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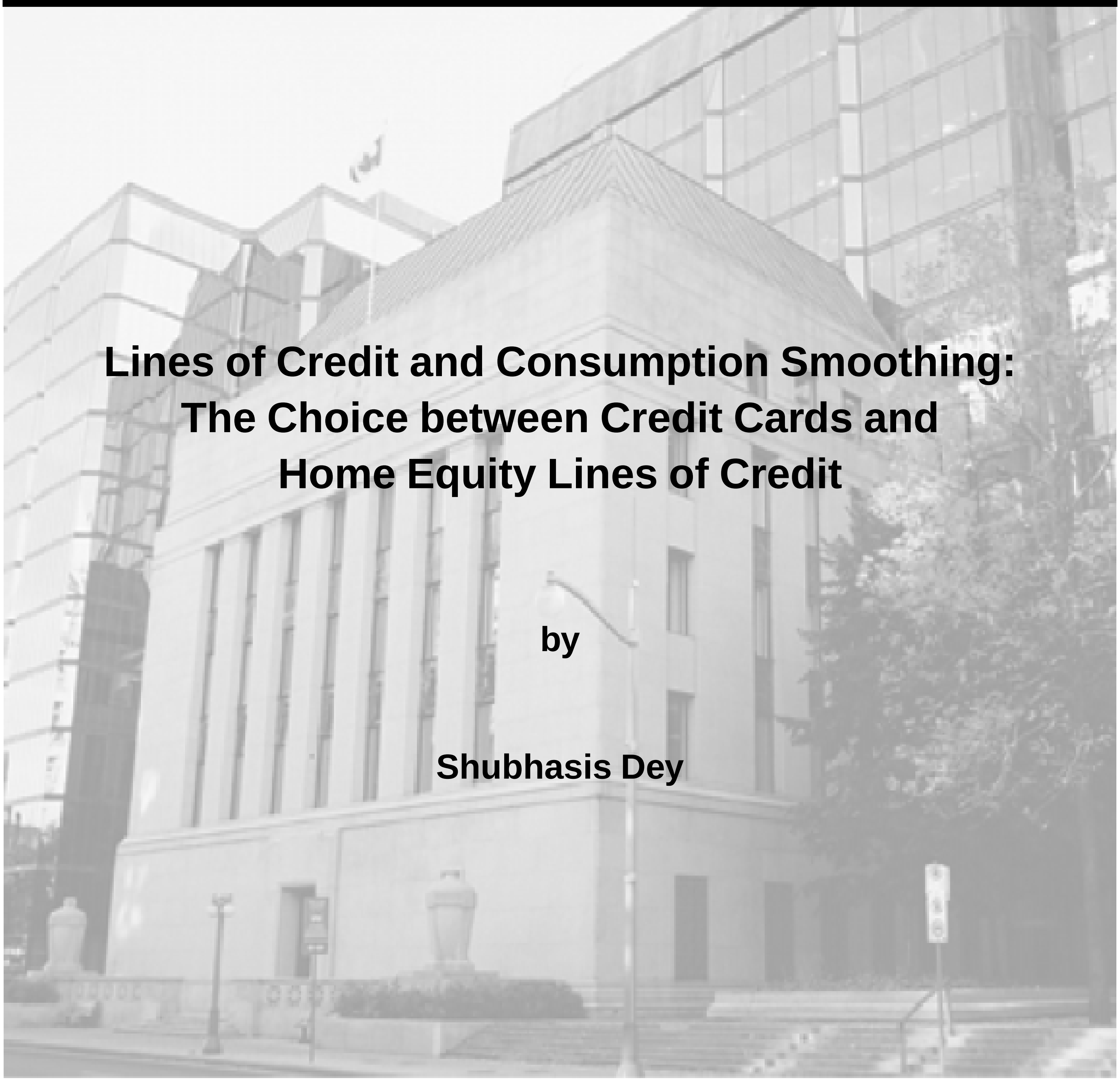

Lines of Credit and Consumption Smoothing: The Choice between Credit Cards and Home Equity Lines of Credit

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Abstract

The author models the choice between credit cards and home equity lines of credit (HELOCs) within a framework where consumers hold lines of credit as instruments of consumption smoothing across state and time. Flexible repayment schemes for lines of credit induce risk-averse consumers with sufficiently high discount rates to underinsure and hold lines of credit instead as a buffer, even when they have access to *full* and *fair* insurance markets. Weighing the fixed upfront fees and higher default costs of HELOCs against the advantages of low and income-tax-deductible interest payments, the author finds a threshold level of *potential* borrowing below which consumers prefer to use credit cards exclusively. Above that threshold, consumers decide to use HELOCs and consolidate all outstanding credit card debt into them; however, a rising probability of default and the resulting loss of equity in the home will put an upper bound on the *potential* HELOC borrowing that will prevent full debt consolidation.

JEL classification: D1, D81
Bank classification: Credit and credit aggregates

Résumé

Postulant que les consommateurs recourent aux lignes de crédit pour lisser leur consommation au fil du temps et des circonstances, l'auteur modélise les conditions qui les amènent à emprunter à l'aide d'une carte de crédit ou d'une marge de crédit garantie par l'avoir propre foncier. La souplesse des plans de remboursement dont sont assorties les lignes de crédit incite les consommateurs peu enclins au risque et ayant un taux d'actualisation assez élevé à s'en servir comme tampon et à ne pas s'assurer contre certains risques, même s'ils ont accès à un marché de l'assurance *complet* et *concurrentiel*. Une fois mis en balance les coûts associés aux marges de crédit garanties par l'avoir foncier (coûts de défaillance plus élevés et frais fixes initiaux) et les avantages qu'elles offrent (taux d'intérêt plus faible et déductibilité des paiements d'intérêts), l'auteur montre qu'en deçà d'un certain seuil d'emprunt *potentiel*, les ménages préfèrent s'en tenir à leur carte de crédit. Passé ce seuil, ces derniers choisissent de transférer à leur marge de crédit tous leurs emprunts sur carte de crédit. Cependant, puisque le risque de défaillance — et donc le risque de perdre l'avoir foncier — augmente avec le montant de dette accumulé sur la marge de crédit, il existe également un plafond au niveau d'emprunt *potentiel* sur la marge de crédit, ce qui empêchera le regroupement de la totalité des dettes.

