10 shows a more thorough parametric stability test. I perform rolling estimations of the dynamic PAC specification over the period 1993 to 2003. Although the expectations parameter appears to decline over time, the parameters appear to be stable.

Following Gosselin and Lalonde (2003), I test the null hypothesis of rational expectations (Table 6). This procedure examines the underlying assumption that expectations are formed to minimize the root mean-squared error (RMSE) of the forecast, conditional on the set of information. One way to verify this assumption is to regress the residual of the PAC model on the growth rate of the variables

28. The inclusion of import prices (or the exchange rate) in the Galí and Gertler framework is not successful.
29. The weight distribution has been calculated and is presented in Figure 11.
30. This is consistent with the recent pass-through trend.

contained in the desired path, to determine whether they have any explanatory power. If the variables contained in the desired path are jointly significant (through an *F*-test), it means that agents are not fully using the information at hand, so that the null hypothesis of rational expectations is rejected. In our case, agents appear to be using all available information, so that the rational expectation assumption is not violated.

Table 6: Polynomial Adjustment Cost: The Dynamic Equation

	1972Q2–2003Q2[a]	**1979Q3–2003Q2**
$p_{t-1} - p^*_{t-1}$	-0.02 (0.007)	-0.02 (0.007)
π_{t-1}	0.53 (0.10)	0.32 (0.10)
π_{t-2}	0.00 -	0.17 -
π_{t-3}	0.17 (0.09)	0.26 (0.10)
$E_t \sum \pi^*_{t+i}$	0.30 -	0.25 -
$\sum_{i=1,4} (gap_{t-i})/4$	0.11 (0.04)	0.11 (0.04)
Δlpm_{t-1}	0.02 (0.01)	-0.00 (0.01)
Sum of squared errors:	0.00	0.00
R^2:	0.91	0.92
LB-Q(1):	0.39	0.43
LB-Q(4):	0.10	0.54
H_0 (Rational expectations)[b]:	0.47	0.46

a. The standard error is in parentheses.
b. The number presented corresponds to the significance level.

5. Forecasting Inflation: A Look at Competing Approaches for the United States

In this section, I evaluate the forecasting properties of the inflation models. Out-of-sample forecasts are obtained for the period 1993Q1 to 2003Q2.[31] This exercise assumes that the forward-looking components in all specifications are exogenous to the systems (i.e., forward-looking variables are known with certainty at any given time *t*). I compare the forecasts of all the different specifications at different horizons to determine which model performs best. Table 7 reports the RMSE results. It appears, at the margin, that the HPC^{gap} is the best-performing forecasting model. The Diebold-Mariano test, which tests the null hypothesis that the RMSEs of competing models are statistically identical, reveals that the performance of the HPC^{gap} specification is better than that of the HPC^{mc}, and statistically as good as that of the alternative PAC model.

Table 7: Root Mean-Squared Error (1993Q1–2003Q2)[a]

N-step-ahead forecast with *N* =	HPC^{gap}	HPC^{mc}	PAC
One	0.0016	0.0018***	0.0018
Two	0.0017	0.0020	0.0020
Four	0.0017	0.0021**	0.0022
Eight	0.0017	0.0026*	0.0020

a. The Diebold-Mariano test rejects the null hypothesis that the RMSEs of competing models are statistically identical at 1 per cent (*), 5 per cent (**), and 10 per cent (***).

Table 8 reports the *p*-value for the out-of-sample encompassing tests between the different models. I test the null hypothesis that it is possible to improve the forecast of a model using the forecast of an alternative model. In most instances, the null hypothesis is rejected, which suggests that none of the competing models can improve the forecast of alternative models. There do appear to be a few interesting exceptions. First and foremost, the test cannot reject the hypothesis that the HPC^{gap} forecast improves on the HPC^{mc} forecast, which suggests that the HPC^{gap} model performs better than its HPC^{mc} counterpart. It would also seem that the HPC^{gap} model improves the PAC model over short periods. Correspondingly, I cannot reject the null hypothesis that the

31. Figures 12, 13, and 14 show the one-, four-, and eight-step-ahead forecasts for the competing models.

one-step-ahead PAC forecast improves that of the HPC^{mc} model at the 10 per cent confidence level. The HPC^{mc} forecast, however, improves the PAC forecast over the same horizon, which suggests that the two forecasts do not embed the same information. It would appear that the HPC^{gap} model performs marginally better than the other two specifications.

