August 1996

Does Inflation Uncertainty Vary with the Level of Inflation?

by

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Acknowledgments

We wish to acknowledge helpful comments from Robert Amano, Walter Engert, Irene Ip, John Kuszczak, David Longworth, Tiff Macklem, Brian O'Reilly, David Rose and Gerald Stuber. The views expressed in this paper are those of the authors and should not be attributed to the Bank of Canada.

ISSN 1192-5434

ISBN 0-662-24986-0

ABSTRACT

The purpose of this study is to test the hypothesis that inflation uncertainty increases at higher levels of inflation. Our analysis is based on the generalized autoregressive conditional heteroscedasticity (GARCH) class of models, which allow the conditional variance of the error term to be time-varying. Since this variance is a proxy for inflation uncertainty, a positive relationship between the conditional variance and inflation would be interpreted as evidence that inflation uncertainty increases with the level of inflation.

We apply GARCH techniques to two models of the inflation process in Canada: a simple autoregressive model and a reduced-form Phillips-curve model. Our findings concerning the link between inflation and its uncertainty are somewhat model-dependent. In the autoregressive case, there is a significant positive relationship between inflation and inflation uncertainty. The estimated relationship is weaker in the reduced-form model, and is not significant at the standard 5 per cent level of significance.

The difference in the strength of the relationship in the autoregressive and reduced-form models makes it difficult to draw firm conclusions about the relationship between inflation and inflation uncertainty. However, given the extreme information assumptions underlying each model, the true relationship may lie somewhere between the two sets of results. By excluding all explanatory variables other than past inflation, the simple autoregressive approach undoubtedly ignores some information that agents would have used to forecast inflation. Accordingly, the autoregressive model will tend to overstate the actual uncertainty faced by agents. Conversely, the reduced-form model may understate the uncertainty that existed, since it implicitly assumes that agents had more information on the structure of the economy than was actually available at each point in time. Future research, covering more low-inflation years and based on alternative models of inflation that explicitly incorporate policy-regime uncertainty, might clarify whether inflation uncertainty increases with the level of inflation.

RÉSUMÉ

L’objectif des auteurs est de tester l’hypothèse que l’incertitude entourant l’inflation s’accroît lorsque le taux d’inflation augmente. Ils fondent leur analyse sur l’utilisation de modèles autorégressifs conditionnellement hétéroscédastiques généralisés (GARCH), lesquels permettent à la variance conditionnelle du terme d’erreur de fluctuer dans le temps. Comme cette variance constitue une approximation de l’incertitude entourant l’inflation, la détection d’une relation positive entre elle et l’inflation viendrait étayer l’hypothèse examinée.

Les auteurs appliquent des techniques GARCH à deux modèles formalisant le processus d’inflation au Canada : un modèle autorégressif simple et un modèle de forme réduite intégrant une courbe de Phillips. Les résultats qu’ils obtiennent relativement au lien entre le niveau de l’inflation et l’incertitude entourant celle-ci dépendent dans une certaine mesure du modèle utilisé. Le modèle autorégressif fait ressortir une relation positive significative entre ces deux variables. La relation estimée est plus faible dans le modèle de forme réduite, et elle n’est pas significative au seuil habituel de 5 %.

Étant donné cet écart entre les résultats obtenus à l’aide du modèle autorégressif et ceux établis à l’aide du modèle de forme réduite, il est difficile de tirer des conclusions fermes sur la relation qui existe entre le niveau de l’inflation et l’incertitude entourant celle-ci. Mais puisque les deux modèles utilisés se fondent sur des hypothèses extrêmes concernant l’information dont les agents économiques disposent, on peut penser que la relation réelle pourrait se situer quelque part entre les deux séries de résultats. En excluant toutes les variables explicatives autres que l’évolution passée de l’inflation, le modèle autorégressif simple omet sans aucun doute de tenir compte de certains renseignements que les agents auraient utilisés pour prévoir le comportement de l’inflation. Par conséquent, il tend à surestimer l’incertitude à laquelle les agents économiques font face. Il se peut inversement que le modèle de forme réduite sous-estime celle-ci, puisqu’il suppose implicitement que les agents possèdent systématiquement plus de renseignements sur la structure de l’économie que cela n’est en fait le cas. De futures recherches, portant sur une plus longue période de faible inflation et se fondant sur d’autres modélisations du processus d’inflation qui tiendraient compte explicitement de l’incertitude liée au régime de politique monétaire, pourraient permettre d’établir si l’incertitude entourant l’inflation s’accentue avec le taux d’inflation.

Contents

1. Introduction

Inflation uncertainty is often cited as a major source of the costs of inflation. Uncertainty about future levels of inflation will distort saving and investment decisions since it causes the real value of future nominal payments to be unknown. These distortions are believed to have adverse effects on the efficiency of resource allocation and the level of real activity (see Fischer 1981; Golob 1993; Holland 1993b).

The random nature of shocks and imperfect knowledge of the structure of the economy means that some inflation uncertainty will exist under any policy regime. Although uncertainty cannot be eliminated, it may be that inflation uncertainty (and therefore its costs) could be minimized by adopting a particular policy regime. In particular, since some theoretical models predict that inflation uncertainty increases with the level of inflation, the costs of inflation uncertainty might be minimized by pursuing a policy of price stability. This conclusion has led to numerous empirical studies since the 1970s on the link between inflation and inflation uncertainty and on the real effects of uncertainty.

The purpose of this study is to develop a benchmark measure of inflation uncertainty in Canada and to examine whether this measure varies systematically with the level of inflation. Our analysis is based on the generalized autoregressive conditional heteroscedasticity (GARCH) class of models, which allow the conditional variance of the error term to be time-varying. This variance can be used as a proxy for inflation uncertainty. Thus, a positive relationship between the conditional variance and inflation would be interpreted as evidence that inflation uncertainty increases with the level of inflation.

We apply GARCH techniques to two models of the inflation process: a simple autoregressive model and a reduced-form Phillips-curve model. Our findings concerning the link between the level of inflation and its uncertainty are somewhat model-dependent. In the autoregressive case, inflation is found to have a strong positive relationship with inflation uncertainty. A significant positive relationship with the level of inflation is not found at the standard 5 per cent level of significance in the Phillips curve case, although there is some evidence from a non-linear specification that uncertainty increases when inflation rises above some moderate level. Future research covering more low-inflation years would help to clarify the relationship closer to price stability.

The organization of the paper is as follows. Section 2 outlines theoretical arguments why a positive relationship may exist between inflation and inflation uncertainty. This theoretical perspective is used in Section 3 to evaluate two alternative approaches to testing for time-varying uncertainty, namely surveys of forecasters and econometric models. After discussing the unit-root and cross-correlation properties of the data in Section 4, the conditional mean of the inflation process is specified in Section 5 as either an autoregressive or a reduced-form Phillips-curve model. The remaining sections report a variety of tests for time-varying inflation uncertainty. In Section 6, we report on “pre-tests” that examine whether the residuals from ordinary least squares estimation of the conditional mean equation have a time-varying conditional variance. In Section 7, models are estimated with GARCH error processes. Direct tests for a positive relationship between inflation and inflation uncertainty are undertaken by including inflation variables in the GARCH process. Section 8 reports evidence on multi-period inflation uncertainty by reporting the historical conditional variance of four-quarter-ahead inflation. Concluding comments are presented in Section 9.

2. Sources of inflation uncertainty

Evans and Wachtel (1993) suggest that the sources of inflation uncertainty can be decomposed into two broad categories: “regime uncertainty” and “certainty equivalence.” With respect to the first category, future inflation may be uncertain because agents are unsure about the characteristics of the current policy regime.[1] However, even if the current policy regime were known in each period (certainty-equivalence), there would still be uncertainty about the structure of the inflation process within each regime.

This decomposition has several interesting implications. For a given country, inflation uncertainty will change over time as agents use new information to update their perceptions of structural parameters and the current policy regime. This suggests that the level of uncertainty during a transition period to price stability will differ from the uncertainty that would prevail once the price stability regime was fully recognized by agents. Another implication of the decomposition is that international differences in average levels of uncertainty could reflect differences in monetary policy regimes.

A number of theoretical models predict that inflation uncertainty will increase at higher rates of inflation. Monetary policy often plays a prominent role in these models. Ball’s (1992) analysis focuses on uncertainty about the monetary policy regime (Okun (1971) and Friedman (1977) expressed similar ideas). In Ball’s model, if there is currently low inflation, agents believe that the monetary authorities will seek to maintain the low inflation, so inflation uncertainty will be low. However, if an unexpected shock raises the current rate of inflation, there is uncertainty about whether the authorities are willing to accept the temporary reduction in output that would accompany a disinflationary policy. This uncertainty concerning future monetary policy causes inflation uncertainty to increase at higher rates of inflation.

Holland (1993a) provided an alternative explanation for a positive relationship between inflation and inflation uncertainty. Whereas Ball considered the effect of regime uncertainty, Holland considered a case in which agents are unsure about the price-level

1. Regime uncertainty can also include uncertainty about the *future* policy regime if there is a possibility that the regime will change.

effects of a given change in the quantity of money.[2] One consequence of this parameter uncertainty is that inflation uncertainty increases at higher rates of expected inflation.

Many empirical studies have focussed on a short-run measure of inflation uncertainty (typically, the one-period ahead measure). However, Ball and Cecchetti (1990) and Evans (1991) have noted that the level of inflation may have different effects on short-run and long-run uncertainty. This idea can be demonstrated most readily by considering the case of a credible price-stability regime. Since monetary policy affects prices with a long lag, current shocks to inflation cannot be reversed by policy in the short run. However, monetary policy would be able to offset the inflationary shock over a longer time horizon. Thus, movement toward a price-stability regime might lead to a greater decline in long-run uncertainty than in one-quarter uncertainty. This is a crucial distinction if most of the real distortions from inflation uncertainty are caused by uncertainty over the longer-run rate of inflation.

These theoretical observations provide some guidance for evaluating the specification and results of empirical models of inflation uncertainty. First, empirical measures of inflation uncertainty may be misleading if the econometric specification does not adequately represent the current policy regime. In addition, when considering the relationship between inflation and its uncertainty, measures of both short-run and long-run uncertainty should be considered.

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2. Uncertainty about the aggregate price effects of changes in the stock of money arise because the length of contracts and the degree of indexation change over time. Inflation uncertainty in the Holland model also depends on the variance of monetary and non-monetary shocks.

3. Inflation and inflation uncertainty: Empirical models

Inflation uncertainty is difficult to measure since it is not directly observed. One strategy in the literature of the 1970s was to define inflation uncertainty as simply the variance of observed inflation. An obvious criticism of this approach is that an increase in the variance of inflation does not imply a corresponding rise in inflation uncertainty if available information allows agents to predict some of the increased volatility. More recent empirical studies have measured inflation uncertainty using proxies obtained from either surveys of forecasters or econometric models of inflation. This literature is discussed below.[3]

3.1 Survey-based proxies for inflation uncertainty

Some studies use proxies for inflation uncertainty constructed from surveys of expectations such as the Livingston survey in the United States. The Livingston survey records the expected rate of inflation for approximately 50 forecasters. Given these point estimates, inflation uncertainty can be proxied by the variance of inflation forecasts across individual forecasters. As noted by Zarnowitz and Lambros (1987), the Livingston survey provides a measure of the heterogeneity of expectations *across* individuals, but it cannot measure the inflation uncertainty of a typical individual since only point estimates of inflation are collected from each forecaster. Using data from the ASA-NBER Survey of Professional Forecasters, they showed that the Livingston measure of uncertainty tends to understate the uncertainty of individual forecasters.[4] Nonetheless, they found that both measures exhibit a positive correlation between U.S. inflation and inflation uncertainty.

