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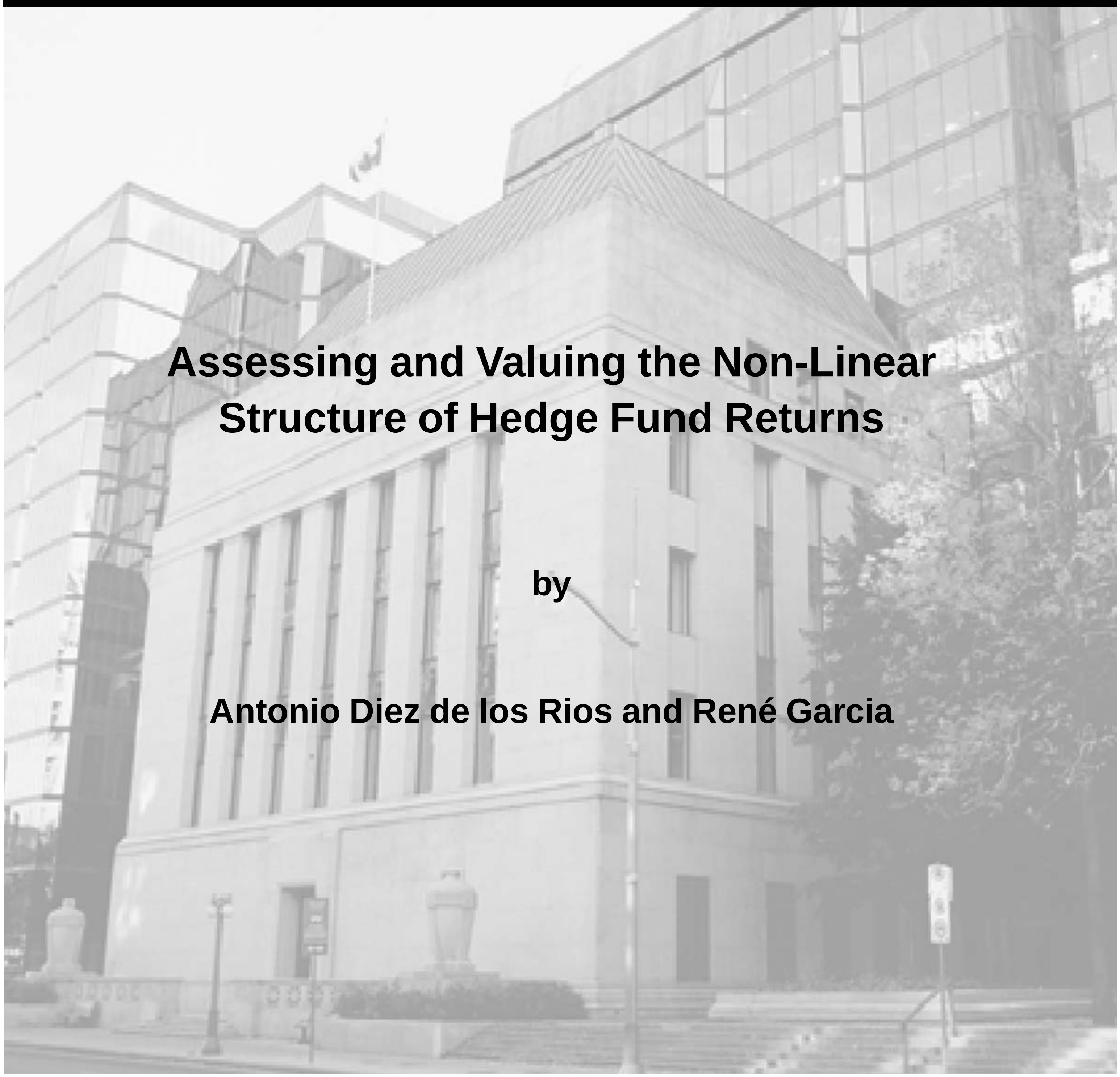

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Abstract

Several studies have put forward the non-linear structure and option-like features of returns associated with hedge fund strategies. The authors provide a statistical methodology to test for such non-linear features with the returns on any benchmark portfolio. They estimate the portfolio of options that best approximates the returns of a given hedge fund, account for this search in the statistical testing of the contingent claim features, and test whether the identified non-linear features have a positive value. The authors find that not all categories of funds exhibit significant non-linearities, and that only a few strategies as a group provide significant value to investors. Individual funds may still provide value in an otherwise poorly performing category.

JEL classification: C1, C5, G1
Bank classification: Econometric and statistical methods; Financial institutions

Résumé

Plusieurs études ont montré que les rendements produits par les stratégies suivies dans la gestion des fonds de couverture ont une structure non linéaire et un profil semblable à celui des rendements associés à l'exercice d'options. Les auteurs proposent ici une méthode statistique destinée à vérifier la présence de non-linéarités par rapport aux rendements de n'importe quel portefeuille de référence. Ils estiment un portefeuille d'options représentant le mieux les rendements d'un fonds de couverture, tiennent compte des résultats de cette recherche dans la vérification statistique de la présence de structures non linéaires, puis déterminent si les non-linéarités décelées ont une valeur positive. Les auteurs constatent que les différentes catégories de fonds ne présentent pas toutes des non-linéarités significatives et qu'une poignée de stratégies seulement, appliquées ensemble, assurent aux investisseurs des gains appréciables. Certains fonds pris individuellement peuvent toutefois générer des profits intéressants, même s'ils font partie d'une catégorie dont les rendements sont faibles.

Classification JEL : C1, C5, G1
Classification de la Banque : Méthodes économétriques et statistiques; Institutions financières

1. Introduction

Since the bursting of the Internet stock bubble in 2000, many pension funds have decided to invest in hedge funds with the hope of improving their performance on a path to full funding of their employee pensions. Assessing the performance of hedge funds has therefore become a topic of major social relevance. Will hedge funds with a typical fee structure of 2 per cent of asset value and a 20 per cent performance fee be able to fulfill institutional investors' expectations? While a cursory look at historical performance suggests that even modest allocations to hedge funds may improve significantly the efficiency of pension fund portfolios, episodes like the near-bankruptcy of Long-Term Capital Management (LTCM) in 1998 have raised questions about the true nature of their risks.

In this paper we pursue two main objectives. First, we want to better characterize and understand the risks associated with the different hedge fund strategies. Second, we want to determine whether, given these risks, the value of the cash flows generated by a fund, net of management fees, is greater than the amount entrusted to the fund manager. Fulfilling these two objectives is not an easy task. Hedge funds often engage in short-selling and derivatives trading, while leveraging their positions. More importantly, hedge funds are not transparent to the investor, since they have no obligation to disclose their positions. Therefore, any assessment of hedge fund performance can only rely on analyzing their ex-post returns. However, the return databases suffer from several biases and they do not go back very far in time.

Recent literature suggests that hedge fund returns exhibit non-linear structures and option-like features, or, in other words, risks that are typically ignored by mean-variance approaches. Fung and Hsieh (2001) analyze trend-following strategies and show that their payoffs are related to those of an investment in a lookback straddle. Mitchell and Pulvino (2001) show that returns to risk arbitrage are similar to those obtained from selling uncovered index put options. Agarwal and Naik (2004) extend these results and show that, in fact, a wide range of equity-oriented hedge fund strategies exhibit this non-linear payoff structure. In particular, they use a stepwise regression procedure to identify the significant risk factors. To account for non-linearities, they include option-based risk factors that consist of returns obtained by buying, and selling one month later, liquid put and call options on the Standard & Poor's (S&P) 500 index.

Two main limitations arise in these studies. For one, they rely on the existence of liquid options to capture the non-linearities. This is a clear drawback, since it precludes investigating the presence of non-linearities with respect to risk factors where liquid option markets do not exist, such as the international equity market, the bond market, or the exchange rate. In addition, these studies identify the significant factors by a stepwise regression approach; that is, adding or deleting them in a sequential way that depends on the value of the F-statistic. Such an approach rules out the possibility of relying on standard statistical inference to determine in a statistically sound way whether the value of the cash flows generated by a fund is greater than the amount entrusted to the fund manager.

We propose a new methodology that goes a long way towards solving the difficulties mentioned above. In particular, our approach allows us to (i) determine the portfolio of options that best approximates the returns of a given hedge fund, (ii) use options on any benchmark portfolio deemed to best characterize the strategies of the fund (and not simply traded options on the S&P 500), (iii) estimate whether the options that best characterize the returns of a particular fund are puts or calls, or both, as well as their corresponding moneyness, (iv) assess whether the presence of the estimated non-linearities are statistically significant, (v) value the performance of a fund by valuing the portfolio of options that have been found to be relevant in characterizing the hedge fund returns, and (vi) provide a reliable test for a positive valuation of the fund.

The starting point of the methodology is based on Glosten and Jagannathan (1994). We estimate a flexible piecewise linear function to capture the potentially non-linear relationship between the returns of a hedge fund and those of benchmark portfolios. The coefficients of such a non-linear regression are interpretable by practitioners, since they correspond to a position on a risk-free asset, a position on a set of benchmark portfolios, several positions on a series of options on these benchmark portfolios, and the effective strikes of such options. Our additional contribution is to propose a valid inference procedure in such a framework. Indeed, standard hypothesis tests to determine whether the coefficients that capture the positions on the options are different from zero are not applicable. When these coefficients are zero, the parameters corresponding to the option strikes are not identified, since any value of the strike will leave the R^2 of the regression unchanged. Thus, the usual critical values of a Student t-test cannot be used to establish whether there exists a non-linear relationship between hedge fund returns and the benchmark returns. To overcome this important problem, we adapt a testing methodology proposed by Hansen (1996) and compute the critical values corresponding to the appropriate asymptotic distribution.

We apply this methodology to several indexes of hedge fund categories, such as convertible arbitrage, fixed-income arbitrage, event driven, equity market neutral, long-short equity, global macro, and managed futures. We compute both equally weighted and value-weighted indexes using the TASS database, which provides net-of-fees monthly returns and net asset value data on 4,606 funds beginning in February 1977. We start our analysis by taking as the market portfolio the Center for Research in Security Prices (CRSP) value-weighted index. We analyze the robustness of our results by correcting for backfilling or lack of reporting biases that affect data on hedge fund returns.

We also extend previous studies by applying the methodology to individual funds within the categories. These studies have mainly considered individual funds in one category or simply indexes.[1] Aggregation of individual funds with different non-linear payoffs may, for example, smooth the index returns and cause an underestimation of the non-linearities. On the contrary, it can also be the case that aggregation of funds with different exposures to the risk factors may create a spurious non-linear pattern and exaggerate the non-linear features actually present in individual hedge funds.

Our findings indicate that using a proper statistical methodology matters. Not all categories exhibit significant non-linearities, even though casual evidence from a scatter plot may be suggestive of an option-like pattern of returns. There is statistical support for rejecting linearity only for convertible arbitrage, fixed-income arbitrage, event driven, and managed futures. These conclusions are robust for both equally weighted and value-weighted indexes of hedge funds. This conclusion differs from Agarwal and Naik (2004), who find evidence of non-linearities in most category indexes. In addition, our results indicate that bias corrections for backfilling and lack of reporting are crucial for valuing funds. They have a definite impact on the test for positive performance. Only the categories of convertible arbitrage and event driven seem to provide value for the investors. The index of managed futures, which comprises a large number of funds, exhibits a poor performance even before accounting for biases.

Looking at individual funds, we confirm that results based only on indexes are misleading. The appearance of non-linear features in hedge fund returns is supported statistically for only a third of the individual funds. Only one fund out of two provides a significant positive performance to its investors. These conclusions emphasize that both testing and disaggregation are important to draw a realistic picture of performance in the hedge fund industry. There are also important variations between the strategies. Arbitrage-based hedge

[1] Mitchell and Pulvino (2001) look at individual funds only in the risk arbitrage category, while Fung and Hsieh (2001) study only trend-following strategies. Patton (2004) and Chan et al. (2005) look at returns from individual funds.

funds, which include convertible arbitrage, fixed-income arbitrage, and event driven, exhibit significant non-linearities (we reject linearity at a confidence level of 10 per cent in about 40 per cent of the cases) and positive performance (the value of about 80 per cent of the funds is significantly greater than 0 at a confidence level of 10 per cent). The directional funds, under which we group global macro, emerging markets, and managed futures, have a lower percentage of significant non-linear features (32 per cent) and do not perform as well (only 30 per cent have a significant positive value). The last grouping includes equity market neutral and long-short strategies. A small percentage shows significant non-linearities (about 15 per cent), but more than 50 per cent of the funds have a significant positive value. We also look at two particular categories at the two extremes. Only 25 per cent of the funds in convertible arbitrage exhibit a significant non-linearity, although 75 per cent offer a positive value to their investors. In managed futures, there is much more evidence of non-linearities (close to 50 per cent), but only 25 per cent of the funds generate a significantly positive performance.

The results remain basically the same when we introduce more diversified indexes (Russell 3000), indexes with international exposure (MSCI world), bond indexes, or other factors, such as implied volatility on the S&P 500 index (VIX). If anything, non-linear effects come out more clearly for certain categories of funds and the value signs remain unchanged. We also test whether our conclusions are robust to the inclusion of the option-based factors used in Agarwal and Naik (2004). We find that our methodology is able to still detect non-linearities when these option regressors are included, and we show that our method can lead to basically the same findings in terms of valuation. Finally, we provide Monte Carlo evidence that our asymptotic tests have good finite-sample properties, an important property given the small sample of returns available in the database.

Other papers have recently proposed statistical tests for the nature of the strategies and the performance of hedge funds. Patton (2004) investigates whether hedge funds in the market neutral category are really market neutral, developing tests for more elaborate notions of market neutrality than the standard correlation-based definition. He finds that a quarter of the funds in this category have a significant exposure to market risk. Bailey, Li, and Zhang (2004) use a stochastic discount factor approach to evaluate hedge funds portfolios based on the style and characteristics of managers. They conclude that a market factor and two option factors (put and straddle) are significantly priced. Finally, Chan et al. (2005) develop a number of new risk measures for hedge funds, such as illiquidity risk exposure and non-linear factor models, and apply them to individual and aggregate hedge-fund returns.

The rest of this paper is organized as follows. In section 2 we explain the contingent claim approach to performance evaluation. Section 3 describes the tests used to assess the presence of non-linearities. Section 4 describes the data and presents results for global indexes, style indexes, and individual funds in several groupings of strategies. Section 5 presents a robustness analysis of the results. Section 6 concludes. Appendix A describes the TASS categories of hedge funds. Appendix B presents the econometric details of our test for non-linearities, and Appendix C provides details of a simulation study performed to assess its finite-sample properties.

2. A Contingent Claim Approach to Performance Evaluation

As noted by Glosten and Jagannathan (1994), the principle behind any evaluation measure is to "assign the correct value to the cashflow (net of management fees) the manager generates from the amount entrusted to him by the investor." For example, this cash flow can be valued using a linear factor model such as the capital-asset-pricing model (CAPM) or the arbitrage pricing theory (APT). The literature, however, has identified several problems with these linear asset-pricing models when used for the task of performance evaluation. First, these models restrict the relationship between risk factors and returns to be linear, and thus do not properly evaluate assets with non-linear payoffs. Second, performance measures based on these linear models, such as Jensen's *alpha* and Sharpe ratios, can be manipulated by taking positions in the derivatives market (see, for example, Jagannathan and Korajczyk 1986 and Goetzmann et al. 2002). These two problems are especially relevant for our work, because several studies have put forward the non-linear structure and option-like features of returns associated with hedge fund strategies and, second, hedge funds usually take positions in derivative securities.

Moreover, these two problems also make difficult an interpretation of the non-linear features potentially present in returns of managed portfolios. For example, it is common practice to divide performance into two components: security selection and market timing. Merton (1981) and Dybvig and Ross (1985) point out that portfolios managed using superior information will exhibit option-like features, even when the portfolio manager does not explicitly trade in options. Henriksson and Merton (1981) introduce one option on an index portfolio, to try to separate the market-timing ability and the stock-picking ability of a portfolio manager. In particular, they propose to run a regression such as

$$X_{p,t+1} = \beta_0 + \beta_1 X_{I,t+1} + \delta_1 \max(0, -X_{I,t+1}) + \varepsilon_{t+1}, \tag{1}$$

where $X_{p,t+1}$ is the excess return on the fund, and $X_{I,t+1}$ on the market portfolio. A positive estimate of β_0 will indicate that the manager has security-selection ability, while a positive δ_1 will measure the market-timing ability of the fund. We estimate this Henriksson and Merton (1981) regression for hedge funds in the TASS database with at least sixty observations. Figure 1 shows a scatter plot of the estimates of β_0 (security selection) against the estimates of δ_1 (market timing). Notice that hedge funds with stock-picking ability ($\beta_0 > 0$) tend to have a "perverse timing activity" ($\delta_1 < 0$), and vice versa. This suggests that, on average, security-selection and market-timing abilities might cancel each other. This result is consistent with previous studies of mutual funds such as Henriksson (1984).

Several reasons have been advanced for this finding. Jagannathan and Korajczyk (1986) suggest an explanation based on the non-linear payoff structure of options and "option-like" securities. Given the nature of the strategies followed by hedge funds, it is specially relevant here. Jagannathan and Korajczyk (1986) show that this negative cross-sectional correlation might come from a manager who has no abilities and who engages in a strategy of writing covered calls on the market. Then, the returns of this fund will show inferior market-timing ability and superior selectivity when the manager is evaluated with the Henriksson and Merton (1981) specification. Therefore, it is difficult to separate market-timing ability and stock-picking ability.[2]

We follow Glosten and Jagannathan (1994) in circumventing these problems by taking into account the possible presence of non-linear structures in the returns of a hedge fund, and in giving a value to the fund manager's abilities regardless of the strategies (e.g., security selecion or market timing) the fund manager follows to generate these returns. Glosten and Jagannathan (1994) suggest approximating the payoff on a managed portfolio using payoffs for a limited number of options on a suitably chosen index portfolio and evaluating the performance of a managed portfolio by finding the value of these options.

[2] In addition, Admati et al. (1986) invoke the difficulty in arriving at consistent theoretical definitions of timing and selectivity abilities.

2.1 Theoretical framework

The analysis in Glosten and Jagannathan (1994) is based on an investor who has to decide whether to invest in a fund for which a portfolio manager promises a return of $R_{p,t+1}$ dollars at time $t+1$ for each dollar invested now (time t), net of management fees. To this end, assume there is a nominally risk-free asset with gross return that, without loss of generality, remains constant at R_f; also, by a no-arbitrage argument consider the existence of a strictly positive stochastic discount factor (SDF) that prices any asset. The SDF is denoted by M_{t+1} and its existence implies that the present value, V_t, of a claim to the portfolio payoff, $X_{p,t+1} = R_{p,t+1} - R_f$, satisfies:

$$V_t = E_t\left[M_{t+1}X_{pt+1}\right], \tag{2}$$

where $E_t\left[\cdot\right]$ denotes expectation with respect to the information available at time t.