Table 8: Encompassing Tests (*p*-values)

N-step-ahead forecast with *N* =	Null hypothesis	
	PAC improves HPC^{mc}	PAC improves HPC^{gap}
One	0.13	0.00
Two	0.02	0.00
Four	0.00	0.00
Eight	0.00	0.00
	HPC^{mc} improves PAC	HPC^{mc} improves HPC^{gap}
One	0.15	0.01
Two	0.03	0.00
Four	0.00	0.00
Eight	0.00	0.00
	HPC^{gap} improves HPC^{mc}	HPC^{gap} improves PAC
One	0.88	0.14
Two	0.77	0.02
Three	0.57	0.00
Four	0.27	0.00

6. Conclusion

The New Keynesian Phillips curve is theoretically appealing, because its purely forward-looking specification is based on a model of optimal pricing behaviour with rational expectations. The observed persistence in inflation, however, suggests that lags as well as leads of inflation would be required in an appropriate empirical specification. The hybrid Phillips curve, which includes the output gap as an explanatory variable, seeks to justify, on the basis of theory, the addition of lags in the New Keynesian Phillips curve. I have shown that the resulting forecast performances are marginally better than those of alternative model specifications (HPC^{mc} and PAC), without losing theoretical consistency, which is central to monetary policy-making.

Nevertheless, a few caveats remain. First, the degree of forward-lookingness appears to be quite sensitive to the sample selection. As I truncate the sample window to account for only the most recent history, the behaviour of agents becomes more forward looking, possibly because of the U.S. monetary authority's growing credibility, given its success in achieving and maintaining low and stable inflation.

It is difficult to estimate a significant relationship between inflation and import prices (or the exchange rate). This difficulty could potentially be attributed to the comparatively closed nature of the U.S. economy. It could also be the case, however, that the absence of such a relationship is not totally unexpected, since the inflation dynamic can ultimately be described in terms of a discounted stream of future output gaps, which would normally include all available pass-through information.

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Figure 1: Personal Consumption Expenditure Inflation

