No-Arbitrage Macroeconomic Determinants of the Yield Curve*

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Abstract

We decompose the interest rate linear dependence on macro and latent factors into a distributed-lag "reaction function" of inflation and real activity, and into orthogonal exogenous shocks. The decomposition is implemented by minimizing the variance of the shocks via dynamic projection of latent factors onto the macro factors. The reaction function explains 80% of the variation in the short rate and 50% of the slope. We relate the unexplained part of the short rate to such liquidity measures as the AAA credit spread, or the Money Zero Maturity. The slope is strongly correlated with public government debt growth. Inflation and liquidity risk premia jointly explain 65% to 85% of the variation in the term premia across the yield curve.

1 Introduction

When economists think about monetary policy, they often envision a central bank which reacts systematically to economic variables that reflect the state of the economy.[1] One way to approximate such a view is to construct a "reaction function" that links the short interest rate with macro variables such as real activity and inflation. This approximation is expected to explain most of the variation in the interest rate. The unaccounted variation is attributed to the exogenous monetary policy shocks – the reaction of a central bank to the information outside of that contained in real activity and inflation. The advantage of this view is in providing an explicit link between the short interest rate and observable macro variables, however, it has limited implications for the rest of the yield curve.

In contrast, the no-arbitrage term structure literature provides rich implications about yields and risk premia, but its links to macro variables is muted. Latent factors are correlated with inflation and real activity and, in practice, explain a major part of the variation in the yield curve. Thus, the standard setup precludes us from answering questions regarding how interest rates and risk premia vary together with observable macro variables. Further, we cannot establish whether other macro variables besides inflation and real activity have an impact on the yield curve. In this paper we propose a new procedure which explicitly maximizes the explanatory power of inflation and real activity for the short interest rate and the whole yield curve in the spirit of the reaction function description.

Specifically, our procedure generates empirically valid relationship between inflation and real activity and interest rates and provides an internally consistent measure of exogenous shocks, which by construction are orthogonal to the current and past values of inflation and real activity. The reaction function part of the rule explains 80%, 52%, and 68% of the variation in the short rate, the slope, and the ten-year term premium, respectively.[2] We find a relationship between the shocks and

[1]See, for example, Christiano, Eichenbaum, and Evans (1999), Clarida, Gali, and Gertler (2000), Eichenbaum and Evans (1995), among many others.

[2]Ang and Piazzesi (2003) in their influential work explain at most 53% of the short rate variation with the two macro variables. The empirical results of that paper focus on the contribution of the macro-shocks to the variance of the forecasting error. Therefore, there is no benchmark for the other numbers that we report. We emphasize that, by construction, the maximum possible amount of variation in all objects that are linear functions of the state variables

such additional macro variables as AAA credit spread, and government debt growth. The former explains the remaining 20% of the short rate and contributes 40% to the variation in the one-year term premium, while the latter mostly captures the remainder of the slope.

The new procedure is useful for at least three reasons. First, no-arbitrage term structure models with latent variables are successful in capturing the dynamics of the yield curve and, in particular, generate time-varying risk premia which are crucial in explaining predictability in the bond markets (Dai and Singleton, 2002; Cochrane and Piazzesi, 2005; among others). Our approach essentially maps these latent variables into macro observables and, therefore, quantifies their contribution to the interest rate dynamics and risk premia variation.

Second, it is desirable to study which other macro variables, besides inflation and real activity, might impact the yield curve. To the best of our knowledge, theoretical models are essentially silent about this issue. Some of the variables might be important for the yield curve but might not affect monetary policy directly.[3] One way to address this issue systematically is to first extract maximum explanatory power from inflation and real activity and then try to relate the residuals to new sources of macro variation. Thus, exogenous shocks provide an avenue of introducing such variables.

Third, as Rudebusch (2002, 2005) has forcefully illustrated, it is impossible to detect the correct reaction function and exogenous shocks by investigating the behavior of the short interest rate alone. Because we use the information from the full yield curve, we can correctly extract both the systematic and exogenous parts of the interest rate rule implicit in the bond prices (conditional on the model that we study).

We allow for a rich correlation structure between the macro and latent factors in our model. We explain this correlation via the macro variables by dynamically projecting the latent factors onto real activity and inflation in a fashion consistent with the model specification. As a result, the spot interest rate becomes a linear function of macro variables, their lags and a set of new "projection residual" latent factors.[4] The new latent factors are exogenous to the information contained in the

is explained via inflation and real activity.

[3]Indeed, if a macro variable is empirically important for a ten-year yield, it does not necessarily imply that this variable is important for a short rate.

[4]Our lag structure is not arbitrary: recursive projection formulas imply the reliance on all lags and the loadings on these lags are optimal as they are selected to minimize the variance of the residuals.

macro variables and their entire history, and, therefore, represent the part of the term structure unexplained by the pre-selected variables (real activity and inflation). Thus, our decomposition allows us to exert maximum pressure on the macro variables to explain the term structure.

We use the panel of eight yields ranging from three months to ten years, with inflation and real activity observed at a monthly frequency from 1970 to 2002 in the empirical implementation. Our projection-based interest rate rule can explain 80% of the short rate variation based exclusively on inflation and real activity and their lags. The quality of the fit deteriorates for slope (50%) and curvature (40%), indicating the need for additional variables in order to explain the whole yield curve. The exogenous residual factors explain the remaining 20% of the level and 50% of the slope, and improve the curvature fit by 10%. These results beg the question of whether these exogenous shocks could be related to other macro fundamentals not captured by inflation and real activity.

Only one of the two residual factors affects the short rate. This factor is correlated with such measures of liquidity as the AAA credit spread and the growth rate of the money zero maturity (MZM) measure of money supply. However, it is difficult to argue that we could use the AAA spread or MZM as one of the systematic determinants of the interest rate rule. Such a specification would imply that a monetary authority systematically reacts to the level of liquidity in the marketplace. However, in practice the causality may go either way. Therefore, we simply rely on correlation with liquidity measures to interpret this factor as a monetary shock.

