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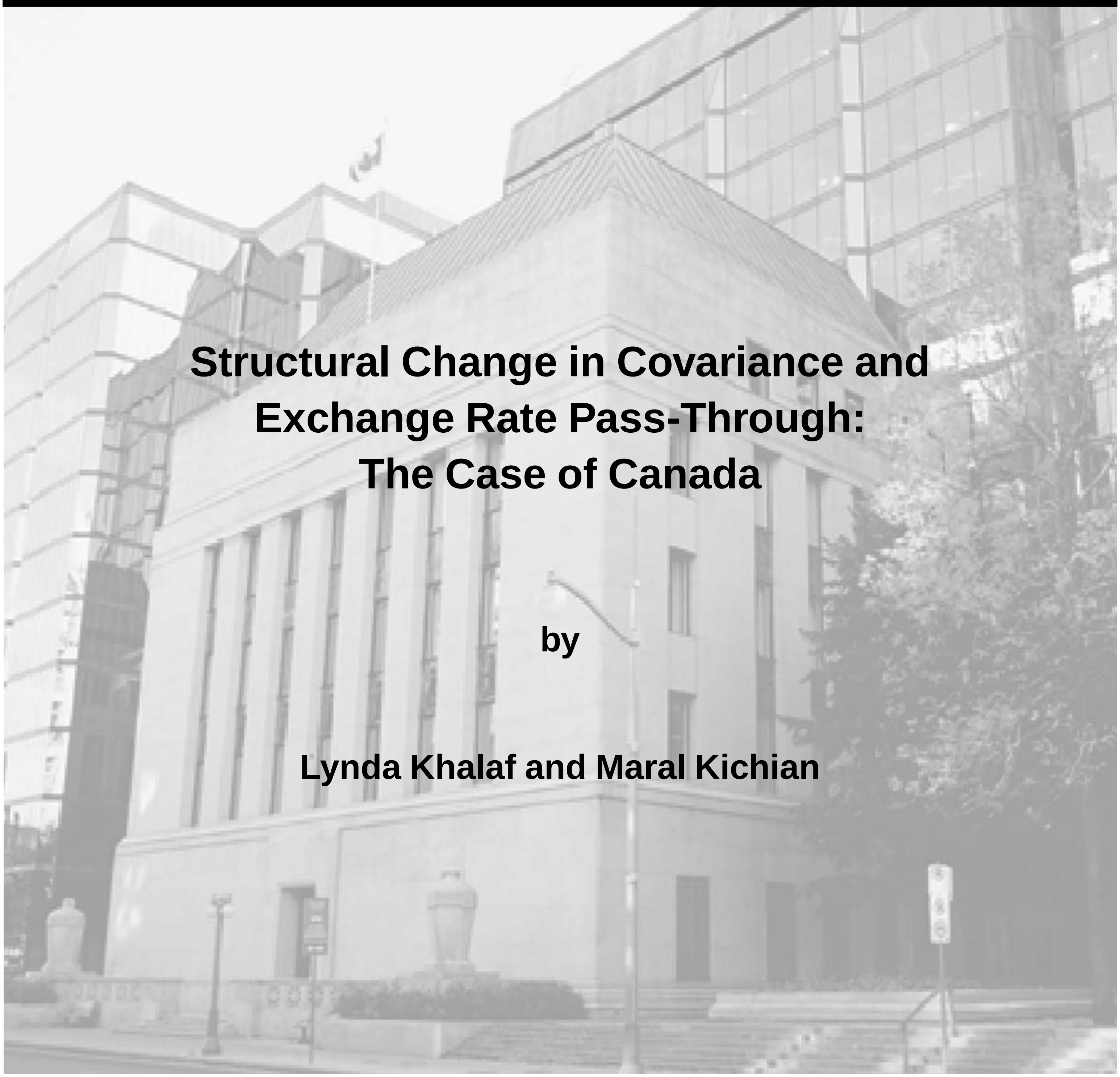

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Structural Change in Covariance and Exchange Rate Pass-Through: The Case of Canada

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Abstract

The authors address empirically the implications of structural breaks in the variance-covariance matrix of inflation and import prices for changes in pass-through. They define pass-through within a correlated vector autoregression (VAR) framework as the response of domestic inflation to an impulse in import price inflation. This approach allows them to examine changes in both the amount and the duration of pass-through.

The authors develop a test to establish the presence of structural breaks in the error covariance matrix of a multivariate system of equations. The test extends the breaks-in-variance/covariance test proposed by Anderson (1971) by accounting for regression covariates and an unknown break date. It is exact for fixed regressors.

The results of the test reveal evidence of breaks in the covariance matrix in 1984Q3 and in 1991Q1. Estimating the VAR and examining impulse responses over the relevant subsamples, the authors find that, while the initial impact of pass-through is quantitatively similar across the two samples, the impact dissipates twice as fast in the later subsample. In other words, the duration of pass-through has declined from about a two-year period to a one-year period. The authors also document important changes over time in the estimated correlation between domestic and import inflation, both in terms of magnitude and sign.

JEL classification: F40, F31, C52, E31
Bank classification: Econometric and statistical methods

Résumé

Les auteures analysent empiriquement les implications de la présence de ruptures structurelles dans la matrice de variance-covariance de l'inflation et des prix à l'importation en ce qui concerne la transmission des variations du taux de change aux prix. Elles ont recours à un vecteur autorégressif (VAR) corrélé où l'incidence de ces variations est représentée par la réaction de l'inflation mesurée par l'indice des prix à la consommation à une modification de l'inflation des prix à l'importation. Cette approche leur permet d'examiner à la fois les changements de degré et de durée qui s'opèrent dans la transmission des mouvements des prix à l'importation.

Afin de vérifier si des ruptures existent dans la matrice de covariance des erreurs d'un système d'équations multivarié, les auteures étendent le test proposé par Anderson (1971) pour déceler les ruptures dans une matrice de variance-covariance en y intégrant des covariables de régression et une date de rupture inconnue. Leur test est exact si les variables de régression sont strictement exogènes.

Les résultats du test révèlent des ruptures au troisième trimestre de 1984 et au premier trimestre de 1991. L'estimation du VAR et l'analyse des profils de réaction obtenus sur les sous-périodes correspondantes amènent les auteures à conclure que, si l'effet initial des variations des prix à l'importation sur l'inflation intérieure est d'ampleur analogue dans les deux sous-périodes, il se dissipe deux fois plus vite dans la seconde : il se fait sentir durant une seule année, contre environ deux auparavant. Les auteures constatent également que l'ordre de grandeur et le sens de la corrélation estimée entre l'inflation intérieure et l'inflation importée se sont tous deux modifiés sensiblement au cours de la période considérée.

