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ABSTRACT

This paper reviews selectively the literature on exchange rate target zones and corresponding methodologies and examines whether they can be used to analyse the inflation-control problem. Given the close correspondence between the exchange rate and inflation-targeting problems, the target-zone literature may help us better understand inflation dynamics and central-bank credibility in the presence of inflation-control targets or ranges. While we find many common elements across the exchange rate and inflation-control target-zone problems, we also find several important differences that make a direct application of the exchange rate target-zone techniques difficult.

RÉSUMÉ

Les auteurs examinent quelques modèles tirés des études portant sur les zones cibles de taux de change, dans l'intention de déterminer si ces modèles peuvent servir à l'analyse de stratégies axées sur la maîtrise de l'inflation. Compte tenu de la grande similitude que présentent la poursuite d'objectifs en matière de taux de change et celle d'objectifs en matière d'inflation, ces études pourraient nous aider à mieux comprendre la dynamique de l'inflation ainsi que les facteurs qui concourent à la crédibilité de la banque centrale lorsque cette dernière vise un taux (ou une fourchette de taux) d'inflation donné. Les deux types de stratégies ont de nombreux points en commun, mais les différences importantes que les auteurs relèvent entre elles rendent difficile l'application directe des modèles de zone cible à l'étude des cibles en matière d'inflation.

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1. Introduction

The goal of price stability is becoming increasingly common among central banks. During the 1990s, this policy framework has been implemented through the application of inflation-control ranges or targets. Inflation-control ranges were first adopted in New Zealand in 1990. Canada followed suit in February 1991 and, since then, Israel (1991), the United Kingdom (1992), Australia, Sweden and Finland (1993), and Mexico and Spain (1994) have all announced formal inflation-control targets. At a practical level, inflation-control ranges are thought to reduce the time-inconsistency problem and facilitate communication of monetary-policy actions. However, in choosing to implement inflation-control ranges, central banks face several important challenges.

While a wide control range gives central banks more flexibility and a higher probability of meeting the target, it does not provide a very good guide for inflation expectations or a basis to establish credibility. Indeed, if there is a lack of credibility, the public may easily perceive the upper limit of any range as the actual target and lengthen the process of achieving price stability. Alternatively, a narrow control range may be useful for reducing inflation expectations and increasing the credibility of the central bank. In practice, however, control and forecasting errors may actually reduce credibility if a narrow target range is missed too often. Moreover, commitment to a rigid inflation target may result in dynamic instability in the economy or policy instrument. Against this backdrop, a better understanding of economic behaviour in the presence of inflation-control targets would be useful.

One line of research that may be useful for understanding the inflation-targeting problem is found in the exchange rate targeting literature. The idea behind an exchange rate target zone is simple: the monetary authority announces that it will allow its currency to float but only within a given range. For example, if the Bank of Canada announces that it will allow the Canada-U.S. exchange rate to float +/- 2 Canadian cents around a central parity of Can.$0.73, then the target zone is between Can.$0.71 and Can.$0.75. The Bank of Canada is then committed to intervene whenever the target bands are threatened. At a superficial level, there appears to be a close mapping from the exchange rate target-zone problem to the inflation-targeting one. Both exchange rate and inflation-targeting problems concern a monetary authority faced with the task of maintaining an economic variable between some prespecified bounds. As well, for both sets of problems, credibility is an important factor in the determination of the dynamic path of the variable under consideration. This paper takes a more detailed look at the relation between the two problems. First, we offer a selective survey of the literature on exchange rate target zones.

Second, we discuss whether the modelling strategies used in the target-zone literature can be used to shed light on the issues concerning inflation targeting.

The organization of the paper is as follows. Section 2 summarizes the first generation of exchange rate target-zone models, developed by Krugman (1988; 1991). In an attempt to reconcile the poor empirical performance of the basic model, Section 3 discusses extensions of Krugman (1991). Section 4 considers the symmetry between target zones for inflation and for the exchange rate. Finally, an outline of the implications for modelling inflation-target zones is included in Section 5.

2. The first generation of exchange rate target-zone models

The first generation of exchange rate target-zone models was initiated by Krugman (1988). This model allows us to study how the behaviour of exchange rates is altered by the presence of a credible governmental commitment to defend the limits of a target zone. In the remainder of this section, we describe the original exchange rate target-zone model and how its main predictions have fared against the data.

2.1 The basic target-zone model

For the purposes of this paper, it will be convenient to express the exchange rate, (domestic currency per unit of foreign currency, in logarithms s) as

$$s(t) = f(t) + \theta E_t[ds(t)]/dt. \tag{2.1}$$

The fundamentals that determine the spot price of foreign exchange are summarized by f. $E_t[ds(t)]/dt$ is the rational expectation of the rate of exchange rate depreciation conditional on the time-t information set and is equal to the domestic-foreign interest-rate differential, $i - i^*$, assuming continuous uncovered interest-rate parity. The parameter θ represents the speed at which the spot exchange rate responds to changes in exchange rate expectations. Equation (2.1) is a general representation of the exchange rate. Alternative target-zone models can be distinguished by the way that the fundamentals are modelled, f in equation (2.1). We follow with a description of the basic target-zone model.

The two fundamental variables that determine the exchange rate in the basic model are the log of the money supply, m, and a shift variable that represents velocity shocks,

v.[1] The money supply is assumed to be under the direct control of the monetary authority.[2] The authority is also precommitted to maintaining the exchange rate within a prespecified range $[\bar{s}, \underline{s}]$. The sole source of exogenous exchange rate fluctuations is v. In terms of equation (2.1), the fundamental determinants of the exchange rate at any moment in time are [3]

$$f(t) = m(t) + v(t). \tag{2.2}$$

There are two central assumptions in Krugman's model. First, the precommitted policy is assumed to be perfectly credible so that the exchange rate boundaries are reflecting. That is, agents believe that the target zone will be constant over time and that the exchange rate will remain within its precommitted range. Second, the monetary authority is assumed to alter the money supply only as the exchange rate reaches a boundary of the target zone; otherwise the supply of money is held constant (that is, only marginal interventions are permitted).[4] Thus, the reaction function of the monetary authority can be expressed as

$$m(t) = \mu - U(t) + L(t), \tag{2.3}$$

where μ represents the constant money-supply rule when the exchange rate is inside the target zone, and $\{U(t)\}$ and $\{L(t)\}$ are non-decreasing continuous variables that increase when the exchange rate is at the upper and lower boundaries (respectively); in other words, $\{U(t)\}$ and $\{L(t)\}$ are the regulator processes that ensure $s \in [\bar{s}, \underline{s}]$.