Note that V_t is the net present value at the margin of a borrowed dollar invested in the managed portfolio, conditional on the information available at time t. For example, consider a hedge fund manager who is a market timer *à la* Merton (1981); that is, a manager who can perfectly forecast whether the return at time $t+1$ on some index $R_{I,t+1}$ will outperform the risk-free return. Using this perfect forecast, the manager invests one dollar in the index if $R_{I,t+1} > R_f$. On the other hand, if $R_f > R_{I,t+1}$, the manager will invest in the risk-free asset (assume, for simplicity and without loss of generality, that short-selling is not allowed). This implies that the hedge fund will have as a return $R_{p,t+1} = \max\left(R_{I,t+1}, R_{f,t+1}\right)$ and an excess return of $X_{p,t+1} = \max\left(R_{I,t+1} - R_f, 0\right)$. If we apply the pricing relationship in (2) to value this fund, we get a net present value

$$V_t = E_t\left[M_{t+1}\max\left(R_{I,t+1} - R_f, 0\right)\right] = C_t, \tag{3}$$

where C_t is the price of a call option with one period to expiration, and exercise price R_f on the index with a current value equal to one. Since the price of a call option cannot be negative, this valuation framework will classify the fund as providing valuable service ($V_t > 0$). The intuition is straightforward: this manager is able to generate the payoffs of a call option with a zero investment (borrow at R_f and invest in the managed portfolio). On the other hand, consider a hedge fund manager with no market-timing ability that buys a call option with the same characteristics as before. If we apply this valuation methodology to this second fund, we will find that it has a zero net present value.

Since the information set available at time t may be complicated, we follow Glosten and

Jagannathan's (1994) suggestion and focus on the average value of V_t given by

$$v = E[V_t] = E[M_{t+1} X_{pt+1}]. \tag{4}$$

This simplification is appropriate in a framework where the hedge fund manager accepts a dollar from the investor at time t and returns $R_{p,t+1}$ dollars at time $t+1$, and where this process is repeated for several periods. We will therefore use the time series of returns of the hedge fund along with the returns on the index to attribute an average value to the fund. We also assume that, even if the manager's abilities change over time, the average ability is still well-defined. Under these simplifications, and under the assumption that the SDF is a function only of the return on some index portfolio, Proposition 1 in Glosten and Jagannathan's (1994) study suggests an evaluation procedure that consists in valuing a contingent claim. For clarity of exposition, we reproduce their result here:

Proposition 1 *Suppose that the SDF, M_{t+1}, is a function solely of the vector of returns, $R_{I,t+1}$, on some index portfolio. Then, the average value of the portfolio $v = E[V_t]$ is the average price of the traded security with payoff $e(R_{I,t+1}) = E[X_{p,t+1} | R_{I,t+1}]$.*

This result implies that X_p can be decomposed into two parts: (i) a payoff that is related to the SDF and that is a function of the return on some index R_I (possibly multidimensional), and (ii) a payoff that is uncorrelated with the SDF and that as a result must have zero mean and zero average price. Therefore, the valuation methodology in Glosten and Jagannathan (1994) consists of selecting the relevant index (or set of indexes), estimating the potentially non-linear relation between the portfolio excess returns and these indexes and, finally, applying contingent claim valuation techniques to arrive at the average value of $e(R_I)$.

2.2 Choosing the functional form

Suppose we have chosen the relevant indexes or risk factors that drive the SDF. The next step would be to choose and estimate a specific functional form for the relation between the portfolio excess returns and the index $e(\cdot)$. This function must be flexible enough to capture the non-linear nature of hedge fund returns. Since any function can be approximated arbitrarily closely by a collection of spline functions, we could use a continuous piecewise linear fit with m "knots," such as:

$$X_{p,t+1} = \beta_0 + \beta_1 R_{I,t+1} + \sum_{i=1}^{m} \delta_i \max(R_{I,t+1} - k_i, 0). \tag{5}$$

Note that the term inside the sum, say $\max(R_{I,t+1} - k_i, 0)$, is the payoff at expiration on an index call option with exercise price k_i when the current value of the index is one. We will also refer to the strike parameter as the "moneyness of the option." In addition, this equation can be interpretable by financial practitioners as it separates the payoffs of the hedge fund into three components. The first term in this equation is related to the payoff of a position in the risk-free asset (say, a one-period bond) that pays one dollar at the end of the period. The second term is related to the return of a position in the index portfolio. Finally, the summation term is related to the payoffs of m call options on the same index portfolio, but with different strikes. The performance of a fund can then be assessed by valuing this particular portfolio of bonds, stocks, and call options. Recently, Vanden (2004) has provided a theoretical support for such a specification. If agents face wealth constraints, the equilibrium SDF may be described by a similar formulation. Another approach that includes non-linearities is proposed by Harvey and Siddique (2000). They add a non-linear term derived from skewness to candidate linear SDFs.

Recall that the value of a dollar for sure received at time $t+1$ is $E[M_{t+1}] = 1/R_f$; the value of $R_{I,t+1}$ received at time $t+1$ is $E[M_{t+1}R_{I,t+1}] = 1$; also note that $E[M_{t+1}\max(R_{I,t+1} - k_i, 0)] = C_i$ is the price of a call option (with one period to expiration, and exercise price k_i, when the current value of the index is one). Following Glosten and Jagannathan (1994), we start by assuming that the return on the index portfolio R_I is lognormally distributed so that the value of the option can be computed using the Black-Scholes formula. This valuation procedure has the advantage of being simple and intuitive. Thus, the value of the portfolio will be:

$$v = \beta_0/R_f + \beta_1 + \sum_{i=1}^{m} \delta_i C_i. \qquad (6)$$

The implementation of this approach requires that the number of options, m, and their strikes $\{k_1, \ldots, k_n\}$ be specified. In previous papers, such as Agarwal and Naik (2004) and Glosten and Jagannathan (1994), these are chosen a priori.[3] Here, we want to let the data determine this level. We will show that this extra degree of flexibility is critical to characterize the strategy followed by hedge funds and to evaluate their performance.

In this context, we want to test the existence of non-linear patterns between hedge fund

[3] Glosten and Jagannathan (1994) set the knot equal to one for a one-knot estimation, as in the Henriksson-Merton method. Agarwal and Naik (2004) do not use the same estimation strategy. They compute the returns of strategies based on options using the observed prices of calls and puts at the money and slightly out-of-the-money for the S&P 500 index.

returns and risk factors. To this end, note that a linear relationship between X_p and the index R_I is nested within the formulation with one option:

$$X_{p,t+1} = \beta_0 + \beta_1 R_{I,t+1} + \delta_1 \max(R_{I,t+1} - k, 0), \tag{7}$$

when $\delta_1 = 0$. However, note that if the model is linear, then the strike of the option is not identified, meaning that any value of k will leave the R^2 of the regression unchanged. The main consequence of such a problem is that the asymptotic distribution of the usual test statistic of the hypothesis that δ_1 is equal to zero is not standard, which means that we cannot rely on a table of known critical values, as is usually done. Therefore, the non-linear pattern in hedge fund returns found in previous papers may just be a statistical artifact due to an ad hoc specification of the number of options (and their strikes), and/or the use of a statistical testing theory that is not valid for the purposes of testing linearity. Still, we can apply the general theory for econometric testing problems involving parameters that are not identified under the null hypothesis developed in Hansen (1996). The next section briefly reviews the estimation and hypothesis testing of this non-linear model.

3. Assessing the Non-Linearities

We are interested in fitting a piecewise linear function such as:

$$X_{p,t} = \beta_0 + \beta_1 R_{I,t} + \sum\nolimits_{i=1}^{m} \delta_i \max(R_{I,t} - k_i, 0) + \varepsilon_t \qquad t = 1, \ldots n,$$

which can be interpreted as a regression equation of the excess return of a hedge fund ($X_{p,t}$) on a constant, the return on an index that drives the SDF ($R_{I,t}$), and m call options on the same index portfolio but with different strikes. Our goal is to optimally determine the number of options m and the positions of the set of strikes $\{k_1, \ldots, k_n\}$ based on the data, instead of setting them a priori as in previous studies.

To determine how many options we need to approximate the returns of a hedge fund, we start by testing whether the linear fit ($m = 0$) provides a better approximation to the description of the data than a model with only one option ($m = 1$). If we cannot reject the hypothesis that the model is linear, we can stop there. Otherwise, we could test whether the fit of a model with two options is better than the fit with only one, and so on. Still, and due to the short history of returns that characterizes hedge fund datasets, we focus only on tests of a linear fit versus one option.

As before, note that when $\delta = 0$ the linear model is nested in the formulation with one option, $m = 1$:

$$X_{p,t} = \beta_0 + \beta_1 R_{I,t} + \delta \max(R_{I,t} - k, 0) + \varepsilon_t \qquad t = 1, \ldots n. \tag{8}$$

When the strike of the option, k, is known a priori, testing the null hypothesis of linearity, $H_0 : \delta = 0$ is straightforward. The parameters β_0, β_1, and δ are first estimated by running an ordinary least squares (OLS) regression, and the usual Wald statistic is then used. The null hypothesis is tested using the fact that this statistic has an approximate chi-square distribution with one degree of freedom (the number of restrictions) in large samples.

However, since hedge funds are not required to be transparent about their strategies, we know little about them. Therefore, letting the data reveal the option that best approximates the returns of a hedge fund can shed light on the specifics of these strategies. This is the empirical approach that we will follow by treating the position of the strike, k, as an unknown to be estimated. In particular, the least-square estimate of k can be found sequentially through concentration. That is, for a given value of the strike of the option, k, we first run an OLS regression as if its value were known. Then, we search over the possible values of k for the one that minimizes the sum of squared errors $\widehat{\varepsilon}_t(k)'\widehat{\varepsilon}_t(k)$ to get the least-square estimate of this parameter. Given this search, the Wald statistic of the null hypothesis $\delta = 0$ does not have a chi-square distribution, because the strike of the option has been chosen in a data-dependent procedure.

Davies (1977, 1987) suggests computing, instead, the Wald test statistic for each possible value of k and focusing on the supremum value of such sequence. We will refer to this statistic as supWald. Again, the problem that we face is that the asymptotic distribution of this test is non-standard and simulation-based methods are necessary for a correct inference. Hansen (1996) shows how to compute the asymptotic distributions of the supWald test (among others) by simulation methods. For completeness, Appendix B provides the details of such simulation methods as well as details on how to compute the corresponding "asymptotic p-values."

We also use this statistical approach to evaluate whether individual funds offer a positive performance for investors. Thus, we first test for the presence of an option-like feature. If a non-linearity is found to be significant, then we test whether the overall value, including the option, is positive.

4. Empirical Results

Our methodology for finding non-linearities will be to estimate the strike instead of setting it a priori. Otherwise, we will follow Glosten and Jagannathan (1994). We also investigate how sensitive the non-linearity and performance results are to corrections for the backfilling and survivorship biases. Finally, we look at the individual funds within the categories. We estimate the non-linearities and the performance at the individual fund level, and then determine whether the results are uniform among the funds, or whether there is a wide cross-sectional dispersion.

4.1 Description of data and construction of hedge fund indexes

Our hedge fund returns are computed from the TASS database, which provides monthly returns and net asset value data on 4,606 funds beginning in February 1977. For building these hedge fund indexes, our sample starts in January 1996 and ends in March 2004.[4] For the individual funds, we use all funds for which at least 60 observations are available. The individual funds are classified into eleven categories: 1) convertible arbitrage; 2) fixed-income arbitrage; 3) event driven; 4) equity market neutral; 5) long-short equity; 6) global macro; 7) emerging markets; 8) dedicated short bias; 9) managed futures; 10) funds of funds; and 11) other. For the sake of saving space, we do not report results for the index on the category "other." Appendix A gives a brief description of the typical strategies followed in each category.

This database also includes, for each fund, an entry date, an exit date (if any), a date for first reporting, reasons for the fund death (if necessary), and lock-up periods. This information is useful for correcting two well-known biases associated with hedge fund data. The first is a backfilling or instant-history bias, whereby the database backfills the historical return data of a fund before its entry into the database. The hypothesis is that a manager will report to the database vendor only after obtaining a good track record of returns over the first periods of the life of the hedge fund (and only if the fund performed well). Consequently, we eliminate all data that precede the fund entry date in the database. A similar approach is used by Fung and Hsieh (2001) and Posthuma and van der Sluis (2003).

The second bias corresponds to survivorship. Many funds disappear from the database during the sample period for various reasons, such as fund liquidation, fund not reporting

[4] We choose to start in 1996 to have a reasonable representation for all categories of funds. Also, the TASS database does not give any information on exited funds prior to 1994.

any longer to the database, no answer from the fund managers, and merger with another fund, to name a few. Not all these reasons have the same consequences in terms of monetary loss for the investor. For example, an outperforming fund may stop reporting data to protect its winning strategy, thus halting the inflow of capital. On the other hand, an underperforming fund has the incentive to not report, in order to hide bad results and avoid investors withdrawing their money. For example, Posthuma and van der Sluis (2003) note that: "[Long-Term Capital Management] lost 92 per cent of its capital from October 1997 to October 1998 and did not report to databases." Therefore, we correct the returns for the survivorship bias by applying a loss of 25 per cent when the indicated reasons for not reporting are fund liquidation, fund not reporting to TASS, managers not answering requests, and other. In all other cases, particularly mergers and dormant funds, we do not apply any loss.[5]

Finally, we use the CRSP value-weighted NYSE, AMEX, and NASDAQ combined index as a market measure, while we compute returns in excess of the 30-day Treasury bill yield from the CRSP RISKFREE files.

4.1.1 Global indexes

We build hedge fund indexes that are representative of the whole industry according to the two basic methodologies used by the main index-producing firms. The Hedge Fund Research (HFR) indexes are equally weighted and therefore give relatively more weight to small hedge funds. The Credit Suisse First Boston/Tremont (TREMONT) indexes are value-weighted (i.e., valued by the net asset value of the fund) and are more representative of larger funds. We construct these two types of global indexes starting from the individual funds in the database. We report the gross returns associated with these two indexes, but we also construct series that are free from the two above-mentioned biases (backfilling and survivorship). Summary statistics are reported in panel a of Table 1.

These descriptive statistics show that annualized mean returns are much lower when the two bias corrections are applied. The corrected returns still have a positive mean, but barely spectacular. As expected, the TREMONT index exhibits higher returns than the HFR index. The corrected indexes exhibit higher standard deviations than the respective gross return indexes. This is explained by the fact that adding losses for the survivorship bias tends to

[5] This loss is applied in the month following the month where the fund stopped reporting. We feel that it is a reasonable assumption given our ignorance regarding the true loss. It should be noted that performance generally deteriorates before a fund stops reporting. Still, this approach is more conservative than the scenario analysis proposed in Posthuma and van der Sluis (2003). Those authors add an extra return of 0 per cent, -50 per cent, and -100 per cent, respectively, for every fund that stops reporting to TASS.

increase dispersion. The corrections also make the fund indexes less Gaussian, since both the skewness and kurtosis increase in absolute value. For gross returns, skewness is almost always positive. However, when the corrections are applied, there is a clear distinction between the equally weighted HFR index and the value-weighted TREMONT index. Skewness of the former becomes negative, while for the latter it remains positive and not significantly different, due to a maximum return that is roughly double the minimum return in absolute value. This tends to show that large funds are less susceptible to the survivorship bias, and that they provide better performance than smaller funds. Finally, all indexes show excess kurtosis, but the effect is most pronounced when the two corrections are applied.

4.1.2 Indexes for fund categories

Looking at the various categories in panel b of Table 1 for the bias-corrected returns, we see ample variation in mean returns. For the equally weighted HFR indexes, dedicated short bias (category 8) and managed futures (category 9) exhibit negative means, while the index for global macro strategies (category 6) is close to zero. This seems to be corrected when we look at value-weighted indexes (TREMONT), where large funds with better performance and less non-reporting funds are given more weight. Indirectly, it suggests that these categories may have a greater number of smaller funds that tend to disappear. This does not seem to be the case for the convertible arbitrage (category 1) and long-short equity (category 5) strategies, since the mean returns are very similar for the HFR and TREMONT indexes.

In terms of standard deviations, the results are less uniform. For the equity market neutral (category 4), the standard deviation of the TREMONT index is more than double the one of the HFR index. A large increase is also noticeable for global macro. Emerging markets (category 7) and dedicated short bias (category 8) strategies exhibit the highest volatility levels. The least volatile are convertible arbitrage and fixed-income arbitrage (category 2) strategies in the TREMONT indexes.

Skewness is almost always negative. The two exceptions are long-short equity and dedicated short bias. Skewness is also more pronounced in the categories than in the global indexes, where averaging tends to make the returns distribution look more symmetric. Similarly, excess kurtosis is generally much higher in the category indexes than in the global indexes. However, for some categories, there are startling differences between the equally weighted and the value-weighted indexes. For the event driven (category 3) index, it is much higher in HFR, while the contrary is true for equity market neutral and, especially, global macro.

We show for comparison the indexes without any bias corrections in panel c of Table 1. The bias corrections lower the means for all categories, but the effect is much more pronounced for some categories than for others. Global macro, emerging markets, and managed futures are the most spectacular, both in HFR and TREMONT. In terms of volatility, the bias corrections tend to push it higher, a little bit more for TREMONT than for HFR. The two bias corrections act in opposite directions. The correction for backfilling tends to take out the higher returns,[6] which lowers dispersion, while the correction for no more reporting adds very negative returns, which increases dispersion.

4.2 Assessing non-linearities and performance in hedge fund indexes

We estimate a one-option specification in (8) for each one of the global hedge fund indexes, each of the indexes in the various categories, and each individual fund within each category. We report three sets of results for each index: (i) the estimated values of the coefficients—respectively, the intercept (β_0), the coefficient of the market index (β_1), the coefficient on the option (δ), and the strike (k); (ii) the test results for the presence of the non-linearity; and (iii) the test results for the fund valuation. We start our analysis by taking the market portfolio (the CRSP value-weighted index) as the only relevant factor driving the SDF. We include other factors in the robustness analysis in section 5.

It is important to discuss the interpretation of the coefficients. Following Glosten and Jagannathan (1994), we estimate a normalized version of equation (8):

$$X_{pt}^* = \beta_0 + \beta_1 R_{It}^* + \delta \max(R_{It}^* - k, 0) + \varepsilon_t, \tag{9}$$

where we define X_{pt}^* as X_{pt}/R_{ft} and R_{It}^* as R_{It}/R_{ft}.

With this transformation, the valuation of the projection of X_{pt}, conditional on the interest rate, is independent of the interest rate. The value of the first two terms is $\beta_0 + \beta_1$, while the value of the third term can be shown (in a Black-Scholes world) to be equal to $\delta(N(d_1) - kN(d_2))$ with: $d_1 = -\log(k)/\sigma + \sigma/2$ and $d_2 = d_1 - \sigma$, where σ denotes the standard deviation of the index returns and $N(\cdot)$ is the standard normal distribution function. With this normalization, the parameter k is a strike on the normalized returns on the market.

[6] This is because funds decide to enter the market after obtaining high returns and choosing how many back years to report. This shows in the maximum, which is often lower in the corrected returns than in the original returns.

With the average value of the monthly interest rate, the at-the-money strike $(1/R_f)$ will be equal to 0.9669. This will be useful for interpreting the estimated values for k. If they are greater than this value, the option will be out-of-the-money.

It should be noted that, even with only one call option, this non-linear specification allows us to capture many meaningful payoff structures. For example, a short position in a put option can be obtained when $\beta_1 > 0$ and $\delta = -\beta_1$. Similarly, the payoffs of a straddle, which involves buying a call and a put with the same strike and expiration date, are obtained when $\beta_1 < 0$ and $\delta = -2\beta_1$. When necessary, we will provide graphs to illustrate these resulting strategies, as well as valuation of the fund as a function of market volatility.