Classification JEL : F40, F31, C52, E31
Classification de la Banque : Méthodes économétriques et statistiques

1. Introduction

Recent studies suggest that, in industrialized countries, pass-through of exchange rate or import price changes into consumer price inflation has declined over time.[1] In general, the amount of pass-through in these studies is represented either by the immediate impact of changes in the exchange rate or in import prices on inflation (short-run pass-through), or by the cumulative effect of such changes (long-run pass-through, calculated as the estimated short-run coefficient divided by one minus the estimated inflation lags). What has received little attention, however, is the duration of pass-through, and whether it too has changed.

Part of the reason for this scant attention is that pass-through has largely been studied within single-equation inflation models, where exchange rate or import price changes are treated as being exogenous. By construction, single-equation approaches emphasize structural breaks in variable means. However, changes in the second moments of inflation and exchange rates, and possible connections between these and declines in pass-through, have also been documented. For instance, using an indirect empirical approach, Gagnon and Ihrig (2004) find significant links between changes in pass-through coefficients of various countries and changes in the inflation variability of those countries. On the theoretical side, Devereux, Engel, and Storgaard (2003) show that the extent of pass-through affects exchange rate volatility, as well as the relationship between exchange rates and various economic variables. In this respect, the more variable the exchange rate, the less are its effects on the economy (the so-called "disconnect" phenomenon).

The above-noted research seems to suggest a role for changes in the variance-covariance matrix of inflation and exchange rates regarding changes in pass-through. In this paper, we investigate this issue empirically, focusing not only on the short- or long-run quantitative effects of pass-through, but also on its duration. Since single-equation models ignore information present in the covariance of the variables of interest, we propose a correlated bivariate vector autoregression (VAR) modelling framework. Accounting for breaks in the mean of the system, we look for breaks in the covariance matrix. In this context, pass-through is defined as the response of inflation over time to an impulse in the innovation of the exchange rate (or import price) variable.

As our second contribution in this paper, we develop a test to uncover breaks in the error covariance matrix of a multivariate linear system (MLS). The test extends the method proposed in Anderson (1971) by allowing for covariates (i.e., for regressors other than a constant) in the equations of the system, and by showing how to obtain a simulation-based cut-off point for the likelihood-ratio (LR) statistic. Two cases are covered: (i) the case where the break date is known, and (ii) the case where it is unknown. Furthermore, the test can look for breaks in the covariance matrix while also allowing for breaks in the mean. We also show that, if the regressors are strongly exogenous, the test is exact in finite samples.

Applications are carried out using Canadian data. First, we uncover evidence of breaks, at the 5 per cent level, in the bivariate VAR error covariance matrix. In particular, the periods 1984Q3 and 1991Q1 exhibit the highest probability of having experienced a break in the covariance matrix of the VAR. In addition, the hypothesis of no break is not rejected at the 5 per cent level prior to 1982Q2, and after 1991Q2. Accordingly, we estimate the

[1]See, for example, Kichian (2001), Leung (2003), Gagnon and Ihrig (2004), Bailliu and Fujii (2004), Murchison (2004), and Bouakez and Rebei (2005).

VAR and examine pass-through by calculating impulse responses over two subsamples: for the period ending in 1982Q2, and for the period starting in 1991Q2. The results show that pass-through has changed over time—not so much quantitatively, but more with respect to its duration—from about a two-years' duration in the first subsample to half that in the second subsample.

In section 2 we summarize the literature on declining pass-through in Canada, and describe our VAR model. Section 3 explains the break test and its application to our data. Section 4 reports the VAR estimation results and discusses the corresponding impulse-response functions. Section 5 offers some conclusions.

2. Canadian Evidence and Our Econometric Model

Several studies suggest that the effect of a change in the exchange rate on Canadian consumer price index (CPI) inflation decreased after 1983–84. The studies differ in their definitions of pass-through, reflecting the general lack of consensus on this issue in the literature, as well as in the estimates obtained. Kichian (2001) uses a backward-looking Phillips curve with time-varying coefficients (and, therefore, changing conditional variances) and measures short-run pass-through as the coefficient on U.S. inflation relative to Canadian inflation. She shows that this coefficient drops from an average value of 0.2 to essentially zero after 1983–84. Leung (2003) uses a backward-looking Phillips curve specification but with fixed coefficients, and measures short-run pass-through as the coefficient on the lag of the first difference in the exchange rate. Finding a structural break in the inflation series in the first quarter of 1984, he estimates the model over 1974Q1–1984Q1 and 1984Q2–2003Q2, respectively. He shows that the coefficient on the lag of the first difference in the exchange rate is about 7 per cent in the first sample, but statistically not different from zero in the second. Bailliu and Fujii (2004) also consider a backward-looking Phillips curve specification, but within the context of a multi-country panel. In this case, short-run pass-through is defined as the coefficient on the contemporaneous change in the exchange rate. Finding breaks in the eighties and nineties in the inflation series of most of the countries considered, Bailliu and Fujii add two terms to their estimation equations: a dummy variable for each of the two decades, multiplied by the change in the exchange rate. They conclude that, while the coefficient on the eighties interaction term is not significant, average short-run pass-through across countries declines from 11 per cent in the seventies to somewhere in the vicinity of 5–6 per cent over the nineties.

The evidence reported so far sheds light on quantitative changes in pass-through, but does not offer any information on whether the duration of the effects of exchange rate shocks on inflation has also changed. Yet, this aspect of pass-through is also interesting to policy-makers, and, in particular, to inflation-targeting monetary authorities, who try to gauge the extent and timing of exchange rate shocks on inflation. One way of obtaining information on the duration is to use models where exchange rate movements are not exogenous to inflation; i.e., by using additional information from the covariance matrix of the system.