Previous work at the Bank has studied the relationship between inflation and inflation uncertainty in Canada from 1975 to 1994 using forecasts of CPI inflation from the Conference Board’s survey of forecasters. These data are comparable to the Livingston data, as they record individual forecasters’ point estimates of future inflation. Following Engle (1983), three alternative proxies for inflation uncertainty were considered: the variance of expectations, the absolute forecast error, and the sum of the variance and

3. See Golob (1993) for a more extensive summary of the empirical literature.

4. The ASA-NBER survey asks each participant to assign probabilities to alternative inflation outcomes.

forecast errors. A positive relationship was found between the variance of (one-year) forecasts and the level of inflation. Forecast errors also tended to be greater in higher inflation periods, although the errors were related primarily to the *change* in inflation rather than the level. These conclusions should be regarded as tentative given the relatively short sample period.

3.2 Econometric measures of inflation uncertainty

Other studies have used the conditional forecast-error variance as a measure of inflation uncertainty. This branch of the literature can be subdivided according to whether the parameters in the inflation equation are assumed to be constant, as in the GARCH class of models, or time-varying.

Fixed-parameter GARCH models

Engle’s (1982) autoregressive conditional heteroscedasticity (ARCH) model uses a conventional inflation equation with fixed parameters but allows the conditional forecast-error variance of inflation to vary over time. Therefore, if we take this variance as a proxy for inflation uncertainty, the ARCH technique models inflation uncertainty as a time-varying process.

The ARCH model specifies the conditional mean of inflation, $\Pi_t | \Psi_{t-1}$, as a function of a vector of explanatory variables, X_{t-1}, while the conditional error variance, h_t, is a function of lagged values of the squared forecast errors; specifically,

$$\Pi_t | \Psi_{t-1} \sim N(\delta X_{t-1}, h_t) \tag{3.1}$$

$$E_{t-1}\varepsilon_t^2 = h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \qquad \alpha_0 > 0, \alpha_i \geq 0 \qquad \text{i = 1,..., q} \tag{3.2}$$

$$\varepsilon_t = \Pi_t - \delta X_{t-1} \tag{3.3}$$

where Ψ_{t-1} is the information set at time *t*-1, ε_t is the one-period forecast error, E_{t-1} is the expectation operator conditional on the information set at time *t*-1, and the vector δ and the α's are parameters which require estimation.

Equation (3.2) simplifies to the standard econometric specification with a constant conditional error variance if $\alpha_1 = \alpha_2 = \ldots = \alpha_q = 0$. Restricting the sum of the α_i (i = 1,..., q) parameters in equation (3.2) to be less than unity is a sufficient condition for a covariance-stationary ARCH process. Non-negativity of the individual α parameters is sufficient (but not necessary) to ensure that the conditional variance does not become negative.

It is evident from equation (3.2) that the conditional variance of inflation will increase if periods with large forecast errors are grouped together. Therefore, if the absolute size of forecast errors tends to increase with the rate of inflation, there will be a positive relationship between inflation and inflation uncertainty.[5]

Empirical applications of the ARCH model often specify long lag processes for the squared residuals, which suggests that shocks have persistent effects on inflation uncertainty. Bollerslev (1986) proposed an alternative approach to modelling persistence. In his GARCH model, the conditional variance is a function of lagged values of both the one-period forecast error and the conditional variance. A linear GARCH(p,q) process is

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{3.4}$$

$$\alpha_0 > 0,\ \alpha_i \geq 0 \qquad i = 1,\ldots,q$$

$$\beta_j \geq 0 \qquad j = 1,\ldots,p$$

The effect of an inflation shock on uncertainty declines geometrically over time through the lagged conditional variance term in equation (3.4). Empirical applications

5. A more direct test of the inflation/inflation uncertainty hypothesis would be to include lagged inflation directly in the specification of the conditional variance. This idea is discussed in Section 7.2.

typically have found that short lags provide adequate representations of the GARCH process. Thus, relative to the ARCH model, the GARCH specification often provides a more parsimonious way of modelling the persistence in uncertainty.

Table 1 summarizes results from some previous studies that examined inflation uncertainty with ARCH or GARCH models. These studies conclude that (short-run) inflation uncertainty is time-varying. However, in most cases there does not appear to be a systematic relationship between uncertainty and the level of inflation.[6] Engle's (1983) study is representative of this group. He found that one-quarter inflation uncertainty remained quite high in the United States during the late 1940s and early 1950s, declined to a lower level in the late 1950s and 1960s, and then rose slightly in the 1970s.[7] He argued that inflation uncertainty did not increase significantly in the 1970s because the rise in inflation was gradual and, therefore, predictable based on available information. In his view, the experience of the 1970s shows that high levels of inflation do not necessarily lead to high inflation uncertainty. Rather, uncertainty is likely to be greatest in periods such as the late 1940s and early 1950s, when sharp fluctuations in actual inflation created uncertainty about future inflation.

More recent studies have examined whether these results are robust to more general specifications of the conditional variance and the conditional mean. A common theme in these studies is that the traditional GARCH specification pays insufficient attention to structural change and agents' perception of the policy regime. These issues are discussed below.

6. An exception to this pattern is Golob (1994), who finds a significant positive relationship between inflation and inflation uncertainty when allowance is made for a time trend. He does not provide an explanation for the role of the time trend.

7. Contrary to these patterns based on the conditional variance of the error term, the variance of actual inflation increased considerably in the 1970s.

TABLE 1. Models with fixed parameters - ARCH and GARCH[a]

Author(s)	Price index	Model and variables in conditional mean	Results
Engle (1982)	U.K. CPI 1958Q2 - 1977Q2	ARCH	Significant ARCH effects.
		lagged inflation; real wages	Inflation uncertainty was substantially higher in 1974-77 than in the late 1960s.
Engle (1983)	U.S. CPI, GNP deflator, and producer price index	ARCH	Significant ARCH effects.
	1947Q4 - 1979Q4	lagged inflation; lagged growth of nominal wages, M1 and import prices; time trend	Inflation uncertainty was slightly greater in the high-inflation 1970s than in the low-inflation 1960s; however, uncertainty in both these periods was well below the levels in the late 1940s and early 1950s.
Engle and Kraft (1983)	U.S. GNP deflator	ARCH	Significant ARCH effects.
	1948Q2 - 1980Q3	lagged inflation	Similar results to Engle (1983) for one-quarter and four-quarter forecast horizons.
Bollerslev (1986)	U.S. GNP deflator	GARCH	Significant GARCH effects.
	1948Q2 - 1983Q4	lagged inflation	Conclusions similar to Engle (1983).
Golob (1994)	U.S. GNP deflator	GARCH	Inflation uncertainty is positively related to inflation after allowing for a time trend.
	1957Q1 - 1993Q4	lagged inflation; time trend	

a. Inflation uncertainty is measured by the conditional variance of the one-quarter-ahead forecast unless otherwise noted.

Fixed-parameter models with asymmetric uncertainty

As shown in equations (3.2) and (3.4), the ARCH and GARCH models assume that positive and negative innovations of equal magnitude will raise the conditional variance by the same amount. However, Brunner and Hess (1993) and Joyce (1995) contend that a positive inflation shock will create more uncertainty about future monetary policy (and therefore future inflation) than a negative shock of equal size. If their view is correct, the symmetric ARCH and GARCH models may provide misleading estimates of inflation uncertainty.[8]

8. Engle and Ng (1993) have developed tests for determining whether the symmetry restriction in the GARCH model is accepted by the data.

Several extensions of the GARCH model allow asymmetries in the conditional variance while maintaining fixed parameters in the inflation equation. In the asymmetric GARCH (AGARCH) model of Engle (1990), a negative shock raises uncertainty by a smaller amount than a positive shock of equal size if the parameter γ_1 is positive in equation (3.5). An AGARCH (1,1) process is:

$$h_t = \alpha_0 + \alpha_1 (\varepsilon_{t-1} + \gamma_1)^2 + \beta_1 h_{t-1}. \tag{3.5}$$

The AGARCH model nests the symmetric GARCH specification if $\gamma_1 = 0$.

Another asymmetric specification is the threshold GARCH (TGARCH) model, which adds a dummy variable to the GARCH process. Negative shocks have a smaller effect on uncertainty if $\gamma_2 < 0$.

$$h_t = \alpha_0 + \alpha_1 \varepsilon^2_{t-1} + \gamma_2 D \varepsilon^2_{t-1} + \beta_1 h_{t-1} \tag{3.6}$$

where D = 0 if $\varepsilon_{t-1} \geq 0$ and D = 1 if $\varepsilon_{t-1} < 0$.

Figure 1 illustrates how inflation uncertainty is affected by forecast errors in the asymmetric models.[9] If positive shocks create more uncertainty than negative shocks of equal magnitude, this relationship is centered about $-\gamma_1$ in the AGARCH model. However, the AGARCH is unusual in that uncertainty is not minimized when the forecast error is zero. In the TGARCH model, the relationship has a steeper slope for positive forecast errors if $\gamma_2 < 0$, and is minimized at a forecast error of zero. The latter characteristic makes TGARCH a more plausible representation of inflation uncertainty than AGARCH.

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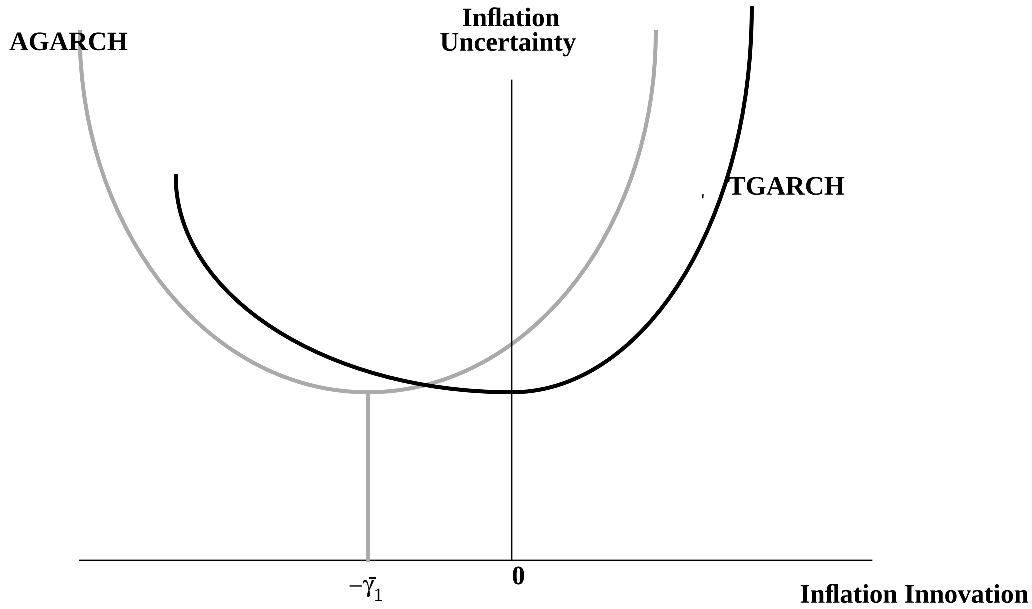
9. Another asymmetric model, the exponential GARCH (EGARCH) model, is most often applied in financial economics. In this model, unexpected positive shocks are deemed “good news” and more good news is better than less. (A common example is an unexpected positive movement in a stock price, even though derivative-market players or short-sellers betting on a fall in the same stock price would consider positive shocks as bad news.) Accordingly, the functional form does not have a well-defined minimum. This makes the EGARCH application to inflation quite troublesome. Intuitively, although negative shocks might generate less uncertainty compared with positive shocks, one would expect uncertainty to be minimized during periods with no shocks.

FIGURE 1
Asymmetric Uncertainty



The models discussed to this point do not include inflation in the specification of the conditional variance. Thus, the hypothesis that uncertainty increases with the rate of inflation can be evaluated only indirectly, by testing for a positive relationship between the estimated conditional variance and inflation. The state-dependent model of Brunner and Hess (1993) provides a direct test of the hypothesis by adding lagged inflation to the AGARCH process:

$$h_t = \alpha_0 + \alpha_1 (\varepsilon_{t-1} + \gamma_1)^2 + \beta_1 h_{t-1} + \varphi (\Pi_{t-1} + \gamma_2)^2 . \qquad (3.7)$$

If the parameter γ_2 in equation (3.7) is non-zero, there is a U-shaped relationship between uncertainty and the level of inflation, with uncertainty minimized at an inflation rate equal to - γ_2.