The impact of the other factor is most pronounced at long maturities. Therefore, it cannot be included in the systematic part of the short interest rate rule. This latent factor is strongly correlated with public government debt growth, which is a monthly counterpart of the quarterly budget deficit. We interpret this element of the model as a fiscal shock.

The identified sources of macro risk allows us to decompose the traditional affine stochastic discount factor into macro-related components. Specifically, we study the bond term premia and their determinants. Inflation and monetary risk premia are significant and jointly explain 65% to 85% of the variation in the term premia, depending on a bond's maturity. The relative contributions of these two factors change over time with monetary shock being more prominent on the short end of the curve. Finally, inflation and fiscal shock contribute most to the Campbell and Schiller (1991) regression slopes pattern, which is interpreted as the violation of the expectation hypothesis.

Our paper is related to the growing literature, influenced by the work of Ang and Piazzesi (2003) (AP henceforth), on the term structure models that incorporate macro variables.[5] Apart from the previously-mentioned work of Rudebusch, our results are most closely related to three specific papers. Evans and Marshall (2002) pursue the similar goal of identifying the macro variables that drive the yield curve in the context of the traditional VAR models. Duffee (2005) focuses on the contribution of macro variables to the term structure as we do. He does it by avoiding specification and estimation of latent variables. This approach offers flexibility by allowing for a potentially large set of models that are consistent with the macro-side of the specification. Such flexibility comes at the cost of partial term-structure implications. Finally, Dai and Philippon (2004) also argue, in the context of a no-arbitrage macro model, but in a different setup, that the budget deficit is an important ingredient of long-maturity bonds. Unlike in this model, the budget deficit in our framework does not affect the short interest rate.

This paper is organized into five sections and three appendixes. Section 2 introduces the theoretical model, describes the projection setup, and discusses relationships to earlier approaches. In section 3 the estimation strategy is discussed and in section 4 the findings are presented. The final section concludes. The appendixes contain technical details.

2 The Model

We develop the theoretical underpinnings of our approach in this section. First, in section 2.1, we discuss how monetary policy is related to interest rate rules and highlight the need for identifying assumptions in the framework of term structure models. Next, in section 2.2 we describe our interest rate rule that is obtained by projecting latent variables onto the macro ones. In section 2.3 we relate our rule to the monetary policy inertia and in section 2.4 we review bond pricing . We discuss the implications of the proposed interest rate rule in section 2.5.

[5]This work includes Ang, Dong, and Piazzesi (2004), Bekaert, Cho, and Moreno (2003), Buraschi and Jiltsov (2005), Diebold, Rudebusch, and Arouba (2005), Gallmeyer, Hollifield, and Zin (2005), Hördahl, Tristani, and Vestin (2005), Law (2004), Rudebusch and Wu (2005), Wachter (2005), among others.

2.1 Monetary Policy Proxies and Identifying Assumptions

Actual monetary policy could be a complicated function of many variables gauging the state of the economy. If the interest rate is the policy instrument, it could be represented as:

$$r_t = \Xi(\Omega_t), \tag{2.1}$$

where Ω_t is a Central Bank's information set and Ξ is the true policy function. An interest rate rule approximates monetary policy via

$$r_t = \xi(\omega_t) + \varepsilon_{rt}, \tag{2.2}$$

where ω_t is a modeler's information set, ξ is a reaction function that explains a large proportion of the variation in the interest rate, and ε_{rt} is a shock that is orthogonal to the elements of ω_t and, in practice, reflects the unaccounted variation in r (see, for instance, Christiano, Eichenbaum, and Evans, 1999, and Eichenbaum and Evans, 1995).

A researcher must take a stand on the reaction function and the information set ω_t, which would automatically imply the structure of exogenous shocks ε_{rt}. Consistent with a large body of work we select ξ to be linear. In terms of the information set, we build on the literature regarding the Taylor rules, which relies on inflation and real activity as the only systematic response variables in interest rate rules.[6] Under this interpretation, other macro variables that could potentially affect the short or long interest rates can come in through the channel of exogenous shocks only; i.e., they command an occasional response from the monetary authority.

The state of the economy is captured by the vector $z_t = (m'_t, x'_t)'$. In particular, the vector of macroeconomic variables m_t is equal to $(g_t, \pi_t)'$, where g_t and π_t are monthly real activity and the inflation rate, respectively. The remaining factors x_t are latent. The latent factors may contain the lags of m_t, other macro variables or other unknown variables. Importantly, the vector z_t fully reflects all available information at time t, so, for instance, one need not consider lags of x_t.

It is customary in the term structure literature to specify the interest rate as a linear function of the state variables (e.g., Dai and Singleton, 2000, or AP):

$$r_t = \delta_0 + \delta'_z z_t = \delta_0 + \delta'_m m_t + \delta'_x x_t. \tag{2.3}$$

[6] Even if a central bank reacts to other macroeconomic indicators, they might be used as real-time information about inflation and real activity.

However, such a specification cannot be immediately interpreted in the context of the monetary policy proxy (2.2). In contrast to the traditional macro VAR analysis, where all state variables are observable, the role of latent variables and whether they represent shocks ε_{rt} or belong to the information set ω_t is not clear.

The interest rate equation (2.3) could be viewed as the "true" monetary policy function (2.1). This interpretation is not appealing because one has to take a very strong stand on how monetary policy is conducted. Moreover, it is hard to link observable variables and the interest rate because latent variables in practice explain a large part of the variation in the interest rates. It seems that one could relax the strong implications of the interest rate rule by appending an error term to (2.3). However, in the presence of multiple latent factors, this error term would serve as one additional latent variable. Thus, one needs additional assumptions in order to interpret particular variables as shocks. For example, this could be implemented by imposing correlation constraints, as was done in AP.