Finally, and since we have searched for the moneyness that best approximates the returns, we test whether the value of the fund is equal to zero using a supWald test. This allows us to account for this search in the statistical testing of the positive performance of the fund.

4.2.1 Global indexes

Panel a of Table 2 provides the estimated values of the coefficients of (9) for the global hedge fund indexes HFR and TREMONT, with and without bias corrections. A first remark is that the estimated coefficients are very similar, in terms of sign and magnitude, for all indexes. The coefficient β_2 is consistently negative, but is not estimated precisely. This is reflected in the p-values of the Wald tests reported in panel b of Table 2.[7] In fact, none of the indexes shows a marginally significant kink. The knot or strike value is estimated very precisely and is always greater than the reference value of 0.9669.

In terms of option strategy, we see that the graphs in Figure 2 for the TREMONT index resemble the payoff structure of selling put options. This result is consistent with what was found for most category indexes by Agarwal and Naik (2004). These graphs also show that bias corrections do not affect the kinks. Another interesting feature in Figure 2 is the comparison between the optimal kink estimated with our procedure and the one estimated by setting k to one. The slope is still broken, especially for the indexes corrected for the two biases, but it is much less pronounced than when chosen optimally.

Panel c of Table 2 provides valuations for the various indexes with and without bias corrections for several market volatilities. There is evidence of a positive valuation even

[7] The p-values deliver the same message even if we set k to one and do not search over the optimal value of the strike (see the Wald k=1 column in panel b of Table 2).

when one corrects for the backfilling bias, but it disappears when the survivorship bias is accounted for. This is very clear in panel a of Figure 3, where the valuations are plotted against volatility. Panels b, c, and d compare, with and without bias correction, the three valuations obtained (i) without any option strategy, the straight line parallel to the volatility axis,[8] (ii) with an option with $k = 1$, the thin line, and (iii) the option with the optimal k, the thick line. Even without bias corrections, the value with the option strategy goes below the linear strategy when volatility is higher than 15 per cent.

A closer look at the various categories reveals the story behind the aggregate indexes. As we will see, there is much variation between the categories.

4.2.2 Indexes for fund categories

We start by reporting the results on the presence of non-linearities for each one of the categories with and without the two bias corrections. Table 3 reports the estimated values for the coefficients of (9) for each category in each type of index: HFR and TREMONT. Table 4 provides results for the linearity test for the same breakdown, and Table 5 shows the corresponding valuation results.

Let us first look at panel a of Table 3, which shows the piecewise linear fit on the raw data without any bias corrections. Estimated values for the non-linear component δ vary across categories. While they are mostly negative, their magnitudes differ considerably. For managed futures and dedicated short bias, the coefficient δ is positive and β_1 is negative, while it is the contrary for the other categories. When we apply the two bias corrections (panel b, Table 3), the estimated values change somewhat but the signs and the relative magnitudes are maintained across categories. In most cases, the estimated value of the strike is greater than the at-the-money strike benchmark, regardless of the bias corrections. This means that the call option is out-of-the-money. Notable exceptions are convertible arbitrage, managed futures, and emerging markets. Results for the other categories alternate between the raw series and the corrected one.

In Figure 4 we present several graphs to illustrate different shapes of non-linear strategies followed by three of the categories: convertible arbitrage, fixed-income arbitrage, and managed futures. Let us comment briefly on the particular strategies involved in these categories and the shapes found for the non-linear features. We start by analyzing the convertible ar-

[8] Under the null hypothesis of linearity, the performance of the fund is given by $\widetilde{\alpha} = \widetilde{\beta}_0 + \widetilde{\beta}_1$, where $\widetilde{\beta}_0$ and $\widetilde{\beta}_1$ are the OLS of estimates of β_0 and β_1 when we impose that $\delta = 0$.

bitrage category. A typical strategy in convertible arbitrage is to be long in the convertible bond and short in the common stock of the same company. Profits are generated from both positions. The principal is usually protected from market fluctuations. The corresponding graph in Figure 4 is suggestive of a short position in a put option, which means that these strategies lose money when the equity index incurs a large fall. In most situations, however, the fund collects a small premium on the price discrepancy.

The shape of the non-linear feature for fixed-income arbitrage resembles that of an inverted straddle. Funds in this category exploit price anomalies between related interest rate securities. They buy undervalued securities and sell short overvalued ones. The corresponding graph in Figure 4 suggests that this strategy makes money when the stock market is calm, since it may be the time when the two securities revert to their fundamental value. On the other hand, a large shock like the Russian crisis creates large losses.

Finally, the managed futures profile in Figure 4 is more illustrative of a straddle. This result is intuitive because funds in this category tend to be trend followers. That is, they buy in an up market and sell (or even take a short position) in a down market. Therefore, large movements up or down are profitable. All these results are consistent with the findings of Agarwal and Naik (2004), but our method allows us to provide clear illustrations of the underlying strategies.

We next look at the linearity test in Table 4, panel a (raw series) and panel b (bias-corrected series), to see whether these apparent non-linearities are statistically significant. There is more support for rejecting linearity than for the global indexes, especially for convertible arbitrage, fixed-income arbitrage, event driven, and managed futures. Without bias correction, this rejection spreads over the HFR and TREMONT indexes. With bias correction, only the p-values of convertible arbitrage and managed futures remain low for the TREMONT value-weighted indexes. We also reject linearity for emerging markets at the 10 per cent level in most cases.

The ultimate test for investors remains a positive value. In Table 5, panels a and b, we report the *alpha* corresponding to a linear projection on the market index, as well as the valuation after accounting for the option feature for various levels of volatility. P-values are also reported between brackets to test whether the performance is significantly different from zero. Without bias correction, the p-values are all close to zero, except for the managed futures and the dedicated short bias strategies. In the latter categories, the linear *alpha* is positive and significantly different from zero, but the performance deteriorates once the non-linear feature is accounted for. For volatilities up to 15 per cent, the valuation is lower and not significantly different from zero. Only high volatilities show that the magnitude and the statistical significance of performance improve markedly. This is consistent with the straddle-like strategies shown in Figure 4 for managed futures. Panel b of Table 5 for valuation with bias corrections provides varied and interesting results. Few categories exhibit a significant positive performance. The two categories that stand out are convertible arbitrage and event driven. The option-like feature improves performance, especially for event driven, where the linear *alpha* in HFR is not significantly different from zero. For both these categories, performance worsens when volatility is high. When more weight is given to large funds (TREMONT index), the event driven and the long-short equity categories show some significant improvement in performance at low levels of volatility.

The graphs in Figure 5 illustrate the performance (corrected for biases) of convertible arbitrage, fixed-income arbitrage, and managed futures. The performance that would be achieved with an *ad hoc* strike set at a value of one is also displayed for comparison purposes. It shows that performance is usually higher with the optimally chosen strike and that the TREMONT index, which weights large funds more heavily, exhibits a higher valuation than the HFR index.

Our results tend to carry the same cautious message as Agarwal and Naik (2004) about the performance of hedge funds. Only a few categories seem to pass the filters of bias corrections and formal statistical inference. Convertible arbitrage stands prominently among the few winners, while the performance of managed futures is simply dreadful. In the next section we look at individual funds in each category and determine how widespread good or bad performance is in these categories. We also look at various strategy groupings.

4.3 Assessing non-linearities and performance in individual funds

Data on individual funds help unveil the reality behind indexes. Aggregation could potentially create either a smoothing effect, which will mask existing non-linearities for each individual fund, or, at the opposite, create spurious non-linear structures that are not present in individual funds. The information in the database will also be used more efficiently. All funds, alive or dead, since the inception of the database in 1977, will be included in the analysis, as long as a fund has been in existence for 60 months.[9] In this section, we look only at raw returns, since we want to maximize the number of funds included. Eliminating

[9] We consider that this is a minimum number of observations required to conduct the type of linearity and value tests we have described in section 3.

backfilled returns would have sensibly reduced the number of funds. We also abstain from introducing a loss when a fund stops reporting for the reasons mentioned above, as we did for the indexes. We want to look at the raw cross-sectional distributions in order to draw a comparison with the raw indexes used in previous studies. This will undoubtedly bias performance upwards for some funds.

All funds, Live Funds, Graveyard Funds In Table 6, we first look at the whole universe of funds in the database since its introduction. Overall, 1,847 funds have 60 observations or more. It should be noted that the total number of funds in the database for our sample period is 4,467. This indicates not so much the large number of exits before a five-year period of operation, but mainly the very large number of entries in the past few years. The rate of growth in the number of funds has averaged 18 per cent over the past ten years and has accelerated considerably in the last two to three years, especially for funds of funds. Of our total funds with 60 observations or more, 1,230 are live funds, meaning that they were still in operation in March 2004, and 617 are graveyard funds, which are no longer reporting to the database. We will conduct the same analysis for these two categories in Tables 7 and 8, respectively.

Tables 6 to 13 are structured in the same way. In panel a, we report the cross-sectional distribution of the linearity test. The cross-sectional distributions for performance and its p-value are presented in panel b. Panel a is based on the p-values for the linearity test; that is, the null hypothesis that the return of the fund has a linear relationship with the return on the market portfolio ($H_0 : \delta = 0$). We compare the results for four tests: the two Wald tests (with or without heteroscedasticity correction) when the moneyness parameter k is set to a value of 1, and the two SupWald tests (with or without heteroscedasticity correction) when this parameter is determined optimally. For panel b, which reports the results for hedge fund performance, we keep this comparison between the set value of k and the optimal value of the threshold. Moreover, all results for value are conditioned on a value for the volatility of the return on the market index, which we assume to be 15 per cent.[10] To summarize the test results, we report the maximum, minimum, and average p-values, but, more interestingly, the number (and percentage) of funds for which the p-value is less than 1 per cent, between 1 per cent and 5 per cent, between 5 per cent and 10 per cent, and above 10 per cent.

The first important result is that we reject linearity for about one-third of the funds. This shows that simply relying on the global indexes is misleading, but also that the non-

[10] This value is a rough estimate of the historical volatility of the market over this period (see Table 1).

linear feature is not a statistical reality for all funds. The result is robust over all tests, whether we search optimally for the k parameter value or not, and whether we account for heteroscedasticity or not. The sup tests reject linearity for 8 per cent to 10 per cent more funds if we set the critical p-value at 10 per cent. Results on the performance tests are also very informative. Only one out of every two funds provides a significant positive value to its investors. It should be stressed that we did not add a loss for the next month after the disappearance of a fund, as we did when we computed the indexes. Again, looking only at the indexes would have been misleading. Without bias corrections, we would have accepted that hedge funds provide positive value to their investors at any level of confidence. When we average, the very good performers push the mean value of the index higher, especially for the value-weighted index. As we can see in panel b of Table 6, the maximum value is very high.

The picture for the linearity test is almost the same for both live and graveyard funds (Tables 7 and 8). Live funds exhibit somewhat less significant non-linearities than the aggregate, and corresponding graveyard funds somewhat more. This is not the case, of course, for performance. For about two-thirds of the live funds, performance is significantly positive at 10 per cent. The percentage falls to one-third for graveyard funds, which means that disappearance from the database is usually associated with bad outcomes. This provides some support for our decision to add a loss when we feel that the disappearance was associated with a negative event. As far as the index is concerned, bad performance over the past few months was not enough to counterbalance stellar performers. The decline in performance over the last months of an individual fund's life is enough to reject an average positive value for this particular fund.

We have seen that there are important differences between the category indexes. We next look at three groupings of our original ten categories: arbitrage, long-short equity, and directional. We leave aside the category funds of funds because it is, by definition, a diversified mixture of managers and strategies.

Arbitrage-based strategies In the TASS database, arbitrage strategies are grouped into three categories: convertible arbitrage, fixed-income arbitrage, and event driven. In the event driven category, the arbitrage is conducted whenever firms are merged, liquidated, bankrupt, or reorganized. Overall, the database contains 328 funds in these three categories. Event driven funds represent about half the number of funds in this grouping. The results in Table 9 show that close to 50 per cent of the funds exhibit a significant non-linearity with respect to the market return. Searching for the optimal moneyness increases the number of significant non-linear strategies by about 30 funds. In terms of performance, a significant positive value is found for close to 80 per cent of the funds. However, the maximum value in panel b is much less (37.95) than what we found for all funds (86.06). The median value is close to 7 per cent a year. There is not much difference between the value at $k = 1$ and at the optimal k.

These results confirm the conclusions of Mitchell and Pulvino (2001) in their thorough study of risk-arbitrage. In particular, they suggest that three parameters, estimated with a piecewise linear regression, should be used in evaluating return series generated by risk arbitrage hedge funds. However, their valuation procedure does not account for the fact that the threshold is determined endogenously. We have provided a statistical procedure that accounts for this search and for uncertainty in the estimation of the parameters in question.

Equity market neutral and long-short strategies In this grouping, the long-short funds are by far the most numerous. Overall, the latter category represents a large percentage (close to 30 per cent) of the hedge fund industry. This strategy corresponds to the original strategy followed by Albert Winslow Jones in 1949. Overall, we do not find overwhelming evidence of non-linearities in this category. Results in Table 10 show that only 15 per cent of the funds exhibit a significant non-linear relationship with the return of the CRSP index. This confirms the results based on the index where the p-values associated with the two categories included in this grouping are among the highest. It also confirms the analysis of Agarwal and Naik (2004), who do not find a significant relationship with their option return index.

In terms of performance, the evidence is mixed. One fund out of two provides a significant positive value to investors. This is where the individual fund analysis provides a picture that is not available when we only look at an index. Without bias corrections, the performance of the index was significantly positive, but it was hiding the fact that a lot of funds in this category disappear. Correcting for biases accounted for this and resulted in a negative overall performance.

Directional strategies This grouping includes three categories: global macro, emerging markets, and managed futures. All strategies associated with these funds involve some kind of bet on market direction. This bet can be based on economic fundamentals or on some technical analysis. A common strategy, studied in detail in Fung and Hsieh (2001), is the trend-following strategy. It consists in buying in an up market and selling in a down market. This strategy is characteristic of managed futures funds, which make up half of the 463 funds included in our grouping. In Table 11, we find that the non-linear feature is present at the 10 per cent level in about 40 per cent of the funds. This, therefore, confirms the evidence in the index for emerging markets and managed futures. For performance, the percentage of p-values below 10 per cent falls to about 30 per cent. The distribution of values in panel b indicates the widespread nature of value in this category, with extreme negative minimum and maximum values.

We conclude this analysis on individual hedge funds by looking more closely at the two categories that exhibit the best and the worst performance.

Convertible arbitrage Table 12 reports results for the category with the best index performance among all categories. This category contains 80 funds. The hypothesis that fund excess returns have a linear relationship with the market portfolio is rejected at the 10 per cent level in about 25 per cent of the cases. A majority of funds thus do not seem to use market fluctuations to build strategies with options. However, these funds still manage to generate a positive performance. About 80 per cent of the funds have a significantly positive performance, for an average volatility of 15 per cent.

Managed futures There are 235 funds claiming to belong to this category (Table 13). There is much more evidence of non-linearities in this category. In about half the cases, we can reject the null hypothesis of linearity at the 10 per cent level. However, these non-linear strategies do not seem to pay off, since about 25 per cent have a significantly positive performance, even though about 80 per cent exhibit a positive valuation. This is where statistical testing in the presence of non-linearities is important. It should be stressed that managed futures represent a percentage of graveyard funds that is about three times the percentage of live funds.

Two main conclusions can be drawn from the results on individual funds. First, the non-linear features in the indexes may overestimate the actual use of non-linear strategies by individual funds. Second, the overall performance of the index is more or less supported by the cross-sectional distributions of individual performances. The quality of the fund within a strategy is also an important factor, since a fund can make profits in a generally losing category, or vice versa.

5. Robustness Analysis

Our core methodology delivers clear results on the presence or absence of non-linearities in the returns of hedge fund indexes and individual hedge funds. However, these results rely on using a particular index to capture the stochastic discount factor with which to value the funds, as well as a specific contingent claim valuation formula. In this section, we assess the robustness of our findings to these choices. First, we compare our results with those obtained in previous studies. Second, we discuss the sensitivity of the results to using the Black-Scholes formula to value the option-like feature. Third, we use several other factors to confirm test results concerning the presence of non-linearities and a positive valuation. Finally, the tests performed to detect non-linearities are asymptotic, but our samples are rather small. Therefore, we perform a Monte Carlo study to check the performance of tests in small samples. The simulation study is described in detail in Appendix C.

5.1 Comparison with previous studies

An alternative approach to assess the presence of non-linearities consists in constructing the returns on a portfolio of options and testing whether this factor explains linearly the returns on a fund (or the corresponding index). This approach is suggested by Agarwal and Naik (2004), who compute four option factors that include highly liquid at-the-money (ATM) and out-of-the-money (OTM) European call and put options on the S&P 500 index.

In this subsection, we incorporate these option-based risk factors into our analysis. We start by running a regression of the normalized hedge fund returns on three normalized factors—the market and the two OTM option factors (call and put options)[11]:

$$X^*_{pt} = \beta_0 + \beta_1 R^*_{It} + \beta_2 R^*_{call,t} + \beta_3 R^*_{put,t} + \varepsilon_t, \tag{10}$$

where the asterisk indicates that the returns on the indexes have been normalized by R_{ft}, as in (9); R_{It} denotes the returns on the CRSP value-weighted index; and $R_{call,t}$ and $R_{put,t}$ denote the returns on the OTM call and put option factors, respectively.

Note that when β_2 and β_3 in equation (10) are jointly equal to zero, the hedge fund

[11]The results in Agarwal and Naik (2004) suggest that ATM option factors do not play a big role in explaining hedge fund returns.

returns are then linearly related to the market return. Therefore, a linearity test amounts to testing $H_0 : \beta_2 = \beta_3 = 0$. Also note that, under this approach, there are no parameters unidentified under the null hypothesis of linearity. Therefore, we can rely for inference on the usual Wald test. With this specification we reject the linearity assumption for all categories of hedge funds, a finding in line with the results in Agarwal and Naik (2004).[12] Therefore, we tend to find more evidence against linearity with this approach than with our procedure. One reason may be that these factors have been built to uncover non-linear patterns. Finding the optimal factors has certainly involved some search from the researchers, which is not accounted for when we use a Wald test.

An important check for our study is to verify whether these two option-based factors can account for all the non-linear patterns found in the hedge fund return data. To do so, we augment equation (10) with a non-linear term:

$$X^*_{pt} = \beta_0 + \beta_1 R^*_{It} + \beta_2 R^*_{call,t} + \beta_3 R^*_{put,t} + \delta \max(R^*_{It} - k, 0) + \varepsilon_t, \qquad (11)$$

and test whether the coefficient in front of this non-linear term, δ, is equal to zero. Again, the parameter k must be estimated and, therefore, the test of the hypothesis $H_0 : \delta = 0$ is non-standard, as explained earlier. Our results indicate that the evidence of non-linearities still remains for fixed-income arbitrage (HFR and TREMONT), global macro (HFR), and managed futures (HFR and TREMONT). Therefore, there remain some non-linearities in the hedge fund return data that cannot be fully explained by Agarwal and Naik's (2004) option-based factors.