TABLE 2. Models with fixed parameters and asymmetric uncertainty[a]

Author(s)	Price index	Model and variables in conditional mean	Results
Brunner and Hess (1993)	U.S. CPI 1947Q1 - 1992Q4 and various subperiods	state-dependent model (SDM) EGARCH	Rejects the symmetry restriction in the GARCH model.
			Finds a significant link between the level of inflation and short-run uncertainty.
		lagged inflation and lagged forecast errors (ARIMA)	Estimates of inflation uncertainty from the EGARCH model are similar to those from the SDM model.
Joyce (1995)	U.K. retail prices 1950Q1 - 1994Q1	GARCH, AGARCH, EGARCH, TGARCH	Rejects the symmetry restriction in GARCH models. Inflation uncertainty is more responsive to positive inflation shocks than to negative shocks.
		lagged inflation	Inflation uncertainty is positively related to lagged inflation in their preferred models. A positive relationship is found in a symmetric GARCH model with lagged inflation in the conditional variance.

a. Inflation uncertainty is measured by the conditional variance of the one-quarter forecast.

The Brunner-Hess and Joyce studies of asymmetric uncertainty find similar results for the United States and the United Kingdom (Table 2). The symmetry restriction in the GARCH model is rejected by the data: inflation uncertainty is more sensitive to positive inflation shocks than to negative shocks. Moreover, contrary to most GARCH studies, these studies find significant positive relationships between the level of inflation and (short-run) inflation uncertainty. Using their asymmetric models, Brunner and Hess estimate that the conditional standard deviation increased from an average of 1 3/4 per cent in the low-inflation period of the late 1950s and 1960s to about 4 per cent in the mid-1970s and 1980. The relationship between inflation and its uncertainty was weaker in a model with symmetric uncertainty.

Models with time-varying parameters

The models of asymmetric uncertainty recognize implicitly that the extent to which an inflation shock raises uncertainty will depend on agents' perception of the policy regime. However, the asymmetries are modelled in a mechanical fashion, with no explicit link to learning or policy credibility. Evans (1991), Evans and Wachtel (1993) and Ricketts and Rose (1995) have studied inflation uncertainty using other techniques that explicitly model changes in the inflation process.

Evans assumed that the inflation process varies over time because of changes in the policy regime and private sector behaviour. This idea was implemented by allowing the parameter vector δ in the inflation equation to be time-varying. Specifically,

$$\Pi_t \big| \Psi_{t-1} \sim N(\delta_t X_{t-1}, h_t + x_{t-1} \Omega_t x_{t-1}) \tag{3.8}$$

$$\delta_t = \delta_{t-1} + v_t \qquad v_t \sim N(0, Q)$$

$$\varepsilon_t = \Pi_t - \delta_t X_{t-1}$$

$\Omega_t =$ conditional covariance matrix of δ given the information available at time t-1.

In this model, inflation uncertainty reflects shocks to inflation (ε_t) as well as unexpected changes in the structure of the inflation process (v_t). The parameter vector δ_t in the inflation equation is updated over time as new information becomes available. If the vector δ_t is known with certainty, Ω_t is the null matrix, and the conditional variance of inflation simplifies to the standard GARCH process (h_t). For the U.S. CPI over the 1960-88 period, Evans found an unexpected *negative* relationship between inflation and short-run uncertainty. However, long-run uncertainty, as measured by the conditional variance of steady-state inflation,[10] was positively related to the level of inflation.

10. Steady-state inflation is defined by the situation in which there are no changes to the parameter vector ($v = 0$) and no shocks to inflation ($\varepsilon = 0$).

Evans and Wachtel (1993) and Ricketts and Rose (1995) model structural change using Markov regime-switching models. This approach postulates that several alternative states (or policy regimes) may be in effect in a given period, and agents use available information to form probabilities that the economy is in each state. Each of these states is characterized by an inflation process with state-dependent values for the mean rate of inflation, inflation persistence and forecast-error variance. Therefore, if the error variances differ across states, changes in inflation uncertainty are associated with shifts in the perceived probabilities of the alternative states.

Table 3 lists results from studies with time-varying parameters and Markov-switching models. Using annual CPI data for the 1954-93 period, Ricketts and Rose found that inflation uncertainty increases during high inflation periods in Canada.[11] Evans and Wachtel (1993) also discovered a significant positive relationship between the level of inflation and long-run inflation uncertainty in the United States. Sauer and Bohara (1995) reported the same conclusion for Germany, but found a positive relationship in the United States for only a high-inflation subperiod.

To summarize, this survey of the empirical literature has shown that different classes of models tend to give conflicting results for the relationship between inflation and inflation uncertainty. Studies based on the GARCH model, with its fixed parameters and symmetric response to inflation shocks, typically have failed to uncover a significant relationship. Models with asymmetries and time-varying parameters, or surveys of expectations, have been more likely to provide evidence that uncertainty increases at higher levels of inflation. Finally, many of these empirical studies have used relatively simple specifications (such as autoregressive processes) for the conditional mean of inflation. Clearly, any conclusions on the relationship between inflation and its uncertainty are conditional on the validity of the underlying models.

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11. In their three-state model, "... the standard deviation of the state with moderate (just over 4 per cent) average inflation is 30 per cent higher than that of the state with low (1.6 per cent) average inflation; and the standard deviation of changes in inflation in the random-walk model (which is assigned to the periods of highest inflation) is five times as large as the result for the low-inflation regime" (Ricketts and Rose 1995, 28). The point estimates of inflation uncertainty in the low- and moderate-inflation states were not statistically significantly different from each other, but were significantly less than the estimate of uncertainty in the (high-inflation) random-walk state.

TABLE 3. Models with time-varying parameters

Author(s)	Price index	Model and variables in conditional mean	Results
Evans (1991)	U.S. CPI 1960M1 - 1988M6	ARCH with time-varying parameters lagged inflation	Significant ARCH effects. Long-run uncertainty strongly related to inflation since the early 1970s. Short-run uncertainty almost unrelated to inflation.
Sauer and Bohara (1995)	U.S. and German GDP deflators 1966Q1 - 1990Q1	time-varying parameters lagged growth rates of inflation, nominal income, M1, and relative oil prices	Significant positive correlation between inflation and uncertainty in Germany; a positive relationship existed in the United States only during the higher-inflation 1966-80 subperiod. Uncertainty about steady-state inflation is lower, less variable and less persistent in Germany than in the United States.
Evans and Wachtel (1993)	U.S. CPI 1955Q1 - 1991Q4	Markov-switching model (2 regimes) 1 AR process and a random-walk state	Inflation uncertainty increased at all fore-cast horizons in 1968 and did not return to the levels of the 1950s and 1960s until 1984.
Kim (1993)	U.S. GNP deflator 1958Q1 - 1990Q4	Markov-switching heteroscedasticity	Long-run uncertainty is positively related to the level of inflation.
Ricketts and Rose (1995)	Canadian CPI 1954 - 1993	Markov-switching model (3 regimes) 2 stationary AR processes and a random-walk state	For Canadian data, uncertainty is positively related to the mean inflation rate across the three regimes, although the difference is not statistically significant between the low- and moderate-inflation states.
	CPI for other G7 countries (various sample periods)		For other G7 countries, higher uncertainty tends to be associated with higher levels of inflation.

This paper has two empirical objectives. First, in accordance with the goal of developing a benchmark measure of inflation uncertainty, a number of fixed parameter GARCH models will be investigated, including the symmetric and asymmetric types. Interest centers around the relationship between the level of inflation and both short- and long-run measures of inflation uncertainty. Results from these fixed-parameter models will be compared with those from the regime-switching study of Ricketts and Rose (1995) to determine whether conclusions on the link between inflation and inflation uncertainty in Canada are model-dependent. In addition, evidence on multi-period inflation uncertainty is examined using the conditional variance of four-quarter-ahead inflation. Section 4 begins the empirical analysis with a brief discussion of the properties of the Canadian data.

4. Properties of the data

Our analysis of inflation uncertainty examines two alternative measures of consumer prices. The variable CPIXFET is the rate of change in the consumer price index excluding food, energy and the effect of indirect taxes. By excluding movements in the sometimes volatile food and energy components and the effect of changes in indirect tax rates, our study attempts to measure inflation uncertainty using a common measure of the “core” rate of inflation. Unfortunately, data availability constrains the analysis of the CPI excluding food, energy and indirect taxes to the period since 1963. Both reduced-form and autoregressive equations are estimated for CPIXFET. In order to gain additional perspective from a longer period, we also estimate an autoregressive equation for total CPI inflation over a sample period beginning in 1916. All equations use quarterly data.

Explanatory variables in the reduced-form CPIXFET equations include measures of demand pressures and supply shocks. Demand pressures are measured by the output gap (YGAP), defined as the percentage deviation of current output from potential output. Import price inflation (MINFL) is measured as the rate of change in the U.S. consumer price index (excluding food and energy) plus the percentage change in the price of foreign exchange ($Can./$U.S.). Supply shocks are represented by the rate of change in real oil prices (POILR), defined as the U.S. dollar price of West Texas Intermediate oil divided by the U.S. GDP price deflator. Finally, we also consider the indirect tax rate for the consumer price index excluding food and energy (TXCPIFE) and the first difference of the indirect tax rate (TXPD).[12]

This section investigates two properties of the data: unit-root tests determine the order of integration of the variables, while vector autocorrelations illustrate data cross-properties. As summarized in Table 4, the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit-root tests indicate that CPI inflation was autoregressive-stationary (I(0)) over the 1916-94 period. The lag-length criterion of the two tests was white-noise errors, which corresponds to an insignificant Ljung-Box Q-statistic. The robustness of this result

12. CPIXFET and TXCPIFE are calculated by the Bank of Canada (see Bank of Canada (1991) for a description of the methodology). The series for potential output is constructed using a version of the multi-variate filtering technique discussed by Laxton and Tetlow (1992).

varies, however, with the sample period. The growth rate of the consumer price index excluding food, energy and indirect taxes contains a unit root over the shorter period beginning in 1963. All other variables are I(0) over the shorter period, with the exception of the level of the indirect tax rate.

TABLE 4. Unit-root test results[a]

Mnemonic	ADF	PP	Result
CPI (1916Q2 - 1994Q3)	- 5.62 *	- 5.90*	stationary
CPIXFET (1963Q3 - 1994Q3)	-1.79	-3.39	non-stationary
YGAP	-3.61 *	-6.42 *	stationary
MINFL	-2.96	-8.14 *	stationary
POILR	-4.89 *	-6.72 *	stationary
TXPCPIFE	-1.50	-1.78	non-stationary

a. A deterministic trend and intercept term are included in the ADF and PP tests.

* The null hypothesis of a unit root is rejected at the 5 per cent significance level (critical value -3.45).

Although the formal unit-root tests suggest that CPIXFET is an I(1) process, the remainder of our empirical work proceeds under the assumption that it is I(0). This decision is based on two considerations. First, the regime-switching model of Ricketts and Rose (1995) suggests that the unit-root process in inflation is a transitory state that tends to occur only during periods of high and variable inflation. Second, these unit-root tests are known to have low power for distinguishing between unit-root and near unit-root states.

Having analysed the unit-root properties of the data, we now examine the sources of turning points and persistence in inflation. Vector autocorrelations (estimated over the short sample period) are a simple way to analyse this stylized information (not shown). These autocorrelations showed a strong persistence in the inflation process; the fourth-quarter lag and contemporaneous inflation were correlated at over 50 per cent. The highest correlation between inflation and the lagged output gap was less than 30 per cent, but it displayed some persistence. The correlation with import inflation peaks at a surprisingly long lag, approximately 15 quarters. The correlation with relative oil prices faded quickly from its peak of under 20 per cent at the first lag. Overall, inflation itself appeared to be the primary source of its own persistence.[13]

Figure 2 summarizes the Canadian inflationary experience from 1963 to 1994 for the CPI excluding food, energy and the effect of indirect taxes. There was an upward trend in this measure of inflation from 1963Q3 to 1969Q4, with an average rate of 3.4 per cent (annualized quarterly rate). The upward trend was, however, relatively smooth; the sample variance in this subperiod was 4.9, less than one-half of the variance from 1963 to 1994.