In order to close the model, a stochastic process must be specified for the exogenous velocity term. In this regard, Krugman assumes that v follows a continuous-time random walk (that is, a Brownian motion without drift). The solution technique for

1. Delgado and Dumas (1991) develop a model that offers a structural interpretation for the velocity shocks.

2. The results can be generalized to a monetary authority that controls an interest rate (see Svensson 1992).

3. The basic model can be derived from a flexible-price monetary model that is summarized by three equilibrium conditions: the domestic money market, foreign money market, and purchasing-power parity. Specifying the demand for real money balances as a function of the log of real income (y) and the nominal interest rate (i), the velocity shock can be expressed as $-m(t)^* - (\kappa y(t) - \kappa^* y(t)^*) + (\theta i(t) - \theta^* i(t)^*)$, where κ is the income elasticity of the demand for money, θ is the interest rate semi-elasticity, and * denotes a foreign variable.

4. A third assumption is that the exchange rate is not subject to speculative bubbles.

this regulated Brownian-motion problem is described in Krugman (1991) and Bertola (1994).[5]

Although simplified in its structure, the basic target-zone model yields interesting predictions with respect to exchange rate dynamics. Consider the dynamics of the exchange range in a free-float regime. Recall that, in this case, the authority holds the money supply constant, and the shift variable follows a continuous-time random walk. Then the aggregate fundamental ($f = m + v$) and the exchange rate (s) also follow random-walk processes. Thus, the expected future change in exchange rate fundamentals is zero and equation (2.1) is satisfied. Note that the random-walk assumption for s is consistent with the empirical findings of Messe and Rogoff (1983) who argue that, in terms of out-of-sample forecasting accuracy, the exchange rate is well characterized by a random-walk process.

Figure 1 plots the exchange rate against the aggregate fundamentals.[6] The 45-degree line (FF) summarizes the equilibrium exchange rate for the free-float case: a shock in v leads to a proportionate change in f and s. The dynamics of the exchange rate in a target-zone regime yield two results that are distinct from the free-float regime. The first result is that a credible target zone stabilizes (reduces the variance of) the exchange rate; that is, exogenous shocks have a smaller impact on the exchange rate in a target-zone regime. When the exchange rate is inside the target zone, the stabilization of the exchange rate is costless to the authority in the sense that the policy actions are identical in the free-float and target-zone regimes (the “honeymoon” effect).

The intuition of this result is straightforward. In the absence of any shocks, so that v is always zero, the target-zone and free-float cases share a unique equilibrium at the origin of Figure 1. However, in the presence of stochastic exchange rate fluctuations, the equilibrium path under a free float differs from that of a target zone. Consider the exchange rate as it approaches a boundary. Under a target zone, the likelihood of a future

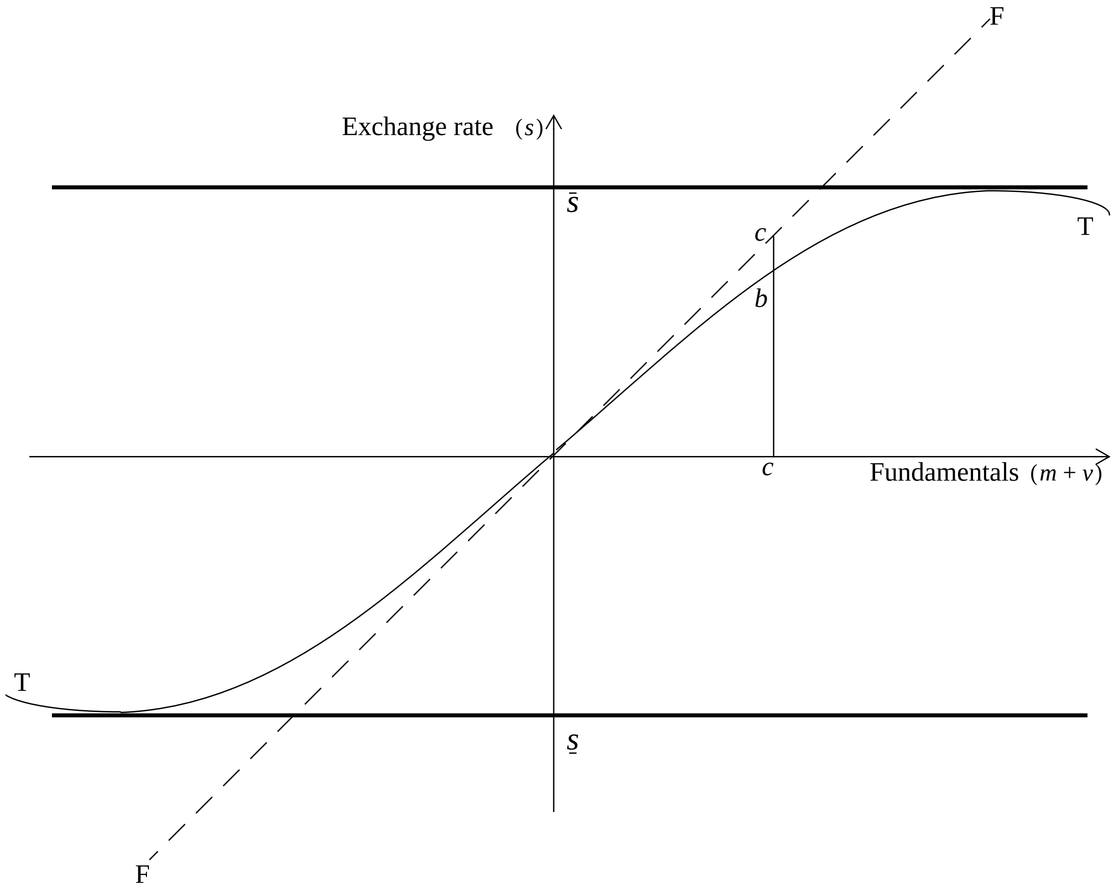
5. More specifically, $dv = \sigma dz$ where σ is a constant representing the standard deviation of the velocity shock and dz is a standard Wiener process. The solution to the model can be represented by $s(t) = f(t) + \theta\mu + A(\exp(\rho v) - \exp(-\rho v))$, where $\rho = (2/\theta\sigma^2)^{1/2}$ and the constant A is uniquely determined by the smooth-pasting condition. The first two terms represent the free-float equilibrium exchange rate. The last term is the non-linearity in the equilibrium exchange rate. For example, the positive root ρ is associated with a fall in the target-zone equilibrium exchange rate relative to the free-float equilibrium. Thus A is negative. Intuitively, a positive root implies that the exchange rate is above central parity. As the exchange rate approaches its upper boundary, agents believe that the regulator process is more likely to be “turned on” following a further increase in the exchange rate. This leads to an instantaneous fall in the equilibrium exchange rate. A symmetric argument applies for a negative root, $-\rho$.