This comparison tells us that our test may be somewhat conservative, but that it might uncover non-linearities undetected by option factors constructed a priori with a set moneyness. However, testing for non-linearities is just a step towards the most important goal, which is to determine whether the funds provide a positive value to the investors. In the next section, we compare the two approaches in terms of valuation.

5.2 Option valuation

In order to value the contingent claims that best approximate the hedge fund returns, we rely on the Black-Scholes formula and a constant average volatility over the sample to value

[12] For space considerations, we do not present these results, but they are available upon request from the authors. It should be noted that we use the bias-corrected returns, whereas Agarwal and Naik (2004) use raw returns without any adjustment.

these contingent claims. To add some confidence to our results we include five values of the volatility in Table 5 to show how fund value varies with the level of volatility. A high volatility may transform a positive performance into a negative one or vice versa. For example, the value for the index of fixed-income arbitrage becomes negative for a volatility of 25 per cent, while the index of managed futures starts showing positive value for a volatility of 20 per cent or more. Earlier, we stated that these results are consistent with the strategies associated with these types of funds.

Of course, volatility varies over time and so will the value of the funds. Ideally, one would like to incorporate this time-varying volatility into the pricing formula of the contingent claim. However, our methodology is based on a regression where the average kink that determines the moneyness is found to characterize the option-like feature in the hedge fund returns. Plugging in a value for volatility gives the average value of the fund over the sample. A clear advantage of the procedure suggested by Glosten and Jagannathan (1994) is that it produces a fund value while incorporating non-linearities with any index chosen as the benchmark portfolio. In the methodology chosen by Agarwal and Naik (2004), one needs to rely on options exchanged on the market to construct the returns on a portfolio of options, and test whether this factor explains linearly the returns on a fund index. Therefore, Agarwal and Naik (2004) limit themselves to options on the S&P 500. Our methodology allows testing for non-linearity and for positive value with any candidate portfolio. The funds could synthetically reproduce such options if they are not exchanged on the market. Finally, it should be noted that to evaluate the performance of hedge funds in the long run, Agarwal and Naik (2004) also rely on the Black and Scholes valuation formula.

Nevertheless, to somehow control for time-varying volatility, we present two different exercises. First, we enter the implied volatility index on the S&P 500 published by the Chicago Board of Options Exchange linearly as a factor, along with the returns on the market index. The presence of this volatility factor can partly capture the hedge fund strategies based on volatility, the variation in the moneyness not accounted for in the non-linear term, and departures from log-normality of hedge fund returns. Therefore, we estimate the following model for the indexes, as in subsection 4.2:

$$X_{pt}^{*} = \beta_0 + \beta_1 R_{It}^{*} + \beta_2 VIX_t^{*} + \delta \max(R_{It}^{*} - k, 0) + \varepsilon_t, \tag{12}$$

where X_{pt}^{*} and R_{It}^{*} are defined as in (9) and VIX_t^{*} as VIX_t/R_{ft}.

Results of the linearity test show that, for many fund categories, the evidence of non-linearity is stronger with VIX added as a factor. For example, this is the case for convertible arbitrage, fixed-income arbitrage, event driven, emerging markets, and funds of funds. On the other hand, evidence of non-linearity disappears for managed futures once we introduce volatility as a factor. This is a very intuitive result, since we see that the strategy followed by these funds resembles a straddle. The relationship between volatility and returns is therefore more complex than the relationship we assume in our one-factor model with one option. Since the VIX is not a return series, it is difficult to draw a valid conclusion about performance given this new characterization of managed futures returns, but the large attrition in this category tells us that our previous results on average valuation are probably a good indicator of performance. The inclusion of a volatility factor in describing the returns seems therefore to be important for most of the categories, but, as expected, its effect is mostly prevalent in categories of funds where performance relies on volatility trading, such as managed futures. We leave further valuation analysis of these categories with volatility strategies for future research.

Second, we can use Agarwal and Naik's (2004) option-based factors to obtain a measure of the overall value of a fund that does not depend on the Black-Scholes formula. In particular, note that equation (10) implies that the performance of a fund is given by $v^{AN} = \beta_0 + \beta_1 + \beta_2 + \beta_3$. In Table 14 we report the valuation of the fund using these option-based factors as well as the p-value that corresponds with the hypothesis that the value of the fund is zero. A first and important finding is that we arrive at the same conclusions regarding positive or negative valuation as in Table 5 (panel b) for volatilities between 15 and 20 per cent. In terms of magnitude, the results are the closest for a volatility of the market equal to 20 per cent. These findings show that using the Black Scholes model to value the option-like features in our procedure does not lead to different conclusions in terms of valuation. Given the possibility of valuing options on any benchmark factor instead of relying on liquid markets, this check provides a comforting reassurance.

5.3 Other market indexes

In Agarwal and Naik (2004), several indexes are used as factors to describe hedge fund returns. To account for the large set of equities in which the hedge funds can invest, these authors use the Russell 3000 index to capture small U.S. firms; and to capture international returns, they include the Morgan Stanley Capital International (MSCI) world return index. By using the CRSP index, as we did until now, we may not have adequately represented the presence of small firms and international diversification in certain strategies. We also include in our analysis a bond return index: the Lehmann Brothers U.S. aggregate bond index.

5.3.1 Russell indexes

For the Russell 3000 index, most of the results (not reported here to conserve space) are both qualitatively and quantitatively the same in terms of detection of non-linearity and valuation. However, the effect of including a larger set of equities is mostly felt in the long-short equity hedge category. The p-values of the linearity test are now more indicative of the presence of a non-linearity in returns, with a value of 0.0010 for the supWald$^{(h)}$ statistic instead of 0.5790 with the CRSP index in the HFR category. The message for valuation is basically the same as before, but all valuations are below the ones obtained with CRSP. Similar results are obtained with the TREMONT category.

To further explore the effect of using strategies based on small and large firms in investment strategies like in the long-short equity hedge, we include two indexes, Russell 1000 and Russell 2000, to capture, respectively, the large and small equities. The Russell 1000 index measures the performance of the 1000 largest companies in the Russell 3000, which represents approximately 92 per cent of its total capitalization. The Russell 2000 index measures the performance of the 2000 smallest companies (approximately 8 per cent of its total capitalization).

Through the signs of exposition to both indexes, we can hope to capture the short and long positions in the strategy. Moreover, we introduce two options, one for each of the indexes. The estimated model is as follows:

$$X_{pt}^{*} = \beta_0 + \beta_1 RU1_t^{*} + \beta_2 RU2_t^{*} + \delta_1 \max(RU1_t^{*} - k_1, 0) + \delta_2 \max(RU2_t^{*} - k_2, 0) + \varepsilon_t, \quad (13)$$

where the asterisk indicates that the returns on the indexes have been normalized by R_{ft}, as in (9), and $RU1_t$ and $RU2_t$ denote the returns on the Russell 1000 and 2000 indexes, respectively. The most interesting results are obtained again for the long-short equity hedge category. In particular, and in line with the rest of the paper, we provide two graphs in Figure 6 to illustrate the shapes of the non-linear strategies of the TREMONT index for this category (the shape of the HFR index is qualitatively similar). Panel a suggests a position on an inverted straddle on the Russell 1000. However, we cannot reject the null hypothesis of $\delta_1 = 0$ (p-value equal to 0.2400 and 0.1080 for the HFR and TREMONT indexes, respectively). The graph in panel b resembles that of a long position in the Russell 2000 index jointly with another long position in an out-of-the-money call on the same index

(k_1 is equal to 1.0694 for both the HFR and TREMONT indexes). We reject that the coefficient δ_2 is equal to zero (p-value equal to 0.0480 and 0.0910 for the HFR and TREMONT indexes, respectively).

These robustness checks with more indexes have shown that we can refine to a certain extent the strategies used by some categories. However, the basic facts that we find relative to category indexes on the presence or absence of non-linearities or the valuation are unchanged for almost all categories.

5.3.2 MSCI world index

We know that certain categories, such as global macro or emerging markets, may include securities from different countries. Also, the equity market neutral strategy may be more related to a world index than to an index like CRSP. Therefore, we run the same regression as in (9) with the MSCI world index instead of the CRSP. Apart from stronger statistical evidence against linearity for the equity market neutral, results are basically the same as for the CRSP, both in terms of evidence on option-like features and of valuation.

5.3.3 Lehmann Brothers U.S. aggregate bond index

Hedge funds in the fixed-income arbitrage category try to exploit price anomalies between related interest rate securities. Therefore, it can be the case that another relevant benchmark to evaluate this category is a bond market factor alongside the equity index. To investigate this issue we estimate the following model:

$$X_{pt}^* = \beta_0 + \beta_1 R_{It}^* + \beta_2 R_{bt}^* + \delta_1 \max(R_{It}^* - k, 0) + \delta_2 \max(R_{bt}^* - k, 0) + \varepsilon_t, \qquad (14)$$

where the asterisk indicates that the returns on the indexes have been normalized by R_{ft}, as in (9), and R_{It} and R_{bt} denote the returns on the CRSP and Lehmann Brothers indexes, respectively.

A first important finding is that the inclusion of the bond factor and the bond-based option does not alter our previous conclusions about non-linearities with respect to the equity index. Results regarding the test of $H_0 : \delta_1 = 0$ are qualitatively the same as before for all categories of funds.

The central issue is to verify whether the bond factor, together with its corresponding option, appears significant in the fixed-income category. When we test the hypothesis that this category is not related to the bond market index ($H_0 : \beta_2 = \delta_2 = 0$), we obtain p-values equal to 0.752 or 0.414 for the HFR and the TREMONT indexes, respectively. Therefore, we can safely conclude that there does not seem to be a missing factor linked to the bond market for the fixed-income category.

Interestingly, there appears to be a role for this factor in the categories that belong to the directional strategies group. In particular, we reject this hypothesis for the global macro (p-values equal to 0.002 and 0.044 for the HFR and TREMONT indexes, respectively), emerging markets (p-value equal to 0.047 for the HFR index), managed futures (p-values equal to 0.001 and 0.050 for the HFR and TREMONT indexes, respectively), and the funds of funds category (p-values equal to 0.050 and 0.046 for the HFR and TREMONT indexes, respectively). Our interpretation of this finding is that these hedge funds can be looking to the bond market to extract information about the overall state of the economy.

5.4 Finite-sample properties of the linearity test

In Appendix C, we briefly describe the experimental setting and the results of two Monte Carlo simulation studies that were done in order to assess the finite-sample properties of the tests used in this paper. One is based on the estimated parameters for the TREMONT index for category 1 (convertible arbitrage), and the second is based on estimates of the TREMONT index for category 9 (managed futures). We investigate two sample sizes: $T = 100$ (which is roughly our sample size for data on indexes) and $T = 60$ (the minimum number of observations that we require to include an individual fund in our study). Our results indicate that the tests have generally good finite-sample size and power properties. Moreover, they are comparable with the case where the position of the knot is set to its true value.

6. Conclusion

We have shown that an approach to optimally searching for non-linearities unveils strategies that look like put selling, straddles, or inverted straddles. However, given the limited information available on hedge fund returns, the statistical evidence is not as overwhelming as previous studies tend to conclude. Even if non-linear strategies are employed, few categories provide a significantly positive value to investors, especially after accounting for the backfilling and survivorship biases. Quality funds can still be found in each category and our methodology helps identify them.

Our findings suggest that prudence should prevail when investing in hedge funds. Pension funds as well as retail investors have increased exposure to funds that engage in active and often difficult-to-decipher strategies. We hope that the tools developed in this paper will help investors recognize funds that offer the risk and return combination ultimately sought in these strategies.

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Table 1
Summary Statistics
Panel a: Indexes

	Mean	Median	S.D.	Skew.	Kurt.	Min.	Max.
One-month interest rate	3.77	4.51	0.49	−0.65	−1.17	0.79	6.23
Value-weighted CRSP index	10.36	19.60	17.34	−0.68	0.22	−189.16	100.71
HFR							
No bias correction	11.96	9.35	6.02	0.20	1.36	−60.12	76.02
Bias 1 corrected	8.60	7.99	7.00	−0.23	2.53	−89.52	83.53
Biases 1 and 2 corrected	4.45	4.41	6.93	−0.38	2.46	−94.49	73.69
TREMONT							
No bias correction	12.87	11.26	5.57	0.63	1.67	−38.99	83.41
Bias 1 corrected	10.19	10.07	6.48	0.72	3.18	−53.93	99.20
Biases 1 and 2 corrected	6.84	6.01	6.60	0.62	2.71	−55.97	94.66

This table shows the means, medians, standard deviations (S.D.), skewness (Skew,), kurtosis (Kurt.), and minimum (Min.) and maximum (Max.) of (annualized) returns for HFR and TREMONT indexes without bias correction, corrected for backfilling bias (Bias 1) and corrected by backfilling and survivorship bias (Biases 1 and 2) during January 1996 to March 2004 (99 observations).

Panel b: Indexes by Categories (Backfilling and Survivorship Biases Corrected)

	Mean	Median	S.D.	Skew.	Kurt.	Min.	Max.
HFR							
C1 Convertible arbitrage	8.14	10.34	4.63	−1.52	7.22	−75.24	44.40
C2 Fixed-income arbitrage	3.33	7.06	4.91	−2.74	11.63	−89.30	34.32
C3 Event driven	6.60	8.11	5.61	−1.38	6.33	−93.35	48.23
C4 Equity market neutral	4.05	3.31	3.55	0.25	1.71	−31.77	50.06
C5 Long-short equity hedge	8.06	6.83	10.57	0.17	1.67	−109.76	125.55
C6 Global macro	0.16	−1.37	6.79	0.21	0.08	−53.15	61.76
C7 Emerging markets	7.70	13.53	17.57	−1.38	5.48	−294.92	152.49
C8 Dedicated short nias	−3.44	−10.50	21.29	0.43	0.84	−186.12	252.03
C9 Managed futures	−3.13	−5.09	9.25	0.22	−0.13	−84.24	80.83
C10 Funds of funds	3.65	5.67	6.18	−0.30	2.75	−81.88	63.75
TREMONT							
C1 Convertible arbitrage	8.45	8.85	4.06	−1.13	4.66	−56.95	46.23
C2 Fixed-income arbitrage	4.87	7.15	3.49	−2.76	12.31	−64.07	26.80
C3 Event driven	11.62	12.87	6.88	−0.13	1.36	−73.73	72.64
C4 Equity market neutral	5.40	8.14	7.72	−2.58	16.56	−164.63	79.96
C5 Long-short equity hedge	10.71	11.38	11.79	0.42	2.95	−108.26	180.37
C6 Global macro	2.22	4.21	11.99	−4.40	34.32	−314.18	98.06
C7 Emerging markets	4.02	18.13	16.89	−1.53	4.83	−236.65	163.96
C8 Dedicated short bias	3.18	−8.17	25.76	0.85	3.03	−261.71	340.94
C9 Managed futures	3.71	5.67	13.20	−0.67	3.31	−200.84	120.96
C10 Funds of funds	5.99	6.65	5.87	−0.47	3.71	−81.43	66.82

This table shows the means, medians, standard deviations (S.D.), skewness (Skew,), kurtosis (Kurt.), and minimum (Min.) and maximum (Max.) of (annualized) returns for HFR and TREMONT indexes corrected by backfilling and survivorship bias, for each one of the categories during January 1996 to March 2004 (99 observations).

Table 1
Summary Statistics
Panel c: Indexes by Categories (No Bias Correction)

	Mean	Median	S.D.	Skew.	Kurt.	Min.	Max.
HFR							
C1 Convertible arbitrage	12.10	12.40	3.72	−0.88	3.43	−44.50	45.20
C2 Fixed-income arbitrage	9.03	11.42	3.70	−3.69	21.25	−76.41	25.80
C3 Event driven	12.10	13.38	4.96	−1.98	9.43	−84.33	48.32
C4 Equity market neutral	10.22	8.45	2.49	0.45	0.16	−9.51	36.63
C5 Long-short equity hedge	16.37	13.84	10.20	0.19	1.56	−102.57	126.69
C6 Global macro	10.02	6.72	5.52	0.65	0.62	−38.58	61.78
C7 Emerging markets	14.90	22.09	16.73	−1.26	4.81	−266.26	158.24
C8 Dedicated short bias	2.69	−5.12	20.99	0.63	0.94	−141.58	272.34
C9 Managed futures	10.17	7.35	9.65	0.20	−0.39	−58.92	93.21
C10 Funds of funds	8.86	6.82	5.31	0.44	1.62	−48.91	65.95
TREMONT							
C1 Convertible arbitrage	12.85	12.74	3.36	−0.55	2.30	−34.50	40.49
C2 Fixed-income arbitrage	6.16	8.06	2.62	−2.04	6.43	−36.93	19.80
C3 Event driven	13.00	14.34	5.83	−1.76	7.74	−95.65	49.05
C4 Equity market neutral	13.91	11.12	3.84	0.54	0.66	−18.04	56.24
C5 Long-short equity hedge	18.73	15.28	10.87	−0.08	3.54	−132.91	161.55
C6 Global macro	11.85	10.25	4.95	0.05	0.86	−43.43	54.82
C7 Emerging markets	13.46	22.57	13.94	−1.55	6.19	−206.42	153.66
C8 Dedicated short bias	7.25	−3.29	19.60	0.40	0.98	−175.59	250.45
C9 Managed futures	12.29	8.70	10.51	0.40	0.05	−57.28	117.95
C10 Funds of funds	10.48	7.29	5.00	0.72	1.47	−28.90	70.23

This table shows the means, medians, standard deviations (S.D.), skewness (Skew,), kurtosis (Kurt.), and minimum (Min.) and maximum (Max.) of (annualized) returns for HFR and TREMONT indexes without bias correction, for each one of the categories during January 1996 to March 2004 (99 observations).

Table 2
Results with Indexes
Panel a: Piecewise Linear Fit

	β_0	β_1	δ	k	$\sigma^2 \times 10^{-3}$
HFR					
No bias correction	−0.2797	0.2860	−0.2417	1.0450	0.1329
	(0.0317)	(0.0325)	(0.2917)	(0.0208)	
Bias 1 corrected	−0.3420	0.3461	−0.2440	1.0331	0.1842
	(0.0519)	(0.0530)	(0.2247)	(0.0245)	
Biases 1 and 2 corrected	−0.3485	0.3492	−0.2583	1.0331	0.1769
	(0.0517)	(0.0528)	(0.2140)	(0.0225)	
TREMONT					
No bias correction	−0.2346	0.2415	−0.1915	1.0471	0.1322
	(0.0282)	(0.0290)	(0.3006)	(0.0273)	
Bias 1 corrected	−0.2575	0.2630	−0.1619	1.0272	0.2222
	(0.0398)	(0.0411)	(0.2515)	(0.0525)	
Biases 1 and 2 corrected	−0.2549	0.2572	−0.1178	1.0272	0.2356
	(0.0414)	(0.0426)	(0.2492)	(0.0721)	

This table shows the results of the following piecewise linear fit for HFR and TREMONT indexes without bias correction, corrected for backfilling bias (Bias 1) and corrected by backfilling and survivorship bias (Biases 1 and 2) during January 1996 to March 2004 (99 obs): $X^*_{p,t+1} = \beta_0 + \beta_1 R^*_{I,t+1} + \delta \max(R^*_{I,t+1} - k, 0) + \varepsilon_{t+1}$; $E\left[\varepsilon^2_{t+1}\right] = \sigma^2$ where $X^*_{p,t+1} = (R_{p,t+1} - R_{f,t})/R_{f,t}$, $R^*_{I,t+1} = R_{I,t+1}/R_{f,t}$. Std. errors (in parentheses) are computed using Chan and Tsay (1998).