Inflation in the early 1970s followed the path set in the 1960s. The remainder of the decade was characterized by shocks. Oil-price inflation began in 1973 and was followed by a run-up in inflation in 1979, which continued until early 1982. In the 1980s, excluding this early period of instability, the moderate average rate of inflation (4.3 per cent) was accompanied by a very low sample variance of 1.6. Inflation over the period 1990Q1 to 1994Q3 was lower, with a mean of 2.3 per cent and variance of 2.1.

As previously indicated, one must be careful not to infer the state of inflation uncertainty from its sample variance. However, one would expect uncertainty to be relatively low over the periods of stable inflation (1963 to 1972, 1983 to 1989, and 1991 to 1994), which are also accompanied by relatively moderate levels. Conversely, it is likely that inflation uncertainty was high during periods such as the 1920s and 1930s, when total CPI inflation was quite volatile (Figure 3).

Having established the basic properties of the data, the next section reports parameter estimates from two specifications of the conditional mean of inflation.

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13. Consideration was also given to the cross-correlations between inflation and money growth (M1, M2 and M2+), nominal wage growth and the 90-day commercial paper rate. The correlation with inflation peaks at about a two-year lag for M1 growth, in contrast to the high short-run correlations between inflation and the lagged values of the other variables.

5. Specification of the conditional mean of inflation

Since inflation appeared to be the primary source of its own persistence, our analysis of the inflation process begins by estimating an autoregressive model. We also estimate a reduced-form model, with the expectation that the conditional variance in this model will be less than that in the autoregressive specification. Various tests of the models, including parameter stability and properties of the residuals, are also reported.

5.1 Autoregressive specification

The optimal lag structure in the autoregressive model was consistently a four-quarter lag length according to the Akaike information criteria. This result was independent of the sample period. Table 5 summarizes the ordinary least squares (OLS) parameter estimates.

In addition to lagged inflation, the equations include a dummy variable for the period surrounding the introduction of the Goods and Services Tax (GST) in 1991Q1. The sharp rise in quarterly inflation in 1991Q1 was greater than what could be attributed to the direct effect of the introduction of the GST. One explanation for this outcome is that retailers timed price increases related to non-tax factors to coincide with the implementation of the GST, leading to a sharp rise in 1991Q1 even in net price indexes like the CPI excluding food, energy and the effect of indirect taxes. According to this view, the GST had an effect on the timing of changes in net prices, with some increases that would otherwise have taken place later in 1991 shifted forward to 1991Q1. To allow for these timing effects, the GST dummy variable has a value of one in 1991Q1 and minus one in 1991Q2. As shown in Table 5, this dummy variable has the expected positive sign and is significant.

A rolling Chow test was performed to assess the stability of the parameter estimates over time (not shown).[14] For the total CPI equation estimated from 1916, the

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14. Estimated from 1963 to 1994, the rolling portion (R) of the Chow test splits the sample into two periods: 1963 to R (S1) and R to 1994 (S2). For each R, an F-test is constructed where the null hypothesis is $\beta_{i,S1} = \beta_{i,S2}, \forall i$.

rolling Chow test rejected the null hypothesis of parameter stability primarily during the volatile 1920s and 1930s and the late 1950s through early 1960s (at the 5 per cent level of significance). For the autoregressive CPIXFET equation estimated from 1963, the rolling Chow test could not reject the null hypothesis of parameter stability except in the early 1980s. The null hypothesis of parameter stability could not be rejected in any period at the 10 per cent level of significance. This result is relevant for later in-sample simulations, which assume parameter stability.

5.2 Reduced-form specification

Our reduced-form equation specifies inflation as a function of lagged inflation, the output gap, imported inflation, relative oil price inflation and the change in the indirect tax rate:

$$\Pi_t = a_0 + A(L)\,\Pi_t + B(L)\,YGAP_t + D(L)\,MINFL_t +$$
$$F(L)\,POILR_t + G(L)\,TXPD_t + \varepsilon_t$$

where $X(L)$ is a polynomial matrix of parameters and L is the lag operator.[15]

The reduced-form model for CPIXFET is estimated from 1963Q3 to 1994Q3. Consistent with the vector autocorrelations, the first three lags of inflation capture its persistence (Table 5). Aggregate demand pressure is captured by the first lag of the output gap. Exclusion tests rejected the first-differenced output gap as an explanatory variable. Import inflation enters the reduced-form specification with one lag.[16] The first lag of relative oil-price inflation, as well as the contemporaneous and first lag of the first difference of the indirect tax rate, also has significant explanatory power for inflation.

15. Duguay (1994) estimated a similar model for Canada.

16. Through the lagged inflation terms in the equation, the long-run effect of exchange rate movements on prices will exceed the initial effect in period t-1. Some previous studies of similar models have included a seven-quarter moving average of import inflation to account for the suspected long lags for exchange rate pass-through to consumer prices. The F-statistic corresponding to this moving-average restriction (that the parameters of seven lags of import inflation are of equal weight and sum to the parameter associated with the moving-average of import inflation) is F(7,106)=2.37. At all reasonable levels of significance, the null hypothesis of the moving-average restriction is rejected. For the sake of parsimony, only the first lag import inflation is included in the final specification.

Since CPIXFET measures inflation in a net price index, the negative-parameter estimate for the contemporaneous change in the indirect tax rate implies that indirect tax changes are partially absorbed by sellers in the short run.

The rolling Chow test could not reject the null hypothesis of parameter stability in the reduced-form model, which suggests that these parameters have been relatively constant over time (not shown).

TABLE 5. OLS parameter estimates[a]

Variable	Autoregressive (CPI) 1916Q2-1994Q3	Autoregressive (CPIXFET) 1963Q3-1994Q3	Reduced-form (CPIXFET) 1963Q3-1994Q3
Constant	0.18 (2.53)[b]	0.15 (1.67)	0.23 (3.32)
Π_{t-1}	0.75 (13.20)	0.44 (4.96)	0.41 (5.70)
Π_{t-2}	-0.12 (-1.80)	0.09 (0.97)	0.11 (1.70)
Π_{t-3}	0.14 (2.03)	0.42 (4.70)	0.24 (3.82)
Π_{t-4}	0.02 (0.29)	-0.08 (-0.85)	--
$YGAP_{t-1}$	--	--	0.07 (5.06)
$MINFL_{t-1}$	--	--	0.07 (3.53)
$POILR_{t-1}$	--	--	0.0007 (2.66)
$TXPD_t$	--	--	-0.49 (-7.31)
$TXPD_{t-1}$	--	--	0.35 (4.50)
GST	1.64 (2.18)	0.71 (2.06)	1.02 (3.93)
Adjusted R^2	0.55	0.63	0.81

a. All models are estimated as a quarterly (non-annualized) per cent.

b. t-statistic in parentheses.

5.3 Model specification tests

Cosimano and Jansen (1988) found that the null hypothesis of a constant variance for the ARCH residuals is more likely to be rejected when the residuals are serially correlated. In order to test for this problem, Table 6 presents Lagrange Multiplier (LM) test results for the autoregressive and reduced-form models estimated from 1963.

TABLE 6. LM test statistics for serial correlation[a]

Model	LM(1)	LM(4)	LM(8)
Autoregressive	0.4e-3	0.24	9.31
Reduced-form	4.55*	9.84	10.54

a. The LM test for serial correlation of order *p* is computed by regressing an AR(*p*) model of the OLS residuals excluding the constant. The no serial correlation null hypothesis restricts the lags of the residuals to be zero. The corresponding test-statistic reported in this table is distributed chi-squared with *p* degrees of freedom.

* Significantly different from zero at the 5 per cent level of significance.

The null hypotheses of fourth- and eighth-order serial correlation are rejected in both models. In the reduced-form model, there is evidence of first-order serial correlation. However, beyond first-order serial correlation, there is little evidence of model misspecification attributable to a moving-average residual. Thus, subsequent tests for time-varying uncertainty are not likely to be biased towards rejecting the constant-variance null hypothesis. The next section uses the OLS residuals to conduct several tests for the presence of time-varying inflation uncertainty.

6. Tests for time-varying inflation uncertainty

The forecast error from a model is an inadequate measure of uncertainty since this approach implies that the average level of uncertainty over periods with large but virtually offsetting positive and negative errors would be comparable to the uncertainty in periods with relatively small forecast errors. Consistent with other studies, we use the squared forecast error (residual variance) as the proxy for uncertainty, since positive and negative errors are not offsetting in this measure.

This section presents two "pre-tests" for time-varying inflation uncertainty using the OLS residuals from the conditional mean equations of the previous section. The first pre-test is a general test for time-varying uncertainty, while the second examines whether uncertainty is a function of inflation. Measures of inflation uncertainty obtained from joint estimation of the conditional mean and conditional variance equations are discussed in Section 7.

6.1 Tests for GARCH effects

Two tests for a non-constant residual variance are considered. First, in the Lagrange Multiplier (LM) test, the squared OLS residuals (ε^2) are regressed against a constant and their lagged values. Given

$$\varepsilon_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \ldots + \alpha_p \varepsilon_{t-p}^2,$$

the constant-variance null hypothesis is defined by the restriction

$$\alpha_1 = \ldots = \alpha_p = 0.$$

As shown by Bollerslev (1986), the LM test for a p^{th} order ARCH is equivalent to a test for GARCH (i,j) where $i + j = p$.

Second, examination of the partial autocorrelation function (PACF) exploits the fact that GARCH models imply second-moment autocorrelation. In this regard, the asymptotic variance of the partial autocorrelation function at lag k converges to $\left(\frac{1}{T}\right)\left(1 + \sum_{j<k} \hat{\rho}_j\right)$. Under the null hypothesis of no GARCH, the asymptotic variance converges to $1/T$.

6.2 GARCH test results

Estimated from 1916, the autoregressive model for total CPI inflation strongly suggests the presence of GARCH residuals, as tests for p equal to 1, 4 and 8 all reject the null hypothesis of a constant variance (Table 7). Consistent with these LM tests, seven lags of the PACF are significantly different from zero. For the shorter sample period, there is evidence of a low-order GARCH effect in the autoregressive model for CPIXFET, but the evidence for a higher-order GARCH process diminishes: LM tests for p equal to 4 and 8 have probability values of only 13 and 15 per cent respectively. Thus, exclusion of the volatile inflation episodes in the 1920s and 1930s from the shorter sample period reduces the evidence of a non-constant variance. Other estimations of the total CPI equation beginning in 1940 and 1954, which eliminated the interwar and Korean War periods respectively, also diminished the evidence in favour of GARCH effects.

The non-constant residual variance found in the autoregressive models may simply reflect the exclusion of relevant information. This possibility is evaluated using the test results for the reduced-form model in Table 7. With the inclusion of additional explanatory variables, all of the LM tests cannot reject the constant-variance null hypothesis, although the presence of one significant lag of the PACF suggests a low-order GARCH process.

Holland (1984) and Cosimano and Jansen (1988) concluded that the non-constant variance in autoregressive models of U.S. inflation is explained by the omission of a single variable (relative commodity prices). In contrast, it appears that no single variable can account for the non-constant variances in the autoregressive Canadian case. As shown by the sensitivity analysis in Table 7, GARCH tests fail to reject the constant variance hypothesis when individual variables are excluded from the reduced-form model.

TABLE 7. Tests for GARCH effects

Model[a] (sample period)	GARCH(1)[b] (p=1)	GARCH(4)[b] (p=4)	GARCH(8)[b] (p=8)	PACF
Autoregressive (1916Q2-1994Q3)	19.49 *	62.56 *	73.34 *	high-order GARCH *
Autoregressive (1963Q3-1994Q3)	7.36 *	8.28	14.90	low-order GARCH *
Reduced-form (1963Q3-1994Q3)	0.96	5.46	9.53	low-order GARCH *
		Reduced-form sensitivity analysis		
Reduced-form (excluding $YGAP_{t-1}$)	0.49	4.33	5.98	--
Reduced-form (excluding $MINFL_{t-1}$)	0.78	2.78	5.27	--
Reduced-form (excluding $POILR_{t-1}$)	0.02	2.30	3.69	--

a. The autoregressive model for the long sample period is based on the total CPI. All other models use CPIXFET.

b. The LM test is distributed chi-squared with *p* degrees of freedom.