6. For simplicity, the target zone is assumed to be symmetric about zero.

policy intervention increases. Thus, forward-looking agents' expectations of future fundamentals change. That is, unlike the free-float case, the expected rate of change in the exchange rate is non-zero. This expectation is embodied in the spot price of foreign exchange instantaneously and "drags" the target-zone curve off the free-float line.[7] Therefore, the slope of the function relating the exchange rate to the fundamentals in a target-zone regime must be less than the slope in a free-float regime.

FIGURE 1

Exchange rate dynamics in a credible target-zone regime



7. For example, consider a shock that increases the aggregate fundamental from the origin to *c* (see Figure 1). In a free-float regime, the exchange rate increases by the same amount *c*. However, in a target zone, agents recognize the increase in the likelihood of a future contraction in the money supply. Thus, agents expect a future appreciation in the exchange rate. This results in an equilibrium exchange rate that is less than *c*, at *b*.

The second result of the basic target-zone model is that the path describing the equilibrium relationship between the exchange rate and fundamentals must be tangent to the boundaries of the target zone. This is known as the “smooth-pasting” condition.[8] The equilibrium path is smooth-pasted because the expected rate of change in the exchange rate increases continuously as the exchange rate approaches a boundary. Otherwise, an arbitrage opportunity would exist; namely, a one-way bet that the exchange rate will move within the target zone. Consider instead the example where the exchange rate hits a boundary in a linear fashion and never leaves the target zone. This cannot be an equilibrium, since a linear exchange rate path implies that agents’ expectations of future exchange rate changes are constant. In a credible target zone, however, agents’ expectations of a future change in the money supply increase as the exchange rate moves infinitesimally toward a boundary.

The main result of the basic target-zone model is the S-shaped relationship between the exchange rate and the fundamentals given in Figure 1 (TT in Figure 1). In the following subsection, we analyse the empirical performance of this basic target-zone model.

2.2 Empirical performance of the basic model

Although the basic target-zone model yields interesting insights with respect to exchange rate dynamics, the principal predictions of the model have been strongly rejected by the data, primarily in Nordic countries in the European Monetary System. This subsection focusses on the empirical performance of two central testable predictions of the model. First, the basic model predicts a non-linear relationship between the exchange rate and its fundamentals (the S-shaped curve). From the smooth-pasting condition, the spot exchange rate is inversely related to its expected future change. Thus, assuming continuous uncovered interest-rate parity, the second prediction of the basic model is a deterministic negative relationship between the spot exchange rate and the interest-rate differential.

Flood, Mathieson, and Rose (1991) investigate whether there is a non-linear relationship between the exchange rate and its fundamentals across six members of the EMS. Assuming uncovered interest-rate parity and specifying a value for the parameter θ

8. Dumas (1991) notes that “smooth pasting” is the condition required when a choice of bands is optimal for some value function. Thus, he argues that a more appropriate term is “value matching,” since the condition matches the value of the exchange rate at values infinitely close to the bands.

in the exchange rate equation, an estimate of aggregate fundamentals is obtained. From equation (2.1), $\hat{f}(t) = s(t) - \hat{\theta}(i - i^*)$. The results of graphical analyses, statistical tests, and out-of-sample forecasting analyses do not suggest the presence of any economically meaningful non-linearities. Messe and Rose (1991) argue that incorporating non-linear effects into the exchange rate equation is not statistically important.[9] Chinn (1991) and Diebold and Nason (1990) use non-parametric approaches to test whether there are any non-linearities in exchange rate behaviour; while some evidence is found in favour of non-linearity, the results are not overwhelming. In short, there is little empirical evidence to support the non-linear relationship between the exchange rate and its fundamentals.

The basic target-zone relationship also predicts a deterministic negative relationship between the spot exchange rate and the interest-rate differential. The intuition behind this is as follows. When the exchange rate is at the upper exchange rate band, domestic currency is weak but will not depreciate further. The exchange rate can either remain at the boundary or drift back to the interior of the range, in which case the currency appreciates. So long as the probability of returning to the interior of the range is greater than zero, the expected rate of currency depreciation (Δs) is negative. Since capital is assumed to be perfectly mobile, this leads to a negative domestic-foreign interest-rate differential. With respect to this prediction, Flood, Mathieson, and Rose (1991) find no clear relationship between the spot exchange rate and the interest-rate differential using graphical analyses and correlation coefficients. More recently, this conclusion has been confirmed by Lindberg and Soderlind (1994) for Swedish data. Again the empirical evidence appears to reject another prediction of the basic target-zone model.

With the goal of reconciling the basic model with the data, subsequent research has incorporated several extensions to the basic model. In the following section, we briefly discuss some extensions and their empirical success.

3. Extensions of the basic model

The pursuit of reconciling the basic target-zone model with the data has followed two general paths. The first path maintains the underlying structure of the basic model but relaxes the principal assumption of perfect credibility. This line of research is motivated by the observation that forward exchange rates are often outside the precommitted target

9. The more general case of estimating non-linear exchange rate models, to which both target-zone and speculative-bubble models apply, is developed by Smith and Spencer (1992).

zone. This is clearly not a second-order issue in the basic model. It is the assumption of perfect credibility that drives the stabilizing features of a target zone. The second path is motivated by the excessively simple structure underlying the basic model. This line of research integrates target zones with alternative underlying economic models, such as sticky-price models. We briefly summarize these two paths below.[10]

3.1 Target-zone models with band realignment: The case of imperfect credibility

Bertola and Caballero (1992) argue that observed target-zone realignments, such as the French franc-German mark exchange rate under EMS, are inconsistent with perfectly credible boundaries. Flood, Mathieson, and Rose (1991) and Svensson (1991b) provide additional evidence to support this view. For instance, Svensson (1991b) argues that the Swedish exchange rate target zone never had credibility within a 5-year horizon and occasionally lacked credibility within a 12-month horizon. This argument is based primarily on the observation that forward rates did not lie within the target zone. In response, Miller and Weller (1991), Bertola and Caballero (1992), and Bertola and Svensson (1993) develop models where the probability of a shift in the central parity of the exchange rate is different from zero. We begin with an intuitive discussion, based primarily on Bertola and Caballero (1992), and then move to a more formal treatment of the model, based on Bertola and Svensson (1993).