Panel b: Tests of Linearity (p-values)

	Wald $k=1$	supWald	Wald$^{(h)}$ $k=1$	supWald$^{(h)}$
HFR				
No bias correction	0.554	0.308	0.580	0.569
Bias 1 corrected	0.195	0.200	0.267	0.387
Biases 1 and 2 corrected	0.143	0.154	0.212	0.324
TREMONT				
No bias correction	0.846	0.560	0.837	0.731
Bias 1 corrected	0.624	0.504	0.564	0.630
Biases 1 and 2 corrected	0.904	0.754	0.889	0.799

This table shows the p-values for the test of the hypothesis $H_0 : \delta = 0$ in the following piecewise linear regression: $X^*_{p,t+1} = \beta_0 + \beta_1 R^*_{I,t+1} + \delta \max(R^*_{I,t+1} - k, 0) + \varepsilon_{t+1}$.

Panel c: Valuation

	α	$\sigma = 5\%$	$\sigma = 10\%$	$\sigma = 15\%$	$\sigma = 20\%$	$\sigma = 25\%$
HFR						
No bias correction	6.4625	7.5627	7.3271	6.5277	5.3666	4.0134
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.003]
Bias 1 corrected	2.8527	4.8480	4.3089	3.1827	1.7965	0.2885
	[0.079]	[0.012]	[0.010]	[0.033]	[0.181]	[0.537]
Biases 1 and 2 corrected	−1.2925	0.8200	0.2492	−0.9431	−2.4107	−4.0072
	[0.421]	[0.801]	[0.957]	[0.571]	[0.276]	[0.189]
TREMONT						
No bias correction	7.6115	8.3771	8.2180	7.6267	6.7387	5.6889
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
Bias 1 corrected	4.9212	6.5778	6.0713	5.2189	4.2368	3.1971
	[0.005]	[0.011]	[0.007]	[0.003]	[0.010]	[0.031]
Biases 1 and 2 corrected	1.5555	2.7611	2.3925	1.7721	1.0574	0.3007
	[0.388]	[0.425]	[0.406]	[0.406]	[0.477]	[0.592]

This table shows the value of the fund for different values of the annual volatility of the market σ. The value of the fund is computed according to the following formula: $v = \beta_0 + \beta_1 + \delta\left[N(d_1) - kN(d_2)\right]$ where $d_1 = -\log(k)/\sigma$ and $d_2 = d_1 - \sigma$. p-values for the hypothesis that the value of the fund is equal to zero, $H_0 : v = 0$, are presented in brackets.

Table 3
Piecewise Linear Fit: Indexes by Category
Panel a: No Bias Correction

	β_0	β_1	δ	k	$\sigma^2 \times 10^{-3}$
HFR					
C1 Convertible arbitrage	−0.2318	0.2478	−0.1915	0.9562	0.0835
	(0.1159)	(0.1243)	(0.1269)	(0.0212)	
C2 Fixed-income arbitrage	−0.0696	0.0755	−0.3081	1.0378	0.1068
	(0.0203)	(0.0207)	(0.1944)	(0.0104)	
C3 Event driven	−0.4334	0.4566	−0.3438	0.9562	0.0941
	(0.1583)	(0.1696)	(0.1720)	(0.0142)	
C4 Equity market neutral	−0.0299	0.0353	−0.1754	1.0591	0.0432
	(0.0148)	(0.0149)	(0.2750)	(0.0231)	
C5 Long-short equity hedge	−0.5165	0.5258	−0.2167	1.0331	0.2708
	(0.0507)	(0.0526)	(0.3064)	(0.0375)	
C6 Global macro	−0.1132	0.1188	−0.2723	1.0468	0.2393
	(0.0332)	(0.0340)	(0.3175)	(0.0251)	
C7 Emerging markets	−1.1186	1.1588	−0.6828	0.9562	1.2848
	(0.5070)	(0.5452)	(0.5573)	(0.0267)	
C8 Dedicated short bias	1.1299	−1.1275	0.2851	1.0272	0.8246
	(0.0947)	(0.0968)	(0.3911)	(0.0495)	
C9 Managed futures	0.5578	−0.5831	0.6741	0.9617	0.6936
	(0.1170)	(0.1281)	(0.1560)	(0.0151)	
C10 Funds of funds	−0.2039	0.2082	−0.2884	1.0468	0.1529
	(0.0335)	(0.0341)	(0.3190)	(0.0172)	
TREMONT					
C1 Convertible arbitrage	−0.1893	0.2033	−0.1383	0.9562	0.0659
	(0.0954)	(0.1026)	(0.1054)	(0.0268)	
C2 Fixed-income arbitrage	−0.0501	0.0534	−0.1443	1.0242	0.0550
	(0.0165)	(0.0166)	(0.0532)	(0.0131)	
C3 Event driven	−0.4028	0.4188	−0.1946	0.9562	0.0968
	(0.2103)	(0.2252)	(0.2271)	(0.0310)	
C4 Equity market neutral	−0.0554	0.0643	−0.4637	1.0591	0.1113
	(0.0262)	(0.0265)	(0.4082)	(0.0141)	
C5 Long-short equity hedge	−0.5540	0.5660	−0.3301	1.0331	0.3420
	(0.0773)	(0.0794)	(0.2630)	(0.0245)	
C6 Global macro	−0.1008	0.1088	−0.1805	1.0260	0.1974
	(0.0411)	(0.0427)	(0.1431)	(0.0246)	
C7 Emerging markets	−0.6780	0.7009	−0.3417	0.9562	1.1537
	(0.5469)	(0.5873)	(0.5995)	(0.0556)	
C8 Dedicated short bias	0.9809	−0.9762	0.3555	1.0272	1.1747
	(0.1007)	(0.1036)	(0.4912)	(0.0500)	
C9 Managed futures	0.6139	−0.6446	0.7556	0.9562	0.8354
	(0.1907)	(0.2080)	(0.2249)	(0.0163)	
C10 Funds of funds	−0.0775	0.0784	0.0850	0.9617	0.1554
	(0.0954)	(0.1029)	(0.1123)	(0.0593)	

This table shows the results of the following piecewise linear fit for SP, HFR, and TREMONT indexes for the different categories and without bias correction during January 1996 to March 2004 (99 observations): $X^*_{p,t+1} = \beta_0 + \beta_1 R^*_{I,t+1} + \delta \max(R^*_{I,t+1} - k, 0) + \varepsilon_{t+1} \qquad E\left[\varepsilon^2_{t+1}\right] = \sigma^2$ where $X^*_{p,t+1} = (R_{p,t+1} - R_{f,t})/R_{f,t}$, $R^*_{I,t+1} = R_{I,t+1}/R_{f,t}$. Standard errors (in parentheses) are computed using Chan and Tsay (1998).

Table 3
Piecewise Linear Fit: Indexes by Category
Panel b: Backfilling and Survivorship Bias Correction

	β_0	β_1	δ	k	$\sigma^2 \times 10^{-3}$
HFR					
C1 Convertible arbitrage	−0.3194	0.3366	−0.2827	0.9562	0.1340
	(0.1888)	(0.2027)	(0.2058)	(0.0216)	
C2 Fixed-income arbitrage	−0.0749	0.0765	−0.3198	1.0346	0.1974
	(0.0383)	(0.0387)	(0.2288)	(0.0166)	
C3 Event driven	−0.2763	0.2804	−0.3344	1.0272	0.1489
	(0.0681)	(0.0690)	(0.1645)	(0.0161)	
C4 Equity market neutral	−0.0559	0.0564	−0.3630	1.0591	0.0946
	(0.0197)	(0.0197)	(0.3944)	(0.0177)	
C5 Long-short equity hedge	−0.5432	0.5457	−0.2452	1.0331	0.3059
	(0.0519)	(0.0538)	(0.3258)	(0.0352)	
C6 Global macro	−0.1674	0.1664	−0.6275	1.0447	0.3612
	(0.0447)	(0.0454)	(0.3735)	(0.0142)	
C7 Emerging markets	−1.1563	1.1896	−0.6687	0.9562	1.3950
	(0.5795)	(0.6231)	(0.6347)	(0.0304)	
C8 Dedicated short bias	1.0124	−1.0144	0.1891	1.0331	1.3727
	(0.1142)	(0.1178)	(0.3790)	(0.0683)	
C9 Managed futures	0.4181	−0.4471	0.5133	0.9617	0.6869
	(0.1117)	(0.1227)	(0.1462)	(0.0206)	
C10 Funds of funds	−0.2644	0.2646	−0.3848	1.0450	0.1948
	(0.0473)	(0.0477)	(0.3214)	(0.0151)	
TREMONT					
C1 Convertible arbitrage	−0.2690	0.2831	−0.2151	0.9562	0.1000
	(0.1321)	(0.1422)	(0.1452)	(0.0227)	
C2 Fixed-income arbitrage	−0.0602	0.0640	−0.1219	0.9947	0.1009
	(0.0217)	(0.0227)	(0.0452)	(0.0211)	
C3 Event driven	−0.2540	0.2606	−0.1554	1.0272	0.2686
	(0.0691)	(0.0700)	(0.2476)	(0.0494)	
C4 Equity market neutral	−0.1643	0.1659	−0.7600	1.0591	0.4479
	(0.0597)	(0.0586)	(0.6101)	(0.0134)	
C5 Long-short equity hedge	−0.5906	0.5966	−0.3497	1.0272	0.4820
	(0.0505)	(0.0513)	(0.4374)	(0.0422)	
C6 Global macro	0.0039	−0.0153	0.2185	0.9658	1.1703
	(0.1025)	(0.1095)	(0.2029)	(0.0639)	
C7 Emerging markets	−0.7052	0.7204	−0.3450	0.9562	1.8907
	(0.6313)	(0.6780)	(0.6927)	(0.0626)	
C8 Dedicated short bias	1.0319	−1.0302	0.4569	1.0346	3.2792
	(0.2070)	(0.2112)	(0.5365)	(0.0412)	
C9 Managed futures	0.6013	−0.6355	0.7523	0.9617	1.3692
	(0.1016)	(0.1125)	(0.1776)	(0.0174)	
C10 Funds of funds	−0.2945	0.3029	−0.1496	0.9562	0.2015
	(0.2046)	(0.2207)	(0.2252)	(0.0550)	

This table shows the results of the following piecewise linear fit for SP, HFR, and TREMONT indexes for the different categories and with backfilling and survivorship bias correction during January 1996 to March 2004 (99 observations): $X^*_{p,t+1} = \beta_0 + \beta_1 R^*_{I,t+1} + \delta \max(R^*_{I,t+1} - k, 0) + \varepsilon_{t+1} \qquad E\left[\varepsilon^2_{t+1}\right] = \sigma^2$ where $X^*_{p,t+1} = (R_{p,t+1} - R_{f,t})/R_{f,t}$, $R^*_{I,t+1} = R_{I,t+1}/R_{f,t}$. Standard errors (in parentheses) are computed using Chan and Tsay (1998).

Table 4
Tests of Linearity: Indexes by Category
Panel a: No Bias Correction

	Wald $k=1$	supWald	Wald$^{(h)}$ $k=1$	supWald$^{(h)}$
HFR				
C1 Convertible arbitrage	0.2848	0.0425	0.4444	0.1740
C2 Fixed-income arbitrage	0.0234	0.0345	0.0399	0.0555
C3 Event driven	0.0003	0.0000	0.0288	0.0220
C4 Equity market neutral	0.5613	0.5675	0.5426	0.4610
C5 Long-short equity hedge	0.7857	0.4385	0.7844	0.6510
C6 Global macro	0.9045	0.5160	0.9012	0.3380
C7 Emerging markets	0.0524	0.0620	0.1514	0.2830
C8 Dedicated short bias	0.5243	0.5880	0.5202	0.6280
C9 Managed futures	0.0275	0.0035	0.0355	0.0000
C10 Funds of funds	0.4114	0.2600	0.4422	0.5150
TREMONT				
C1 Convertible arbitrage	0.4421	0.1110	0.5549	0.2615
C2 Fixed-income arbitrage	0.0301	0.0750	0.0036	0.0145
C3 Event driven	0.0312	0.0480	0.2407	0.3115
C4 Equity market neutral	0.3158	0.1845	0.3239	0.1680
C5 Long-short equity hedge	0.2892	0.2090	0.3406	0.3505
C6 Global macro	0.3731	0.3740	0.3878	0.4210
C7 Emerging markets	0.2845	0.5015	0.4344	0.7145
C8 Dedicated short bias	0.3161	0.5555	0.2848	0.5390
C9 Managed futures	0.0500	0.0050	0.0930	0.0000
C10 Funds of funds	0.7383	0.7640	0.7521	0.7330

This table shows the p-values for the test of the hypothesis $H_0 : \delta = 0$ in the following piecewise linear regression: $X^*_{p,t+1} = \beta_0 + \beta_1 R^*_{I,t+1} + \delta \max(R^*_{I,t+1} - k, 0) + \varepsilon_{t+1}$.

Panel b: Backfilling and Survivorship Bias Correction

	Wald $k=1$	supWald	Wald$^{(h)}$ $k=1$	supWald$^{(h)}$
HFR				
C1 Convertible arbitrage	0.1188	0.0145	0.3315	0.1955
C2 Fixed-income arbitrage	0.0335	0.0915	0.0540	0.1095
C3 Event driven	0.0134	0.0075	0.1068	0.0650
C4 Equity market neutral	0.2085	0.2945	0.1465	0.1985
C5 Long-short equity hedge	0.4933	0.3845	0.4686	0.5790
C6 Global macro	0.1367	0.0405	0.1145	0.0545
C7 Emerging markets	0.0429	0.0855	0.1435	0.2935
C8 Dedicated short bias	0.9778	0.9570	0.9787	0.9420
C9 Managed futures	0.0647	0.0455	0.0441	0.0015
C10 Funds of funds	0.0862	0.1190	0.1361	0.2695
TREMONT				
C1 Convertible arbitrage	0.2182	0.0275	0.3767	0.1475
C2 Fixed-income arbitrage	0.1020	0.2455	0.0077	0.0185
C3 Event driven	0.8060	0.6180	0.8500	0.7375
C4 Equity market neutral	0.2811	0.3350	0.1612	0.0875
C5 Long-short equity hedge	0.4585	0.2095	0.3902	0.3335
C6 Global macro	0.9747	0.7715	0.9698	0.7715
C7 Emerging markets	0.6233	0.6940	0.7049	0.8750
C8 Dedicated short bias	0.9869	0.8110	0.9892	0.5925
C9 Managed futures	0.0643	0.0300	0.0441	0.0015
C10 Funds of funds	0.2203	0.4715	0.3685	0.6405

This table shows the p-values for the test of the hypothesis $H_0 : \delta = 0$ in the following piecewise linear regression: $X^*_{p,t+1} = \beta_0 + \beta_1 R^*_{I,t+1} + \delta \max(R^*_{I,t+1} - k, 0) + \varepsilon_{t+1}$.

Table 5
Valuation: Indexes by Category
Panel a: No Bias Correction

	α	$\sigma = 5\%$	$\sigma = 10\%$	$\sigma = 15\%$	$\sigma = 20\%$	$\sigma = 25\%$
HFR						
C1 Convertible arbitrage	7.6367	9.1635	8.9960	8.4067	7.5394	6.5228
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
C2 Fixed-income arbitrage	5.0670	7.1259	6.6235	5.3625	3.7143	1.8762
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.062]
C3 Event driven	7.0258	9.7673	9.4665	8.4083	6.8509	5.0255
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.037]
C4 Equity market neutral	6.2443	6.5276	6.4732	6.1246	5.4765	4.6366
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
C5 Long-short equity hedge	9.3726	11.1446	10.6658	9.6657	8.4347	7.0955
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
C6 Global macro	5.6318	6.7384	6.5076	5.6596	4.3911	2.8943
	[0.002]	[0.007]	[0.005]	[0.002]	[0.005]	[0.014]
C7 Emerging markets	6.9216	12.3663	11.7689	9.6673	6.5743	2.9491
	[0.128]	[0.012]	[0.016]	[0.049]	[0.285]	[0.836]
C8 Dedicated short bias	5.8760	2.9587	3.8506	5.3518	7.0812	8.9123
	[0.090]	[0.374]	[0.271]	[0.121]	[0.063]	[0.055]
C9 Managed futures	6.9684	0.6507	1.5706	4.0785	7.4498	11.2535
	[0.037]	[0.288]	[0.210]	[0.072]	[0.020]	[0.005]
C10 Funds of funds	3.9041	5.0765	4.8320	3.9335	2.5898	1.0041
	[0.007]	[0.002]	[0.000]	[0.002]	[0.029]	[0.140]
TREMONT						
C1 Convertible arbitrage	8.4091	9.5119	9.3909	8.9652	8.3388	7.6045
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
C2 Fixed-income arbitrage	2.2729	3.9088	3.3811	2.5769	1.6769	0.7353
	[0.011]	[0.000]	[0.000]	[0.003]	[0.042]	[0.273]
C3 Event driven	7.4228	8.9748	8.8045	8.2055	7.3238	6.2905
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.018]
C4 Equity market neutral	9.8197	10.5688	10.4249	9.5031	7.7894	5.5687
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]
C5 Long-short equity hedge	11.6081	14.3074	13.5780	12.0545	10.1792	8.1391
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.001]
C6 Global macro	7.6297	9.5565	8.9538	7.9803	6.8723	5.7050
	[0.000]	[0.000]	[0.000]	[0.000]	[0.000]	[0.002]
C7 Emerging markets	6.7862	9.5106	9.2117	8.1601	6.6125	4.7985
	[0.115]	[0.037]	[0.039]	[0.066]	[0.224]	[0.565]
C8 Dedicated short bias	9.3207	5.6836	6.7955	8.6671	10.8233	13.1061
	[0.023]	[0.211]	[0.131]	[0.044]	[0.013]	[0.008]
C9 Managed futures	8.9322	2.9069	3.5680	5.8937	9.3165	13.3282
	[0.015]	[0.196]	[0.109]	[0.029]	[0.010]	[0.009]
C10 Funds of funds	5.7602	4.9634	5.0794	5.3957	5.8209	6.3006
	[0.000]	[0.001]	[0.000]	[0.000]	[0.001]	[0.004]

This table shows the value of the fund for different values of the annual volatility of the market σ. The value of the fund is computed according to the following formula: $v = \beta_0 + \beta_1 + \delta\,[N(d_1) - kN(d_2)]$ where $d_1 = -\log(k)/\sigma$ and $d_2 = d_1 - \sigma$. p-values for the hypothesis that the value of the fund is equal to zero, $H_0 : v = 0$, are presented in brackets.