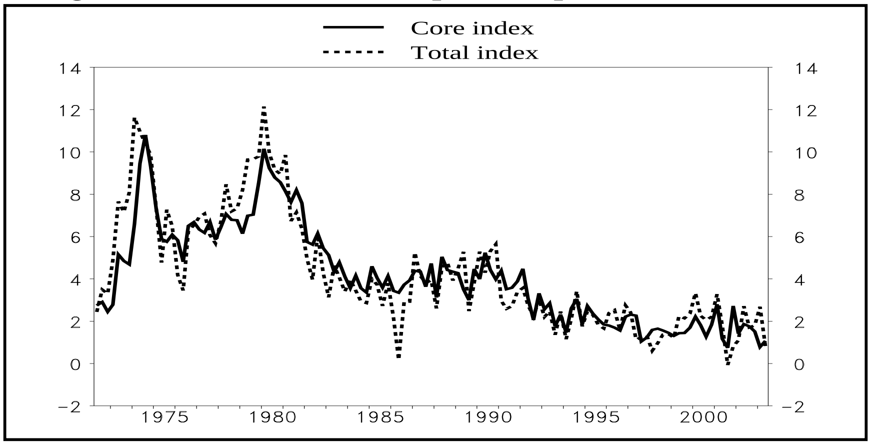


Figure 2: Output Gap

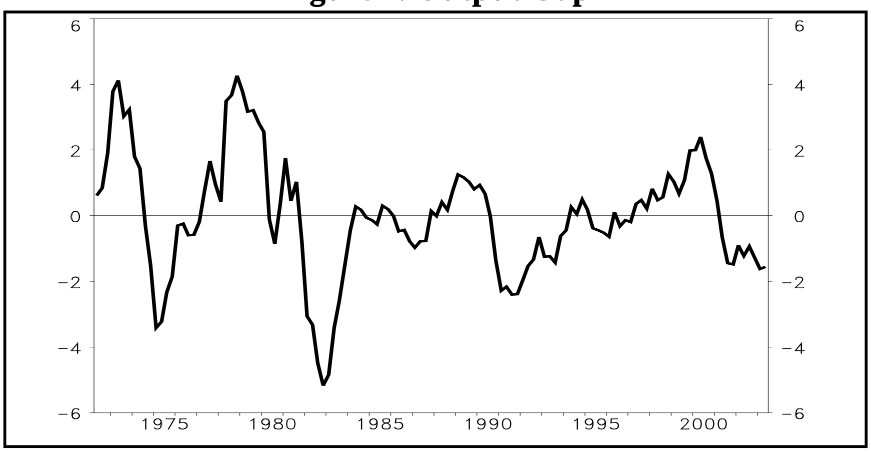


Figure 3: Marginal Cost

Figure 4: Wage Inflation

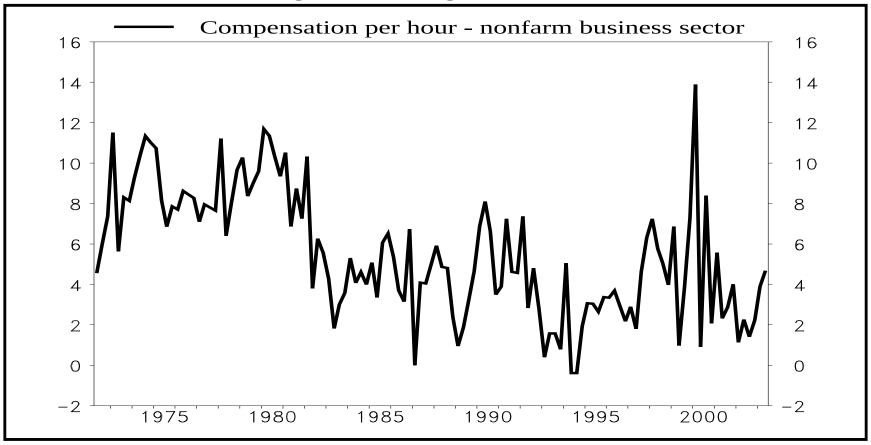


Figure 5: Parameter Stability of the HPC^{mc}

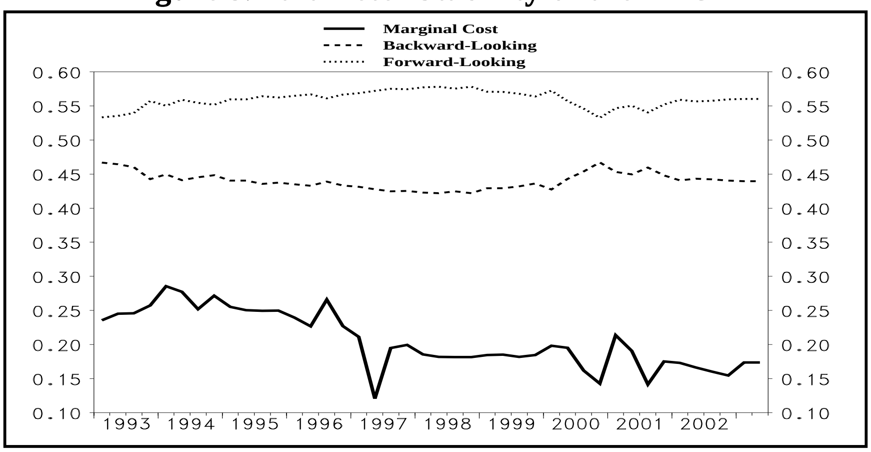


Figure 6: Parameter Stability of the HPC^{gap}

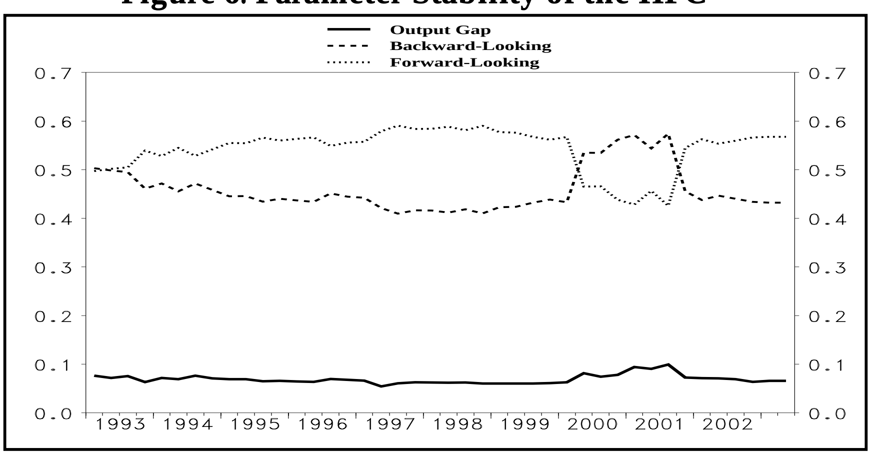


Figure 7: Observed and Desired Price Path

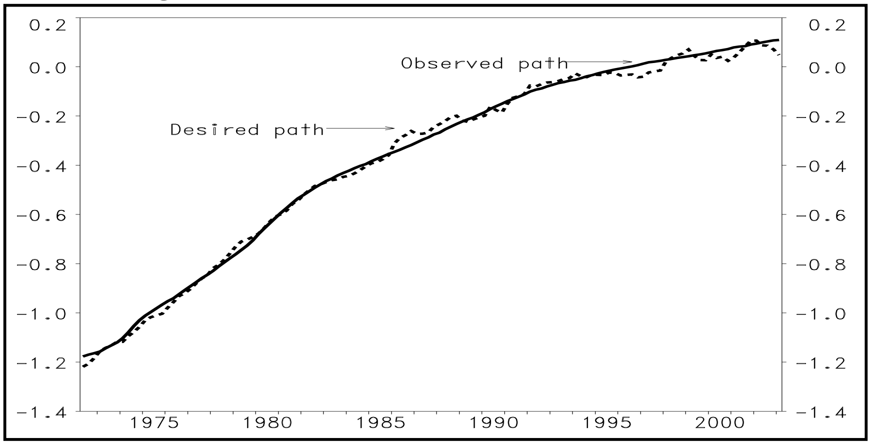


Figure 8: Gap Between the Observed and Desired Price Path

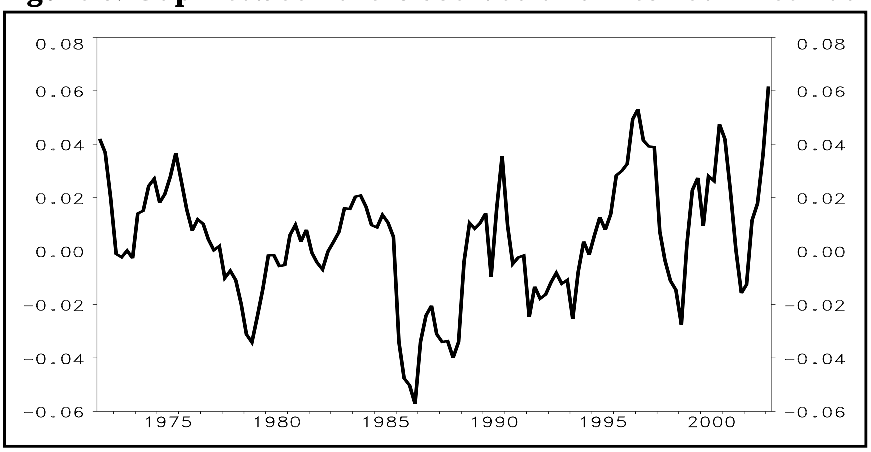


Figure 9: Parameter Stability of the Desired Price Path

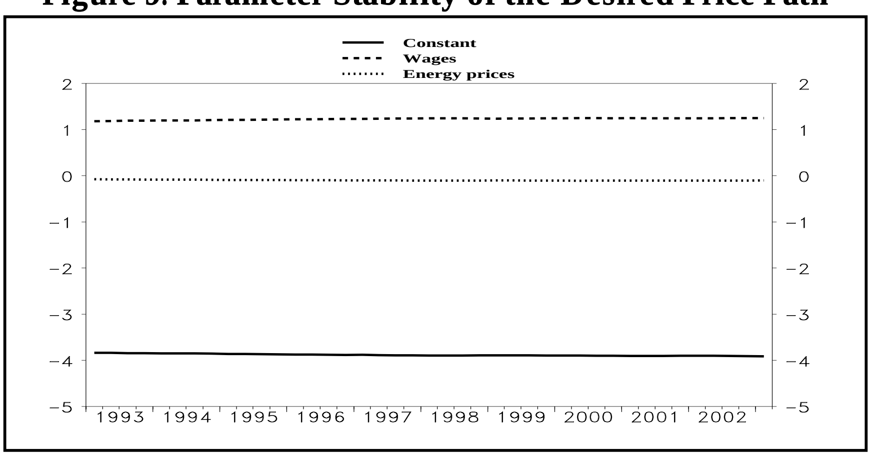