Ang, Dong, and Piazzesi (2004) (ADP henceforth) show, in the context of a model with one latent factor, that one and the same interest rule can be interpreted as a simple, forward-looking, or backward-looking rule, depending on additional assumptions like the forecasting horizon or whether exogenous shocks depend on current or past innovations in the state variables. The ADP analysis implies that, in the absence of structural restrictions, additional identifying assumptions are required in order to settle on a particular interest rate rule in the form (2.2). The question of which version of the interest rate rule is ultimately selected is important because the interpretation of how the systematic policy affects interest rates and the magnitudes of exogenous shocks will be different.

Our identification assumption is based on the choice of the information set ω_t. If we assume that an econometrician knows the macro variables m_t, it is natural to assume that she knows their entire history $M_t = \{m_t, m_{t-1}, \ldots, m_0\}$. Our identifying assumption is that $\omega_t = M_t$. As one will see in the next subsection, this assumption implies a general way of constructing exogenous shock ε_{rt} regardless of the size of the latent state x_t. All that is required is the specification of the joint dynamics of m_t and x_t.

2.2 Interest Rate Rule

We assume that the state vector z_t follows a VAR(1) process

$$
\begin{aligned}
z_t &= \mu + \Phi z_{t-1} + \Sigma \epsilon_t && (2.4)\\
&= \begin{bmatrix} \mu^m \\ \mu^x \end{bmatrix} + \begin{bmatrix} \Phi^{mm} & \Phi^{mx} \\ \Phi^{xm} & \Phi^{xx} \end{bmatrix} \begin{bmatrix} m_{t-1} \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} \Sigma^{mm} & \Sigma^{mx} \\ \Sigma^{xm} & \Sigma^{xx} \end{bmatrix} \begin{bmatrix} \epsilon_t^m \\ \epsilon_t^x \end{bmatrix}, && (2.5)
\end{aligned}
$$

where $\epsilon_t \sim N(0, I)$. We denote the vector of parameters controlling the dynamics of state by $\Theta = (\mu, \Phi, \Sigma)$. The block representation will be useful for later discussions. Thus, our setup (2.3), (2.4) is generally similar to that of AP and ADP. The difference is in our identifying assumption $\omega_t = M_t$. We believe that our identification assumption is attractive because, as we detail below, it allows for an internally consistent approach towards understanding monetary policy rules, the nature of shocks, and their impact on risk premia.

Our identifying assumption allows us to decompose the latent factors into a macro-related component and an innovation component. We construct the decomposition by dynamically projecting the latent factors onto the macro factors. The projection residuals, which are by construction orthogonal to the macro variables and their entire history represent the true exogenous shocks.

To be more specific, we can rewrite the interest rate equation (2.3) as:

$$r_t = \delta_0 + \delta_m' m_t + \delta_x' x_t = \delta_0 + \delta_m' m_t + \delta_x' \hat{x}(M_t) + \delta_x' f_t, \tag{2.6}$$

where $\hat{x}(M_t)$ denotes the linear projection of x_t onto M_t; and f_t is the residual of x_t, which is orthogonal to M_t

$$f_t = x_t - \hat{x}(M_t). \tag{2.7}$$

By definition, the linear projection $\hat{x}$ has a simple functional form that fits nicely into the overall linear structure of the model and could be thought of as having a VAR structure

$$\hat{x}(M_t) = c(\Theta) + \sum_{j=0}^{t} c_{t-j}(\Theta) m_{t-j}, \tag{2.8}$$

where the matrices c are functions of parameters Θ that control the dynamics of the state variables. The dependence of the coefficients c on the model parameters allows us to avoid overparameterization, a problem in all multiple-lag studies. Appendix A provides the details of the procedure.

Our identification of the monetary policy proxy has appealing properties. First, the procedure maximizes the explanatory power of M_t by construction because linear projection is optimal in our setting. The resulting linear function of M_t has a natural interpretation as a "backward-looking" reaction function:

$$\xi(\omega_t) = \delta_0 + \delta_m' m_t + \delta_x' \hat{x}(M_t). \tag{2.9}$$

Second, given our choice of the information set M_t, the new latent factors f_t will be exogenous, i.e., $\delta_x' f_t = \varepsilon_{rt}$ in (2.2).

An important question is how our approach could be interpreted in the context of structural models. While a full analysis is beyond the scope of this paper, we can provide one interpretation. Recent research suggests that the actual monetary policy Ξ in (2.1) could be forward-looking. A forward-looking rule with conditioning on the full-information set Ω_t could be rewritten as a backward-looking approximating rule ξ by projecting the expectations on the available information $\omega_t = M_t$. In this regard, the work of Rudebusch and Svensson (1999) provides both a normative and positive analysis of the forward-looking rules, whose implementation is similar to that of the backward-looking rules.

2.3 Monetary Policy Inertia

ADP show that, when a factor x_t is a scalar, the interest rate rule (2.3) can be rewritten in the equivalent form

$$r_t = \widetilde{\delta}_0 + \delta_m' m_t + \widetilde{\delta}_m' m_{t-1} + \widetilde{\delta}_r r_{t-1} + \widetilde{\epsilon}_t, \tag{2.10}$$

where tilde highlights parameters which are functions of the model's original parameters. The case $\widetilde{\delta}_m = 0$ corresponds to the traditional monetary policy inertia specification, which is empirically successful (for the details and references, see Rudebusch, 2002). Similarly, Piazzesi (2003), in a no-arbitrage model which explicitly accounts for the Fed decision-making process, finds that the implied interest rate rule incorporates a response to the two-year yield. Such rules imply an adjustment of the interest rate target which suggests policy inertia, or interest rate smoothing behavior of the monetary authority. However, Rudebusch (2002) questions this interpretation because (2.10) implies a counterfactually strong forecastability of the interest rates. He conjectures that "the illusion of monetary policy inertia" reflects persistent shocks.