Table 5
Valuation: Indexes by Category
Panel b: Backfilling and Survivorship Bias Correction

	α	$\sigma = 5\%$	$\sigma = 10\%$	$\sigma = 15\%$	$\sigma = 20\%$	$\sigma = 25\%$
HFR						
C1 Convertible arbitrage	3.5636	5.8174	5.5701	4.7002	3.4199	1.9193
	[0.018]	[0.000]	[0.000]	[0.002]	[0.061]	[0.359]
C2 Fixed-income arbitrage	−0.5568	1.9012	1.2567	−0.1672	−1.9517	−3.9075
	[0.734]	[0.148]	[0.318]	[0.853]	[0.482]	[0.274]
C3 Event driven	1.4879	4.9092	3.8632	2.1027	0.0744	−2.0730
	[0.351]	[0.004]	[0.007]	[0.092]	[0.652]	[0.559]
C4 Equity market neutral	0.0121	0.5985	0.4859	−0.2357	−1.5771	−3.3156
	[0.992]	[0.401]	[0.537]	[0.849]	[0.714]	[0.449]
C5 Long-short equity hedge	1.0038	3.0096	2.4676	1.3356	−0.0579	−1.5737
	[0.618]	[0.351]	[0.376]	[0.514]	[0.850]	[0.883]
C6 Global macro	−4.2076	−1.3082	−1.9315	−4.0234	−7.0496	−10.5709
	[0.065]	[0.474]	[0.340]	[0.102]	[0.020]	[0.005]
C7 Emerging markets	−0.5245	4.8079	4.2228	2.1646	−0.8646	−4.4150
	[0.912]	[0.278]	[0.418]	[0.807]	[0.662]	[0.391]
C8 Dedicated short bias	−0.7777	−2.3244	−1.9065	−1.0335	0.0411	1.2101
	[0.861]	[0.889]	[0.882]	[0.884]	[0.974]	[0.982]
C9 Managed futures	−6.3996	−11.2096	−10.5093	−8.5998	−6.0331	−3.1370
	[0.051]	[0.006]	[0.006]	[0.014]	[0.083]	[0.381]
C10 Funds of funds	−1.5713	0.1801	−0.1950	−1.4675	−3.3157	−5.4698
	[0.358]	[0.755]	[0.953]	[0.494]	[0.228]	[0.148]
TREMONT						
C1 Convertible arbitrage	3.8877	5.6028	5.4146	4.7526	3.7783	2.6363
	[0.002]	[0.000]	[0.000]	[0.000]	[0.008]	[0.083]
C2 Fixed-income arbitrage	1.0743	3.3023	2.4902	1.6598	0.8249	−0.0117
	[0.360]	[0.070]	[0.113]	[0.243]	[0.505]	[0.978]
C3 Event driven	6.3482	7.9378	7.4518	6.6339	5.6915	4.6938
	[0.001]	[0.001]	[0.001]	[0.001]	[0.006]	[0.069]
C4 Equity market neutral	0.7526	1.9803	1.7443	0.2337	−2.5749	−6.2145
	[0.781]	[0.272]	[0.383]	[0.652]	[0.860]	[0.712]
C5 Long-short equity hedge	3.5331	7.1105	6.0168	4.1759	2.0551	−0.1902
	[0.173]	[0.102]	[0.099]	[0.142]	[0.293]	[0.506]
C6 Global macro	−2.4550	−4.7508	−4.3471	−3.4266	−2.2610	−0.9782
	[0.561]	[0.703]	[0.660]	[0.594]	[0.517]	[0.492]
C7 Emerging markets	−2.7315	0.0193	−0.2826	−1.3443	−2.9069	−4.7384
	[0.610]	[0.952]	[0.895]	[0.704]	[0.622]	[0.629]
C8 Dedicated short bias	5.6145	2.1009	3.0223	5.0574	7.6077	10.4025
	[0.426]	[0.694]	[0.597]	[0.449]	[0.356]	[0.352]
C9 Managed futures	0.5035	−6.5469	−5.5203	−2.7215	1.0407	5.2856
	[0.913]	[0.335]	[0.455]	[0.768]	[0.672]	[0.267]
C10 Funds of funds	0.9778	2.1709	2.0400	1.5795	0.9017	0.1072
	[0.573]	[0.174]	[0.201]	[0.384]	[0.809]	[0.952]

This table shows the value of the fund for different values of the annual volatility of the market σ. The value of the fund is computed according to the following formula: $v = \beta_0 + \beta_1 + \delta\,[N(d_1) - kN(d_2)]$ where $d_1 = -\log(k)/\sigma$ and $d_2 = d_1 - \sigma$. p-values for the hypothesis that the value of the fund is equal to zero, $H_0 : v = 0$, are presented in brackets.

Table 6
Results with All Funds
Panel a: Cross-sectional Distribution of Linearity Tests (p-values)

	Wald $k = 1$	supWald	Wald$^{(h)}$ $k = 1$	supWald$^{(h)}$
Max p-value	0.999	1.000	0.999	1.000
Average p-value	0.380	0.331	0.394	0.330
Min p-value	0.000	0.000	0.000	0.000
Number and % of funds				
p-value >10%	1334	1228	1425	1244
	72.23%	66.49%	77.15%	67.35%
5% < p-value < 10%	123	151	145	158
	6.66%	8.18%	7.85%	8.55%
1% < p-value < 5%	188	198	171	195
	10.18%	10.72%	9.26%	10.56%
p-value < 1%	202	270	106	250
	10.94%	14.62%	5.74%	13.54%

This table shows the value of the cross-sectional distribution of the p-values for the linearity tests of those funds with at least 60 observations (1847 funds). Each p-value is for the hypothesis that the return of the fund has a linear relationship with the return of the market portfolio ($H_0 : \delta = 0$).

Panel b: Cross-sectional Distribution of Hedge Fund Performance

	Value $k = 1$	Value
Average value	6.925	6.933
Standard deviation value	9.092	9.311
Quartiles min value	−84.713	−90.345
25%	2.596	2.492
50%	6.165	6.227
75%	10.546	10.670
Max value	99.713	86.061
Max p-value	0.999	0.995
Average p-value	0.222	0.222
Min p-value	0.000	0.000
Number and % of funds (Value<*0*)		
p-value < 1%	10	9
	0.54%	0.49%
1% < p-value < 5%	16	17
	0.87%	0.92%
5% < p-value < 10%	10	7
	0.54%	0.38%
p-value >10%	223	230
	12.07%	12.45%
Number and % of funds (Value>*0*)		
p-value >10%	604	603
	32.70%	32.65%
5% < p-value < 10%	123	131
	6.66%	7.09%
1% < p-value < 5%	206	209
	11.15%	11.32%
p-value < 1%	655	641
	35.46%	34.71%

This table shows the cross-sectional distribution of the (annualized) performance (in %) and the cross-sectional distribution of the p-values for the hypothesis that the value of the fund is equal to zero for those funds with at least 60 observations (1847 funds). The value of the fund is computed under the assumption that the annual volatility of the return on the market index is $\sigma = 15\%$.

Table 7
Results with Live Funds
Panel a: Cross-sectional Distribution of Linearity Tests (p-values)

	Wald $k=1$	supWald	Wald$^{(h)}$ $k=1$	supWald$^{(h)}$
Max p-value	0.999	1.000	0.999	1.000
Average p-value	0.403	0.344	0.420	0.353
Min p-value	0.000	0.000	0.000	0.000
Number and % of funds				
p-value >10%	925	850	987	884
	75.20%	69.11%	80.24%	71.87%
5% < p-value < 10%	84	96	88	99
	6.83%	7.80%	7.15%	8.05%
1% < p-value < 5%	101	120	99	110
	8.21%	9.76%	8.05%	8.94%
p-value < 1%	120	164	56	137
	9.76%	13.33%	4.55%	11.14%

This table shows the value of the cross-sectional distribution of the p-values for the linearity tests of those live funds with at least 60 observations (1230 funds). Each p-value is for the hypothesis that the return of the fund has a linear relationship with the return of the market portfolio ($H_0 : \delta = 0$).

Panel b: Cross-sectional Distribution of Hedge Fund Performance

	Value $k=1$	Value
Average value	8.278	8.281
Standard deviation value	7.307	7.533
Quartiles min value	−27.495	−23.254
25%	4.133	3.921
50%	7.208	7.225
75%	11.087	11.186
Max value	52.304	52.171
Max p-value	0.999	0.995
Average p-value	0.160	0.158
Min p-value	0.000	0.000
Number and % of funds (Value<*0*)		
p-value < 1%	1	2
	0.08%	0.16%
1% < p-value < 5%	4	3
	0.33%	0.24%
5% < p-value < 10%	0	0
	0.00%	0.00%
p-value >10%	74	78
	6.02%	6.34%
Number and % of funds (Value>*0*)		
p-value >10%	341	344
	27.72%	27.97%
5% < p-value < 10%	92	88
	7.48%	7.15%
1% < p-value < 5%	162	170
	13.17%	13.82%
p-value < 1%	556	545
	45.20%	44.31%

This table shows the cross-sectional distribution of the (annualized) performance (in %) and the cross-sectional distribution of the p-values for the hypothesis that the value of the fund is equal to zero for those live funds with at least 60 observations (1230 funds). The value of the fund is computed under the assumption that the annual volatility of the return on the market index is $\sigma = 15\%$.

Table 8
Results with Graveyard Funds
Panel a: Cross-sectional Distribution of Linearity Tests (p-values)

	Wald $k = 1$	supWald	Wald$^{(h)}$ $k = 1$	supWald$^{(h)}$
Max p-value	0.999	0.995	0.999	0.986
Average p-value	0.333	0.303	0.344	0.285
Min p-value	0.000	0.000	0.000	0.000
Number and % of funds				
p-value >10%	409	378	438	360
	66.29%	61.26%	70.99%	58.35%
5% < p-value < 10%	39	55	57	59
	6.32%	8.91%	9.24%	9.56%
1% < p-value < 5%	87	78	72	85
	14.10%	12.64%	11.67%	13.78%
p-value < 1%	82	106	50	113
	13.29%	17.18%	8.10%	18.31%

This table shows the value of the cross-sectional distribution of the p-values for the linearity tests of those graveyard funds with at least 60 observations (617 funds). Each p-value is for the hypothesis that the return of the fund has a linear relationship with the return of the market portfolio ($H_0 : \delta = 0$).

Panel b: Cross-sectional Distribution of Hedge Fund Performance

	Value $k = 1$	Value
Average value	4.227	4.245
Standard deviation value	11.411	11.651
Quartiles min value	−84.713	−90.345
25%	−0.745	−1.004
50%	3.596	3.548
75%	8.679	8.999
Max value	99.713	86.061
Max p-value	0.995	0.994
Average p-value	0.345	0.349
Min p-value	0.000	0.000
Number and % of funds (Value<*0*)		
p-value < 1%	9	7
	1.46%	1.13%
1% < p-value < 5%	12	14
	1.94%	2.27%
5% < p-value < 10%	10	7
	1.62%	1.13%
p-value >10%	149	152
	24.15%	24.64%
Number and % of funds (Value>*0*)		
p-value >10%	263	259
	42.63%	41.98%
5% < p-value < 10%	31	43
	5.02%	6.97%
1% < p-value < 5%	44	39
	7.13%	6.32%
p-value < 1%	99	96
	16.05%	15.56%

This table shows the cross-sectional distribution of the (annualized) performance (in %) and the corresponding cross-sectional distribution of the p-values for the hypothesis that the value of the fund is equal to zero for those graveyard funds with at least 60 observations (617 funds). The value of the fund is computed under the assumption that the annual volatility of the return on the market index is $\sigma = 15\%$.

Table 9
Results with Arbitrage Funds
Panel a: Cross-sectional Distribution of Linearity Tests (p-values)

	Wald $k = 1$	supWald	Wald$^{(h)}$ $k = 1$	supWald$^{(h)}$
Max p-value	0.994	0.976	0.993	0.972
Average p-value	0.282	0.235	0.298	0.233
Min p-value	0.000	0.000	0.000	0.000
Number and % of funds				
p-value >10%	194	178	204	178
	59.15%	54.27%	62.20%	54.27%
5% < p-value < 10%	26	26	33	24
	7.93%	7.93%	10.06%	7.32%
1% < p-value < 5%	41	50	49	52
	12.50%	15.24%	14.94%	15.85%
p-value < 1%	67	74	42	74
	20.43%	22.56%	12.81%	22.56%

This table shows the value of the cross-sectional distribution of the p-values for the linearity tests of those funds with at least 60 observations and that belong to the category convertible arbitrage, fixed-income arbitrage, or event driven (328 funds). Each p-value is for the hypothesis that the return of the fund has a linear relationship with the return of the market portfolio ($H_0 : \delta = 0$).

Panel b: Cross-sectional Distribution of Hedge Fund Performance

	Value $k = 1$	Value
Average value	7.161	7.125
Standard deviation value	6.283	6.470
Quartiles min value	−22.525	−23.491
25%	4.220	4.133
50%	6.865	6.883
75%	9.833	9.712
Max value	39.005	37.951
Max p-value	0.961	0.993
Average p-value	0.090	0.097
Min p-value	0.000	0.000
Number and % of funds (Value<*0*)		
p-value < 1%	1	2
	0.30%	0.61%
1% < p-value < 5%	3	2
	0.91%	0.61%
5% < p-value < 10%	0	0
	0.00%	0.00%
p-value >10%	15	13
	4.57%	3.96%
Number and % of funds (Value>*0*)		
p-value >10%	48	55
	14.63%	16.77%
5% < p-value < 10%	10	14
	3.05%	4.27%
1% < p-value < 5%	33	33
	10.06%	10.06%
p-value < 1%	218	209
	66.46%	63.72%

This table shows the cross-sectional distribution of the (annualized) performance (in %) and the corresponding cross-sectional distribution of the p-values for the hypothesis that the value of the fund is equal to zero for those funds with at least 60 observations and that belong to the category convertible arbitrage, fixed-income arbitrage or event driven (328 funds). The value of the fund is computed under the assumption that the annual volatility of the return on the market index is $\sigma = 15\%$.

Table 10
Results with Equity Market Neutral/Long-Short Funds
Panel a: Cross-sectional Distribution of Linearity Tests (*p*-values)

	Wald $k=1$	supWald	Wald$^{(h)}$ $k=1$	supWald$^{(h)}$
Max p-value	0.998	1.000	0.998	1.000
Average p-value	0.487	0.436	0.484	0.428
Min p-value	0.000	0.000	0.000	0.000
Number and % of funds				
p-value >10%	513	496	519	478
	85.50%	82.67%	86.50%	79.67%
5% < p-value < 10%	34	34	38	48
	5.67%	5.67%	6.33%	8.00%
1% < p-value < 5%	34	38	26	33
	5.67%	6.33%	4.33%	5.50%
p-value < 1%	19	32	17	41
	3.17%	5.33%	2.83%	6.83%

This table shows the value of the cross-sectional distribution of the p-values for the linearity tests of those funds with at least 60 observations and that belong to the category equity market neutral or long-short equity hedge (600 funds). Each p-value is for the hypothesis that the return of the fund has a linear relationship with the return of the market portfolio ($H_0 : \delta = 0$).

Panel b: Cross-sectional Distribution of Hedge Fund Performance

	Value $k=1$	Value
Average value	8.704	8.705
Standard deviation value	8.414	8.797
Quartiles min value	−17.474	−15.671
25%	3.917	3.600
50%	8.153	8.109
75%	12.350	12.435
Max value	63.664	71.489
Max p-value	0.997	0.995
Average p-value	0.208	0.208
Min p-value	0.000	0.000
Number and % of funds (Value<*0*)		
p-value < 1%	1	0
	0.17%	0.00%
1% < p-value < 5%	2	3
	0.33%	0.50%
5% < p-value < 10%	1	1
	0.17%	0.17%
p-value >10%	57	64
	9.50%	10.67%
Number and % of funds (Value>*0*)		
p-value >10%	219	208
	36.50%	34.67%
5% < p-value < 10%	49	54
	8.17%	9.00%
1% < p-value < 5%	90	87
	15.00%	14.50%
p-value < 1%	181	183
	30.17%	30.50%

This table shows the cross-sectional distribution of the (annualized) performance (in %) and the cross-sectional distribution of the p-values for the hypothesis that the value of the fund is equal to zero for those funds with at least 60 observations and that belong to the category equity market neutral or long-short equity hedge (600 funds). The value of the fund is computed under the assumption that the annual volatility of the return on the market index is $\sigma = 15\%$.

Table 11
Results with Directional Funds
Panel a: Cross-sectional Distribution of Linearity Tests (p-values)

	Wald $k=1$	supWald	Wald$^{(h)}$ $k=1$	supWald$^{(h)}$
Max p-value	0.999	0.995	0.999	0.987
Average p-value	0.337	0.300	0.357	0.292
Min p-value	0.000	0.000	0.000	0.000
Number and % of funds				
p-value >10%	313	274	348	288
	67.60%	59.18%	75.16%	62.20%
5% < p-value < 10%	34	46	35	42
	7.34%	9.94%	7.56%	9.07%
1% < p-value < 5%	65	65	55	57
	14.04%	14.04%	11.88%	12.31%
p-value < 1%	51	78	25	76
	11.02%	16.85%	5.40%	16.42%

This table shows the value of the cross-sectional distribution of the p-values for the linearity tests of those funds with at least 60 observations and that belong to the category global macro, emerging markets, or managed futures (463 funds). Each p-value is for the hypothesis that the return of the fund has a linear relationship with the return of the market portfolio ($H_0 : \delta = 0$).

Panel b: Cross-sectional Distribution of Hedge Fund Performance

	value $k=1$	value
Average value	6.502	6.545
Standard deviation value	12.729	12.924
Quartiles min value	−84.713	−90.345
25%	0.165	0.264
50%	5.133	4.924
75%	12.322	11.999
Max value	99.713	86.061
Max p-value	0.999	0.994
Average p-value	0.342	0.338
Min p-value	0.000	0.000
Number and % of funds (Value<*0*)		
p-value < 1%	8	7
	1.73%	1.51%
1% < p-value < 5%	6	7
	1.30%	1.51%
5% < p-value < 10%	6	4
	1.30%	0.86%
p-value >10%	92	93
	19.87%	20.09%
Number and % of funds (Value>*0*)		
p-value >10%	213	210
	46.00%	45.36%
5% < p-value < 10%	36	38
	7.78%	8.21%
1% < p-value < 5%	41	42
	8.86%	9.07%
p-value < 1%	61	62
	13.18%	13.39%

This table shows the cross-sectional distribution of the (annualized) performance (in %) and the cross-sectional distribution of the p-values for the hypothesis that the value of the fund is equal to zero for those funds with at least 60 observations and that belong to the category global macro, emerging markets, or managed futures (463 funds). The value of the fund is computed under the assumption that the annual volatility of the return on the market index is $\sigma = 15\%$.

Table 12
Results with Category 1: Convertible Arbitrage
Panel a: Cross-sectional Distribution of Linearity Tests (p-values)

	Wald $k=1$	supWald	Wald$^{(h)}$ $k=1$	supWald$^{(h)}$
Max p-value	0.994	0.952	0.986	0.916
Average p-value	0.433	0.331	0.447	0.320
Min p-value	0.002	0.001	0.003	0.001
Number and % of funds				
p-value >10%	68	62	69	61
	85.00%	77.50%	86.25%	76.25%
5% < p-value < 10%	3	8	5	5
	3.75%	10.00%	6.25%	6.25%
1% < p-value < 5%	8	5	4	9
	10.00%	6.25%	5.00%	11.25%
p-value < 1%	1	5	2	5
	1.25%	6.25%	2.50%	6.25%

This table shows the value of the cross-sectional distribution of the p-values for the linearity tests of those funds with at least 60 observations and that belong to the category convertible arbitrage (80 funds). Each p-value is for the hypothesis that the return of the fund has a linear relationship with the return of the market portfolio ($H_0 : \delta = 0$).