Indeed, evidence suggests that the variances of inflation and of exchange rates have changed over time, with possible implications for pass-through. Some of these implications may have resulted from shifts in monetary policy by way of a larger emphasis on inflation control.[2] For example, Monacelli (2004) uses a sticky-price model with incomplete pass-through and endogenous monetary policy to show that the choice of monetary policy has implications for short-run changes in relative prices and inflation variability, and thus changes in the degree of pass-through. Similarly, both Murchison (2004) and Bouakez and Rebei (2005) propose dynamic stochastic general-equilibrium (DSGE) models for Canada that are partly estimated and partly calibrated, and with various types of real and nominal rigidities. Both report that sufficient increases in policy aggressiveness can decrease pass-through.[3] Devereux, Engel, and Storgaard (2003), using an open-economy model of endogenous exchange rate pass-through, show that the relationship between exchange rate volatility and economic structure can be importantly affected by the extent of pass-through, and that the latter is related, among other things, to the relative stability of monetary policy between trading countries.

On the empirical side, Gagnon and Ihrig (2004) estimate a backward-looking Phillips curve equation for various countries over two different subsamples. In almost all cases, the coefficient on import price inflation, which is taken to be their measure of short-run pass-through, is found to be higher before the mid-eighties than after.[4] Furthermore, Gagnon and Ihrig find significant links between changes in pass-through coefficients and changes in first and second moments of inflation. In particular, the change in inflation variability is found to have a stronger impact on changes in pass-through.

These observations seem to suggest that, quite separately from interactions at the first-moment level, some dimension of the change in the nature of the relationship between inflation and exchange rate or import price changes may be due to the behaviour of their variances and covariances. We therefore propose a correlated and unrestricted VAR model and examine whether any aspect of pass-through is affected by changes in the covariance matrix.

The parsimonious VAR set-up is chosen because of the possibility of subsequently conducting estimations on subsamples of data. We consider pass-through based on import price inflation, similar to Gagnon and Ihrig (2004).[5] The system that we consider is given by:

$$\begin{aligned}
\pi_t &= \alpha_{10} + \alpha_{11}(L)\pi_t + \alpha_{12}(L)\Delta p_t^m + \phi_{11}(L)\Delta p_t^c + \phi_{12}(L)\Delta p_t^o + \epsilon_{1t}, \qquad (1)\\
\Delta p_t^m &= \alpha_{20} + \alpha_{21}(L)\pi_t + \alpha_{22}(L)\Delta p_t^m + \phi_{21}(L)\Delta p_t^c + \phi_{22}(L)\Delta p_t^o + \epsilon_{2t}, \qquad (2)
\end{aligned}$$

where inflation is denoted by π_t, the nominal exchange rate is e_t, foreign price is given by p_t^*, real oil prices are denoted as p_t^o, commodity prices are given by the index p_t^c, import price is defined as $p_t^m = (e_t + p_t^*)$, and the first-difference operator is Δ, so that $\Delta x_t = x_t - x_{t-1}$. The residuals of the two equations are assumed to be contemporaneously correlated, and the variance-covariance matrix is given by Σ.

[2]For early work on this topic, see Taylor (2000).

[3]Murchison considers pass-through within the context of a Phillips curve, and Bouakez and Rebei examine impulse responses of inflation to various structural shocks that affect the exchange rate.

[4]In the case of Canada, the long-run pass-through coefficient has a value of 0.3 before 1985, and 0.04 thereafter.

[5]Results using the first difference of the exchange rate instead of import price inflation yield quantitatively similar results and are available upon request.

3. A Test for Breaks in the Covariance Matrix

In this section, we propose and apply a new test to determine whether there are any breaks in the variance-covariance matrix, Σ, of the residuals of the VAR. Our test is an extension of the LR procedure proposed in Anderson (1971) that tests the equality of covariance matrices across subsamples. More precisely, the original Anderson test is applicable to a system of equations where the only regressor in every equation is a constant. For a known break point, Anderson proposes an LR test, and shows that the test statistic is distributed as a χ^2. We make two extensions to the test. First, we generalize the model by including covariates (i.e., regressors other than the constant) in the equations. In this case, and for a known break point, the LR statistic for testing breaks in the residual variance-covariance matrix is no longer distributed as a χ^2. We show how to obtain a cut-off point (or p-value) for the LR statistic under these conditions, and demonstrate that the cut-off point is exact when the regressors are strictly exogenous. Second, for the case where the break point is unknown, we show how to derive a sup-LR test, and how to obtain a simulation-based cut-off point (or minimum p-value) for this test statistic.

The following subsections provide the general outline of our test, and summarize the steps involved—as we apply them to our VAR model (1)-(2); formal theorems and proofs are given in the appendix. We also report the results of a small Monte Carlo (MC) experiment that documents the size and power of our test.

3.1 Test framework

Consider a break point T_B. Let T denote the size of the full sample, and let $T_{T_B,i}$, $i = 1, 2$, be the sizes of the first and second subsamples, corresponding to the periods before and after the break date, respectively. To test our model for a break-in-variance (or scale) that occurs at time T_B, obtain ordinary least squares (OLS) estimates of the residual covariance matrix over the full sample, $\widehat{\Sigma}$, and over the two subsamples, $\widehat{\Sigma}_{(T_B,1)}$ and $\widehat{\Sigma}_{(T_B,2)}$, respectively—the first subsample including the observations prior to date T_B, and the second including observations from date T_B to the sample endpoint. The tested null hypothesis is $H_0 : \Sigma_1 = \Sigma_2$ and the test statistic we consider is:

$$LR(T_B) = T \ln(\det(\widehat{\Sigma})) - \sum_{i=1}^{2} T_{T_B,i} \ln(\det(\widehat{\Sigma}_{(T_B,i)})).$$

In the appendix, we show that the null distribution of the test statistic is invariant to the regression coefficients and the error covariance. This allows us to derive an MC p-value (or a parametric bootstrap) by simulation. The procedure we apply can be summarized as follows.

(i) Calculate the test statistic from the observed data. In the process, save the constrained estimates (i.e., imposing stability) of the VAR coefficients and of the estimated residual covariance matrix.

(ii) Using the latter estimates, and drawing errors from the normal distribution, obtain N simulated samples from model (1)-(2); since the parameter estimates used to derive these samples were obtained while imposing stability, the N simulated samples satisfy, by construction, the null hypothesis under test. For each simulated sample, calculate the associated LR statistic.

(iii) Obtain an MC p-value based on the rank of the observed statistic relative to the simulated ones; the exact formulae are given in the appendix. The latter p-value is then referred to the desired (e.g., 5 per cent) significance level.

To implement the test with an unknown break point, consider a number of potential break points. Based on these, obtain a supremum statistic, sup LR, sweeping over all potential break dates (as in Andrews 1993; the date that yields the highest test statistic is the likeliest break date). In this case, the derivation of an MC p-value (or a parametric bootstrap) follows the same basic steps as when the break date is known; see the appendix for details.