Our projection procedure decomposes a vector x of any dimension into a component associated with the macro variables and a new component f. On the one hand, the lagged macro component $\hat{x}(M_t)$ achieves the objective of interest rate smoothing. On the other hand, because f is orthogonal to the macro variables and their entire history by construction, f can be interpreted as exogenous shocks. This orthogonality also helps to disentangle the explanatory power of macro variables and that of the residual factors f. Our decomposition

$$r_t = \delta_0 + \delta_x' c(\Theta) + \delta_m' m_t + \delta_x' \sum_{j=0}^{t} c_{t-j}(\Theta) m_{t-j} + \delta_x' f_t, \tag{2.11}$$

implied by (2.6)-(2.8), is fundamentally different from a seemingly related one obtained via recursive substitution of r in (2.10),

$$r_t = \widetilde{\delta}_0 \sum_{j=0}^{\infty} \widetilde{\delta}_r^j + \delta_m' m_t + \left(\widetilde{\delta}_r \delta_m' + \widetilde{\delta}_m'\right) \sum_{j=1}^{\infty} \widetilde{\delta}_r^{j-1} m_{t-j} + \sum_{j=0}^{\infty} \widetilde{\delta}_r^j \widetilde{\epsilon}_{t-j}. \tag{2.12}$$

The error part is not constructed optimally in this MA(∞) representation. Moreover, while $\widetilde{\epsilon}_t$ is orthogonal to the contemporaneous macro variables, the whole error term is correlated with the macro-component. As a result, it is difficult to asses the explanatory power of the macro variables versus the residual, as this will depend on the order of conditioning. Finally, our approach applies to any number of latent factors. Thus, the two decompositions have different properties and interpretations.

2.4 No-Arbitrage Bond Valuation

The specification of the state variables combined with the interest rate specification in (2.3) allows us to complete the usual affine no-arbitrage framework by specifying the stochastic discount factor ξ_t

$$\log \xi_t = -r_{t-1} - \frac{1}{2}\Lambda_{t-1}' \Lambda_{t-1} - \Lambda_{t-1} \epsilon_t, \tag{2.13}$$

where the market prices of risk follow the essentially-affine specification (Duffee, 2002)

$$\Lambda_t = \Lambda_0 + \Lambda_z z_t. \tag{2.14}$$

Therefore, yields on zero-coupon bonds are linear in the state variables,

$$\begin{aligned} y_t(\tau) &= -\frac{1}{\tau}\log E_t\left(\prod_{s=t+1}^{t+\tau}\xi_s\right) = a^Q(\tau) + b^Q(\tau)'z_t \\ &\equiv \underbrace{a^P(\tau) + b^P(\tau)'z_t}_{\text{Short rate expectations}} + \underbrace{a^{TP}(\tau) + b^{TP}(\tau)'z_t}_{\text{Term premium}}, \end{aligned} \tag{2.15}$$

where τ is the respective maturity, and a^Q and b^Q solve recursive equations with boundary conditions $a^Q(1) = \delta_0$ and $b^Q(1) = \delta_z$ (see, e.g., Bekaert and Grenadier, 2001). In particular, the one-month yield coincides with the short rate, $y_t(1) = r_t$. The last line decomposes the yields into the expectations of the future short rates and the term premium. The former component is equal to the usual factor loadings computed under the assumption of zero market prices of risk.

The risk premia Λ_t and bond yields y_t can be equivalently expressed via the history of macro variables M_t and shocks f_t based on (2.7) and (2.8). Therefore, our identifying assumption allows one to think in terms of observable macro variables not only regarding the interest rate, but the risk premia as well. The projection procedure maximizes the explanatory power of macro variables m with respect to the latent variables x, and, therefore, with respect to both the interest rates and risk premia as they are linear functions of x. This property will allow us to fully address the question of how observable variables affect the dynamics of the yield curve and the variation in the risk premia.

2.5 Discussion

Our approach provides an opportunity for the macro variables to explain as much of the variation in the interest rates and risk premia as possible. Put differently, it replaces the traditional latent term structure variables with the pre-selected macro variables to the maximum extent possible. The natural question that arises is what implications this approach has for term structure modeling.

First, we can characterize the variation of yields and risk premia in terms of observable factors. Our approach ensures that the characterization is complete. The following sections provide examples of such applications.

Second, the explicit construction of exogenous shocks allows for consideration of additional sources of variation in the yield curve. We can do this in a systematic fashion by isolating the effects that are truly novel relative to the initial macro variables. Examining the impact of these shocks on the interest rates should provide ideas about the observable sources of these shocks. We provide an analysis along these lines later in the paper.

Third, our method provides an interest rate forecasting tool. Indeed, using the dynamics of the full state z_t we obtain:

$$E(z_{t+\tau}|z_t) = \mu\,(I-\Phi)^{-1}\,(I-\Phi^{\tau}) + \Phi^{\tau} z_t. \tag{2.16}$$

Therefore, the interest rate forecast based on the observable macro variables is equal to:

$$E(r_{t+\tau}|M_t) = \delta_0 + \delta_z'\mu\,(I-\Phi)^{-1}\,(I-\Phi^{\tau}) + \delta_z'\Phi^{\tau}\begin{pmatrix} m_t \\ \hat{x}_t\,(M_t)\end{pmatrix}. \tag{2.17}$$

Naturally, the same argument would apply to all yields via (2.15). By construction of $\hat{x}_t(M_t)$, the forecast is minimum-variance. We leave exploration of this direction to future research.

Fourth, because our approach generically applies to any number of observable and latent factors, the accumulated evidence should prompt developments of more elaborate structural models. Such models should motivate the use of new macro variables and establish how they affect interest rates in a way that is both theoretically justified and empirically valid.

3 Empirical Setup

We start this section with a brief description of the dataset and then proceed with the estimation methodology. We conclude with the description of our parameter identification strategy.