* Significant at the 5 per cent level of significance.

6.3 Inflation as a source of time-varying uncertainty

We now consider the first of our direct tests for the existence of a relationship between inflation and inflation uncertainty. Similar to Engle (1982), a “pre-test” assumes that inflation uncertainty is proxied by the squared OLS residuals from the quarterly autoregressive or reduced-form models, and these squared residuals are regressed on a constant and lagged inflation. Three alternative inflation variables are considered: the level of inflation, the absolute change in inflation, and squared inflation. Intuitively, we would expect low levels of inflation to be associated with stable inflation and thus low uncertainty. The absolute change in inflation is considered in order to test whether inflation uncertainty is more closely related to changes in inflation, as opposed to the level of inflation. Finally, squared inflation is used to test for a non-linear relationship. Since the primary focus of this paper is the relationship between the trend level of inflation and uncertainty, the inflation variables are defined using the lagged year-over-year rate of inflation ${}^{y}\Pi_{t-1}$ rather than a more volatile shorter-term measure such as the lagged one-quarter rate.

Table 8 shows that the level of inflation and squared inflation are statistically significant in both autoregressive models. The relationships are particularly strong over the longer sample period. In the reduced-form model, the level of inflation is almost significant at the 5 per cent level using a one-tail test. The change in year-over-year inflation is insignificant in the equations estimated over the shorter sample period.

TABLE 8. Tests of inflation as the source of time-varying uncertainty[a]

Model (sample period)	${}^{y}\Pi_{t-1}$	$\lvert\Delta^{y}\Pi_{t-1}\rvert$	${}^{y}\Pi^{2}_{t-1}$
Autoregressive (1916Q2-1994Q3)	0.46[b] (2.93)[c]	3.34 (7.37)	0.14 (3.50)
Autoregressive (1963Q-1994Q3)	0.14 (3.63)	0.26 (1.00)	0.03 (3.46)
Reduced-form (1963Q3-1994Q3)	0.04 (1.63)	-0.04 (-0.32)	0.01 (1.37)

a. The dependent variable (the squared OLS residuals from the models shown in Table 5) is regressed on a constant and the measure of year-over-year inflation. Since the dependent variable in the model is the non-annualized quarterly rate of inflation, the lagged inflation variable in Table 8 is expressed as a non-annualized quarterly equivalent.

b. The absolute level of inflation was used in the estimation over the long sample period (which includes periods of deflation in the 1920s and 1930s).

c. t-statistic in parentheses.

The pre-test evidence from the autoregressive model suggests there is a strong positive relationship between inflation and inflation uncertainty. The relationship is somewhat weaker in the reduced-form model. In order to discriminate between these results, it is important to contrast the implicit information assumptions underlying each model. By excluding all variables other than past inflation, the simple autoregressive approach ignores some of the information that agents would have used to forecast inflation. This suggests that the autoregressive model will tend to overstate the actual uncertainty faced by agents.

In contrast, the reduced-form results may understate the uncertainty that existed, since the model implicitly assumes that agents had more information on the structure of the economy than was actually known at each point in time. For example, the output gap variable in the equation is based on a series for potential output that is constructed from a methodology that includes a two-sided filter of actual output. In addition, the uncertainty measures are calculated using parameter estimates obtained from estimation over the entire sample period. Thus, the reduced-form equation exploits information that was not fully known to agents at the time, which suggests that it provides lower-bound estimates of inflation uncertainty. Inflation uncertainty is investigated further in Section 7 using the GARCH model.

7. Inflation and its uncertainty: GARCH estimation

We now specify a GARCH process for the conditional variance of inflation and jointly estimate this process together with either the autoregressive or reduced-form version of the conditional mean equation. The lag structures of the symmetric GARCH processes are determined so as to satisfy two conditions. First, the conditional variance coefficients must be non-negative in order to ensure a positive conditional variance over time. In addition, wide-sense (covariance) stationarity, where the sum of the variance coefficients is less than one, is needed to ensure a non-explosive conditional variance (see theorem 2 in Engle (1982), or theorem 1 in Bollerslev (1986)). Among the models satisfying these two conditions, the most parsimonious GARCH(p,q) specification was selected.

As discussed in Section 3.2, past empirical studies have found a pronounced relationship between inflation and inflation uncertainty in asymmetric GARCH models (see Table 2). These models allow negative shocks to have different effects on uncertainty than positive shocks of equal absolute magnitude. Two of these asymmetric models, namely AGARCH and TGARCH, are also considered in this section.

7.1 Maximum likelihood estimation

Table 9 summarizes the maximum likelihood parameter estimates of the conditional variance equations.[17] The significant parameter estimates in the autoregressive models provide evidence of time-varying uncertainty. The one-quarter measures of inflation uncertainty constructed from the autoregressive models are quite persistent (Figures 4a-4b), as would be expected given the parameter estimates in the 0.8-0.9 range for the lagged conditional variance. These results suggest that uncertainty was very high during the volatile period of the 1920s and 1930s and the high-inflation episode

17. The maximum likelihood estimates of the parameters in the conditional mean equation (not reported) were, as expected, very similar to the OLS estimates. Most models converged after approximately 20 iterations using the Berndt, Hall, Hall and Hausman algorithm. OLS parameter estimates proxied the initial values while the initial values of the GARCH parameters were 0.05. Alternative starting values did not change the results.

of the late 1970s and early 1980s. More recently, inflation uncertainty has been relatively low and stable.

TABLE 9. Parameter estimates of the conditional variance equations

Variables	Autoregressive (1916Q2-1994Q3) GARCH	Autoregressive (1963Q3-1994Q3) GARCH	Reduced-form GARCH	Reduced-form AGARCH[a]	Reduced-form TGARCH[b]
constant	0.006 (1.46)[c]	0.003 (0.46)	0.08 (1.31)	0.08 (2.25)	0.08 (1.22)
ε^2_{t-1}	0.10 (3.21)	0.17 (1.91)	0.28 (1.30)	0.47 (2.21)	0.25 (0.87)
h_{t-1}	0.89 (30.46)	0.82 (10.49)	0.002 (0.003)	-0.15 (-0.74)	0.04 (0.06)
γ_i		--	--	-0.11 (-1.20)	0.06 (0.14)

a. Refer to equation (3.5).
b. Refer to equation (3.6).
c. t-statistic in parentheses.

In contrast to the autoregressive case, the conditional variance parameters in the reduced-form GARCH model (Table 9, column 3) are not significant at the standard levels, which would suggest that inflation uncertainty is not time-varying. However, as mentioned previously, the reduced-form model may understate uncertainty given its strong assumptions about the information available to agents, so there is still interest in the uncertainty measures constructed from this model. As shown in Figure 5, one-quarter uncertainty is far less persistent than in the autoregressive case, and is generally smaller in size. This result indicates that much of the forecast-error variance in the autoregressive model reflects the exclusion of the additional variables included in the reduced-form model. As expected, the inclusion of relative oil prices and other explanatory variables in the reduced-form model gives a much lower measure of inflation uncertainty in the 1970s, relative to the autoregressive alternative.

Parameter estimates of the asymmetry parameters (γ) in the AGARCH and TGARCH models are insignificant (Table 9). Therefore, contrary to findings for other countries, the Canadian data do not support the hypothesis that negative inflation shocks have a smaller effect on uncertainty than positive shocks of equal magnitude. As a result, further analysis in this study is restricted to the symmetric GARCH specifications.

7.2 Lagged inflation in the conditional variance equation

We now consider direct tests for a relationship between inflation and inflation uncertainty by adding one of three alternative measures of lagged inflation to the conditional variance equation. These measures include the level of inflation, the absolute change in inflation and squared inflation.[18] The GARCH process with lagged inflation is

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} + \varphi(Z_{t-1}) \tag{7.1}$$

where $Z = {}^{y}\Pi \;\; or \;\; |\Delta^{y}\Pi| \;\; or \;\; {}^{y}\Pi^2$ and ${}^{y}\Pi$ is the year-over-year rate of inflation. Each of these alternative definitions of Z is considered for the reduced-form model, while attention is restricted to the lagged level of inflation in the autoregressive case. The lags for the non-inflation terms in the GARCH process may differ from those in Table 9, as optimal lag profiles are reconsidered for all terms in the conditional variance.

Table 10 summarizes parameter estimates in the conditional variance when the GARCH processes are defined by equation (7.1). Consistent with the pre-test results from Section 6, there is a significant positive relationship between the level of inflation and uncertainty in the autoregressive model. With the exception of an unexpected negative parameter for the absolute change in inflation, there are no significant parameters in the conditional variance processes of the reduced-form models. For comparison with other results, Figure 6 shows the one-quarter uncertainty from the reduced-form GARCH models with and without the level of lagged inflation in the conditional variance.

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18. In Section 7.3, the conditional variance contains forward-looking measures of expected inflation rather than the backward-looking measures used here.

TABLE 10. Parameter estimates for conditional variance equations containing lagged inflation[a]

Variables	Autoregressive GARCH	Reduced-form GARCH	Reduced-form GARCH	Reduced-form GARCH	Reduced-form GARCH
constant	-0.002 (0.06)[b]	0.08 (1.31)	0.04 (0.71)	0.11 (5.40)	0.06 (0.98)
ε^2_{t-1}	0.45 (1.98)	0.28 (1.30)	0.21 (1.02)	0.24 (1.21)	0.23 (1.12)
ε^2_{t-2}	--	--	--	0.04 (0.29)	--
h_{t-1}	--	0.002 (0.003)	0.04 (0.05)	--	0.001 (0.001)
${}^y\Pi_{t-1}$	0.12 (3.07)	--	0.03 (0.84)	--	--
$\lvert\Delta {}^y\Pi_{t-1}\rvert$	--	--	--	-0.21 (3.49)	--
${}^y\Pi^2_{t-1}$	--	--	--	--	0.01 (0.82)

a. All models cover the 1963Q3-1994Q3 period.

b. t-statistic in parentheses.

7.3 Expected inflation in the conditional variance equation

We now consider models in which the conditional variance specification includes alternative inflation variables constructed from the eight-quarter-ahead expectation of inflation from a Markov regime-switching model (Π^M).[19] This series for expected inflation is quite volatile in the late 1970s, is relatively stable in the 4-5 per cent range over much of the 1980s, and falls sharply in the early 1990s (Figure 7).

19. The series for the eight-quarter expectation of inflation was constructed from a Markov regime-switching model of quarterly CPI inflation estimated by Nick Ricketts using data from 1954Q1 to 1994Q4. His model has two autoregressive states and a random-walk state. For each period, the eight-quarter expectation of inflation was generated for each state. (The two stationary states were found to converge to their long-run levels within eight quarters.) Π^M was then calculated as the weighted sum of expected inflation in the three states, where the weights are the one-quarter conditional probabilities of the respective states. Thus, the variance in the series Π^M reflects the variance of expected inflation in the unit-root state and the variances of the conditional probabilities in the stationary states.

With Π^M included in the conditional variance equation, $Z_t = \Pi_t^M \; or \; \left|\Delta\Pi_t^M\right| \; or \; \left(\Pi_t^M\right)^2 \; or \; \left(\Pi_t^M + \gamma_1\right)^2$ in equation (7.1). Following Brunner and Hess (1993), the last definition of Z imposes a U-shaped relationship between the level of expected inflation and inflation uncertainty, with uncertainty minimized at an inflation rate equal to - γ_1.[20]

Overall, the evidence for time-varying uncertainty is stronger when expected (rather than lagged) inflation enters the conditional variance. Consistent with previous results, there is a significant positive relationship between the level of expected inflation and the conditional variance in the autoregressive model (Table 11, column 1). In addition, contrary to previous results, uncertainty is time-varying in the reduced-form model with expected inflation in the conditional variance process: parameter estimates for the second lag of the forecast-error variance are statistically significant in the equations with Π_{t-1}^M and $\left(\Pi_{t-1}^M\right)^2$. The level of expected inflation, the change in expected inflation, and squared expected inflation all have a positive effect on inflation uncertainty in the reduced-form model, although these parameter estimates are insignificant at the standard 5 per cent level.