Classification JEL : D1, D81
Classification de la Banque : Crédit et agrégats du crédit

1. Introduction

Consumers in the United States today have a choice of two major lines of credit: credit cards and home equity lines of credit (HELOCs). Over 72 per cent of American households have at least one bank-type credit card, and about 15 per cent of homeowners have a HELOC, although the rate of growth of HELOCs is currently much greater than that of credit cards (U.S. Federal Reserve Board 2001; Aizcorbe, Kennickell, and Moore 2003). A line of credit, as opposed to a traditional loan, extends a fixed amount of credit to a borrower, and it is then the borrower's decision regarding when and how much of that credit to utilize. Previous literature in this area has considered the transactions and the precautionary motives for holding lines of credit and their use within the life-cycle hypothesis. In this paper, I further explore the precautionary motive or the motive to use lines of credit as a buffer against uninsured or uninsurable risks. I work within a set-up of an insurance market that offers consumers *full* and actuarially *fair* insurance. The framework shows how lines of credit can be used as a hedge against optimally uninsured risks. Given the repayment scheme, the rate of interest on lines of credit, and the probability of a wealth loss, even if a consumer has access to *full* and actuarially *fair* insurance, it may be optimal to leave a portion of the risks uninsured and hold lines of credit instead as a buffer. If the wealth loss occurs and the consumer resorts to borrowing on lines of credit to cover for the uninsured part, the benefit of not paying a full insurance premium and repaying the debt on lines of credit over time will outweigh the cost if the consumer has a sufficiently high discount rate. When the consumer optimally chooses to buy full insurance, lines of credit may still be used to smooth out consumption across time periods. I therefore investigate the principal motives for borrowing on lines of credit in a way that has not been previously done in the literature.

Facing a choice between credit cards and HELOCs, a consumer must weigh the costs and advantages of each. Although HELOCs have the advantage of lower and tax-deductible interest rates, their non-interest costs are a disadvantage. I will show that the choice of line of credit to use depends on the amount that the consumer *potentially* wishes to borrow. Since some consumers will hold lines of credit as a buffer against optimally uninsured risks, a part of the borrowing on lines of credit will be state-dependent; hence, there is a need to differentiate between *actual* and *potential* borrowing. Below some threshold level of *potential* borrowing, the costs associated with HELOCs outweigh the interest rate and income tax advantages, and at that point consumers prefer to use credit cards exclusively.

I also examine why a consumer who has incurred the fixed cost of obtaining a HELOC[1] and has not yet fully utilized the line would continue to borrow on high-rate credit cards—a puzzling phenomenon that is often observed in the data.[2] I find that a rising probability of default and the resulting loss of equity in the home impose an upper bound on the *potential* HELOC borrowing that the consumer wishes to undertake, which in turn will optimally prevent full debt consolidation into HELOC. Moreover, since it is argued that consumers take out lines of credit in anticipation of the wealth losses they may incur, which is to say that they care about the *potential* debt to be carried on their credit lines, the upper bound on the *potential* HELOC borrowing also explains why some consumers have *both* types of credit lines and carry positive debt only on their credit cards.

This paper is organized as follows. Section 2 discusses previous research and provides background on lines of credit. Section 3 introduces the theoretical model. Section 4 offers some conclusions.

2. Previous Research and Background

Ausubel (1991) was among the first to initiate research on the credit card market. He argues that some consumers borrow on high-rate credit cards because they underestimate their likelihood of carrying balances. Brito and Hartley (1995), however, propose that consumers borrow on high-rate credit cards because alternative consumer loans involve transactions costs. Mester (1994) argues that low-risk borrowers leave the credit card market in favour of low-interest collateralized loans, thereby making the credit card client pool riskier and preventing interest rates from falling. Park (1997) explains the downward stickiness of credit card interest rates using the option value of credit lines. He shows that the interest rate that produces zero

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[1] Such as an appraisal fee, lawyer fee, or points.

[2] See the *U.S. Survey of Consumer Finances*, available at <http://www.federalreserve.gov/pubs/oss/oss2/scfindex.html>.

profit for card issuers is higher than interest rates on most other loans because rational cardholders borrow more money when they become riskier. An empirical paper by Calem and Mester (1995) finds evidence for consumers' reluctance to search for lower rates due to high search costs in this market. Cargill and Wendel (1996) suggest that, since there are a large number of convenience users in the credit card market, even modest search costs could keep the majority of consumers from seeking out lower interest rates. Gross and Souleles (2002) utilize a unique new dataset on credit card accounts to analyze how people respond to changes in credit supply. They find that increases in credit limits generate an immediate and significant rise in debt, consistent with the buffer-stock models of precautionary saving, as cited in Deaton (1991), Carroll (1992), and Ludvigson (1999). Agarwal, Ambrose, and Liu (Forthcoming) test the relationship between credit quality and credit line utilization. Their empirical analysis supports the fact that borrowers with higher expectations of future credit quality deterioration originate credit lines and utilize them at lower rates to preserve financial flexibility. Dey and Mumy (2005) emphasize the difference between credit limit and credit utilization, and point out the possible credit misallocation that could result from the information asymmetry generated by this difference.