3.1 Data

We use a monthly time series of macro and bond data from 1970 to 2002. We use the CPI and help wanted index (HWI) taken from FRED to proxy for the price level and real activity, respectively.[7] We compute annual log-changes in the CPI index to construct our measure of inflation. The differencing frequency is selected to mitigate the noise in month-to-month changes. Because the HWI

[7] The index Help Wanted Advertising in Newspapers is used by AP and Dai and Philippon (2004). It is a leading indicator of real activity. Its advantage is that it is stationary and, hence, can be used as is. We have also considered linearly detrended per capita employment as a proxy for real activity with largely similar results.

is of order 100 (its 1987 level is precisely 100), we divide it by 10 to obtain an order of magnitude similar to those of yields and inflation.

Our choice of macro variables is not the only possible one. Apart from alternative reported statistics, the literature has suggested various principal components based aggregate measures of inflation and real activity (see AP; Boivin and Bernanke, 2003; Law, 2004; among others). These studies are motivated by the presence of measurement noise in any given stand-alone statistic. These considerations are definitely important, but they are less critical in the context of our study. Because we are not estimating a structural model, measures of "true" real activity and inflation are not required. Effectively, we assume that the market participants react specifically to the CPI and HWI.

We use an unsmoothed Fama-Bliss approximation of the zero-coupon bond prices with maturities of three and six months and one, two, three, five, seven, and ten years.[8] It is important to measure the full yield curve because its slope is correlated with the macro environment (Estrella and Hardouvelis, 1991; Estrella and Mishkin, 1998). Moreover, using rich yield data helps to identify the risk premia.

One concern that always arises in the interest rate studies with a relatively long data span is that of structural stability, which is related to the monetary experiment of 1979-82. One strategy is to consider the pre- and post-experiment samples separately as, for instance, in Duffee (2005). However, because of the persistence of the interest rates, it is desirable to study long data spans. AP and ADP, in settings similar to ours, find that sub-samples yield results similar to the ones from the full data set. Perhaps, the most appealing approach is to consider a regime-switching model which explicitly accounts for the structural changes due to either the conduct of monetary policy, or business cycles (see, for instance, Ang and Bekaert, 2004, Bikbov, 2005 or Dai, Singleton, and Yang, 2003). We proceed with our analysis with these caveats in mind.

We provide a preliminary descriptive analysis of the relationship between the yields and macro variables in Table 1. We implement univariate regressions and unrestricted VARs in order to establish the explanatory power of inflation and real activity with respect to level, slope and curvature of the term structure. The omitted variables problem is always a potential issue in such an anal-

[8]We are grateful to Robert Bliss for providing us with the data.

ysis, especially in the case of regressions. We provide the numbers as a simple benchmark for our analysis, which should be interpreted with care. The reported R^2s indicate that the two macro variables can explain more than 50% of the level, especially when their lags are used as well. The contemporaneous macro variables seem to robustly explain about 40% of the slope. There is a lot of variation in the curvature, depending on the particular methodology.

3.2 The Econometric Method

We estimate our term structure model via maximum likelihood with the Kalman filter following Bikbov and Chernov (2005), Duffee and Stanton (2004), and de Jong (2000), among others.[9] We place estimation errors on all yields so that the latent factors are not associated with pre-specified maturities. We assume that the macro variables are observed without error. There are many compelling arguments in favor of introducing macro measurement errors. However, since the literature is in the early stages of combining macro variables and the term structure, our model is likely to be misspecified. As a result, our model with measurement error may generate fitted macro variables that look much different from the original series.

3.2.1 Observation Equations

The state equation in the state-space system is the Gaussian VAR(1) described in (2.4). The observation equations can be represented in the following way:

$$y_t \quad = \quad a + b_m^{Q\prime} m_t + b_x^{Q\prime} x_t + u_t, \tag{3.1}$$

where y represents the vector of eight yields of maturities from one month to ten years. The right-hand side of the equation is an expanded version of the no-arbitrage expression for the yield in (2.15). The measurement errors are denoted by u. We assume the simplest possible structure of the errors; that they are independent and normally distributed with zero mean and standard deviation σ_u (for each individual element of the vector u). We need not specify a more flexible error structure because these variables are introduced in addition to the VAR shocks that we considered earlier.

[9] Other important estimation strategies applied to term structure models include, but not limited to, exact inversion likelihood of Chen and Scott (1993), closed-form approximate likelihood of Aït-Sahalia and Kimmel (2002), simulated maximum likelihood of Brandt and He (2002), and Bayesian MCMC of Collin-Dufresne, Goldstein, and Jones (2003).

After estimating a model and filtering the state variables, we can use the projection (2.8) to construct our interest rate decomposition (2.6). We will denote the filtered state variables by $\hat{x}(M_t, Y_t)$, where, as before, the capital letters M and Y denote the entire history of m and y, respectively, up to time t.[10] The estimated orthogonalized residual is $\hat{f}_t = \hat{x}(M_t, Y_t) - \hat{x}(M_t)$.

3.2.2 Number of Factors and Identification

We estimate a model with a total of four factors; i.e., $x_t = (x_{1t}, x_{2t})'$. First, the principal component analysis (available upon request) suggests at least four factors to explain the joint variation in the macro variables and the yield data. Second, we have estimated a three-factor model and discovered that it had no potential to capture the slope of the yield curve. Therefore, we must identify the maximally flexible four-factor model with two observable factors.

Dai and Singleton (2000) show that if all factors are latent in the Gaussian system, the parameters of the model are identified if $\mu = 0$, Φ is lower triangular, Σ is diagonal, and $\delta = 1$. We also know from the macro literature that if all factors are observable, then μ, Φ, and δ are free, and Σ is lower triangular. As we have a mixture of observed and latent factors, we have to combine the insights from the two strands of the literature.