The non-linear Brunner-Hess specification (Table 11, column 5) has some interesting results. The parameter estimate for the inflation variable (φ) is not individually significant, but φ and γ_1 are jointly significant.[21] The negative parameter estimate for γ_1 implies that uncertainty is minimized when expected inflation is about 1.25 per cent at a non-annualized quarterly rate (about 5 per cent annualized). The suggestion that uncertainty is minimized when annualized inflation is near 5 per cent should be interpreted very carefully, since it is specific to the sample period considered, which had relatively few observations with inflation significantly below the 4-5 per cent range (see Figure 7). The Markov regime-switching model of Ricketts and Rose (1995) also found that uncertainty was a non-linear function of inflation.[22] A longer sample period with additional low-inflation years might clarify whether inflation uncertainty falls significantly when an economy moves from moderate inflation towards price stability.

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20. The Brunner-Hess model did not converge to a unique solution when it was estimated using the lagged inflation variable considered in the previous section.

21. The test for the joint significance of φ and γ_1 has a p-value of 0.004.

TABLE 11. Parameter estimates for conditional variance equations containing expected inflation[a]

Variables	Autoregressive GARCH	Reduced-form GARCH	Reduced-form GARCH	Reduced-form GARCH	Reduced-form GARCH
constant	0.02 (0.39)[b]	0.05 (1.56)	0.04 (2.66)	0.05 (2.09)	0.05 (1.37)
ε^2_{t-1}	0.29 (1.47)	0.01 (0.07)	0.35 (1.75)	0.006 (0.04)	0.33 (1.56)
ε^2_{t-2}	0.26 (1.66)	0.37 (2.00)	--	0.41 (2.19)	--
h_{t-1}	--	--	--	--	0.05 (0.12)
Π^M_{t-1}	0.08 (2.24)	0.02 (1.00)	--	--	--
$\lvert\Delta\Pi^M_{t-1}\rvert$	--	--	0.11 (1.51)	--	--
$\left(\Pi^M_{t-1}\right)^2$	--	--	--	0.01 (1.21)	--
$\left(\Pi^M+\gamma_1\right)^2$	--	--	--	--	0.04 (0.88)
γ_1					-1.27 (3.30)

a. All models cover the 1963Q3-1994Q3 period.

b. t-statistic in parentheses.

Figures 8a and 8b show the estimates of one-quarter uncertainty from the reduced-form models with GARCH processes containing the level of expected inflation or the Brunner-Hess non-linear specification. Since the latter model implies that uncertainty is minimized when expected inflation is in the 5 per cent range, its measure of uncertainty shows a moderate upward movement in the low-inflation 1990s relative to the low level reached in the late 1980s.

22. As noted in footnote 11, inflation uncertainty in the high-inflation state was considerably greater than in the medium-inflation state (which had a mean inflation rate of about 4 per cent). The point estimate of inflation uncertainty was even lower in the low-inflation state, but was not statistically different from estimated uncertainty in the medium-inflation state.

In summary, the findings in Sections 6 and 7 provide some evidence that inflation uncertainty decreases at lower rates of inflation. All results from the autoregressive GARCH model are consistent with this positive relationship. The evidence is not as strong for the reduced-form model, although it has been argued that this model may tend to understate uncertainty, given its strong implicit assumptions concerning the amount of information known to agents. When comparing these results for Canada with those for other countries, it should be noted that most previous studies used specifications for the conditional mean of inflation with less structure than the reduced-form model examined in this paper.

8. Multi-period inflation uncertainty

The preceding discussion has focussed on the relationship between inflation and the one-quarter-ahead measure of inflation uncertainty. As noted in Section 3, the adverse real effects of uncertainty may be related more to a longer-run measure of uncertainty than to the short-run measure. We now use the GARCH models estimated in Section 7 to examine the uncertainty of longer-run inflation as measured by the conditional variance of four-quarter-ahead inflation.

8.1 Methodological review

To illustrate the methodology to be used, consider the multi-period forecast error for an AR(4) model of inflation:

$$\Pi_t = B_0 + B_1\Pi_{t-1} + B_2\Pi_{t-2} + B_3\Pi_{t-3} + B_4\Pi_{t-4} + \varepsilon_t .$$

In this case, the s-period forecast error is

$$e_{t+s} = B_1 e_{t+s-1} + B_2 e_{t+s-2} + B_3 e_{t+s-3} + B_4 e_{t+s-4} + \varepsilon_{t+s} \tag{8.1}$$

where $e_{t+s} = \Pi_{t+s} - E_t\Pi_{t+s}$ and E_t is the expectation operator conditional on the information set at time t. By recursive substitution, (8.1) can be represented as

$$e_{t+s} = \varepsilon_{t+s} + b_1\varepsilon_{t+s-1} + b_2\varepsilon_{t+s-2} + b_3\varepsilon_{t+s-3} + b_4\varepsilon_{t+s-4} + \ldots + b_{s-1}\varepsilon_{t+1} \tag{8.2}$$

where

$$b_j = \sum_{i=1}^{s-1} B_i b_{j-1}$$

$$b_0 = 1$$

$$b_j = 0 \text{ for } j < 0 .$$

Assuming mean-zero forecast errors and no covariance between inflation innovations, and abstracting from the variances of parameter estimates, the conditional variance of equation (8.2) (i.e., the *s*-period conditional variance of inflation) is

$$E_t e_{t+s}^2 = \sum_{j=0}^{s-1} b_j E_t \varepsilon_{t+s-j}^2 . \tag{8.3}$$

Notice that the right-hand side of equation (8.3) requires forecasted values of the conditional variance. These values are easy to generate in the GARCH case since, for a given set of starting values, an *s*-period dynamic simulation of the conditional variance can be constructed (in-sample and/or out-of-sample).

8.2 Four-quarter uncertainty and confidence intervals

Figure 9a shows in-sample estimates of four-quarter inflation uncertainty from the autoregressive and reduced-form GARCH models containing lagged inflation in the conditional variance.[23] The general trends are similar in the two cases. However, as expected given the parameter estimates in Table 10, uncertainty is considerably larger and more volatile in the autoregressive case. Uncertainty is greatest during the high-inflation period of the late 1970s and early 1980s, is relatively stable at a lower level during the second half of the 1980s, and falls further during the low-inflation years of the early 1990s.

Longer-run measures of inflation uncertainty with the autoregressive and reduced-form models are presented in Figure 9b. These figures report the conditional standard deviation of out-of-sample forecasts from 1994Q4 assuming a value of 2 per cent (annualized) for the lagged inflation term in the conditional variance. Once again, at all forecast horizons, uncertainty is significantly greater in the autoregressive case. Moreover,

23. In this case, the multi-period conditional variance of inflation is dependent upon forecasted values of inflation. To generate an inflation forecast for the reduced-form equation, forecasted values of the exogenous right-hand side variables are required for all forecast horizons greater than one quarter ($s > 1$). Our in-sample simulations assume perfect foresight of these exogenous variables (i.e., actual data are used). Since policy makers also face uncertainty about the exogenous variables, the overall level of inflation uncertainty is greater than the following results would suggest.

whereas multi-period uncertainty reaches close to its steady state within eight quarters in the reduced-form model, it continues to rise even beyond the 16-quarter horizon in the autoregressive alternative.

To assess the width of uncertainty around the four-quarter-ahead inflation forecast, an *s*-period 95 per cent *confidence interval* is approximated by the prediction of inflation plus/minus (approximately) two conditional standard deviations of inflation, where the conditional standard deviation is the square root of equation (8.3).[24] Then, a 95 per cent *confidence range* is defined as the difference between the upper and lower limits of the confidence intervals (or four times the square root of equation (8.3)). The first column in Table 12 shows the 95 per cent four-quarter confidence ranges from the reduced-form model using the GARCH specification *without* inflation in the conditional variance. The confidence range averages about 6.5 per cent, with relatively little variation in the size of the range across individual periods. Unfortunately, the confidence ranges remain high even with inflation in the conditional variance process (column 2). However, the inclusion of inflation in the conditional variance allows considerably more flexibility in the size of the bands over time, as shown by the decrease in the average confidence range from almost 7 per cent in the 1970s to about 5.75 per cent during the lower-inflation 1990s.

The final column in Table 12 shows the four-quarter confidence ranges from the autoregressive model *with* lagged inflation in the conditional variance. The average confidence range of 11.2 per cent is much greater than the 6.6 per cent average from the reduced-form models. Consistent with Figure 9a, the confidence range also shows considerably greater volatility over time, falling from an average of 12.2 per cent in the 1970s and 1980s to about 8.5 per cent in the low-inflation 1990s.

24. The primary obstacle in constructing multi-period confidence intervals is that the normality assumption is violated in the GARCH case. According to Monte-Carlo experiments of Baillie and Bollerslev (1992), the degree of violation is an increasing function of s. Hence, for small values of s, the normality assumption is a close approximation of the true distribution. (One should see a large increase in the number of outliers if the normality assumption is grossly violated as s increases.)

TABLE 12. Statistics on confidence ranges: Four-quarter horizon[a]

Decade	Statistics[b]	Reduced-form GARCH(1,1)	Reduced-form GARCH(1,1) as a function of inflation	Autoregressive GARCH(0,1) as a function of inflation
1960s	H	6.82	6.79	13.01
	Av	**6.64**	**6.13**	**9.60**
	L	6.59	5.54	6.16
1970s	H	7.06	8.57	18.15
	Av	**6.65**	**6.94**	**12.19**
	L	6.60	5.64	8.07
1980s	H	6.79	8.06	16.52
	Av	**6.64**	**6.78**	**12.19**
	L	6.60	5.78	8.47
1990s	H	6.67	6.62	11.02
	Av	**6.62**	**5.74**	**8.53**
	L	6.59	4.92	5.49
1960s - 1990s	**Av**	**6.64**	**6.57**	**11.21**

a. The statistics are based on annualized quarterly rates of inflation and 95 per cent confidence ranges. The reduced-form GARCH (1,1) model is $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1}$; the reduced-form GARCH(1,1) estimated as a function of inflation is $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1} + \varphi(\Pi_{t-1})$, where inflation is the year-over-year rate of inflation. Parameter estimates for these models are shown in Table 10. (Notice that the four-quarter expected variance in the latter case is a function of the in-sample three-quarter-ahead prediction of inflation.)

b. Confidence ranges are computed for each quarter from 1963 to 1994. The statistics represent the high (H), average (Av) and low (L) confidence range in the decade.

9. Conclusions

Theoretical models have suggested a number of reasons why inflation uncertainty may increase at higher levels of inflation. To test for such a relationship in Canada, measures of one-quarter and multi-period inflation uncertainty were constructed from both autoregressive and reduced-form GARCH models. For one-quarter uncertainty, a strong positive relationship with the level of inflation was found in the autoregressive models. Results from the reduced-form case, which may represent the lower-bound estimates of uncertainty, provide weaker evidence that uncertainty increases at higher levels of inflation. GARCH models with lagged inflation included as a variable in the conditional variance displayed considerable intertemporal variation in multi-period uncertainty.

As discussed in the literature review, the credibility of monetary policy is an important determinant of inflation uncertainty. Thus, one potential area for future research is to consider alternative models of inflation that explicitly represent policy-regime uncertainty. In addition, future estimation over a sample period covering more low-inflation years would help to identify the link between inflation and inflation uncertainty near price stability. Another possible area of study is to evaluate the real effects of inflation uncertainty using the estimates of uncertainty presented in this study.