Consider the following simple exposition of a model with imperfect credibility. To relax the assumption of perfect credibility, market participants must be uncertain whether the central bank will defend a target zone as the exchange rate reaches a boundary. A priori, suppose that the market believes with probability p that the central bank will intervene and defend the target zone and with probability $1-p$ that it will not defend the zone.[11] At an exchange rate boundary, the central bank chooses to defend or not. If the central bank chooses to defend, the size of the intervention is such that the exchange rate returns to central parity. Conversely, in choosing not to defend, the size of the intervention is such that the new central parity is twice the old parity. Thus, for simplicity, the size of intervention is symmetric about the decision to defend. The difference between the basic

10. Another line of research relaxes the assumption of marginal interventions. For example, Klein and Lewis (1991) allow the central bank to follow a non-constant money-supply rule within a target zone and illustrate that, depending on the aggressiveness, intramarginal interventions increase the stabilizing features of a credible target zone. This result, however, is driven by the assumption of perfect credibility.

11. Although we present this probability as constant, one can imagine a scenario where market participants update this probability over time.

model and the imperfect credibility extension can be summarized by the following monetary-policy reaction function:

$$m(t) = \mu + p[-\hat{U}(t) + \hat{L}(t)] + (1-p)[\hat{U}(t) - \hat{L}(t)], \tag{3.1}$$

where $\{\hat{U}(t)\}$ and $\{\hat{L}(t)\}$ are non-decreasing continuous variables that increase when the exchange rate is at the upper and lower boundaries. With probability *p*, the exchange rate returns to central parity, while with probability 1-*p*, the central bank realigns central parity.

In this model, the exchange rate dynamics obviously depend on the probability of intervention. Bertola and Caballero (1992) describe these dynamics by varying the probability of the central bank defending.[12] First, consider the case where *p* is equal to one. In this case, the imperfect-credibility model is identical to the basic model, with the exception that intervention in the former is discrete. The curve TT in Figure 2 illustrates this case. In the opposite extreme, suppose *p* is equal to zero. In this case, markets believe that the current target zone will be abandoned once the exchange rate reaches a boundary. The likelihood that the central bank will realign central parity increases as the exchange rate deviates from central parity. Forward-looking market participants therefore accelerate the shift in central parity and the target zone is destabilizing. The curve DD in Figure 2 illustrates this case. Finally, suppose that the central bank randomizes between the decision to defend so that *p* is equal to one-half. According to the central bank's reaction function, the expected change in the money supply in this case is equal to the constant money-supply rule, μ.[13] This is the same central-bank rule followed in the free-float case of the basic model. Thus, the line FF in Figure 2 corresponds to the randomizing case.

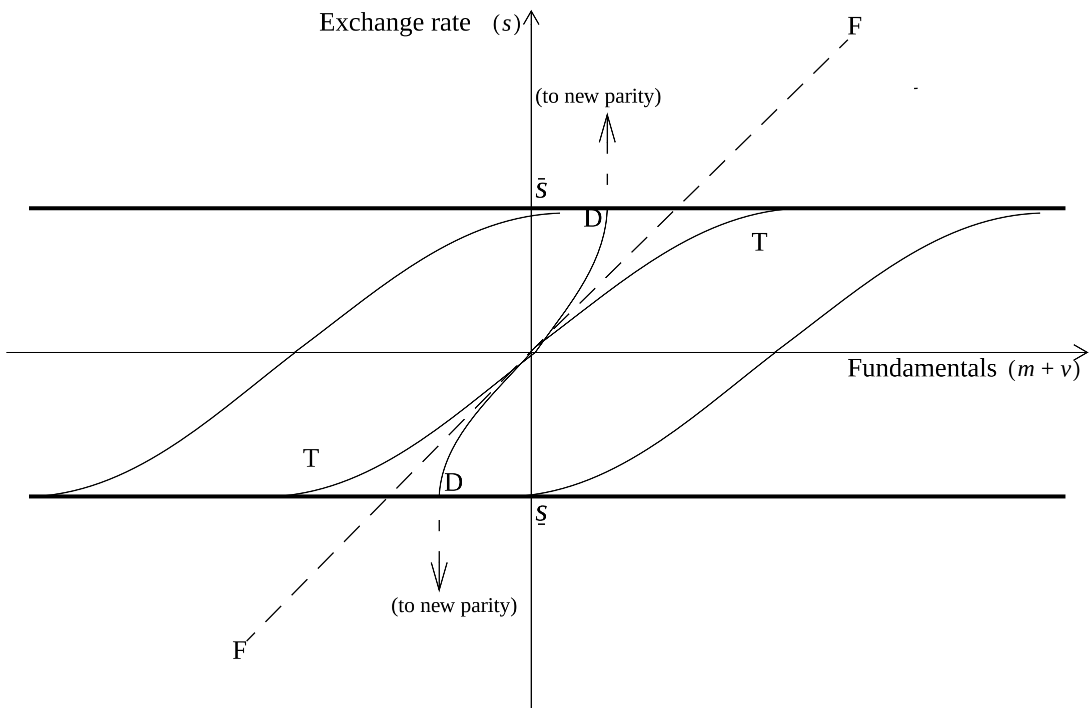
Clearly, the stabilizing features of a target zone under imperfect credibility depend on whether or not the market believes the policy precommitment. Thus, in contrast to the honeymoon effect, a target zone can have destabilizing features under the assumption of imperfect credibility (a "divorce" effect).

12. The technical details of this model map to the basic model quite closely. See Bertola and Caballero (1992) for this exposition.

13. This result is clearly attributable to the assumption that a central-bank intervention is symmetric about the decision to defend. The qualitative features of the model are not dependent on this assumption.

FIGURE 2

Exchange rate dynamics under imperfect credibility



As an alternative means of addressing the issue of imperfect credibility, Bertola and Svensson (1993) extend the basic target-zone model by incorporating stochastic realignment risk (that is, stochastic jumps in central parity) into the model.[14] In this model, stochastic realignment risk is the risk of a shift in the exchange rate bands, that is, a "jump" in central parity. Unlike the basic model, the rate of change expected in the exchange rate by forward-looking agents has two components: the expected rate of change in the exchange rate within the target zone or the expected deviation of the exchange rate from central parity, and the expected rate of change in central parity or the expected rate of realignment.