We set μ^m, Φ, δ_0, and δ_m to be free. We restrict μ^x in such a way that the long-run mean of the factors x is equal to zero, i.e.:

$$e_i'(I - \Phi)^{-1}\mu = 0, \tag{3.2}$$

where e_i's are vectors of zeros with a one in positions corresponding to the factors x. Σ is

$$\Sigma = \begin{bmatrix} \sigma_{gg} & 0 & 0 & 0 \\ \sigma_{\pi g} & \sigma_{\pi\pi} & 0 & 0 \\ \sigma_{1g} & \sigma_{1\pi} & \sigma_{11} & 0 \\ \sigma_{2g} & \sigma_{2\pi} & 0 & \sigma_{22} \end{bmatrix}. \tag{3.3}$$

These restrictions imply that we have to set $\delta_x = 1$. Finally, since all the risk premia parameters are identified in the case of the all-latent model as long as there are more yields than factors, these parameters will be identified when some of the factors are observable.

[10]Note the difference between the filter of the latent variable x based on all observables, $\hat{x}(M_t, Y_t)$, and the projection of the latent variable x onto the history of macro variables, $\hat{x}(M_t)$, discussed in the section 2.2.

3.2.3 Estimation of the Risk Premia

Risk premia are hard to estimate in practice, despite their theoretical identification. Typically, one encounters multiple local optima that have similar likelihood values, but imply dramatically different estimates of the risk premia. Additionally, a rich specification of market prices of risk might be a reason for concern, because they could be compensating for the misspecification of the factors dynamics instead of measuring the compensation for risk.

In order to mitigate these issues, we augment the standard log-likelihood function, $\mathcal{L}$, with a penalization term which is proportional to the variation of the term premium in (2.15):

$$\mathcal{L}_p = \mathcal{L} - \frac{1}{2\sigma_p^2} \sum_{\tau} (a^{TP}(\tau))^2 + b^{TP}(\tau)' \cdot \mathrm{Diag}(\mathrm{var}(z_t)) \cdot b^{TP}(\tau), \tag{3.4}$$

where σ_p controls the importance of the penalization term, and the Diag operator creates a diagonal matrix out of a regular one. If market prices of risk are equal to zero, the term premium will be equal to zero as well. Therefore, $\mathcal{L}_p$ imposes an extra burden on the model to use the risk premia as a last resort in fitting the yields.

In practice we take $\sigma_p = 300$, which introduces a modest modification to the original log-likelihood. Nonetheless, it helps to stabilize the likelihood and simplifies the search for the global optimum. In particular, this setup helps us to avoid very large values of risk premia.

4 Results

We split the discussion of the results into two parts. Section 4.1 discusses the estimated latent variables, how well the model fits the yield curve, and it contrasts different versions of the interest rate rule implied by the model. Section 4.2 identifies the latent variables with observable macro variables other than inflation and real activity and discusses their interactions.

4.1 The Model Properties

4.1.1 Parameter Estimates

Table 2 presents the estimated parameters. Because asymptotic standard errors are of a concern in the context of such persistent time series as interest rates, we compute the confidence bounds via the parametric bootstrap (Conley, Hansen, and Liu, 1997). Specifically, we simulate 1000 paths from the estimated model and re-estimate it along each path. This procedure yields a finite sample distribution of parameter estimates, which subsequently allows us to determine the confidence intervals.

While some parameters are individually insignificant, they appear to be jointly significant based on our parameter elimination routine. Following Dai and Singleton (2002) (DS2 henceforth), we restricted some of the insignificant parameters to zero, if this restriction did not lead to a notable decline in the value of the log-likelihood function. The remaining parameters are therefore important for the model fit. The insignificance of the individual parameters stems from the fact that we are estimating a large model and the data might not be sufficiently informative about each of the parameters, even if they are theoretically identified.[11]

Christiano, Eichenbaum, and Evans (1999) caution that the estimated parameters are difficult to interpret because they represent a convolution of the parameters of the actual interest rate rule, and the parameters of the projection of the missing data onto the econometrician's dataset. Moreover, the values of parameters associated with the latent variables depend on the particular identification scheme. For example, the magnitude of the lower-right block of Σ depends on the restricted value of δ_x. Impulse response functions and related diagnostics represent the proper way of assessing the model implications. We discuss this in later sections.

4.1.2 The Model Fit

First, we highlight the value of σ_u, the standard deviation of the error in the yield observation equation (3.1), which is equal to 0.16. This implies that the model values the bonds within 32 basis points ($2\sigma_u$). Specifically, as indicated in panel (a) of Table 3, the average absolute pricing error ranges from 6.2 basis points for the one-year yield to 33 basis points for the ten-year yield. In this regard the results are consistent with other macro studies (see ADP for a discussion).

[11]Still, the insignificance of the factor loadings δ_0 and δ_m is somewhat disturbing. We conjecture that this effect is the manifestation of general difficulties of estimating the means. This problem is subdued in latent factor models because δ_0 is typically fixed at the sample mean and the rest of δ's are restricted to the value of one. Because two of our factors are observable, we have to estimate the factors' means, and the respective loadings. Moreover, δ_0 cannot be fixed as easily.

We conduct a more thorough evaluation of the model performance by checking how well it fits certain moments. Again, we use the parametric bootstrap strategy and compute the finite sample distribution of the model-implied moments.[12] Panel (b) of Table 3 reports the results.

We see that the model successfully captures many important aspects of the data, such as means, standard deviations, kurtosis and autocorrelations. The difference between the fitted and actual means of the yields is quite large, at least relative to the extant studies. This happens for two reasons. First, we do not assume precisely observed yields. Second, we estimate the coefficient δ_0 in the interest rate rule, while most studies fix it to match unconditional mean of the short yield exactly.

The model struggles primarily with explaining the skewness of many of the variables and the curvature. The former is not surprising as a Gaussian model is incapable of generating non-normal skewness. In the case of curvature, the differences between the data and the model moments are significant. We intentionally selected a parsimonious model to investigate first-order effects in this paper and to minimize already formidable difficulties associated with a large set of parameters. The curvature fit can be improved by adding another latent factor. We do not pursue such extensions here, because the curvature, which explains at most one percent of the yield curve variation, does not have a first-order effect.