In the appendix, we show that when the regressors are fixed, the obtained MC p-values are exact. For regular VAR models (i.e., for dynamic specifications), the test procedure can provide a good approximate level-correct p-value.[6]

3.2 A Monte Carlo exercise

We report a small-scale simulation study to document the size and power of our test in multivariate regressions. We consider two designs: (i) a model with dimensions close to our empirical VAR, having two equations, 60 observations, and 12 regressors, and (ii) a model with larger dimensions: five equations, 100 observations, and 12 regressors. We denote these two models MLR(2) and MLR(5), respectively. The regressors include an intercept and 11 variates drawn as standard normal. The regression coefficient is set to zero (because of location-scale invariance, there is no loss of generality).

To set up a simple design, we proceed as in Dufour and Khalaf (2002): under the null hypothesis, the errors are independently generated as $i.i.d.N(0, \Sigma)$ with $\Sigma = GG'$, and where the elements of G are drawn (once) from a normal distribution. Under the alternative, the errors in the first subsample are drawn (once) as $i.i.d.$ $N(0, \Sigma_1)$ with $\Sigma_1 = G_1 G_1'$, and where the elements of G_1 are drawn (once) from a normal distribution; the errors in the second subsample are independently drawn (once) as $i.i.d.$ $N(0, \Sigma_2)$ with $\Sigma_2 = g \times G_2 G_2'$, where the elements of G_2 are drawn (once) from a normal distribution (independently from G_1). The parameter g is a scale term that serves to assess the power of the test; i.e., as a response to varying scale deviations across the samples.

In all cases, breaks at the first third, midpoint, and last third of the sample are considered. The potential break dates (as in Dufour et al. 2004) sweep a window of 11 observations, centred at the break date $\pm$ up to five observations. The results (empirical rejections for a test with 100 MC replications and 1,000 simulations) are summarized in Table 1.

The first row of Table 1 shows that the test size is very close to 5 per cent. In addition, even with the small samples that we consider, the test displays very good power. In the case of the five-equation system, the power of the test is one no matter where the break occurs in the sample, and no matter how different the covariances are among the two subsamples. In

[6]Exact extensions in dynamic models (such as the ones proposed in Dufour and Kiviet 1996, 1998) are conceptually feasible, and are a worthy research objective beyond the scope of this paper.

Table 1: MC Experiment to Assess the Size and Power of the Test

		MLR(2)			MLR(5)		
Break window		$T/3$	$T/2$	$2T/3$	$T/3$	$T/2$	$2T/3$
$\Sigma_1 = \Sigma_2 = \Sigma$	$g = 1$	0.032	0.046	0.026	0.030	0.035	0.062
$\Sigma_1 \neq \Sigma_2$	$g = 1$	0.097	0.929	0.247	1.000	1.000	1.000
	$g = 10$	0.400	1.000	1.000	1.000	1.000	1.000
	$g = 20$	0.968	1.000	1.000	1.000	1.000	1.000

Notes: g is a design scale parameter that controls the extent to which Σ_1 is different from Σ_2. The entries under the headings $T/3$, $T/2$, and $2T/3$ are empirical rejections.

the case of the two-equation model (i.e., for MLR(2)), and for $g = 1$, the test performs best when the break occurs in the middle of the sample, and worst when the break is in the first third of the sample. However, when $g = 20$ (where Σ_1 is much more different than Σ_2), the power is one (or almost one) no matter where the break occurs.

3.3 Test application

To keep the model in equations (1) and (2) general yet parsimonious, we include one autoregressive lag for each endogenous variable, and the first and second lags of inflation for commodities and real oil prices, in addition to their contemporaneous values. Dummy variables—conformable with the periods before and after a potential break date—are also added. These are applied to all of the coefficients of the mean to account for any breaks in the first moments.

Casual inspection of the inflation series and evidence from regime-switching models of inflation suggest the existence of two episodes where inflation shifts to a lower-mean and lower-variance situation. The first break likely occurs between 1982–84, where inflation transits from a high-level and high-variance series to a moderate level and moderate variance. The second break likely occurs around the period 1990–91, after the adoption of inflation targeting by the Canadian monetary authority, when the level and variance of inflation decline even further. Since two distinct breaks may have occurred, and our test assumes the existence of one break, we divide our sample into two and apply the break test to each subsample separately. The break test is considered to be unknown over a given window of time. First, we test over the 1982–84 window, in a subsample that ends in 1989 (i.e., before the potential second break), and then we test over the 1990–91 window, in the subsample that starts in 1985 (i.e., after the first break has taken place).

Our data are at a quarterly frequency and at annualized rates, and extend from 1967Q1 to 2005Q2. Canadian (domestic) and U.S. (foreign) inflation are calculated using CPI prices. Our nominal exchange rate variable is the bilateral exchange rate between Canada and the United States (defined as the Canadian price of one U.S. dollar). The Bank of Canada commodity price index is used to construct the variable p_t^c, and real oil prices are measured by deflating the West Texas Intermediate oil price (in U.S. dollars) by the U.S. CPI.

Tables 2 and 3 report results of the break tests. The tables show that the LR value is

Table 2: Testing for Breaks Over 1982Q1–1984Q4

Break date	LR value
1982Q1	4.28
1982Q2	4.25
1982Q3	12.61
1982Q4	11.90
1983Q1	16.86
1983Q2	16.52
1983Q3	14.74
1983Q4	19.26
1984Q1	17.53
1984Q2	17.94
1984Q3	22.12
1984Q4	20.17

p-val for sup-LR test = 0.039
Likeliest break date: 1984Q3

Table 3: Testing for Breaks Over 1990Q1–1991Q1

Break date	LR value
1990Q1	23.57
1990Q2	25.21
1990Q3	25.22
1990Q4	26.80
1991Q1	28.42
1991Q2	5.69
1991Q3	6.24
1991Q4	7.19

p-val for sup-LR test = 0.026
Likeliest break date: 1991Q1

highest in 1984Q3, and that the Monte Carlo sup-LR test p-value associated with a break at this date is significant at the 5 per cent level (at 0.039). There is also evidence of a break in the second subsample. Over the tested window, the LR value is highest in 1991Q1, and the associated Monte Carlo sup-LR test p-value is 0.026. Tests applied to the 1980Q1–1982Q2 and 1991Q2–1992Q4 windows yield no evidence of breaks, even at the 10 per cent level.