Panel b: Cross-sectional Distribution of Hedge Fund Performance

	value $k=1$	value
Average value	6.755	6.927
Standard deviation value	5.997	6.461
Quartiles min value	−17.737	−21.250
25%	3.720	3.225
50%	5.753	5.864
75%	9.427	9.712
Max value	23.792	26.260
Max p-value	0.961	0.884
Average p-value	0.092	0.090
Min p-value	0.000	0.000
Number and % of funds (Value<*0*)		
p-value < 1%	0	1
	0.00%	1.25%
1% < p-value < 5%	1	0
	1.25%	0.00%
5% < p-value < 10%	0	0
	0.00%	0.00%
p-value >10%	4	3
	5.00%	3.75%
Number and % of funds (Value>*0*)		
p-value >10%	12	12
	15.00%	15.00%
5% < p-value < 10%	2	4
	2.50%	5.00%
1% < p-value < 5%	7	6
	8.75%	7.50%
p-value < 1%	54	54
	67.50%	67.50%

This table shows the cross-sectional distribution of the (annualized) performance (in %) and the cross-sectional distribution of the p-values for the hypothesis that the value of the fund is equal to zero for those funds with at least 60 observations and that belong to the category convertible arbitrage (80 funds). The value of the fund is computed under the assumption that the annual volatility of the return on the market index is $\sigma = 15\%$.

Table 13
Results with Category 9: Managed Futures
Panel a: Cross-sectional Distribution of Linearity Tests (p-values)

	Wald $k=1$	supWald	Wald$^{(h)}$ $k=1$	supWald$^{(h)}$
Max p-value	0.989	0.995	0.990	0.987
Average p-value	0.305	0.278	0.321	0.255
Min p-value	0.000	0.000	0.000	0.000
Number and % of funds				
p-value >10%	148	126	166	128
	62.98%	53.62%	70.64%	54.47%
5% < p-value < 10%	18	24	17	21
	7.66%	10.21%	7.23%	8.94%
1% < p-value < 5%	41	41	36	34
	17.45%	17.45%	15.32%	14.47%
p-value < 1%	28	44	16	52
	11.92%	18.72%	6.81%	22.13%

This table shows the value of the cross-sectional distribution of the p-values for the linearity tests of those funds with at least 60 observations and that belong to the category managed futures (235 funds). Each p-value is for the hypothesis that the return of the fund has a linear relationship with the return of the market portfolio ($H_0 : \delta = 0$).

Panel b: Cross-sectional Distribution of Hedge Fund Performance

	value ($k=1$)	value
Average value	6.968	6.734
Standard deviation value	11.803	11.597
Quartiles min value	−39.724	−37.350
25%	1.018	1.069
50%	5.108	4.710
75%	11.680	11.167
Max value	99.713	86.061
Max p-value	0.998	0.971
Average p-value	0.390	0.371
Min p-value	0.000	0.000
Number and % of funds (Value<*0*)		
p-value < 1%	2	1
	0.85%	0.43%
1% < p-value < 5%	2	3
	0.85%	1.28%
5% < p-value < 10%	2	2
	0.85%	0.85%
p-value >10%	42	41
	17.87%	17.45%
Number and % of funds (Value>*0*)		
p-value >10%	129	127
	54.89%	54.04%
5% < p-value < 10%	19	18
	8.09%	7.66%
1% < p-value < 5%	20	23
	8.51%	9.79%
p-value < 1%	19	20
	8.09%	8.51%

This table shows the cross-sectional distribution of the (annualized) performance (in %) and the corresponding cross-sectional distribution of the p-values for the hypothesis that the value of the fund is equal to zero for those funds with at least 60 observations and that belong to the category managed futures (235 funds). The value of the fund is computed under the assumption that the annual volatility of the return on the market index is $\sigma = 15\%$.

Table 14
Comparison with Agarwal and Naik's (2004) Approach

	Valuation
HFR	
C1 Convertible arbitrage	2.541
	[0.132]
C2 Fixed-income arbitrage	−0.678
	[0.716]
C3 Event driven	0.113
	[0.950]
C4 Equity market neutral	−0.239
	[0.836]
C5 Long-short equity hedge	−1.023
	[0.564]
C6 Global macro	−3.994
	[0.115]
C7 Emerging markets	−3.761
	[0.475]
C8 Dedicated short bias	0.872
	[0.841]
C9 Managed futures	−4.843
	[0.154]
C10 Funds of funds	−2.568
	[0.165]
C11 Other	−2.810
	[0.298]
TREMONT	
C1 Convertible arbitrage	2.799
	[0.044]
C2 Fixed-income arbitrage	0.676
	[0.594]
C3 Event driven	5.000
	[0.024]
C4 Equity market neutral	0.260
	[0.923]
C5 Long-short equity hedge	0.697
	[0.752]
C6 Global macro	−1.532
	[0.710]
C7 Emerging markets	−6.047
	[0.315]
C8 Dedicated short bias	7.188
	[0.355]
C9 Managed futures	3.037
	[0.502]
C10 Funds of funds	0.155
	[0.940]
C11 Other	1.009
	[0.731]

This table presents results for the valuation of the corresponding hedge fund category index computed according to the following formula $v = \beta_0 + \beta_1 + \beta_2 + \beta_3$ where these coefficients have been computed according to the following regression: $X^*_{pt} = \beta_0 + \beta_1 R^*_{It} + \beta_2 R^*_{call,t} + \beta_3 R^*_{put,t} + \varepsilon_t$; where the asterisk indicates that the returns on the indexes have been normalized by R_{ft}, R_{It} denote the returns on the CRSP value-weighted index, and $R_{call,t}$ and $R_{put,t}$ denote the returns on the OTM call and put option factors, respectively, in Agarwal and Naik (2004). The corresponding p-values for the hypothesis that the value of the fund is equal to zero, $H_0 : v = 0$, are presented in brackets.

Figure 1
Selectivity versus Market Timing in Henriksson-Merton Regressions

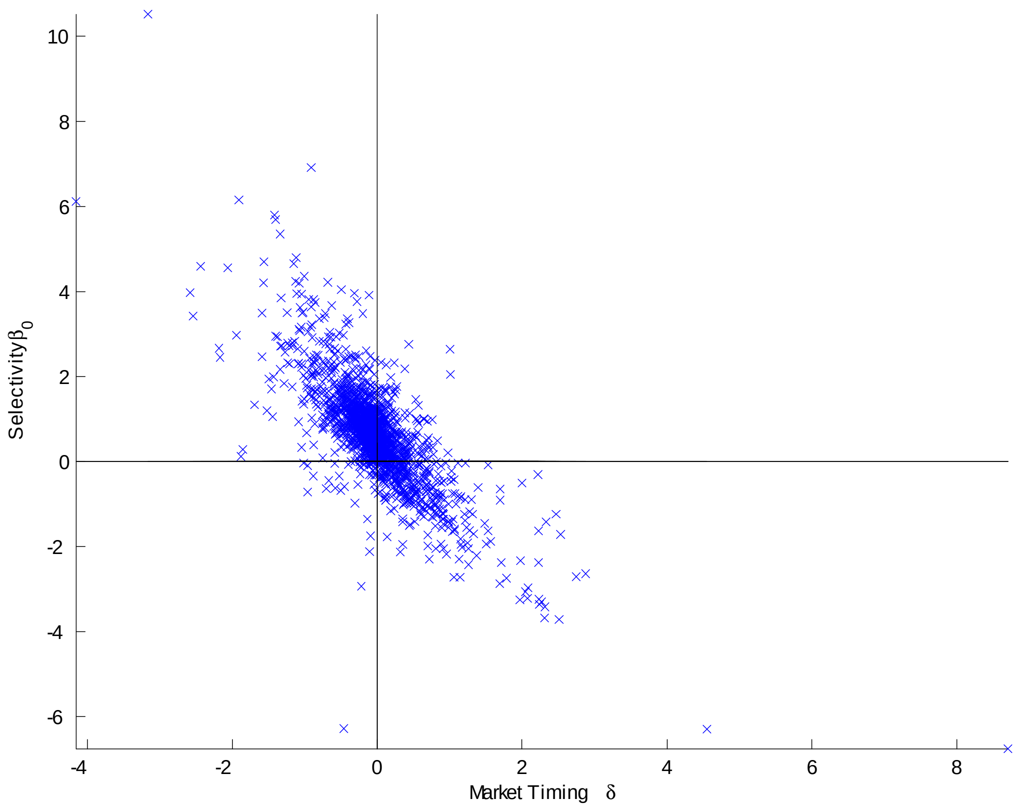


Figure 2
Piecewise Linear Fit: TREMONT Index

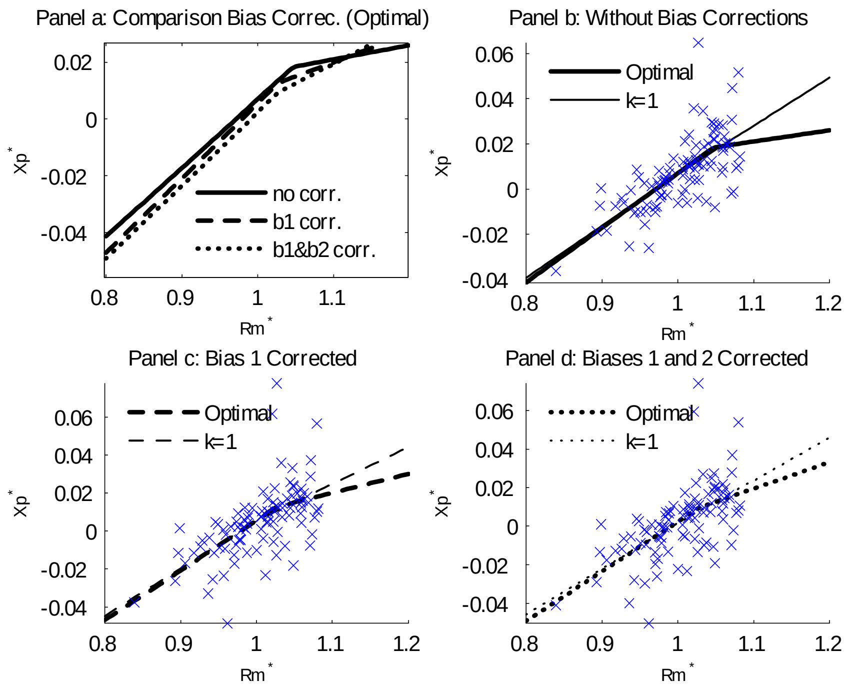


Figure 3
Valuation: TREMONT Index

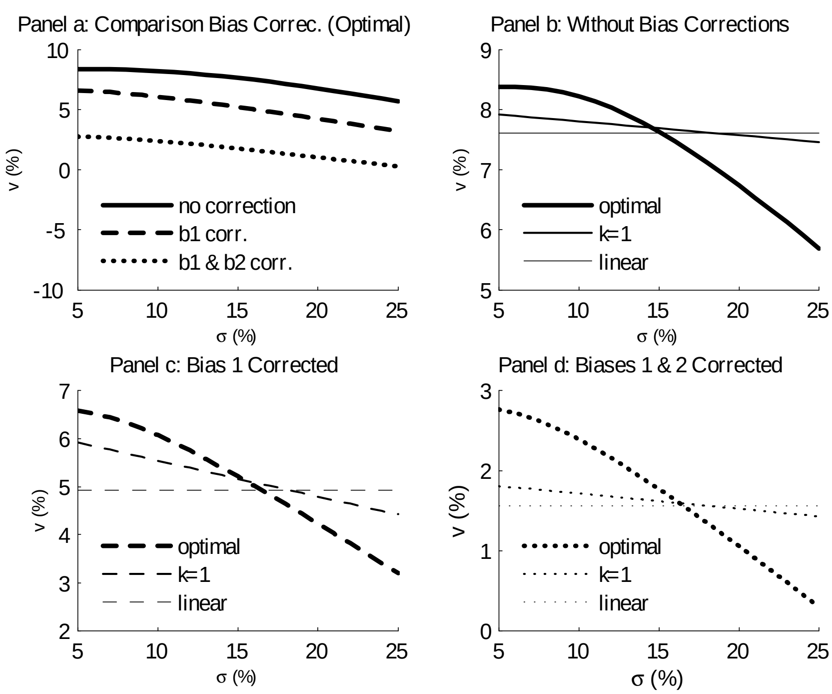
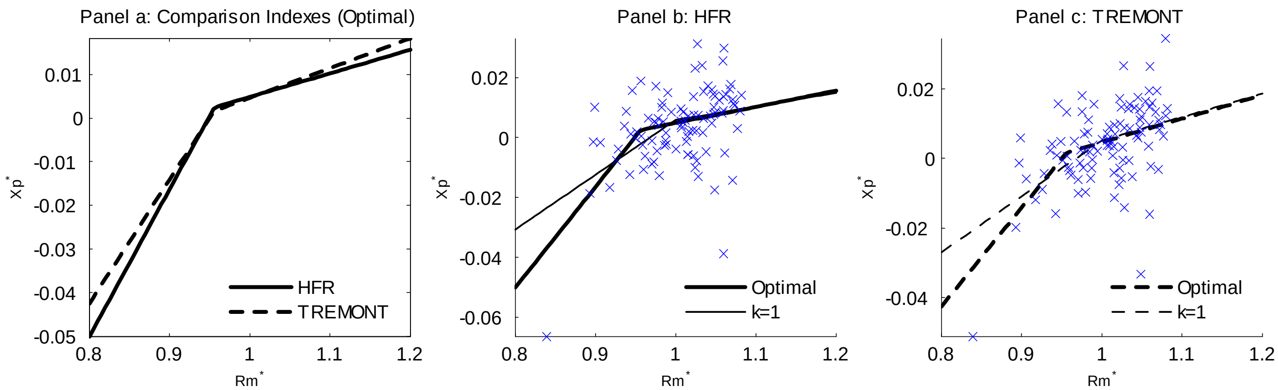


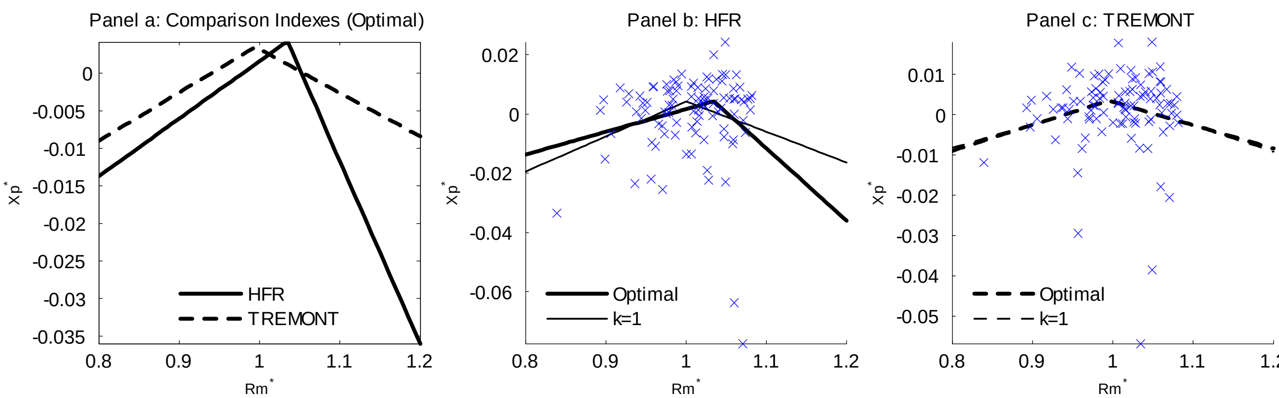
Figure 4

Piecewise Linear Fit: Categories

C1 Convertible Arbitrage (Backfilling and Survivorship Bias Correction)



C2 Fixed-Income Arbitrage (Backfilling and Survivorship Bias Correction)



C9 Managed Futures (Backfilling and Survivorship Bias Correction)

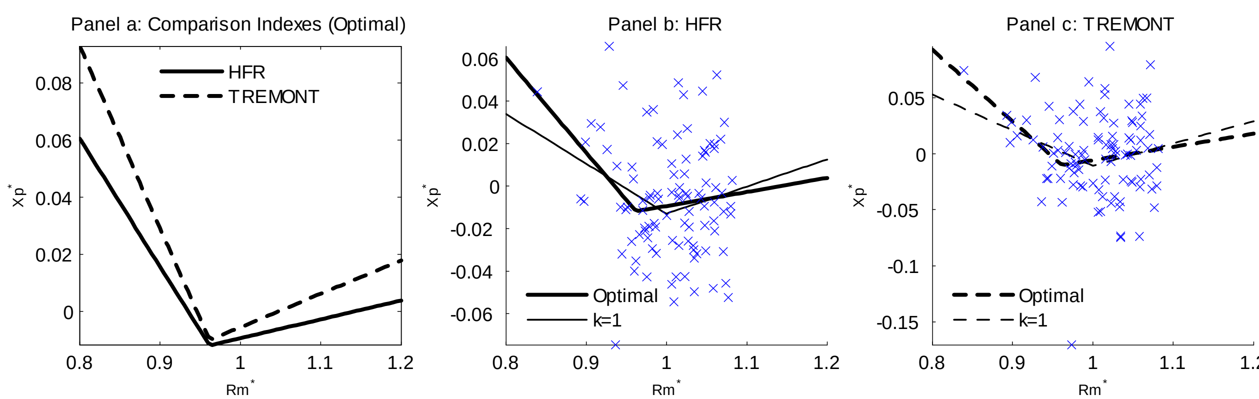
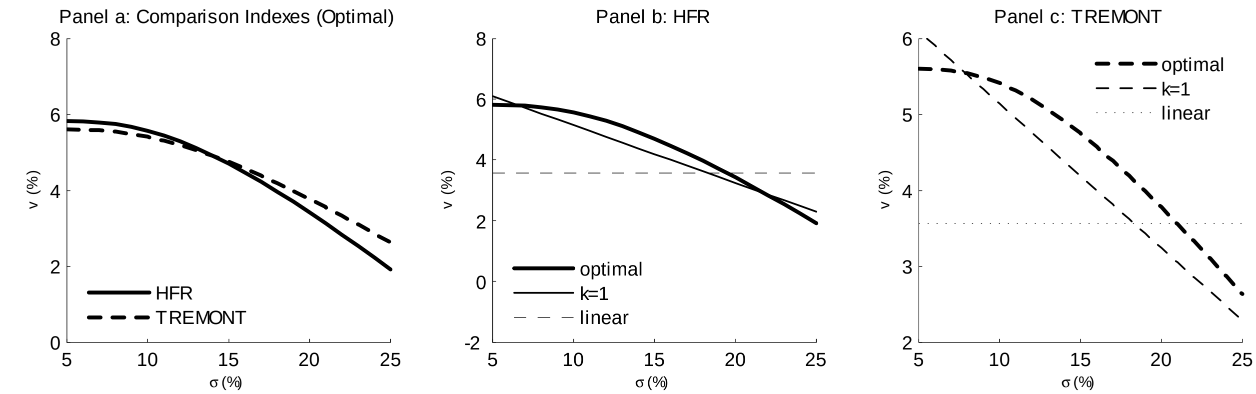
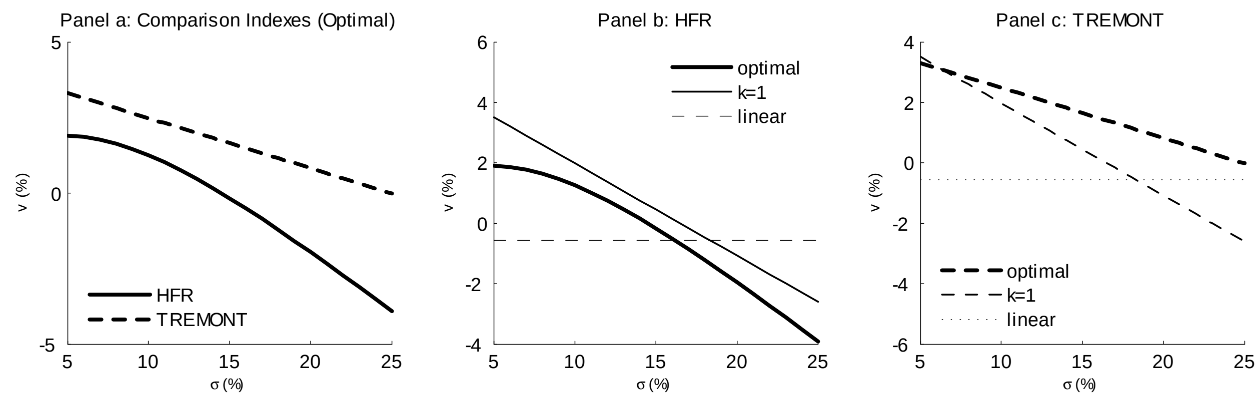