General features of the HELOC market are documented by Canner, Fergus, and Luckett (1988); DeMong and Lindgren (1990); Canner and Luckett (1994); Canner, Durkin, and Luckett (1998); and Aizcorbe, Kennickell, and Moore (2003). Eugeni (1993) examines the tax and interest advantages of HELOCs. Motivations for using HELOCs are also investigated by Chen and Jensen (1985), who analyze HELOCs in the life-cycle consumption-saving plan; Delaney (1994), who explains HELOC use as a way to tap into the forced savings induced by mortgage payments; and Salandro and Harrison (1997), who empirically determine the demand for HELOCs as a function of financial and socioeconomic characteristics. Agarwal et al. (Forthcoming) provide an empirical analysis of home equity loan and line performance. They find that households with equity loans are relatively more sensitive to changes in interest rates, whereas those with equity lines are more sensitive to appreciation in property value. Finally, Dey and Dunn (2005) show how banks use the loan-to-value ratio as a signal of borrower quality to sort and price risk in the HELOC market.

2.1 Lines of credit as an alternative to full insurance

Beyond their standard use as instruments of consumption smoothing across time periods, lines of credit can play a role in consumers' risk management. Most risks are financed by traditional insurance; consumers, however, often do not buy insurance to cover themselves against wage-income risks and all possible medical risks, even if there are insurance markets that potentially provide *full* and *fair* insurance. Consumers may choose to leave these risks uncovered simply because they have access to lines of credit. A major motive for holding lines of credit is to replace traditional insurance. Eisenhauer (1994) examines the role that lines of credit can play in consumers' risk management. He models a framework with *full* but *unfair* insurance, where consumers optimally leave some risks uninsured and resort to lines of credit to cover the uninsured losses. His model does not consider the ability to repay debt on lines of credit over time; i.e., repayment flexibility. In the next section, I will show how being asked to repay only a small fraction of the outstanding debt induces consumers who face *full* and *fair* insurance and have sufficiently high rates of discount to optimally keep some wealth losses uninsured and hold lines of credit instead as a buffer.

3. The Theoretical Model

I model a typical consumer's choice of borrowing on lines of credit as a lifetime utility maximization problem in an environment of risk.[3] I consider three different scenarios:

(i) The consumer can borrow *only* on credit cards.

(ii) The consumer can borrow *only* on HELOCs.

(iii) The consumer can borrow on credit cards *and* HELOCs.

We consider two time periods: 0 and 1. The world begins with the beginning of period 0 and ends with the end of period 1. The consumer knows right from the very beginning that in period 0 (the time during which the consumer is assumed to be working full-time) they face a possible reduction of wealth, such as a wage-income risk or health risk. In period 1 (the time during which the consumer is secured against income or medical risk by some public safety net[4]),

[3] We exclude convenience use or the use of lines of credit solely for transaction purposes.

[4] Such as the social security safety net.

consumption is determined by a certain income, government-provided medical benefits, the consumer's net worth,[5] and the consumer's bankruptcy status. Hence, in the beginning of period 0, a consumer faces four possible states of the world: no-default-loss, no-default-no-loss, default-loss, and default-no-loss. The consumer has an instantaneous utility function, U, such that $U' > 0$, $U'' < 0$. The variables used in the model are defined in Table 1.

Table 1: Definition of Variables

W_t – Wealth of the consumer at time t	t – Income tax rate; $0 < t < 1$
L – Loss; $L > 0$	d –Discount factor; $0 < d < 1$
q – Insurance coverage	a – Required rate of repayment; $0 < a < 1$
p – Probability of the Loss	r_C – Credit card interest rate
s^C – Probability of default on credit cards	r_H –HELOC interest rate
s^H – Probability of default on HELOCs	t – Fixed cost of HELOC; $t > 0$

The market for lines of credit that I deal with is realistically assumed to have the following institutional background. Since the world begins with the beginning of period 0, the consumer starts with a wealth endowment of W_0 (which includes their net worth and the wage income), and in period 1 their wealth is equivalent to a certain and predetermined income, government-provided medical benefits, and their net worth. Therefore, the periodic wealth of the consumer (W_t) is exogenously determined. Credit cards are unsecured lines of credit: if the consumer fails to repay any outstanding credit card debt, they will lose all their non-exempt assets, which usually consist of their entire financial wealth. Hence, the value of the wealth retained by the consumer after defaulting on credit card debt in period 1 is given by $g_C W_1$, where $0 < g_C < 1$. Moreover, the legal default procedure discharges all outstanding debt that the consumer may have at the time of default.[6] In the event of a default on HELOCs, the equity in the home also becomes non-exempt from creditors; HELOCs therefore have higher default

[5] This is the difference between gross assets and liabilities.

[6] The assumption is that consumers declare bankruptcy under the legal protection of Chapter 7 of the bankruptcy code, where the bankrupt consumers get a fresh start.

costs, because the consumer can only retain wealth of amount $g_H W_1$ in period 1, where $0 < g_H < 1$ and $g_H < g_C$. The interest rate on credit cards (r_C) is greater than the interest rate on HELOCs (r_H).[7] Moreover, I assume that the consumer can lend at the same rate as they borrow.[8] The required rate of repayment (a) is the same for HELOCs and credit cards.[9] I also assume that the consumer pays back at the required rate, which is again a simplifying assumption, though not very unrealistic for the majority of HELOC borrowers and some credit card borrowers. I further assume that insurance is *full* and actuarially *fair*, so that the insurance premium = pq.