4.1.3 Orthogonalized Residuals, f_t

Table 4 reports the correlations of the filtered latent variables x, $\hat{x}(M_t, Y_t)$ and its estimated orthogonalized residual $\hat{f}_t$. We see that both fs are different from their x counterparts (the correlations are 0.22 and 0.31 for the first and second pairs, respectively). The low correlations imply that inferring the impact of macro variables with x as the latent factors is very different from using f. Moreover, the correlation between the traditional latent factors level, slope, and curvature and either xs or fs are not as strong as in the latent factor models. The strongest relationship is between the slope and x_1 or f_1 (the correlation is above 0.60 in both cases). The fact that the correlations with the slope are nearly identical means that factor f_1 is more important for explaining the slope than is x_1. We

[12] In principle, we should have taken the parameter uncertainty and data sampling error into account when computing the confidence intervals. This would widen the intervals further.

will highlight this effect later.

These results indicate that our orthogonalization procedure was worth pursuing, as it leads to new latent variables, that are substantively different from the ones typically studied in the literature. It appears that real activity and inflation variables have a bigger potential to explain the yield curve than would appear by considering the interest rate rule (2.3) directly. The next natural question is which fraction of the yield curve variability is explained by output, inflation, and their lags contained in the projection $\hat{x}(M_t)$.

4.1.4 Do Real Activity and Inflation Explain the Short Interest Rate?

In this section we evaluate how well different implementations of the interest rate rule explain the variation in the short end of the curve. We use "theoretical R^2" as a simple measure of fit. In contrast to the regular R^2, which is a side product of an OLS estimation, our measure is computed based on the parameter values obtained via an ML estimation. Because the factors f_t are orthogonal to the history of the macro variables M_t, there are no issues associated with attributing the explanatory power to latent versus macro factors. We compute the fraction of the interest rate variance explained by the reaction function (2.9) as:

$$R_M^2 = \frac{\mathrm{var}(\xi(M_t))}{\mathrm{var}(r_t)}, \tag{4.1}$$

where the numerator is computed based on the estimated model and the denominator is computed from the data. The fraction of the interest rate variance explained by the full model is:

$$R_z^2 = \frac{\mathrm{var}(\delta_z' z_t)}{\mathrm{var}(r_t)}. \tag{4.2}$$

Panel (a) of Table 5 reports the theoretical R^2 for the full model and a nested, macro-only specification.

These theoretical R^2 are different from the variance decomposition that is often computed in the course of a VAR analysis. Variance decomposition measures the contribution of the shocks in the variables to the variance of the forecasting error. Thus, this measure speaks more to model properties rather than to model's fit. The shock contributions always add up to 100%. However, R^2 is not guaranteed to be equal to 100%. Variance decomposition will always be the same for a given dynamics of the state z_t, regardless of the choice of one of the equivalent representations of the interest rule. For this reason and because this analysis was conducted in the earlier work (AP, ADP), we do not focus on variance decompositions here. For completeness, we report them for one (infinite) horizon in panel (b) of Table 5.

If we consider the macro-component, the projection-based rule $\xi(M_t)$ explains the majority of the variation in the short rate: $R^2_M = 80\%$. Panel (a) of Figure 1 complements our observations about the strong relationship. This result is in stark contrast to the previous no-arbitrage literature: AP report an $R^2 = 45\%$ based on contemporaneous macro variables only and $R^2 = 53\%$ for an interest rate rule involving both macro variables and their lags and latent variables. Dai and Philippon (2004) report a high R^2 of 95% for their interest rate rule, however, it includes the contemporaneous Fed Funds rate.

Going back to Table 1, the result is stronger than the numbers implied by the unrestricted univariate regression. The VAR R^2 is higher, but it reflects the contribution of the lagged interest rates as well. We would like to highlight two points regarding the comparison of the fit. First, because we use a fully specified model, biases in parameters and R^2 due to omitted variables are not a concern. Second, because of our decomposition into the macro component and orthogonal latent factors, we are able to separate the contributions of the two. Hence, our R^2 of 80% reflects the maximal possible explanatory power of the macro variables themselves. We will revisit these points when we discuss the overall term structure fit.

Panel (b) of Figure 1 illustrates the weights of the macro variables and their lags in the systematic interest rate rule $\xi(M_t)$. The highest loading for real activity is at its contemporaneous value. The weights decline from there forward in an exponential fashion. The inflation loadings peak at the second lag and then decline similarly to real activity. We note that the weights die out quite slowly, which implies a need for lags well beyond the traditional twelve. Implementing such a lag structure in the VAR framework would be difficult because of parameter proliferation.

Finally, after fully incorporating the latent variables in (2.6), the R^2_z is close to 100% in Table 5. Despite a clear success of our strategy, there is still room for improvement, as we should be able to explain the remaining 20% of the variation in r. This is the purpose of the new latent factors f, which were omitted from our rule $\xi(M_t)$.

There are clear periods when the actual interest rate is above, or below, the systematic rule

$\xi(M_t)$; i.e., the effective policy is more aggressive, or passive (panel (a) of Figure 1). The latent factor f_2 accounts for the difference. Figure 2 complements this observation by showing that the factor loadings, or responses of the yield curve, to one standard deviation moves in the factors implied by the model. We see that factor f_2 operates mostly on the short end of the curve. It might seem surprising that f_1 does not affect the short rate. Indeed, our identification scheme imposes a unit loading on factor f_1 in the interest rate rule (2.6). In fact, the impact of a factor on the interest rate is controlled by its loading and by the variance of the factor's innovation. In our case, the volatility of the innovation in f_1 is so small that it has a negligible effect on the short interest rate.[13]

The deviations between the interest rate rule and the short yield in Figure 1 seem to correspond to our intent of interpreting the latent residual factors as exogenous shocks. Perhaps, the most striking deviation from the macro-based rule that we observe is during the period from November 1982 to Spring 1986. Goodfriend (1993) associates this period with the Fed establishing its credibility by handling the inflation scare of 1983 – 1984, when it aggressively increased the funds rate by three percent during this period. A more recent yet similar episode pertains to the "soft landing" of 1994, when the Fed hiked the interest rate by three quarters of a percentage point at once. Blinder and Yellen (2001) cite the years 1990 through 1993 as a period of passive policy. The Fed was accused of not cutting the interest rate in a sufficiently proactive fashion during and after the recession of 1990-91. Their policy was characterized by long pauses in interest changes and by small cuts at a time. The deviations of the three-month yield from the macro-based rule in Figure 1 reflect this situation accurately.