A more formal treatment of the Bertola-Svensson model is straightforward. It will be useful to define central parity as c and the deviation of the exchange rate from central parity as x. Note that $s(t) \equiv x(t) + c(t)$. Then, defining the expected instantaneous rate of

14. In the Bertola-Caballero model, realignments can occur only at an exchange rate boundary. However, in the Bertola-Svensson model, realignments are independent of the deviation of the exchange rate from central parity.

change in the exchange rate within the target zone as $E_t[dx(t)]/dt$ and the expected instantaneous rate of change in central parity as $E_t[dc(t)]/dt$, the expected instantaneous rate of change in the exchange rate is [15]

$$E_t[ds(t)]/dt = E_t[dx(t)]/dt + E_t[dc(t)]/dt. \tag{3.2}$$

When one substitutes equation (3.2) into equation (2.1), the exchange rate can be expressed as

$$x(t) = h(t) + \theta E_t[dx(t)]/dt, \tag{3.3}$$

where the aggregate fundamental is $h(t) = f(t) - c(t) + \theta E_t[dc(t)]/dt$. The representation of this model is similar to the basic model: deviations of the exchange rate from central parity, $x(t) \equiv s(t) - c(t)$, are explained by the aggregate fundamental and the expected instantaneous rate of change in the deviation of the exchange rate from central parity. However, in this model, the aggregate fundamental is not continuous because c follows a jump process. Thus, unlike the basic model, there are two exogenous sources of variance in the exchange rate: velocity shocks and shifts in central parity.

In order to close the model, a stochastic process must be specified for the two state variables, v and $E_t[dc(t)]/dt$. As in Krugman (1991), for analytic convenience, v is assumed to follow a continuous-time random walk.[16] The expected rate of change in central parity is equal to the probability of a shift in central parity multiplied by the expected size of the shift. Between realignments, the stochastic component of $E_t[dc(t)]/dt$ is also assumed to follow a continuous-time random walk.[17]

There are four central predictions of the Bertola-Svensson model, two that are similar to those of the basic model and two that are novel. First, the Bertola-Svensson

15. In the basic target-zone model, $E_t[ds(t)]/dt = E_t[dx(t)]/dt$.

16. While there are two state variables, the representation of the exchange rate in (3.3) is summarized by one state variable, the aggregate fundamental. The solution to the model, analogous to Krugman (1991), is represented as $s(t) = f(t) + \theta\mu + A(\exp(\rho k) - \exp(-\rho k))$, where ρ is a characteristic root consistent with an equilibrium exchange rate, $k = v - c + \theta E_t[dc]/dt$, and the constant A is uniquely determined by the smooth-pasting condition (see Bertola and Svensson (1993)).

17. Werner (1995) develops an alternative realignment model where the probability of realignment is an increasing function of the deviation of the exchange rate from central parity. The stabilization properties associated with the target zone are inversely related to the band width. In contrast, Svensson (1991a) finds that interest-rate differentials predict reversion of the exchange rate toward central parity for Sweden. This mean-reversion property implies that the realignment risk is independent of the position of the exchange rate in the target zone.

model predicts that, within a target zone, the exchange rate displays reversion towards central parity; and second, for a given rate of change in central parity, the relationship between the exchange rate and the fundamentals is non-linear. The two novel predictions are that exchange rate target zones can, in some instances, be destabilizing and that the negative relationship between the expected deviation of the exchange rate from central parity and the interest-rate differential is subject to stochastic shifts in central parity. This differs from the negative deterministic relationship predicted by the basic model.

Empirically, there are two interesting applications of the model. First, the model is able to reconcile the relationship between the spot exchange rate and interest-rate differentials. For example, Lindberg and Soderlind (1995) estimate a model with imperfectly credible bands and intramarginal interventions for Sweden. They argue that the positive correlation between the spot exchange rate and interest-rate differential can be explained by expected shifts in central parity. Second, an estimate of the probability of realignment is a straightforward way of quantifying the credibility of a policy precommitment. This estimate can be interpreted as agents' expectations that the current target zone will collapse. For example, Rose and Svensson (1991) estimate the expected rate of realignment for the French franc-German mark exchange rate during EMS. Conditional on a prespecified size of realignment, the authors also estimate a probability of realignment. Such an application is particularly attractive in the case of inflation, where relatively few quantitative measures of credibility exist.

3.2 Target-zone models with sticky prices

The second path for reconciling the basic model with the data integrates target zones with alternative underlying economic models. Miller and Weller (1991) deviate from the monetary model of fundamentals, relying instead on a model where prices respond sluggishly to current excess demand. The specific model that is used is a stochastic version of Dornbusch's (1976) overshooting model. The advantages of the sticky-price approach over the basic model are that it describes the dynamics of both output and real exchange rates as well as nominal exchange rates. In addition, the sticky-price model allows one to distinguish between the effects of imposing real and nominal exchange rate bands.

To achieve an understanding of the underlying structure of the sticky-price target-zone model, we start by comparing it with the basic model. Recall that the basic model gives rise to an exchange rate equation of the form

$$s(t) = m(t) + v(t) + \theta E[ds(t)]/dt, \tag{3.4}$$

where the velocity term, $v(t)$, is assumed to follow a Brownian-motion process, and the log of the money supply, $m(t)$, is used as a regulator by the central bank. In contrast, the exchange rate equation in the sticky-price model can be written as

$$s(t) - p(t) + p(t)^* = e(t) = \theta_1 M(t) + \theta_2 E[de(t)]/dt, \tag{3.5}$$

where $M = m - p$ and $dM(t) = (\gamma_1 M(t) + \gamma_2 e(t))dt + \sigma dz$. Notice that equations (3.4) and (3.5) are essentially identical. The important difference comes from the process driving the money supply. In the basic model, the money supply is determined by the monetary authority. In contrast, in the sticky-price model, the money supply is a function of an endogenously determined drift term that is dependent upon both its own current value and the (log of the) real exchange rate. Thus, within the sticky-price model, the monetary authority can impose a real exchange rate target zone by announcing suitably chosen upper and lower limits for M at which discrete adjustments to m occur.

Interestingly, Miller and Weller (1991) find the same type of stabilizing impact as Krugman (1991). One important distinguishing feature, however, is the implications for policy actions. To generate mean reversion in the basic model, it is necessary to assume that the monetary authority undertakes intramarginal interventions that push the exchange rate towards central parity. Given the structure of the basic model, it is necessary to assume that the intervention is unsterilized; otherwise it would have no effect on the money supply or the exchange rate. In the sticky-price model, mean reversion is generated by the price-adjustment process, so there is no need to make such assumptions about policy actions.

Unfortunately, there is very little empirical work testing the quantitative predictions of the sticky-price model. There is, however, some qualitative evidence which suggests that the sticky-price model is more consistent with the data than the basic model. Sutherland (1994) shows that the sticky-price model can give rise to an exchange rate distribution with a central hump, whereas the basic model is unable to generate such a distribution. Also, similar to the case of imperfect credibility, the model generates a noisy relationship between interest-rate differentials and the exchange rate. This result is not very surprising, since Sutherland (1994) shows that, in terms of nominal variables, the sticky-price and basic models are observationally equivalent when the basic model is augmented with marginal intervention and stochastic probabilities of realignments.