3.1 The choice of borrowing on credit cards

Suppose that the consumer has only credit cards available for borrowing. This would occur if the consumer does not own a home with positive equity or does not own a home at all. Let $(1 - a)B$[10] denote the amount of credit card debt incurred in period 0, irrespective of whether wealth loss occurs. A consumer can *potentially* carry a debt of amount $D = (1 - a)(B + (L - q))$ on their credit card in period 0, where $(1 - a)(L - q)$ is the extra borrowing that the consumer may need to cover for any shortfall as a result of a wealth loss, L. The probability of default is given by $s^C = (1 - p)\ Prob(W_1 < (1 + r_C)(1 - a)B) + p\ Prob(W_1 < (1 + r_C)(1 - a)(B + (L - q)))$, such that $s_B{}^C > 0$, $s_{BB}{}^C > 0$, $s_q{}^C < 0$, and $s_{qq}{}^C < 0$. The consumer's discounted expected lifetime utility in the event of no default on credit card debt is given by

$$V^{NDC} = V_0{}^C + d\ V_1{}^{NDC}, \text{ where}$$

$$V_0{}^C = (1 - p)\ U(W_0 - pq + (1-a)B) + p\ U(W_0 - pq + (1-a)B - a(L-q)) \text{ and}$$

$$V_1{}^{NDC} = (1-p)\ U(W_1 - (1+r_C)(1-a)B)$$

$$+ p\ U(W_1 - (1+r_C)(1-a)(B+(L-q))).$$

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7 This is usually the case except for introductory credit card offers, which are excluded from this analysis.
8 This is a simplifying assumption, invoked for the sake of completeness.
9 The required rate of repayment is usually between 2–5 per cent of outstanding balances on both types of credit lines.
10 If B is negative, then the consumer saves the amount $-(1 - a)B$. In other words, I assume that savers get back a part of their savings ($-aB$) for consumption in period 0.

The consumer's discounted expected lifetime utility in the event of a default on credit card debt is given by

$$V^{DC} = V_0^{\,C} + d\ V_1^{\,DC}, \text{ where}$$

$$V_1^{\,DC} = (1 - p\ )\ U(g_C W_1)$$

$$+\ p\ U(g_C W_1) = U(g_C W_1).$$

The consumer's discounted expected lifetime utility is given by

$$V^C = (1 - s^C)\ V^{NDC} + s^C\ V^{DC}$$

$$= V_0^{\,C} + d\ [(1 - s^C)\ V_1^{\,NDC} + s^C\ V_1^{\,DC}].$$

The consumer then faces the following optimization problem:

A. Maximize V^C by choosing q and B.

Differentiating V^C with respect to q and setting it to zero gives

$$p\ (1 - p\ )\ U'(C_{N_0}^{C}) = p\ (\alpha - p\ )\ U'(C_{L_0}^{C}) + p d\,(1 - a\,)(1 + r_C)\,(1 - s^C)\ U'(C_{NDL_1}^{C})$$

$$-\ d\ s_q^{\,C}\ [V_1^{\,NDC} - V_1^{\,DC}] \qquad (1)$$

The variable $C_{j_k}^{i}$ represents consumption, where the superscripts C and H represent, respectively, credit cards and HELOCs, the subscripts L and N indicate loss and no-loss, the subscripts NDL and NDN indicate no-default-loss and no-default-no-loss states, and the subscripts 0 and 1 represent the time periods.

Using equation (1),

$$\frac{U'(C_{N_0}^{C})}{U'(C_{L_0}^{C})} < 1 \Leftrightarrow d < \frac{p(1-a\,)U'(C_{L_o}^{C})}{p(1+r_C)(1-a\,)(1-s^C)U'(C_{NDL_1}^{C}) - s_q^{C}[V_1^{NDC} - V_1^{DC}]} = d^{\,AC}.$$

The concavity of the utility function implies that

$$\delta < \delta^{AC} \Leftrightarrow q^* < L.$$[11]

Differentiating V^C with respect to B gives

$$B^* > 0, \text{ if } \left.\frac{\partial V^C}{\partial B}\right|_{B=0} > 0, \text{ that is, if}$$

[11] The superscript * represents the optimal level of consumer's choice variable when using credit cards. See the appendix for a numerical example.

$$d < \frac{[(1-a)[(1-p)U'(C^C_{N_0}) + pU'(C^C_{L_0})]]\big|_{B=0}}{[(1+r_C)(1-a)(1-s^C)[(1-p)U'(C^C_{NDN_1}) + pU'(C^C_{NDL_1})] + s^C_B[V_1^{NDC} - V_1^{DC}]]\big|_{B=0}}$$

$$\Rightarrow \qquad d < d^{BC}.$$

The following results are obtained:

(i) If $d < Min\,[d^{AC}, d^{BC}]$, then $q^* < L$ and $B^* > 0$.

(ii) If $d^{BC} < d < d^{AC}$, then $q^* < L$ and $B^* = 0$.

(iii) If $d^{AC} < d < d^{BC}$, then $q^* = L$ and $B^* > 0$.

(iv) If $d > Max\,[d^{AC}, d^{BC}]$, then $q^* = L$ and $B^* = 0$.

Given the repayment scheme, the rate of interest on credit cards, and the probability of the wealth loss, if the consumer has a sufficiently low discount factor (as described in results (i) and (ii)), then they will optimally want to underinsure (even in the presence of *full* and actuarially *fair* insurance) in period 0 and hold credit cards instead as a buffer. If the consumer experiences the wealth loss and resorts to borrowing on credit cards to cover for the uninsured part, the benefit from not paying the full insurance premium and repaying the credit card debt over time will outweigh the interest and expected default costs when the consumer's discount factor is lower than d^{AC}. Results (iii) and (iv) describe the case where the consumer fully insures against the wealth loss, L. Borrowings on credit cards can occur (as shown in results (i) and (iii)) irrespective of whether the wealth loss occurs, if the consumer's discount factor lies below d^{BC}.