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Appendix of figures

FIGURE 2
CPI Inflation Excluding Food, Energy and Indirect Taxes

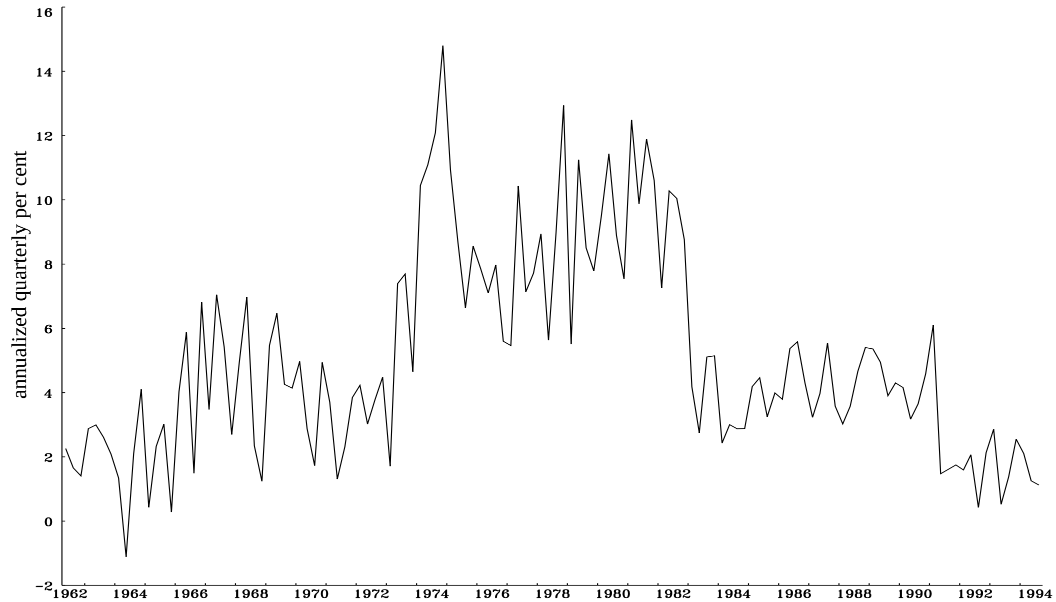


FIGURE 3
Total CPI Inflation (from 1916)

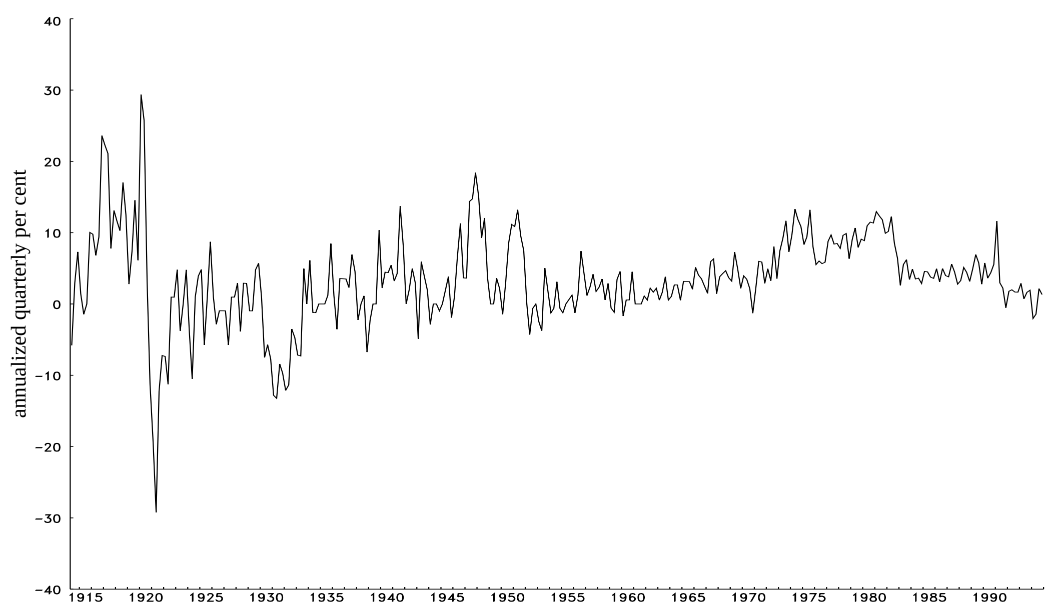
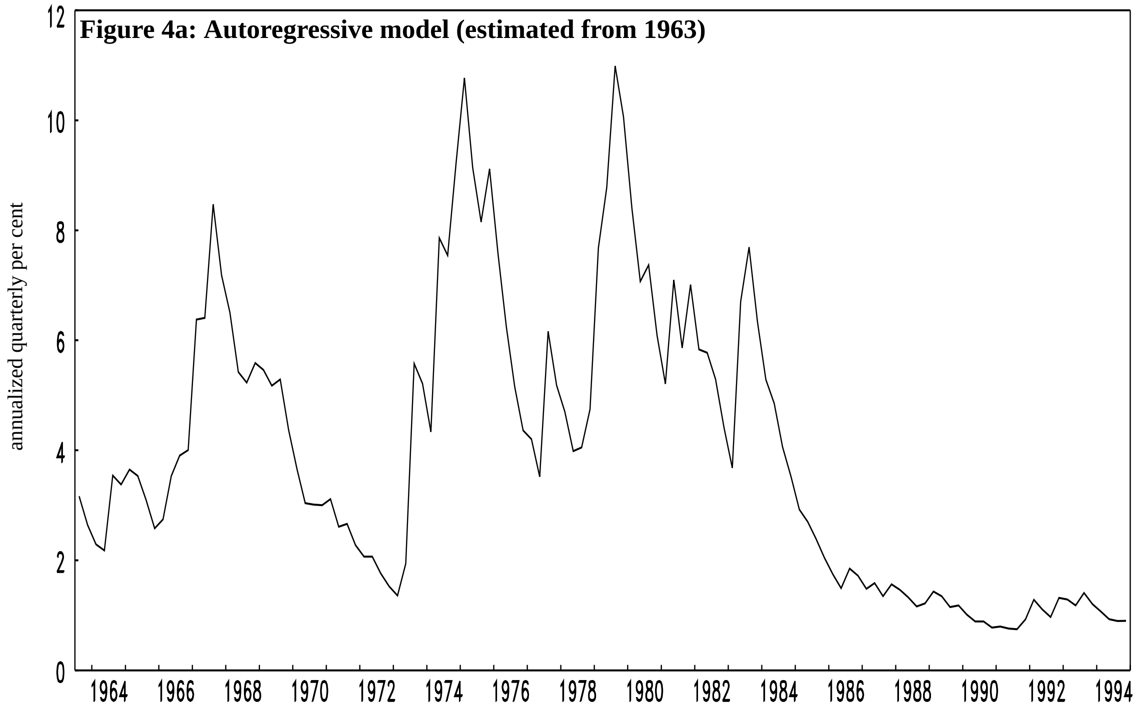


FIGURE 4
One-Quarter Conditional Variance: Autoregressive Models



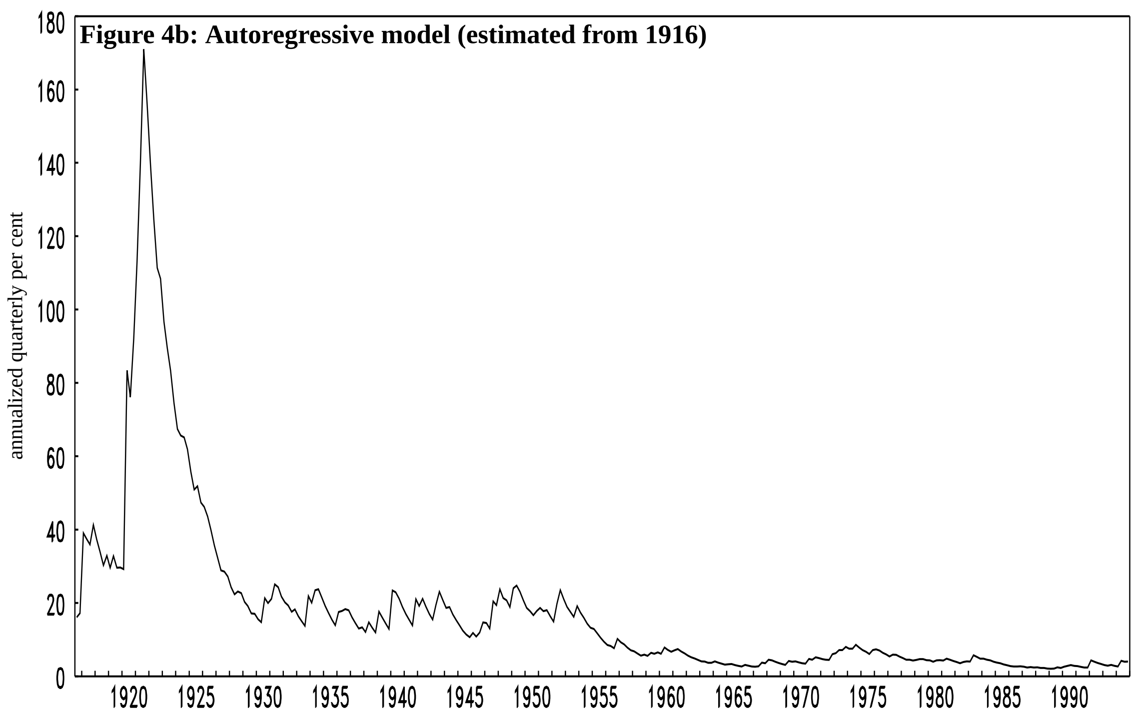


FIGURE 5
One-Quarter Conditional Variance: Reduced-Form Model
(Reduced-form model: estimation period 1963Q3 to 1994Q3)

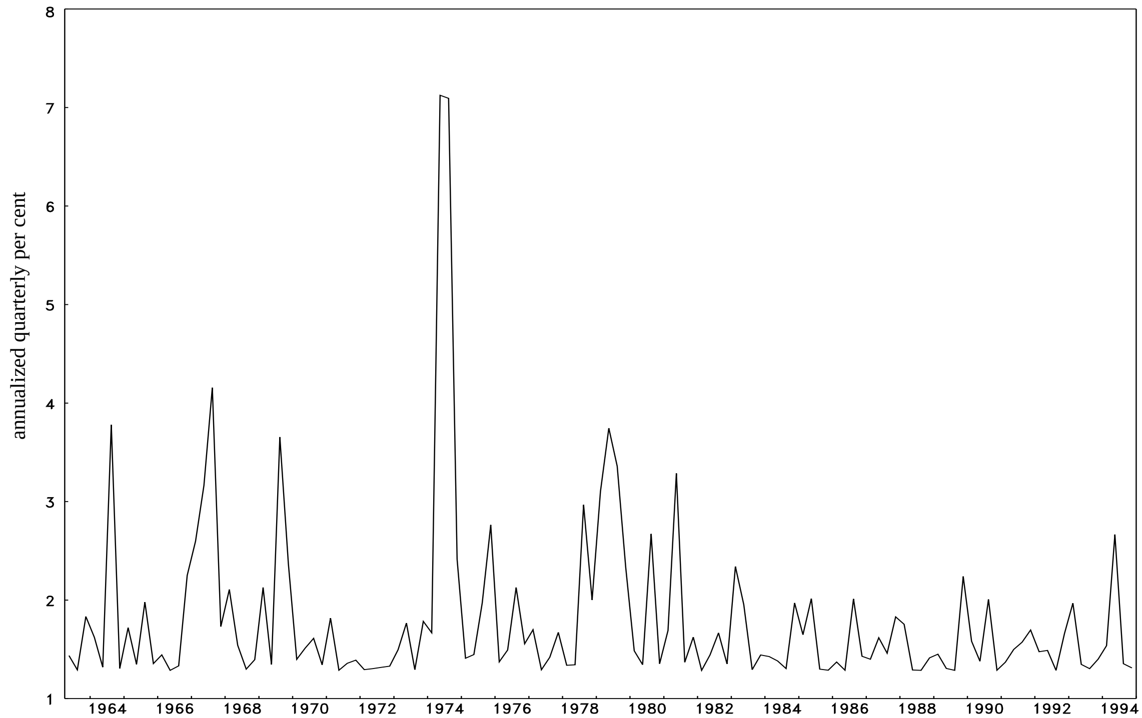


FIGURE 6
Conditional Variance Comparison of GARCH Specifications
(Reduced-form model: estimation period 1963Q3 to 1994Q3)