4. The symmetry between target zones for the exchange rate and for inflation

It would be surprising if there were not a relationship between target zones for inflation and for the exchange rate. After all, they both have the monetary authority using a policy instrument to keep an economic variable between a set of bands. Somewhat surprisingly, this symmetry has not been extensively studied (one exception is Gerlach 1994). In this section, potential relationships between inflation-targeting problems and exchange rate targeting problems are explored.

4.1 A simple model of inflation targeting

To provide some concreteness on how the literature on exchange rate ranges can be applied to inflation targeting, this section presents a simple example. This both illustrates the flavour of the results that will come from treating the inflation-targeting problem as one similar to exchange rate bands, and suggests some areas for future research. It should be emphasized at the outset that the example is very basic and, while it provides some valuable insights, it should not be taken too seriously. To begin, a simple model is developed and solved for given inflation ranges. This results in a S-shaped function that corresponds to Figure 1 in Section 2. Following this, the optimal width of the inflation range is considered.

In this example, inflation is modelled from Cagan's (1956) model of money demand. Following Blanchard and Fischer (1989, Chapter 4), the demand for real money balances is given by

$$m(t) = \frac{M(t)}{P(t)} \propto e^{-\alpha \mathrm{E}_t \pi(t)}, \tag{4.1}$$

where m is real money balances, M is nominal money balances, P is the price level, α is the elasticity of money demand to expected inflation, and π is inflation. The derivation of this model assumes constant output growth and real interest rate, so the elasticity parameter can be thought of as the elasticity of money demand with respect to the nominal interest rate. Taking logs and differentiating with respect to time gives

$$\pi(t) = \mu(t) + a\mathrm{E}_t\frac{\partial}{\partial t}\pi(t), \tag{4.2}$$

where μ is the growth in the money supply, which is assumed to follow a Wiener process. That is:

$$\mathrm{d}\mu = 0\mathrm{d}t + \sigma\mathrm{d}z + \mathrm{d}L - \mathrm{d}U, \tag{4.3}$$

using the notation described in Section 2.[18] Note that the zero drift in the growth rate of the money supply is compatible with a target inflation rate of zero (this assumption is made for simplicity). Thus, it is assumed that the monetary authority regulates the growth in money supply so it lies in the interval $[-\beta, \beta]$. As in the exchange rate case, this will ensure that inflation also lies within some range—the monetary authority chooses β so that the width of the inflation range is what they want.

This system of equations can be solved, subject to a value-matching (what Krugman (1991) calls a smooth-pasting) boundary condition, for inflation as a function of the growth in the money supply. The resulting function exhibits the “S” shape of the exchange rate literature described in Section 2.

It is useful to put some numbers into the problem. The variance of the money supply is chosen so that changes in inflation would have a standard deviation of about 1 percentage point per year ($\sigma = 0.01$) if there were no intervention.[19] The range width for the growth rate of the money supply is set to plus or minus 8.5 percentage points ($\beta = 0.085$), which leads to inflation being between plus or minus 1 percentage point of the target.[20] The elasticity of money demand to changes in inflation expectations is given

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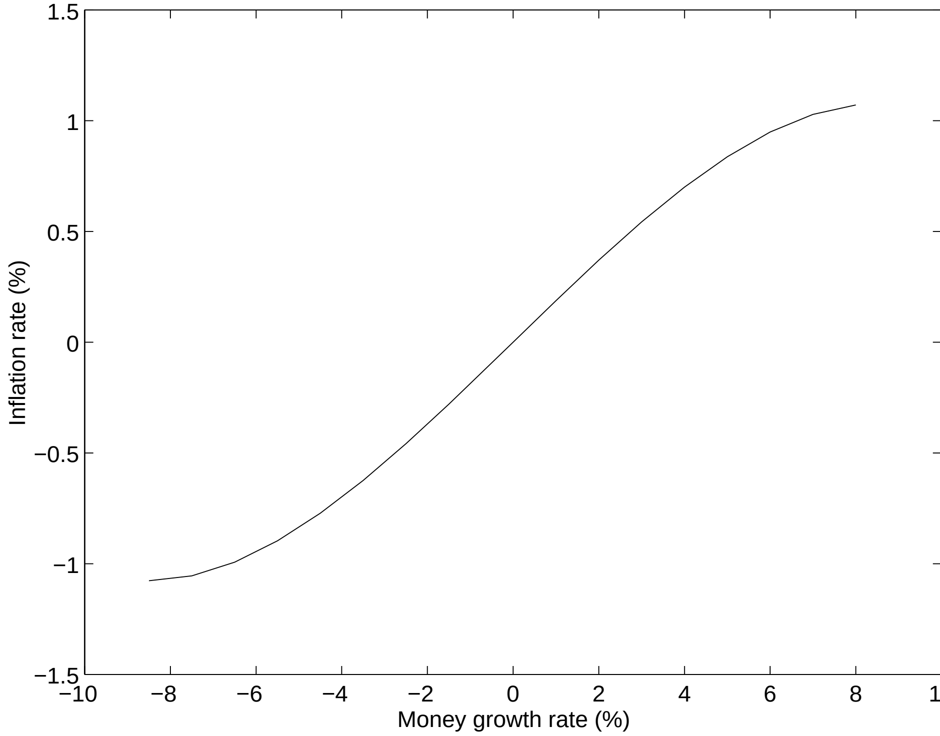
18. Note that, by design, this model is very similar to those used in the exchange rate target-zone literature.

19. This is less than, say, the standard deviation of inflation (CPIXFE) about its trend obtained from using the HP filter. This gives a standard deviation of about 1.3 percentage points. It is also less than the historical standard deviation of changes in inflation, which is about 3.5 percentage points per year.

20. Historically, the standard deviation of the growth rate of M1 divided by the CPIXFE has been about 5.3 percentage points. Around its (HP) trend, this falls to about 4.3 percentage points.

FIGURE 3

Inflation with a money growth bandwidth of 8.5 per cent



by $\alpha = 0.31$ (the elasticity of money demand to nominal interest rates in Black, Macklem, and Poloz 1994). Figure 3 illustrates the "S" under this calibration.

One of the questions that is important for a central bank is optimal range width. This topic is pursued in the remainder of this subsection.[21]

To consider questions of optimality, it is necessary to specify a loss function and then choose instruments to minimize this loss. It is assumed that the central bank seeks to minimize the variance of inflation plus the cost incurred when the ranges are hit. This cost is assumed to be proportional to the changes in the money supply necessary to keep inflation within the ranges. That is, the central bank chooses the range width to minimize

$$L(\mu;\beta) = E_t[(\int_t^\infty \pi^2 e^{-\rho s} ds + \int_t^\infty c e^{-\rho s}(dL_s + dU_s)) \mid \mu_t = \mu]. \tag{4.4}$$

21. This section draws on Dumas (1991) and Miller and Zhang (1994).

In equation (4.4), the first integral is the loss associated with the variance of inflation and the second is the loss from hitting the bands. The constant c is the (proportional) cost of intervention.