3.2 The choice of borrowing on HELOCs

Suppose that the consumer has only HELOCs available for borrowing. HELOCs have low and income-tax-deductible interest rates; however, they carry non-interest costs, such as fixed or upfront costs (t). The new probability of default is given by $s^H = (1 - p)\,Prob(W_1 < (1 + r_H(1 - t))(1 - a)B) + p\,Prob(W_1 < (1 + r_H(1 - t))(1 - a)(B + (L - q)))$, such that $s_B{}^C > s_B{}^H > 0$, $s_q{}^C < s_q{}^H < 0$, $s_{BB}{}^C > s_{BB}{}^H > 0$, and $s_{qq}{}^C < s_{qq}{}^H < 0$. The consumer's discounted expected lifetime utility in the event of no default on HELOC debt is given by

$$V^{NDH} = V_0{}^H + d\, V_1{}^{NDH}, \text{ where}$$

$$V_0^{H} = (1 - p\ )\ U(W_0 - pq + (1 - a\ )B - t) +$$

$$p\ U(W_0 - pq + (1 - a\ )B - a\,(L - q) - t) \text{ and}$$

$$V_1^{NDH} = (1 - p\ )\ U(W_1 - (1 + r_H\,(1 - t))(1 - a\ )B\ ) +$$

$$p\ U(W_1 - (1 + r_H\,(1 - t))(1 - a\ )(B + (L - q))\ ).$$

The consumer’s discounted expected lifetime utility in the event of a default on HELOC debt is given by

$$V^{DH} = V_0^{H} + d\ V_1^{DH}, \text{ where}$$

$$V_1^{DH} = (1 - p\ )\ U(g_H W_1)$$

$$+\ p\ U(g_H W_1) = U(g_H W_1).$$

The consumer’s discounted expected lifetime utility is given by

$$V^{H} = (1 - s^{H})\ V^{NDH} + s^{H}\ V^{DH}$$

$$= V_0^{H} + d\ [(1 - s^{H})\ V_1^{NDH} + s^{H}\ V_1^{DH}].$$

Here, the consumer faces the following optimization problem:

A. Maximize V^H by choosing q and B.

Differentiating V^H with respect to q and setting it to zero gives

$$p\ (1 - p\ )\ U'(C^{H}_{N_0}) = p\ (\alpha - p\ )\ U'(C^{H}_{L_0}) + pd\,(1 - a\ )(1 + r_H\,(1 - t))\ (1 - s^{H})\ U'(C^{H}_{NDL_1})$$

$$- d\ s_q^{H}\ [V_1^{NDH} - V_1^{DH}] \qquad (2)$$

Using equation (2),

$$\frac{U'(C^{H}_{N_0})}{U'(C^{H}_{L_0})} < 1 \Leftrightarrow$$

$$d < \frac{p(1 - a\ )U'(C^{H}_{L_0})}{p(1 + r_H\,(1 - t))(1 - a\ )(1 - s^{H}\ )U'(C^{H}_{NDL_1}) - s_q^{H}\,[V_1^{NDH} - V_1^{DH}\ ]} = d^{\,AH}\ .$$

The concavity of the utility function implies that

$$\delta < \delta^{AH} \Leftrightarrow q^{**} < L\,.$$ [12]

Differentiating V^H with respect to B gives

[12] The superscript ** represents the optimal level of consumer’s choice variable when using HELOCs.

$B^{**} > 0$, if $\left.\frac{\partial V^H}{\partial B}\right|_{B=0} > 0$, that is, if

$d < d^{BH}$, where

$d^{BH} =$

$$\frac{\left.[(1-a)[(1-p)U'(C^H_{N_0}) + pU'(C^H_{L_0})]]\right|_{B=0}}{\left.[(1+r_H(1-t))(1-a)(1-s^H)[(1-p)U'(C^H_{NDN_1}) + pU'(C^H_{NDL_1})] + s^H_B[V_1^{NDH} - V_1^{DH}]]\right|_{B=0}}.$$

The following results are obtained:

(i) If $d < Min\ [d^{AH}, d^{BH}]$, then $q^{**} < L$ and $B^{**} > 0$.

(ii) If $d^{BH} < d < d^{AH}$, then $q^{**} < L$ and $B^{**} = 0$.

(iii) If $d^{AH} < d < d^{BH}$, then $q^{**} = L$ and $B^{**} > 0$.

(iv) If $d > Max\ [d^{AH}, d^{BH}]$, then $q^{**} = L$ and $B^{**} = 0$.

Given the repayment scheme, the rate of interest on HELOCs, and the probability of the wealth loss, if the consumer has a sufficiently low discount factor (as described in results (v) and (vi)), then they will optimally want to underinsure in period 0 and hold HELOCs instead as a buffer. If the consumer experiences the wealth loss and resorts to borrowing on HELOCs to cover for the uninsured part, the benefit from not paying the full insurance premium and repaying the HELOC debt over time will outweigh the interest and expected default costs when the consumer's discount factor is lower than d^{AH}. Results (vii) and (viii) describe the case where the consumer fully insures against the wealth loss, L. Borrowings on HELOCs can occur (as shown in results (v) and (vii)) irrespective of whether the wealth loss occurs, if the consumer's discount factor lies below d^{BH}.