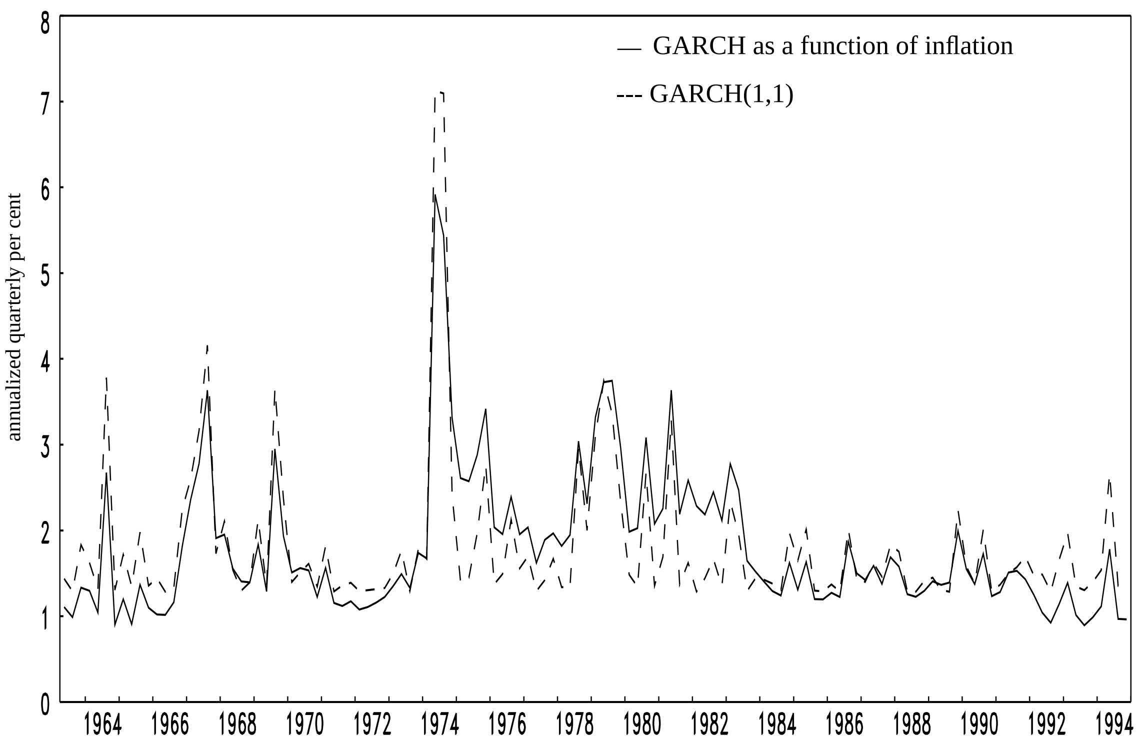


FIGURE 7
Eight-Quarter Expected Inflation from the Markov-Switching Model

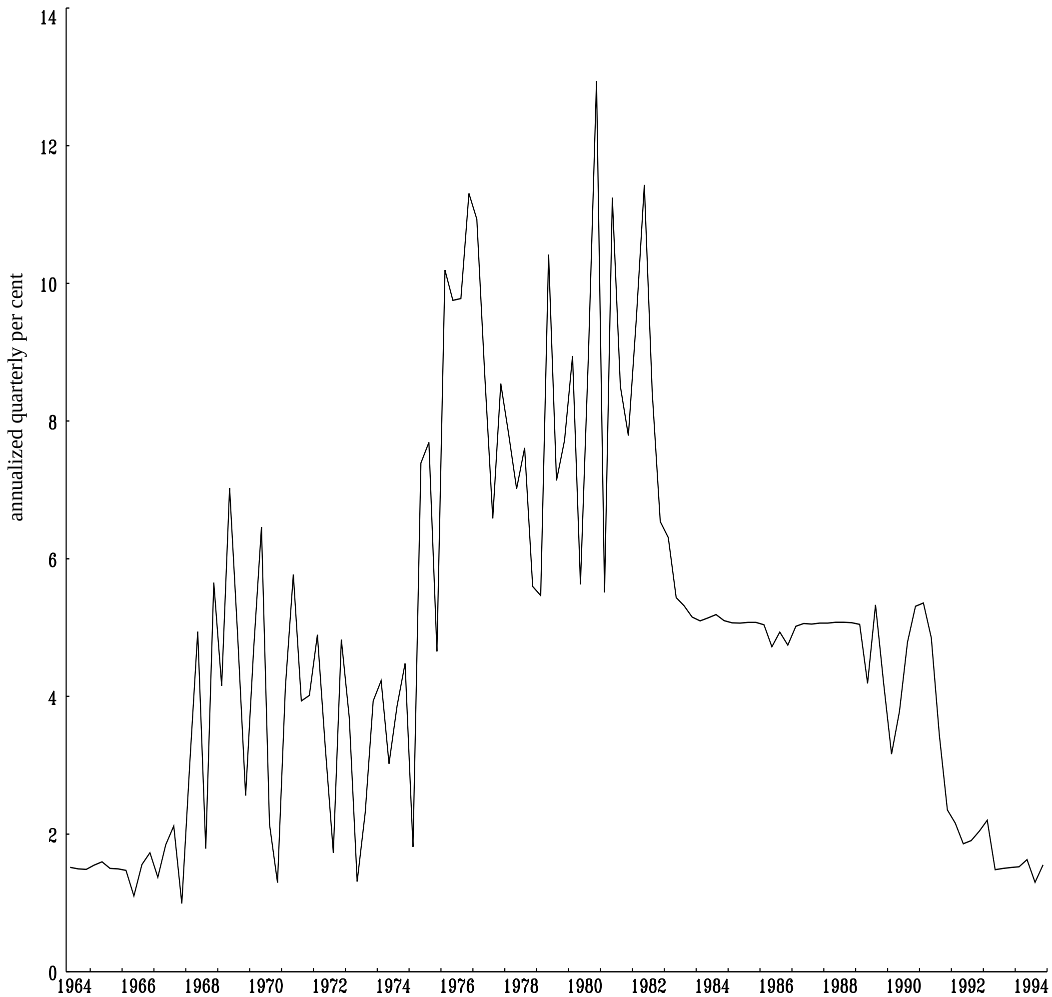
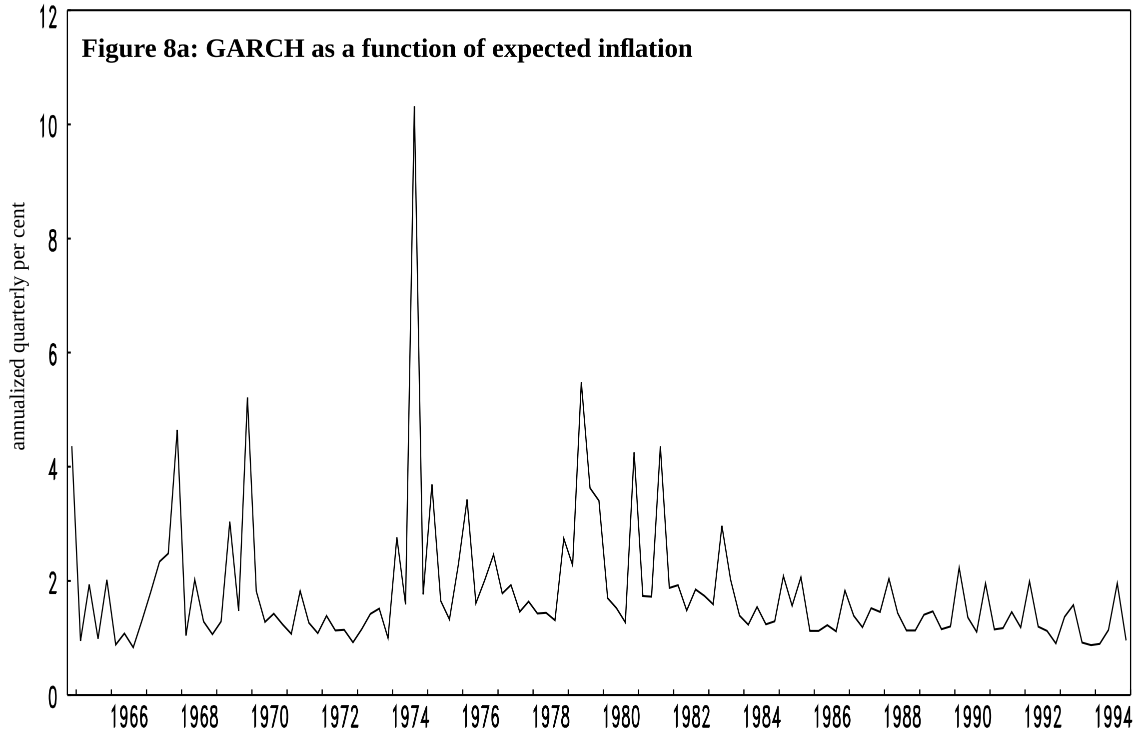


FIGURE 8
One-Quarter Conditional Variance: Reduced Form Models
(Estimation period 1964Q2 to 1994Q3)



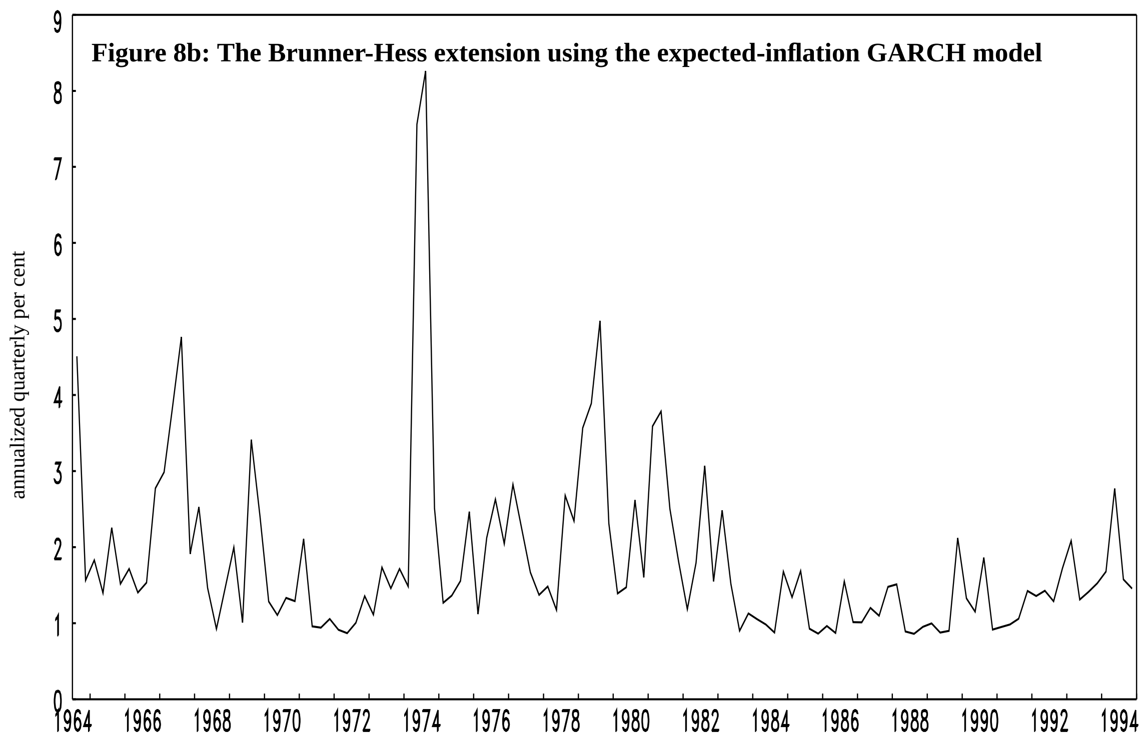


FIGURE 9
Multi-Period Conditional Standard Deviation