Figure 10: Parameter Stability of the Dynamic PAC Equation

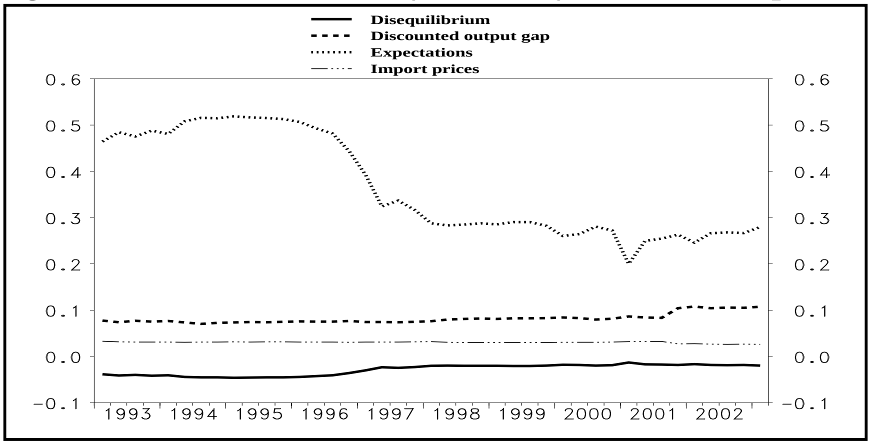


Figure 11: Decomposition of the Expectation Term

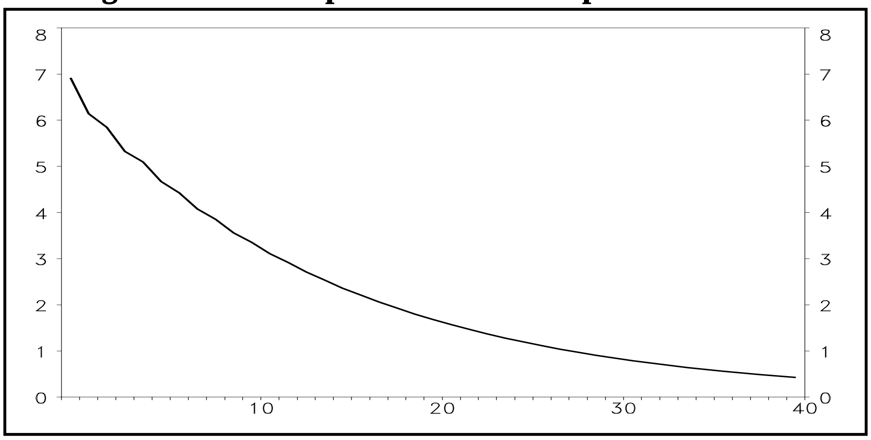


Figure 12: Forecasting Properties: One-Step-Ahead

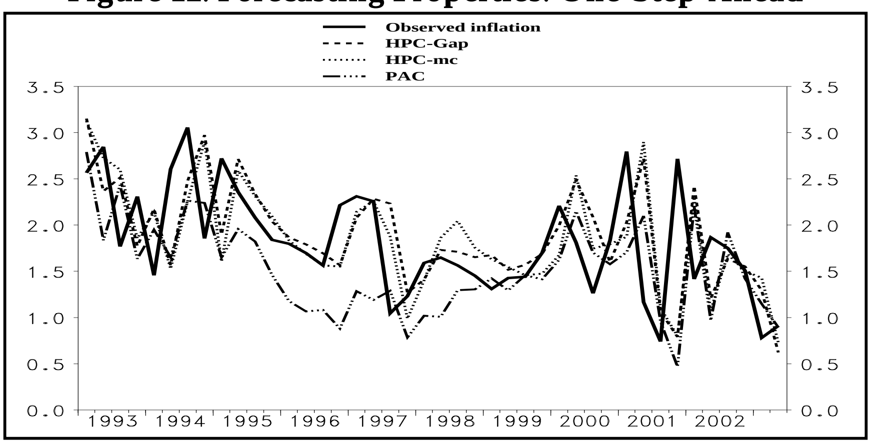


Figure 13: Forecasting Properties: Four-Steps-Ahead

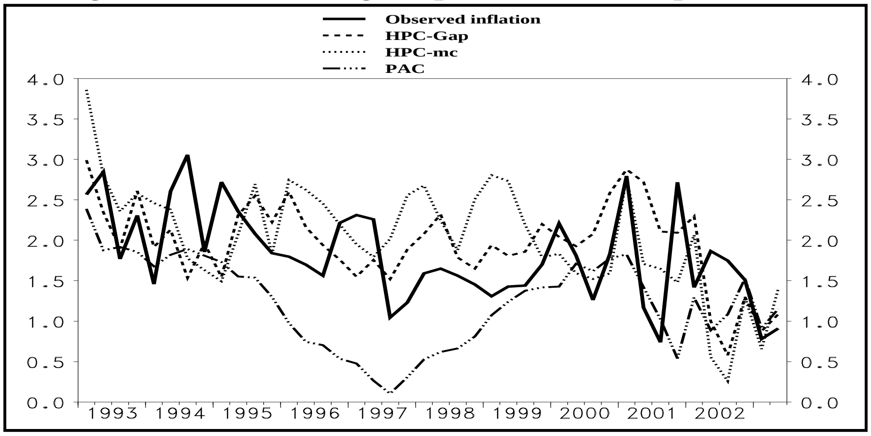
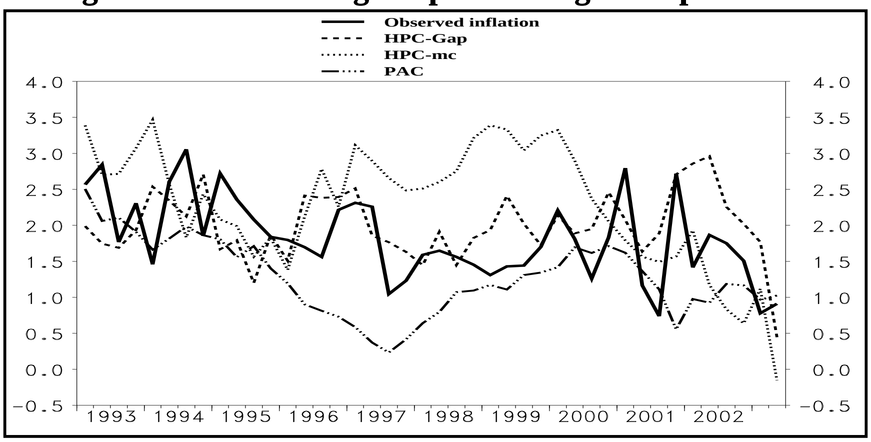


Figure 14: Forecasting Properties: Eight-Steps-Ahead



Appendix: The Link Between the Equilibrium Deviation in Marginal Cost and Output