3.3 The choice of borrowing with access to credit cards and HELOCs

Suppose that the consumer has access to both credit cards and HELOCs. The consumer is faced with two decisions: (i) whether the low-interest but otherwise costly HELOC should actually be acquired, and (ii) if a HELOC is acquired in addition to a credit card, the amount of debt to put on each. In conducting this analysis, it is important to remember the costs and benefits associated with the two borrowing instruments the consumer has access to. HELOCs have lower and income-tax-deductible interest charges. They also, however, involve fixed costs, and since they are secured by the equity in the home, defaulting on them incurs higher costs in terms of loss of wealth in period 1 (i.e., $g_H W_1 < g_C W_1$).

The discounted expected lifetime utility from not acquiring a HELOC and *potentially* carrying a debt of amount $D = (1 - a)(B + (L - q))$ on a credit card is given by

$$V^C = (1 - s^C)\, V^{NDC} + s^C\, V^{DC}$$

$$= V_0^C + d\,[(1 - s^C)\, V_1^{NDC} + s^C\, V_1^{DC}].$$

For $D = 0$, $s^C = 0$ and $V^C = U(W_0 - pL) + dU(W_1)$.

The discounted expected lifetime utility from *potentially* borrowing the same amount of D only on a HELOC is given by

$$V^H = (1 - s^H)\, V^{NDH} + s^H\, V^{DH}$$

$$= V_0^H + d\,[(1 - s^H)\, V_1^{NDH} + s^H\, V_1^{DH}].$$

Similarly, for $D = 0$, $s^H = 0$ and $V^H = U(W_0 - pL - t) + dU(W_1)$.

Hence, $\forall\ t > 0$ and for $D = 0$, $V^H - V^C < 0$. In other words, given the positive fixed costs of obtaining HELOCs, the consumer will never consider borrowing on a HELOC unless they anticipate a positive desired debt.

I next draw upon the following sufficient conditions that are assumed to represent the situation for a consumer who has access to both types of lines of credit, thus generating empirically observed consumer behaviour.

Condition 1: $\dfrac{\partial(V^H - V^C)}{\partial D} > 0, \forall 0 \le D < \overline{D}.$

To explore what Condition 1 actually entails with respect to a consumer's utility calculations, I first redefine the default probabilities and the discounted lifetime utility functions in terms of the *potential* debt, D. The probability of default for credit cards is given by $s^C = (1 - p)\, Prob(W_1 < (1 + r_C)(D - (1 - a)(L - q))) + p\, Prob(W_1 < (1 + r_C)D)$, such that $s_D{}^C > 0$ and $s_{DD}{}^C > 0$. Similarly, the probability of default for HELOCs is given by $s^H = (1 - p)\, Prob(W_1 < (1 + r_H\,(1$

$-\ t))(D-(1-a)(L-q))) + p\ Prob(W_1 < ((1+r_H(1-t))D)$, such that $s_D{}^C > s_D{}^H > 0$ and $s_{DD}{}^C > s_{DD}{}^H > 0$. Also,

$$V_0{}^C = (1-p)\ U(W_0 - pq + D - (1-a)(L-q))$$
$$+\ p\ U(W_0 - pq + D - (L-q)),$$
$$V_1{}^{NDC} = (1-p)\ U(W_1 - (1+r_C)(D-(1-a)(L-q)))$$
$$+\ p\ U(W_1 - (1+r_C)D),$$
$$V_1{}^{DC} = U(g_C W_1), \text{ and}$$
$$V^C = V_0{}^C + d\ [(1-s^C)\ V_1{}^{NDC} + s^C\ V_1{}^{DC}].$$

Similarly,

$$V_0{}^H = (1-p)\ U(W_0 - pq + D - (1-a)(L-q) - t)$$
$$+\ p\ U(W_0 - pq + D - (L-q) - t),$$
$$V_1{}^{NDH} = (1-p)\ U(W_1 - (1+r_H(1-t))(D-(1-a)(L-q)))$$
$$+\ p\ U(W_1 - (1+r_H(1-t))D),$$
$$V_1{}^{DH} = U(g_H W_1), \text{ and}$$
$$V^H = V_0{}^H + d\ [(1-s^H)\ V_1{}^{NDH} + s^H\ V_1{}^{DH}].$$

Imposing Condition 1 is equivalent to assuming that $\forall 0 \le D < \overline{D}$,

$$(1+r_C)(1-s^C)[(1-p)U'(C^C_{NDN_1}) + pU'(C^C_{NDL_1})] -$$
$$(1+r_H(1-t))(1-s^H)[(1-p)U'(C^H_{NDN_1}) + pU'(C^H_{NDL_1})] >$$
$$s^H_D[V_1{}^{NDH} - V_1{}^{DH}] - s^C_D[V_1{}^{NDC} - V_1{}^{DC}].$$

In other words, Condition 1 implies that $\forall 0 \le D < \overline{D}$, the gain in expected utility owing to lower and income-tax-deductible interest payments from putting a dollar into HELOCs instead of credit cards in the event of no default is greater than the loss of expected utility, due to higher default costs associated with HELOCs.

I next impose Condition 2, which implies that a rising probability of defaulting on HELOCs, and thereby losing the equity in the home, as well, eventually makes HELOCs unfavourable for consolidating debt away from credit cards:

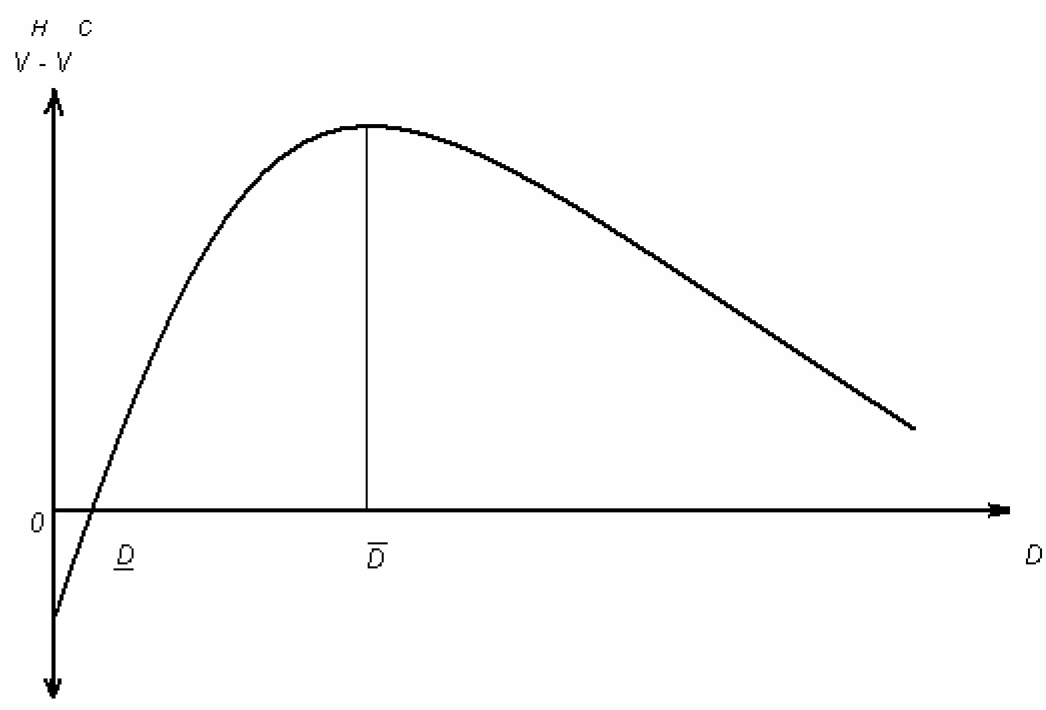
Condition 2: $\frac{\partial(V^H - V^C)}{\partial D} \leq 0, \forall D \geq \overline{D}$.

I then impose a final condition:

Condition 3: There exists a particular level of borrowing, $\underline{D}$, such that $0 < \underline{D} < \overline{D}$, $(V^H - V^C)\big|_{D=\underline{D}} = 0$ and $V^H - V^C > 0, \forall D > \underline{D}$.

Condition 3 assumes that, starting from a negative value at $D = 0$, $V^H - V^C$ becomes zero at $\underline{D} \in (0, \overline{D})$ and stays positive at all values of D after that. Figure 1 illustrates the trade-offs facing the consumer that Conditions 1 to 3 delineate.

Figure 1: The Choice of Lines of Credit



If the consumer has actually acquired a credit card and a HELOC, then the new discounted expected lifetime utility from *potentially* borrowing D is given by

$$V^B = V_0^B + d\,[(1 - s^B)\, V_1^{NDB} + s^B\, V_1^{DB}],$$

where

$$V_0^B = (1 - p)\, U(W_0 - pq + D^C + D^H - (1-a)(L-q) - t)$$
$$+\, p\, U(W_0 - pq + D^C + D^H - (L-q) - t),$$

$$V_1^{NDB} = (1 - p)$$
$$U(W_1 - (1+r_C)(D^C - (1-a)(L-q^C)) - (1+r_H(1-t))(D^H - (1-a)(L-q^H)))$$
$$+\, p\, U(W_1 - (1+r_C)D^C - (1+r_H(1-t))D^H),$$

$$V_1^{DB} = (1 - s^H)\, V_1^{DC} + s^H\, V_1^{DH}.$$

In the expressions above, $D^C = (1 - a)(B^C + (L - q^C))$ is the credit card debt, $D^H = (1 - a)(B^H + (L - q^H))$ is the HELOC debt, and $D^C + D^H = D = (1 - a)(B + (L - q))$. Moreover, B^C and B^H are state-independent credit card and HELOC borrowings, respectively, such that $B^C + B^H = B$. Therefore, the insurance coverages q^C and q^H are such that $(L - q^C) + (L - q^H) = L - q$. Finally, the new probability of default when the consumer has taken out *both* lines of credit is given by

$$s^B = (1-p)\, Prob(W_1 < (1+r_C)(D^C - (1-a)(L-q^C)) +$$
$$(1+r_H(1-t))(D^H - (1-a)(L-q^H))) +$$
$$p\, Prob(W_1 < (1+r_C)D^C + (1+r_H(1-t))D^H), \text{ such that}$$
$$s^B_{D^C} > s^B_{D^H} > 0 \text{ and } s^B_{D^C D^C} > s^B_{D^C D^H} > s^B_{D^H D^H} > 0.$$

From the foregoing analysis of consumer's choice under the three different scenarios, given my assumptions and using Conditions 1, 2, and 3, I obtain the following results:

(i) $D > 0$, if $d < Max\,[d^{AC}, d^{BC}]$,

(ii) $\ni \overline{D} > \underline{D} > 0$, such that $(V^H - V^C)\big|_{D=\underline{D}} = 0$,

(iii) For $D < \underline{D}$, $V^H - V^C < 0$,

(iv) For $D > \underline{D}$, $V^H - V^C > 0$, and

(v) For $D \geq \overline{D} > \underline{D}$, $V^B\big|_{D^C = D - \overline{D}, D^H = \overline{D}} \geq V^H > V^C$.