This loss function is different from others commonly used when considering optimal control of a monetary economy. Often, for example, the variance of output is considered. Nevertheless, the loss function does have an intuitive basis—the monetary authority is penalized for variance in the inflation rate as well as hitting the bands.

The first stage of solving the loss function, (4.4), is to calculate inflation as a function of money growth and substitute this for π. This function is simply the "S" described above. The next stage is to solve for the function L. It can be shown (for example, in Malliaris and Brock 1982) that this loss function satisfies the Hamilton, Jacobi, and Bellman equation:

$$\rho L = \frac{1}{2}\sigma^2 L_{\mu\mu} + \pi^2 . \qquad (4.5)$$

This, together with the boundary condition

$$L'(-\beta;-\beta) = L'(\beta;\beta) = c , \qquad (4.6)$$

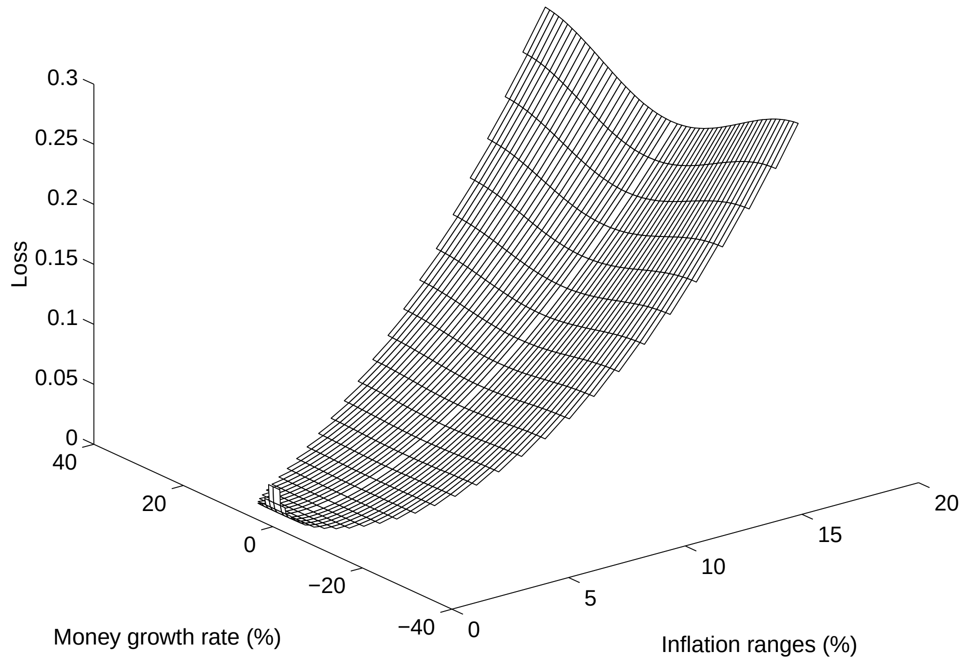
provides enough information to solve (4.4) for a given range width. The boundary condition (4.6) is a value-matching condition and implies that at the bands the change in the loss function is exactly c, ruling out any discrete jumps in the money-supply process—a discrete jump would imply a change in the loss function greater than c as the cost of hitting the bands is proportional to the size of any jump.

The third and final stage is to consider the range width that minimizes the loss function just derived. Denoting the solution to (4.5) subject to (4.6) by $L^*(\mu;\beta)$, then one way to consider finding the optimal range width is to simply plot L^* for different values of μ and β. This is done in Figure 4 with the proportional cost set to 0.005 ($c = 0.005$) and a discount rate of 5 per cent ($\rho = 0.05$).[22] The proportional cost is chosen so that the optimal inflation-range width is 1 percentage point, and the discount rate is similar to others used in the literature. The height of each point in Figure 4 represents the expected

22. A simple finite-difference method is used to solve both for inflation as a function of money growth and for the loss function as a function of money growth. Miller and Zhang (1994) provide a closed-form expression for the corresponding case of exchange rate targeting.

FIGURE 4

The loss function for different range widths



loss given the current value of monetary growth and the width of the range. Each line for a given range width is U-shaped, reflecting the fact that, when monetary growth (and hence inflation) is close to exceeding the range, the expected loss is higher.

A general property of the solution to this kind of problem is that the optimal range width is the same regardless of the value of money growth.[23] Put another way, the monetary authority has no incentive to change the width of the range as the inflation rate approaches them.

Under these conditions, the loss function can be easily minimized numerically. This occurs when β is 0.085 and inflation lies within 1 percentage point of its target. Clearly, the "optimality" result hinges on the judicious choice of c. As c increases, so that the cost of hitting the bands relative to the cost of inflation variability increases, the optimal range width, not surprisingly, increases. Alternatively, Dumas (1991) shows that the loss function is minimized when the changes in the band width do not affect the loss

23. See Dumas (1991).

function at the boundary; that is, when

$$\left. \frac{d}{d\mu} L^{*}(\mu;\beta) \right|_{\mu = \beta} = 0 . \tag{4.7}$$

Dumas (1991) refers to (4.7) as a smooth-pasting condition.

As noted in the opening comments of this section, this example of inflation targeting is very stylized and not meant to be taken seriously from a policy perspective. Nevertheless, it illustrates how the literature on exchange rate target-zone models can be applied to learn more about inflation-control ranges.

5. Implications for future research

Modelling exchange rate target zones and inflation-rate ranges have much in common: they both deal with a monetary authority regulating a policy variable to keep a random-target variable within some fixed range. Given that the techniques used to model exchange rate bands are simple and elegant, their application to research aimed at better understanding inflation-target ranges appears a sensible and logical step. However, despite the similarity between the two questions, there are fundamental differences that make the application of the exchange rate target-zone literature to inflation-rate ranges difficult.

First, modelling the monetary authority's control of inflation as a continuous process is not without its problems. Incorporating lags, for example, is difficult in this set-up. As well, the assumption that the monetary authority is able to control inflation instantaneously is tenuous at best. Similarly, the instantaneous equilibrium adjustment assumed in the structure of the basic model precludes the distinction between short-run and long-run reactions to monetary-policy actions. Although monetary policy may affect foreign exchange markets instantaneously, this assumption is more difficult to motivate in the case of inflation. Nonetheless, in the literature reviewed in this paper, the alternative underlying structures of the exchange rate do not appear to change the fundamental message of target zones.