Accordingly, two distinct subsamples are chosen on which to examine impulse responses: 1967Q1–1982Q2 and 1992Q2–2005Q2. The interim period is discarded, since it is too short a period on which to run meaningful estimations.

4. Estimation and Pass-Through

The VAR model is estimated over the two selected subsamples using ordinary least squares, and is orthogonalized using a Cholesky decomposition.[7] The results (Table 4) generally show some fairly high adjusted R-squares (0.78 for the inflation equation in the 1967Q1–1982Q2 subsample, and 0.60 for the equation for import price inflation in the 1992Q2–2005Q2 subsample), although the regression fit is less good for the other cases. Comparing the regression coefficients over the two subsamples, four interesting results emerge. First, the coefficient on the inflation autoregressive term decreases importantly (from 0.58 to 0.23), confirming the fact that inflation is much less persistent in the second subsample. On the other hand, the persistence of import price inflation does not change at all across the two subsamples. Second, changes in real oil prices have a bigger impact on both dependent variables in the 1992Q2–2005Q2 subsample than they do in the 1967Q1–1982Q2 subsample. Third, whereas commodity price changes affect CPI inflation more and import price inflation less in the earlier subsample, the reverse is true in the later subsample. Fourth, the estimated coefficient of Δp^m_{t-1} in the inflation equation (the short-run pass-through coefficient) is the same for both subsamples.

We find that the estimated variance-covariance matrix of the system changes importantly from one subsample to the other. The estimated error variances are 2.87 for inflation and 31.00 for import inflation in the 1967Q1–1982Q2 subsample. These change to 1.79 and 38.55, respectively, in the 1992Q2–2005Q2 subsample. More dramatic is the change in the covariance between these variables, with respect to both sign and magnitude, from a value of 2.31 to -0.69. The resulting correlation coefficient changes, from 0.24 over the earlier subsample to a value of -0.08 over the later subsample.

To determine what these changes in the covariance structure imply for the duration of pass-through, we calculate impulse-response functions over the two subsamples. Figures 1 and 2 show the responses of the dependent variables to a one-standard-deviation factorized shock to the innovation in each of the variables. Two standard-error (i.e., 90 per cent) Monte Carlo confidence bands are also calculated.[8] It can be seen that, although the initial quantitative impact of pass-through is the same for both subsamples, the duration of the impact has indeed changed over time. In other words, over both subsamples, 5 per cent of the innovation to import inflation gets transmitted, upon impact, to inflation, but the

[7]In each case, exogenous variables that are not significant at the 10 per cent level are omitted from the regressions. In addition, the usual diagnostic checks are applied, to ensure that there is no residual autocorrelation and that joint normality of residuals is not rejected.

[8]For the Monte Carlo exercise, 1,000 replications are used.

Table 4: VAR Estimation Results

Sample: 1967Q1 to 1982Q2

	π_t *equation*	Δp_t^m *equation*
Constant	1.50 (0.49)	1.41 (1.61)
π_{t-1}	0.58 (0.08)	-0.01 (0.27)
Δp_{t-1}^m	0.05 (0.04)	0.33 (0.12)
Δp_t^o	0.01 (0.003)	-0.02 (0.01)
Δp_t^c	0.14 (0.05)	0.56 (0.18)
Adjusted R^2	0.78	0.37

Sample: 1992Q2 to 2005Q2

	π_t *equation*	Δp_t^m *equation*
Constant	1.11 (0.29)	0.94 (1.35)
π_{t-1}	0.23 (0.10)	0.09 (0.46)
Δp_{t-1}^m	0.05 (0.03)	0.34 (0.12)
Δp_t^o	0.02 (0.004)	-0.08 (0.02)
Δp_{t-2}^o	0.01 (0.004)	-0.01 (0.02)
Δp_t^c	0.08 (0.04)	1.57 (0.20)
Δp_{t-1}^c	-0.09 (0.05)	-0.79 (0.24)
Adjusted R^2	0.31	0.60

Note: Standard errors are in parentheses.

duration of pass-through decreases over time: from approximately two years in the earlier subsample to about one year in the later subsample.[9]

We obtain the well-known result that a shock to inflation dies out much quicker over the 1991–2005 period compared with the 1967–1982 period. We also find, however, that a shock to import inflation dies out at about the same rate over the two subsamples.

In sum, our analysis reveals three important changes in the Canadian economic environment. First, inflation persistence is very different in the later subsample compared with the earlier subsample. Second, the correlation between inflation and import inflation innovations has changed importantly over time, decreasing from a fairly strong correlation to almost zero.[10] Third, although the quantitative short-run impact of pass-through is no different in the 1991–2005 subsample than over the 1967–1982 subsample, the duration of pass-through has decreased by almost half. The latter result differs from findings within single-equation contexts, where it is concluded that short-run pass-through has declined over time.

Our results lend some support to Devereux, Engel, and Storgaard (2003), who suggest that, under some conditions, a more stable inflation environment renders the exchange rate more volatile, and that, in turn, this will cause more of a "disconnect" between the exchange rate and other macroeconomic variables (in our case, inflation).

5. Conclusion

In this paper, we have developed a new test to establish the presence of structural breaks in the error covariance matrix of a multivariate system of equations. The method extends the breaks-in-variance/covariance test proposed in Anderson (1971) by accounting for regression covariates and an unknown break date. Moreover, we have shown that the test is exact for fixed regressors.