The results that capture the borrowing decisions on lines of credit within a framework of consumption smoothing across state and time can be summarized as follows. The consumer desires to borrow on lines of credit, such as credit cards, if they are sufficiently impatient (i.e., if $d < Max\ [d^{AC}, d^{BC}]$). The upfront fixed costs associated with HELOCs prevent the consumer from using them for small amounts of *potential* debt; i.e., for $D < \underline{D}$. The consumer uses credit cards exclusively for all borrowing needs below $\underline{D}$. For all *potential* debt levels below a certain upper bound (i.e., $\forall D < \overline{D}$), transferring balances away from high-interest credit cards and into HELOCs is beneficial. Therefore, for these intermediate levels of *potential* debt, the consumer prefers to use HELOCs exclusively for all their borrowing needs. Finally, for very large levels of *potential* debt (i.e., for $D \geq \overline{D} > \underline{D}$), a rising probability of defaulting on HELOCs and thereby losing the equity in the home, as well, eventually makes HELOCs unfavourable for consolidating debt away from credit cards. In these situations, I find that the consumer is induced to carry debt on both credit cards and HELOCs.

Moreover, I often find in the data that there are consumers who have taken out *both* credit cards and HELOCs and yet are carrying a positive balance only on their credit cards. Since the *potential* debt to be carried is the relevant variable for the choice of lines of credit, a consumer with *both* lines of credit and a positive balance carried on credit cards alone could be considered to anticipate a very large level of *potential* debt (i.e., $D \geq \overline{D} > \underline{D}$), so that transferring balances away from high-interest credit cards and into HELOCs is not beneficial.

4. Conclusions

This paper has modelled the choice consumers have between the two major lines of credit—credit cards and HELOCs—in a framework where consumers hold lines of credit as instruments of consumption smoothing across state and time. Even when consumers have access to *full* and actuarially *fair* insurance, it may be optimal for them to leave a portion of the risks that they face uninsured and hold lines of credit instead as a buffer. For a consumer with a sufficiently high discount rate, if the anticipated wealth loss actually occurs and they resort to borrowing on lines of credit to cover for the uninsured part, the benefit of not paying a full insurance premium and repaying the debt on lines of credit over time will outweigh the cost.

Given the costs and advantages of the different lines of credit, I find that the choice between credit cards and HELOCs depends on the amount of borrowing that consumers *potentially* wish to undertake. For small levels of *potential* borrowings, my theoretical model shows that consumers will prefer to use credit cards exclusively, and for intermediate levels they will use only HELOCs. This finding includes those who replace their credit card debt by fully consolidating into HELOCs. When the *potential* borrowing level reaches a certain threshold, however, a rising probability of defaulting on HELOCs and thereby losing the equity in the home, as well, will eventually make HELOCs unfavourable for consolidating debt away from credit cards. In these situations, I find that consumers will be induced to carry debt on both credit cards and HELOCs. Thus, I find a rational explanation for the observed phenomenon of consumers carrying debt on both HELOCs and credit cards. Moreover, since I argue that the consumers take out lines of credit in anticipation of wealth losses that they may incur, which is to say that they care about the *potential* debt to be carried on their lines, the upper bound on the *potential* HELOC borrowing also explains why some consumers have *both* lines of credit and carry positive debt only on their credit cards.

Previous research has focused on the role lines of credit play in smoothing consumption across time periods, and the role they play in precautionary purposes when the insurance market is either not *full* or not *fair*. I have delineated a precautionary role for lines of credit even in an environment of *full* and actuarially *fair* insurance. Moreover, I have explained why a consumer who has incurred the fixed cost of obtaining a HELOC and has not yet fully utilized the line would continue to borrow on high-rate credit cards. If the use of lines of credit continues to grow at the current unprecedented rate, their full incorporation into traditional economic models will become critical to a complete understanding of the complex behaviour underlying consumption, savings, and demand for money.

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Appendix

In this appendix, I numerically solve the consumer's utility maximization problem when they have only credit cards to use as a buffer against uninsured wealth losses. For my numerical example, I set the values of the model's variables using the U.S. *Survey of Consumer Finances, 2001* (Aizcorbe, Kennickell, and Moore 2003). I take a = 0.03, r_C = 0.13, and $U = \frac{C^{1-s}}{1-s}$, with s = 2. I also assume that B = \$1000. Using the *Survey of Consumer Finances, 2001*, I take the median credit card borrowing for 2001 to be \$1,900, and the median income of a household to be \$40,000. Assuming that the household faces only wage-income risk, I set L = \$40,000 and p as the average unemployment rate of 2001, which is 0.05. Using the assumed value of B and the fact that $(1 - a)(B + (L - q))$ = \$1,900, I get q = \$39,000. The median household net worth in 2001 is \$86,000. Therefore, I have W_0 = \$126,000. I set W_1 as the sum of the median net worth and the median income of the age group 65–74 of the year 2001, which is \$204,000. The median value of financial assets of this age group is \$51,000. Hence, $g_C = 1 - \frac{\$51,000}{\$204,000}$ = 0.75. Using the values of the variables set as above, I can write the probability of default, $s^C = f(q, p, W_0, W_1, r_C, B, L, a) = a - \frac{q^2}{b}; q \in [0, \$40,000]$ and where the constants $a > 0$ and $b > 0$, thus satisfying $s_q^C < 0$ and $s_{qq}^C < 0$. Assuming that (i) s^C at q = \$39,000 is equal to the bankruptcy rate of 2001, which is 0.005, and (ii) if q = \$40,000, the median credit card debt and the bankruptcy rate of 2001 would be half their current levels, I get a = 0.05 and b = \$31,600,000. The insurance premium pq = (0.05)\$39,000 = \$1,950 and s_q^C at q = \$39,000 is –0.003. Hence, the sufficient condition for having the optimal q = \$39,000 < L = \$40,000, which is

$$d < \frac{p(1-a)U'(C^C_{L_o})}{p(1+r_C)(1-a)(1-s^C)U'(C^C_{NDL_1}) - s^C_q[V_1^{NDC} - V_1^{DC}]} = d^{AC},$$

becomes

$$d < d^{AC} = 0.64.$$

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