Second, it is arguably inappropriate to model inflation bands as reflecting barriers which inflation never crosses. Unanticipated large shocks may warrant the monetary authority to allow inflation to leave the range on occasion. Indeed, the inflation range announced in New Zealand are subject to contingency clauses. If this is deemed to be a sufficiently serious problem, there are other techniques better suited, such as standard control theory.

Despite these difficulties, the exchange rate target-zone literature does offer some very interesting ideas about bands in general. In particular, the principle of “honeymoon” and “divorce” effects are likely to apply with inflation ranges. Such effects depend critically on agents’ expectations of the likelihood that the central bank will defend a pre-announced target zone. Empirically, an estimate of this likelihood appears to be a suitable candidate for a measure of central-bank credibility.

In sum, avenues of future research can rely on the exchange rate target-zone literature to understand both how the economy may change with the introduction of inflation ranges and how credibility may be measured.

References

Bertola, Giuseppe. 1994. “Continuous-time Models of Exchange Rates and Intervention.” In *Handbook of International Macroeconomics*, edited by Fredrick van der Ploeg, Cambridge: Blackwell, 251-298.

Bertola, Giuseppe and Ricardo Caballero. 1992. “Target Zones and Realignments.” *American Economic Review* 82: 520-536.

Bertola, Giuseppe and Lars Svensson. 1993. “Stochastic Devaluation Risk and the Empirical Fit of Target-Zone Models.” *Review of Economic Studies* 60: 689-712.

Black, Richard, Tiff Macklem and Stephen Poloz. 1994. “Non-Superneutralities and Some Benefits of Disinflation: A Quantitative General-Equilibrium Analysis.” In *Economic Behaviour and Policy Choice Under Price Stability*, 477–516. Proceedings of a conference held at the Bank of Canada, Ottawa, Canada, October 29-31, 1993.

Blanchard, Olivier J. and Stanley Fischer. 1989. Lectures on Macroeconomics. Cambridge: MIT Press.

Cagan, Phillip. 1956. “The Monetary Dynamics of Hyperinflation.” In *Studies in the Quantity Theory of Money*, edited by Milton Friedman, 23–117. Chicago: University of Chicago Press.

Chinn, Menzie. 1991. “Some Linear and Nonlinear Thoughts on Exchange Rates.” *Journal of International Money and Finance* 10: 214-230.

Delgado, Francisco and Bernard Dumas. 1992. “Target zones, broad and narrow.” In *Exchange rate targets and currency bands*, edited by Paul Krugman and Marcus Miller, 35–56. Cambridge: Cambridge University Press.

Diebold, Francis and James Nason. 1990. “Nonparametric Exchange Rate Prediction?” *Journal of International Economics* 28: 315-332.

Dornbusch, Rudiger. 1976. “Expectations and Exchange Rate Dynamics.” *Journal of Political Economy* 84: 1161-1176.

Dumas, Bernard. 1991. “Super Contact and Related Optimality Conditions.” *Journal of Economic Dynamics and Control* 15: 675-685.

Flood, Robert and Peter Garber. 1983. “A Model of Stochastic Process Switching.” *Econometrica* 51: 537-551.

Flood, Robert P., D. J. Mathieson, and Andrew K. Rose. 1991. “An Empirical Exploration of Exchange Rate Target-Zones.” *Carnegie-Rochester Series on Public Policy* 35: 7-65.

Gerlach, Stefan. 1994. “On the Symmetry Between Inflation and Exchange Rate Targets.” *Economic Letters* 44: 133-137.

Klein, Michael and Karen Lewis. 1991. “Learning About Intervention Target Zones.” Working Paper No. 3674. NBER

Krugman, Paul. 1988. “Target Zones and Exchange Rate Dynamics.” Working Paper No. 248. NBER

——————. 1991. “Target Zones and Exchange Rate Dynamics.” *Quarterly Journal of Economics* CVI: 669-682.

Lindberg, Hans and Paul Soderlind. 1994. “Testing the Basic Target Zone Model on Swedish Data: 1982-1990.” *European Economic Review* 38: 1441-1469.

——————. 1995. “Intervention Policy and Mean Reversion in Exchange Rate Target Zones: The Swedish Case.” *Scandinavian Journal of Economics* 96: 499-513.

Malliaris, A. G. and W. A. Brock. 1982. *Stochastic Methods in Economics and Finance*. New York: North-Holland.

Messe, Richard and Kenneth Rogoff. 1983. “Empirical Exchange Rate Models of the Seventies: Do they Fit Out of Sample?” *Journal of International Economics* 14: 3-24.

Messe, Richard and Andrew Rose. 1991. “Nonlinear, Nonparametric, Nonessential Exchange Rate Estimation.” *American Economic Review* 80:192-196.

Miller, Marcus and Paul Weller. 1991. “Exchange Rate bands with Price Inertia.” *Economic Journal* 101: 1380-1399.

Miller, Marcus and Lei Zhang. 1994. “Optimal Target Zones: How an Exchange Rate Mechanism Can Improve Upon Discretion.” Discussion Paper 1031. Centre for Economic Policy Research.

Mundaca, Gariela. 1990. “Intervention Decisions and Exchange Rate Volatility in a Target Zone.” Norges Bank Working Paper.

Rose, Andrew and Lars Svensson. 1991. “Expected and Predicted Realignments: The FF/DM Exchange Rate During EMS.” Working Paper No. 3685. NBER.

Smith, Gregor and Michael Spencer. 1992. “Estimation and Testing in Models of Exchange Rate Target Zones and Process Switching.” In *Exchange rate targets and currency bands*, edited by Paul Krugman and Marcus Miller, 211–239. Cambridge University Press.

Sutherland, Alan. 1994. “Target Zone Models with Price Inertia: Solutions and Testable Implications.” *Economic Journal* 104: 96-112.

Svensson, Lars. 1990. “The Simplest Test of Target Zone Credibility.” Working Paper No. 3394. NBER.

——————. 1991a. “Term Structure of Interest Rate Differentials in Target Zones.” *Journal of Monetary Economics* 28: 87-116.

——————. 1991b. “The Simplest Test of Target Zone Credibility.” *IMF Staff Paper* 38: 655-665.

——————. 1992. “An Interpretation of Recent Research on Exchange Rate Target Zones.” *Journal of Economic Perspectives* 6: 119-144.

Werner, Alejandro. 1995. “Exchange rate target zones, realignments, and the interest rate differential: Theory and evidence.” *Journal of International Economics* 39: 353-367.

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