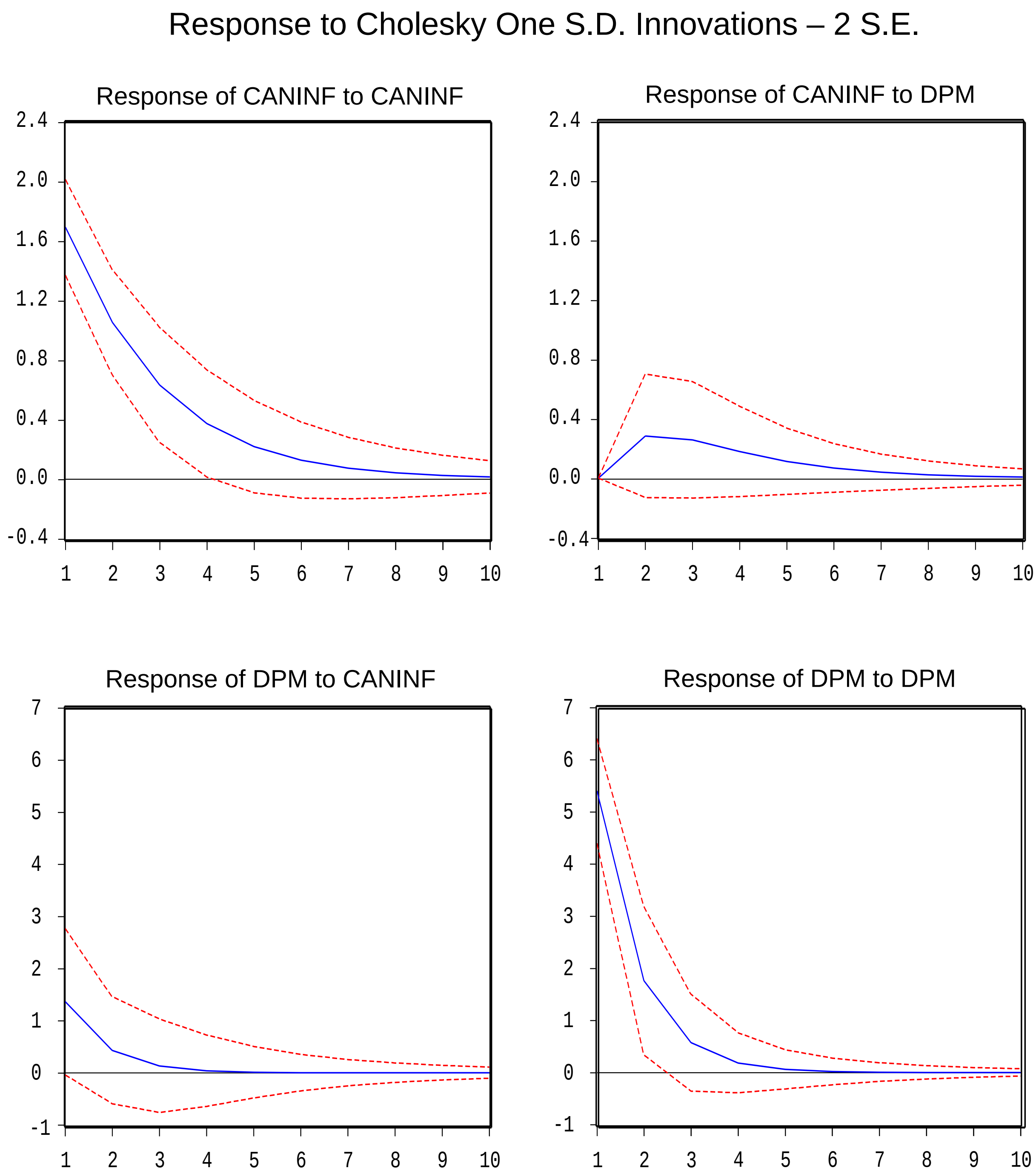
The test was applied to Canadian data to examine changes in the quantity and duration of pass-through. More precisely, we studied the implications of structural breaks in the variance-covariance matrix of inflation and import prices for changes in pass-through. We defined pass-through within a correlated vector autoregression framework as the response of domestic inflation to an impulse in import price inflation. This approach allowed us to examine changes in both the amount and the duration of pass-through.

We found evidence of breaks in the covariance matrix in 1984Q3 and in 1991Q1. Estimating the VAR and examining impulse responses over the relevant subsamples, we found that, while the initial impact of pass-through is quantitatively similar across the two samples, this impact dissipates twice as fast in the later subsample. In other words, the duration of pass-through has declined from about a two-year period to a one-year period. We also documented important changes over time in the estimated correlation between domestic and import inflation, both in terms of magnitude and sign.

[9] Although the 90 per cent confidence bands around our estimated impulse responses are relatively wide, the 95 per cent bands likely do not include zero.

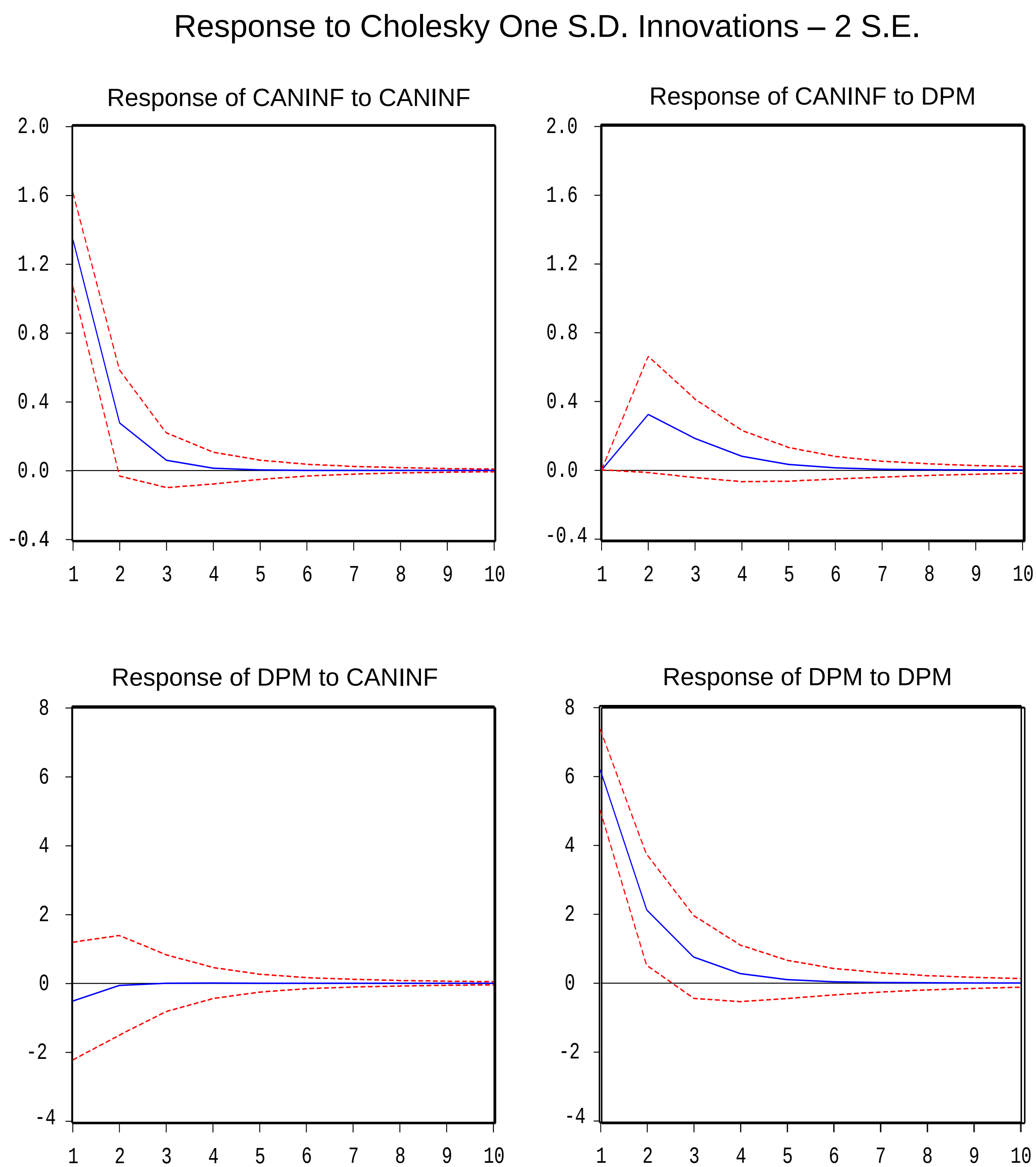
[10] Indeed, the -0.08 correlation is not statistically different from zero according to a Breusch-Pagan test.