Figure 5
Valuation: Categories
C1 Convertible Arbitrage (Backfilling and Survivorship Bias Correction)



C2 Fixed-Income Arbitrage (Backfilling and Survivorship Bias Correction)



C9 Managed Futures (Backfilling and Survivorship Bias Correction)

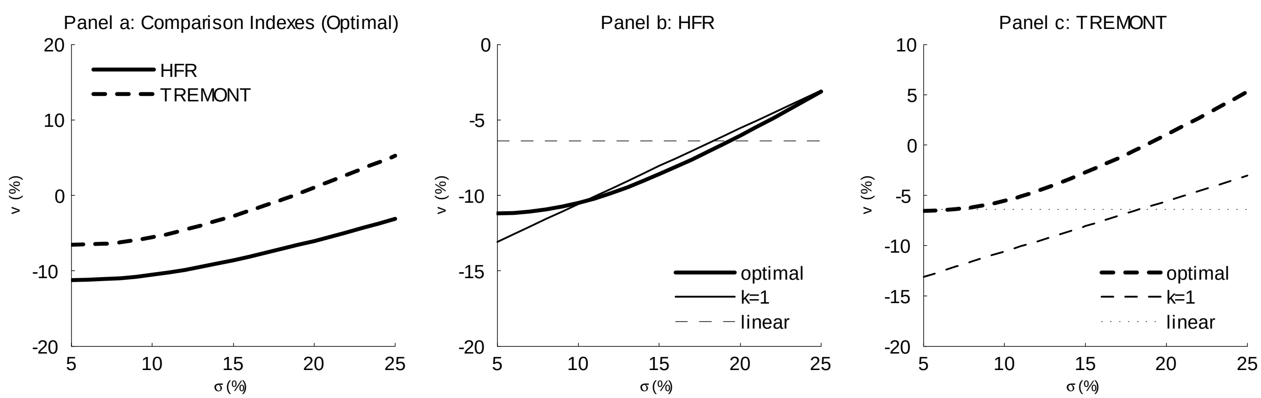
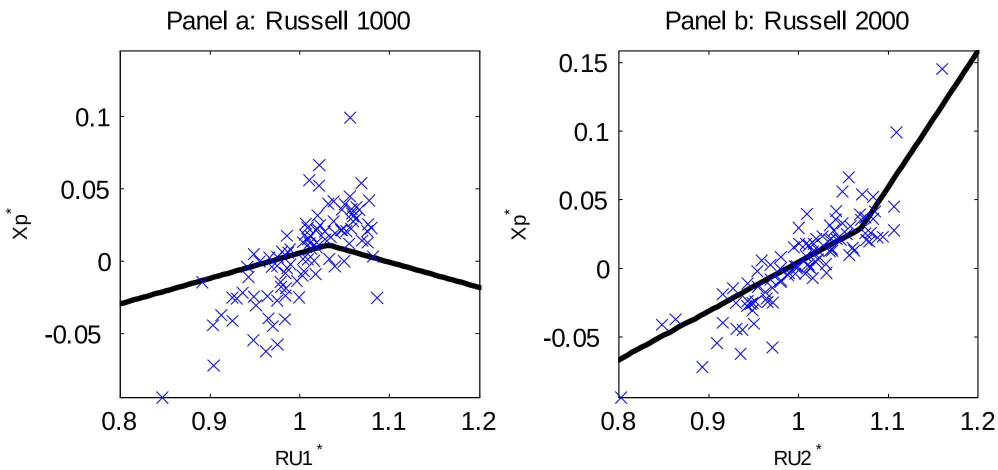


Figure 6
Piecewise Linear Fit Russell: TREMONT Index



Appendix A: Brief Definitions of TASS Hedge Fund Categories

These definitions are based on Lhabitant (2004).

C1 Convertible arbitrage

A typical strategy in this category is to be long in the convertible bond and short in the common stock of the same company. Profits are generated from both positions. The principal is usually protected from market fluctuations.

C2 Fixed-income arbitrage

The goal is to exploit price anomalies between related interest rate securities, such as interest rate swaps, U.S. and non-U.S. government bonds, and mortgage-backed securities.

C3 Event driven

This strategy aims at making profits by using price movements related to special pending events such as mergers, liquidations, bankruptcies, or reorganizations. In risk arbitrage, the hedge fund manager usually invests long in the stock of the company being acquired and short in the stock of the acquiring company.

C4 Equity market neutral

This investment strategy aims at balancing long and short positions to ensure a negligible market exposure in a broad sense. A fund may be neutral to a specific exchange rate, a stock index, a series of interest rates, or other factors.

C5 Long-short equity

Long/short strategies involve the combined purchase and sale of two securities. The main source of return comes from the spread in performance between the stocks on the long side (which should appreciate in value) and the shorted stocks (which should decrease in value). The strategies can be based on value, growth, or size.

C6 Global macro

Global macro funds do not hedge anything. They make directional bets based on their forecasts of market directions according to economic trends or particular events. They are not specialized, and carry long and short positions in any of the major world capital or derivative markets. The portfolios include stocks, bonds, currencies, and commodities. Most funds invest globally in both developed and emerging markets.

C7 Emerging markets

These funds take positions in all types of securities in emerging markets around the world. Investments in emerging market equities are primarily long, since many emerging markets do not allow short selling and no viable futures markets exist to hedge market risk.

C8 Dedicated short bias

Dedicated short hedge funds seek to profit from a decline in the value of stocks by taking short positions. These funds are rare nowadays, since they migrated to the long/short category, where they still have a systematic short bias.

C9 Managed futures

These funds, often referred to as commodity trading advisers (CTAs), invest in financial and commodity futures markets and currency markets around the world. A large proportion are trend followers (buy in an up market and sell in a down market). Others use discretionary (judgmental) or systematic (based on technical information) strategies.

C10 Funds of funds

Managers of these funds allocate capital to several hedge funds. Investors in these gain exposure to many different managers and strategies.

C11 Other

This category consists of the funds that announce using more than one strategy and cannot be classified into these other categories.

Appendix B: Testing for Linearity

We are interested in fitting piecewise linear functions such as:

$$y_t = \beta_0 + \beta_1 x_t + \sum_{i=1}^{m} \delta_i \max(x_t - k_i, 0) + \varepsilon_t \qquad t = 1, \dots n,$$

where ε_t is a real-valued martingale sequence. This equation is just a general formulation of the one that appears in (5). In particular, y_t is the excess return of a hedge fund and x_t is the return on an index that drives the SDF (e.g., the market).

To determine the number and position of the knots (that is, the m and the k_i's, respectively) we start by testing whether the linear fit ($m = 0$) provides a better approximation to the description of the data than a model with only one option ($m = 1$). Note that when $\delta = 0$, the linear model is nested in the formulation $m = 1$:

$$y_t = \beta_0 + \beta_1 x_t + \delta \max(x_t - k, 0) + \varepsilon_t \qquad t = 1, \dots n.$$

We rewrite this specification as:

$$y_t = \mathbf{x}_t(k)'\mathbf{b} + \varepsilon_t \qquad t = 1, \dots n,$$

where $\mathbf{x}_t(k) = [1, x_t, \max(x_t - k, 0)]'$ and $\mathbf{b} = [\beta_0, \beta_1, \delta]'$.

If the strike of the option k were known a priori, the testing problem would be the usual one where the following heteroscedasticity-robust Wald statistic could be used:

$$T_n(k) = n\widehat{\mathbf{b}}(k) R \left[R'\widehat{\mathbf{V}}(k) R' \right]^{-1} R'\widehat{\mathbf{b}}(k), \tag{B1}$$

where $\widehat{\mathbf{b}}(k) = \left[\sum_{t=1}^{n} \mathbf{x}_t(k)\mathbf{x}_t(k)'\right]^{-1} \left[\sum_{t=1}^{n} \mathbf{x}_t(k) y_t\right]$ and R is the vector (or, more generally, the matrix) that, applied to the vector $\mathbf{b}$, selects the parameter of interest, δ; that is, $R = (0, 0, 1)'$. The robust estimate of the covariance matrix $\widehat{\mathbf{V}}(k)$ is of the usual form $\widehat{\mathbf{M}}(k,k)^{-1}\widehat{\mathbf{K}}(k,k)\widehat{\mathbf{M}}(k,k)^{-1}$, where

$$\widehat{\mathbf{K}}(k_1, k_2) = \frac{1}{n} \sum_{t=1}^{n} \left[\widehat{\mathbf{s}}_t(k_1)\widehat{\mathbf{s}}_t(k_2)'\right],$$

$$\widehat{\mathbf{M}}(k_1, k_2) = \frac{1}{n} \sum_{t=1}^{n} \left[\mathbf{x}_t(k_1)\mathbf{x}_t(k_2)'\right],$$

with $\widehat{\mathbf{s}}_t(k) = \mathbf{x}_t(k)\left[y_t - \mathbf{x}_t(k)'\widehat{\mathbf{b}}(k)\right]$ being the regression score under the alternative. The Wald statistic $T_n(k)$ will have an approximate chi-square distribution with one degree of freedom (the number of restrictions) in large samples.

Note that the OLS estimate, $\widehat{\mathbf{b}}(k)$, and the value of the Wald test, $T_n(k)$, will then vary according to the choice k. Therefore, we treat the value of k as an unknown and its value is estimated in a data-dependent procedure. In particular, the least-square estimate of k can be found sequentially through concentration. That is, for a given value of the strike of the option k, we run an OLS regression as in the case where k is known. We then search over the possible values of k for the one that minimizes the sum of squared errors $\widehat{e}_t(k)'\widehat{e}_t(k)$ to get our estimate of this parameter. Following Hansen (1996, 1999), we restrict our search to the observed values of x_t. Moreover, since the point-wise statistics are ill behaved for extreme values k, we further restrict the search to values of x_t lying between the τth and $(1-\tau\text{th})$ quantiles of its distribution, being $\tau = 0.15$.

However, the chi-square distribution for the Wald test statistic is invalid, since k is chosen in a data-dependent procedure. Instead, we follow Davies (1977, 1987), who suggests computing the Wald test statistic, $T_n(k)$, for each possible value of k and then focusing on the supremum value of such a sequence; that is, $T_n = \sup_k T_n(k)$. This statistic is known as "supWald." Again, the problem that we face is that the asymptotic distribution of this test is non-standard. In particular, using an appropriate asymptotic theory for random functions (known as empirical process theory), Hansen (1996) derives the asymptotic distributions of this test under the null hypothesis and provides a simulation method to compute the various distributions. He shows that the test statistic sequence, $T_n(k)$ (e.g., the Wald test for each possible value of the strike k), converges in distribution to the following process:

$$T_n(k) \to_d T(k),$$
$$T(k) = \mathbf{S}(k)'\mathbf{M}(k,k)^{-1}R\left[R'\mathbf{V}(k)R'\right]^{-1}R'\mathbf{M}(k,k)^{-1}\mathbf{S}(k),$$

where $\mathbf{S}(k)$ denotes a mean zero Gaussian process with a covariance kernel $\mathbf{K}(k_1,k_2)$,[1] such that $\mathbf{S}_n(k) = (1/\sqrt{n})\sum_{t=1}^{n}\mathbf{s}_t(k)$ converges in distribution to $\mathbf{S}(k)$. This implies that the supWald statistic T_n converges to $T = \sup_k T(k)$, and Hansen (1996) proposes calculating the asymptotic distribution of this statistic T through simulation.

The algorithm for that purpose is the following. Let J be the number of simulations used

[1] Which means that, for any $\{k_1, k_2, \ldots k_l\}$, $\{\mathbf{S}(k_1), \mathbf{S}(k_2), \ldots \mathbf{S}(k_l)\}$ is multivariate normal with mean zero and covariances $E\left[\mathbf{S}(k_i)\mathbf{S}(k_j)'\right] = \mathbf{K}(k_i,k_j)$.

to approximate the asymptotic distribution of the statistic. Then, for $j = 1, \ldots, J$, execute the following steps:

(i) generate $\{v_{tj}\}_{t=1}^{n}$ i.i.d. $N(0,1)$ random variables,

(ii) set $\mathbf{S}_n^j(k) = \frac{1}{\sqrt{n}} \sum_{t=1}^{n} \widehat{\mathbf{s}}_t(k) v_{tj}$,

(iii) set $T_n^j(k) = \mathbf{S}_n^j(k)' \widehat{\mathbf{M}}(k,k)^{-1} R \left[R' \widehat{\mathbf{V}}(k) R' \right]^{-1} R' \widehat{\mathbf{M}}(k,k)^{-1} \mathbf{S}_n^j(k)$,

(iv) set $T_n^j = \max_k T_n^j(k)$.

Again, we follow Hansen (1996, 1999) and set $J = 2000$ in our empirical exercise. This gives a random sample $\left\{ T_n^1, \ldots, T_n^J \right\}$ of observations of the conditional distribution of the statistic. Finally, we can compute the percentage of these artificial observations which exceed the actual test statistic T_n to compute an "asymptotic p-value" such as:

$$\widehat{p}_n^J = \frac{1}{J} \sum_{t=1}^{n} \left\{ T_n^j \geq T_n \right\},$$

and, as usual, if the value of this "asymptotic p-value" $\widehat{p}_n^J$ falls below the usual 10 per cent, 5 per cent, or 1 per cent value, then we will reject the null hypothesis of linearity at that level.

Appendix C: Monte Carlo Simulation Study

The Monte Carlo experiment is conducted for tests of size 10 per cent, 5 per cent, and 1 per cent, but we will refer only to those of size 5 per cent, since no qualitative differences are observed. The number of replications is set to $N = 2000$, the number of internal simulations to compute the p-values is set to $J = 1000$, and the search is restricted to the values of x_t lying between the τth and $(1 - \tau\text{th})$ quantiles, being $\tau = 0.15$.

In a similar spirit to the simulation study in Hansen (1996), we compare test statistics using four different covariance matrices: (i) standard Wald, (ii) standard LM, (iii) heteroscedasticity-consistent LM, and (iv) heteroscedasticity-consistent Wald. In particular, we compute Davies' (1977, 1987) supremum test, which is the one we use in the main text, but we also compute Andrews and Ploberger's (1994) average and exponential average tests for each one of the covariance matrices. These authors suggest that superior local power can be constructed by computing an average or an exponential average of the Wald test statistic (aveWald and expWald tests, respectively) over the parameter space admissible for k. The asymptotic distribution of such a statistic can be computed by replacing step number (iv) in the simulation of the p-values with the corresponding (exponential) average of the random sample $\left\{T_n^1, \ldots, T_n^J\right\}$ of observations of the statistic. We also include the Wald and LM tests for the case where the knot value is known and set to its true value. As a reminder, the moneyness of the option that best approximates hedge fund returns is not known a priori, so we include them only for comparison purposes.

The market return R_{It}^* and the error term are generated from two independent Gaussian distributions.[1] The hedge fund return is generated according to the piecewise linear function in equation (9). The values of β_0, β_1, k, and σ^2 are set to their corresponding estimates, while the variance of R_{It}^* is set to the unconditional variance of this variable during the period January 1996 to March 2004. In order to assess the finite-sample size of these tests, we start by setting $\delta = 0$ (null hypothesis of linearity).

We find that the asymptotic approximation in Hansen (1996), which we use in our main text, delivers good size properties for the "standard tests" (those without heteroscedasticity correction) and the heteroscedasticity-consistent LM test: the proportion of rejections is around 5 per cent for a 5 per cent size test, regardless of the sample size and the choice of the

[1] Regarding the presence of autocorrelation, we did a thorough check of the presence of first-order autocorrelation in the residuals of both the linear and non-linear models for all categories, for the indexes as well as for individual funds. We did not find any strong evidence of remaining autocorrelation.

Monte Carlo design. However, the heteroscedasticity-robust Wald test tends to over-reject for our sample sizes: approximately 11 per cent of rejections for the $\sup\text{Wald}^{(h)}$ test when the sample size is $T = 100$ and 15 per cent when the sample size is $T = 60$. These results are consistent with the simulation study in Hansen (1996). Still, this size distortion is similar to the one we find for the heteroscedasticity-robust Wald test when the position of the "kink" is known and set to the true value: 10 per cent when $T = 100$ and 12 per cent when $T = 60$.

To assess finite-sample power, we set $\delta = \widehat{\delta}$ and $\delta = 2\widehat{\delta}$ while maintaining fixed the remaining parameters. As expected, the number of rejections (power) increases with δ and the sample size. In particular, we find that, for a sample size of $T = 100$, the proportion of rejections is close to 50 per cent when $\delta = \widehat{\delta}$ and close to 90 per cent when $\delta = 2\widehat{\delta}$. It is also worth mentioning that the finite-sample power of the Andrews and Ploberger (1994) average and exponential average tests tends to be smaller than their supremum test counterpart. Therefore, we drop them from our analysis. Our results suggest that the heteroscedasticity-robust Wald test is the most powerful test across all the statistics computed in this simulation study, while the standard Wald and LM tests rank second and third, respectively. On the other side of the spectrum, the heteroscedasticity-robust LM test is the most conservative. While one possible explanation for this finding could reside in the evidence of overrejection found for the $\sup\text{Wald}^{(h)}$ test, that is not the case, since we also find the same ranking when computing size-adjusted power for these tests. Again, our results indicate that the loss of power with respect to the benchmark of a known knot set to its true value is small. For example, for the TREMONT category 1 design, we find that the proportion of rejections for the $\text{Wald}^{(h)}$ test when the knot is known and set to the true value is 94 per cent, while the proportion of rejections for the $\sup\text{Wald}^{(h)}$ is 92 per cent.

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