Using the notion that real marginal cost is the difference between nominal marginal cost and the price level ($mc_t = nmc_t - p_t$), which implies that equilibrium deviation in real marginal cost is equal to $mc_t^* = nmc_t - p_t^*$, I determine that the equilibrium price deviation is the reverse of the equilibrium deviation in marginal cost:

$$p_t^* - p_t = mc_t - mc_t^* . \tag{A1}$$

Then, since firms will demand labour at the log real wage,

$$w_t - p_t = mc_t + z_t , \tag{A2}$$

where z_t is the productivity of labour, and, since households supply labour at the real wage,

$$w_t - p_t = \alpha y_t + \gamma l_t . \tag{A3}$$

These two equations imply that

$$mc_t = (\alpha - 1)y_t + (\gamma + 1)l_t \text{, or} \tag{A4}$$

$$mc_t - mc_t^* = (\alpha - 1)\tilde{y}_t + (\gamma + 1)\tilde{l}_t , \tag{A5}$$

in deviation form, where $\tilde{y}_t$ and $\tilde{l}_t$ are the output and employment gap, respectively.

Making use of the fact that the long-term trends in output and employment are cointegrated (i.e., $\tilde{y} = \tilde{l}$), then:

$$mc_t - mc_t^* = (\alpha + \gamma)\tilde{y}_t . \tag{A6}$$

The new Keynesian hybrid Phillips curve can also be a function of the output gap, and be written as:

$$\pi_t = \lambda\eta gap_t + \gamma_f E_t\{\pi_{t+1}\} + \gamma_b \pi_{t-1} , \tag{A7}$$

where $\eta = (\alpha + \gamma)$.

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