Figure 1: Impulse Responses, 1967Q1–1982Q2



Note: In Figures 1 and 2, CANINF is Canadian CPI inflation and DPM is import inflation.

Figure 2: Impulse Responses, 1992Q2–2005Q2



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Appendix
The Break Test: Derivation and Finite-Sample Theory

Consider the multivariate linear regression (MLR) model:

$$Y = XB + U, \tag{3}$$

where $Y = [Y_1, \ ... \ , \ Y_n]$ is a $T \times n$ matrix of observations on n dependent variables, X is a $T \times k$ full-column rank matrix of fixed regressors, and $U = [U_1, \ \ldots \ , \ U_n] = [V_1, \ \ldots \ , \ V_T]'$ is the $T \times n$ matrix of error terms. Our main statistical results require that the rows of U (i.e., the vectors $V_t, \ t = 1, \ \ldots \ , \ T$) satisfy the following:

$$V_t = JW_t \ , \ t = 1, \ \ldots \ , \ T \, , \tag{4}$$

where J is an unknown, non-singular matrix and the distribution of the vector $w = vec(W_1, \ \ldots \ , \ W_T)$ (i) is fully specified (i.e., does not depend on any unknown parameter), or (ii) is specified up to an unknown nuisance parameter. Let

$$\Sigma = JJ'.$$

In particular, assumption (4) is satisfied when

$$W_t \overset{i.i.d.}{\sim} N(0, I_n) \, , \quad t = 1, \ \ldots \ , \ T \, , \tag{5}$$

in which case Σ gives the variance/covariance of V_t. In matrix form, and setting $W = [W_1, \ \ldots \ , \ W_T]'$, (4) may be rewritten as $W = U(J^{-1})'$; i.e., $U = WJ'$. We report general distributional results that require *no further regularity assumptions* on the error terms.

In this set-up, the break test procedure is as follows. Partition the observed X and Y matrix conforming to the break date into $X_{(T_B,1)}$ and $Y_{(T_B,1)}$, and $X_{(T_B,2)}$ and $Y_{(T_B,2)}$. In other words, $X_{(T_B,1)}$ and $Y_{(T_B,1)}$ include the observations on the regressors and dependent variables prior to date T_B, and $X_{(T_B,2)}$ and $Y_{(T_B,2)}$ include the observation from date T_B to the sample endpoint. For further reference, let $T_{T_B,1}$ and $T_{T_B,2} = T - T_{T_B,1}$ denote the size of each subsample, respectively.

As will become clear, this test is applicable when $T_{T_B,1}$ and $T_{T_B,2}$ both exceed k to allow running the MLR over both subsamples. In this context, the quasi-likelihood-ratio (QLR) -based procedure of Anderson leads to the following statistic:

$$LR(T_B) = T \ln(\det(\widehat{\Sigma})) - \sum_{i=1}^{2} T_{T_B,i} \ln(\det(\widehat{\Sigma}_{T_B,i})), \tag{6}$$

where

$$\widehat{\Sigma} = \widehat{U}'\widehat{U}/T, \quad \widehat{U} = Y - X\widehat{B}, \quad \widehat{B} = (X'X)^{-1}X'Y$$

$$\begin{aligned}
\widehat{\Sigma}_{(T_B,i)} &= \widehat{U}'_{(T_B,i)}\widehat{U}_{(T_B,i)}/T_{T_B,i}, \\
\widehat{U}_{(T_B,i)} &= Y_{(T_B,i)} - X_{(T_B,i)}\widehat{B}_{(T_B,i)}, \\
\widehat{B}_{(T_B,i)} &= (X'_{(T_B,i)}X_{(T_B,i)})^{-1}X'_{(T_B,i)}Y_{(T_B,i)} \\
i &= 1, 2.
\end{aligned}$$

In location-scale models (with no covariates), an asymptotic p-value for the statistic may be obtained if T_B is known, using the approximation for its null distribution due to Anderson (1971):

$$LR(T_B) \sim \chi^2\left((n\,(n+1)\,/2) + n\right). \tag{7}$$

Let us first examine the null distribution of $LR(T_B)$ when T_B is known.

Under (4) and (3), the statistic defined by (6) is distributed like

$$LR(T_B) = T\ln(\det(\widehat{S})) - \sum_{i=1}^{2} T_{T_B,i}\ln(\det(\widehat{S}_{(T_B,i)})), \tag{8}$$

where

$$\begin{aligned}
\widehat{S} &= W'MW/T \\
\widehat{S}_{(T_B,i)} &= W'_{(T_B,i)}M_{(T_B,i)}W_{(T_B,i)}/T_{T_B,i}
\end{aligned}$$

$$\begin{aligned}
M &= I - X(X'X)^{-1}X, \\
M_{(T_B,i)} &= I - X_{(T_B,i)}(X'_{(T_B,i)}X_{(T_B,i)})^{-1}X_{(T_B,i)},
\end{aligned}$$

and $W_{(T_B,i)}$ are obtained by partitioning the matrix W in (4) conforming to the subsamples T_B, i.

The latter distributional result is obtained as follows. Under the null hypothesis,

$$\begin{aligned}
T\ln(\det(\widehat{S})) &= T\ln(\det(U'MU)) \\
&= T\ln(\det((J)(J^{-1})U'MU(J^{-1})'(J)')) \\
&= T\ln(\det((J)W'MW(J)')) \\
&= T\ln\left[\det(J)\det(W'MW)\det(J')\right] \\
&= T\left[\ln(\det(J)) + \ln(\det(W'MW)) + \ln(\det(J'))\right].
\end{aligned}$$

Similarly,

$$\begin{aligned}
T_{T_B,i}\ln(\det(\widehat{S}_{(T_B,i)})) &= T_{T_B,i}\ln(\det(U'_{(T_B,i)}M_{(T_B,i)}U_{(T_B,i)})) \\
&= T_{T_B,i}\left[\ln(\det(J)) + \ln(\det(W'_{(T_B,i)}M_{(T_B,i)}W_{(T_B,i)})) + \ln(\det(J'))\right],
\end{aligned}$$

so that

$$\begin{aligned}
T_{T_B,1}\ln(\det(\widehat{S}_{(T_B,1)})) + T_{T_B,2}\ln(\det(\widehat{S}_{(T_B,2)})) &= T_{T_B,1}\left[\ln(\det(W'_{(T_B,1)}M_{(T_B,1)}W_{(T_B,1)}))\right] \\
&\quad + T_{T_B,2}\left[\ln(\det(W'_{(T_B,2)}M_{(T_B,2)}W_{(T_B,2)}))\right] \\
&\quad + (T_{T_B,1} + T_{T_B,2})\ln(\det(J)) \\
&\quad + (T_{T_B,1} + T_{T_B,2})\ln(\det(J')).
\end{aligned}$$

On recalling that $T_{T_B,1} + T_{T_B,2} = T$, we see that $\ln(\det(J))$ and $\ln(\det(J'))$ are evacuated by subtraction from the expression for the test statistic, to yield the pivotal quantity (8).

This shows that the null distribution of $LR(T_B)$ depends on neither B nor J (and thus not on Σ), and may easily be simulated if draws from the distribution of $W_1, \ldots, W_T$ are available. Thus, a Monte Carlo exact test procedure may be easily applied using the above theorem and the procedures from Dufour (2006). The simulation-based algorithm that allows us to obtain an MC size-correct exact p-value can be summarized as follows. Let $LR^0(T_B)$ denote the observed value of test statistic, calculated from the observed data set. Draw $W^j = [W_1^j, \ldots, W_T^j]$, $j = 1, \ldots, N$, as in (4), and compute the pivotal quantity (8):

$$LR^j(T_B) = T\ln(\det(\widehat{S}^j)) - \sum_{i=1}^{2} T_{T_B,i}\ln(\det(\widehat{S}^j_{(T_B,i)})),$$

where

$$\begin{aligned}\widehat{S} &= W^{j\prime}MW^j/T,\\ \widehat{S}_{(T_B,i)} &= W^{j\prime}_{(T_B,i)}M_{(T_B,i)}W^j_{(T_B,i)}/T_{T_B,i}.\end{aligned}$$

This leads to N simulated values of the test statistic $LR^j(T_B)$, $j = 1, \ldots, N$. Under the null hypothesis, $LR^0(T_B)$, $LR^1(T_B), \ldots, LR^N(T_B)$ are exchangeable. Given the latter series, compute

$$\hat{p}_N\left(LR(T_B)\right) = \frac{N\hat{G}_N\left(LR^0(T_B)\right) + 1}{N+1}, \tag{9}$$

where $N\hat{G}_N\left(LR^0(T_B)\right)$ is the number of simulated values greater than or equal to $LR^0(T_B)$. The MC critical region is: $\hat{p}_N\left(LR(T_B)\right) \leq \alpha$. Since the distribution of the statistic is continuous, then

$$\mathsf{P}\left[\hat{p}_N\left(LR(T_B)\right) \leq \alpha\right] = \alpha \tag{10}$$

under the null hypothesis when $\alpha(N+1)$ is an integer; see Dufour (2006). This formally demonstrates that the test so described will be size correct.

To obtain a test for an unknown break date, it is usual practice to run the latter test over a window of possible break dates; denote this subset of potential break dates J_B. Then, a combined statistic can be derived as in Dufour et al. (2004) as follows:

$$LR_{\sup} = \max_{T_B \in J_B}\left\{LR\left(T_B\right)\right\}, \tag{11}$$

where $LR\left(T_B\right)$ is the statistic defined in (6). Under the null hypothesis, the statistics corresponding to each T_B are jointly pivotal, so the above-defined MC test procedure can be applied to the sup-test and will also yield an exact test procedure. For completion, we summarize the simulation-based algorithm associated with the joint test. Let $LR^0_{\sup}$ denote the observed value of test statistic, calculated from the observed data set. Draw $W^j = [W_1^j, \ldots, W_T^j]$, $j = 1, \ldots, N$, as in (4), and for each draw compute the pivotal quantity:

$$LR^j_{\sup} = \max_{T_B \in J_B}\left\{T\ln(\det(\widehat{S}^j)) - \sum_{i=1}^{2} T_{T_B,i}\ln(\det(\widehat{S}^j_{(T_B,i)}))\right\},$$

where

$$\begin{aligned}\widehat{S} &= W^{j\prime}MW^j/T,\\ \widehat{S}_{(T_B,i)} &= W^{j\prime}_{(T_B,i)}M_{(T_B,i)}W^j_{(T_B,i)}/T_{T_B,i}.\end{aligned}$$

This leads to N simulated values of the test statistic $LR_{\sup}^{j}$, $j = 1\ ,\ \ldots\ ,\ N$. Given the latter series, the MC p-value of the sup-test is

$$\hat{p}_N\left(LR_{\sup}\right) = \frac{N\hat{G}_N\left(LR_{\sup}^{0}\right) + 1}{N + 1}, \tag{12}$$

where $N\hat{G}_N\left(LR_{\sup}^{0}\right)$ is the number of simulated values greater than or equal to $LR^{0}(T_B)$. A test based on the latter p-value is exact because, under the null hypothesis, the combined statistics are jointly pivotal, and so $LR_{\sup}^{0}$, $LR_{\sup}^{1}$, $\ldots$, $LR_{\sup}^{N}$ are exchangeable. Refer to Dufour et al. (2004) for related results on sup-type break tests in univariate models (they do not consider the multivariate case).

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