4.1.5 Do Real Activity and Inflation Explain the Yield Curve?

In this section we establish which fraction of the variation in the yield curve is explained by various versions of the interest rate rule. We summarize the yield curve by its first three principle components: level $y_t(3)$, slope $y_t(120) - y_t(3)$, and curvature $y_t(3) + y_t(120) - 2 \cdot y_t(24)$. Note that evaluating the fit quality of the principle components is a more stringent exercise than evaluating

[13]At longer horizons the loadings $b^Q(\tau)$ magnify the effect and make contribution of f_1 progressively more important.

the fit of certain yields. Because of interest rate persistence, success in explaining the short rate will translate into success in explaining the rest of the curve. Therefore, the key challenge to the model is to evaluate its ability to explain features of the curve that are not directly related to its level.[14]

Similar to the previous section, we consider two versions of the interest rate rule, which are based on different implementation of our specifications in (2.6). In the first case, we omit the orthogonalized residuals f. In the second case, we use the full state vector. The reported R^2 represent theoretical values based on the estimated model parameters, rather than the ones obtained from OLS. Indeed, because no-arbitrage theory allows for the computation of any bond yield based on each specification of r, we can compute the theoretical values of R^2 for all principal components, which are linear combinations of yields.

Table 5 reports these R^2. As we observed in the previous section, our model is very successful in explaining the level based on the macro variables only. There is a large difference in how the two interest rate rules explain the slope of the term structure. First, our full model is successful in capturing the slope of the curve as it explains 97%. Second, the ability of real activity and inflation to capture the shape of the curve deteriorates. While the two macro variables could explain 80% of variation in the level, they explain only 52% of the slope. Still, by construction, this is the maximum of the slope variation that can be explained by our two macro variables.

Figure 3 complements this discussion by contrasting the macro-based and the observed slopes in panel (a) and by showing the weights of macro variables and their lags in the macro-based expression for the slope in panel (b). In contrast to the level, the weights die out almost immediately. This observation implies that the slope is mostly determined by the contemporaneous macro variables. This explains a "robust" explanatory power of macro variables with respect to the slope. In Table 1, the R^2s range between 40% and 45%, regardless of the implementation, and our no-arbitrage model delivers a similar 52%.

As is the case with the interest rate rule in Figure 1, it is instructive to view the difference between the actual and macro-based slope not as model error, but as a response of the yield curve

[14]Moreover, since the likelihood was constructed based on yields, the errors will be small. However, if the model is misspecified, the errors will have complicated correlation structure. This will be revealed by evaluating the linear combinations of yields, e.g., the principle components.

to the developments beyond those related to real activity and inflation. Indeed, factor f_1 accounts for almost the entire difference between the actual and macro-based slope in panel (a) of Figure 3. This could be seen from Figure 2 as well; factor f_1 loadings indicate that the factor operates mostly on the long end of the curve. This property of f_1 complements our observations about its role in Table 4.

It is interesting to contrast the plot of the slope with two U.S. government budget-related episodes highlighted by Blinder and Yellen (2001). First, President Clinton introduced a budget reduction package in February 1993, immediately after the recession of 1990-91. According to Blinder and Yellen, the unprecedented nature of this package awarded it instantaneous credibility. This perception was reflected in the 1.5% drop in the long interest rate from late 1992, when Clinton started advertising the package, to late 1993. Figure 1 shows that both macro-based and actual short interest rate remained flat during this time period. The macro-based slope in Figure 3 is flat as well. Therefore, the change in the actual slope may be attributed to the changes in the fiscal policy. Second, another Clinton budget agreement dated November 1999 "...shifted the norm for fiscal policy fundamentally by declaring the Social Security surplus off-budget..., the fiscal bar was thus raised enormously." Similar to the first episode, the 2% drop in the slope over the period from November 1999 to October 2000 is attributable mostly to the fiscal policy or, in the context of our model, to factor f_1.

While the explanatory power of the various variables follows the same pattern for the curvature, our model fails to generate the realistic pattern (the full model explains only 55%). This outcome is consistent with the model diagnostic results reported earlier. While this problem could be remedied by introducing an extra latent variable, we decided to leave out the model refinements since the curvature explains less than 1% of the whole term structure.

4.2 What Are the Exogenous Shocks?

4.2.1 The Candidates

We want to relate the new latent factors f_1 and f_2, which we interpret as exogenous shocks, to macro factors other than inflation and real activity.[15] The nature of the two shocks is different. The shock f_1 does not affect the short interest rate but makes an important contribution to the long rates. Moreover, the shock f_2 affects the whole yield curve, although the major impact is on the short end. These properties and the examples of shocks discussed in the previous sections indicate, at least anecdotally, that f_1 and f_2 could be thought of as a fiscal shock and a monetary shock, respectively. In this section we would like to explore this avenue further and check whether our shocks f are correlated with some measures of fiscal and monetary activity.

The two closely related factors that might cause a monetary shock are financial stability and liquidity. Indeed, the analysis in Mishkin and White (2003) suggests that periods of financial instability are associated with liquidity crunches. Furthermore, Mishkin and White (2003) discuss that the Fed might react to such measures of financial instability as a large rise in interest rates for defaultable securities. Hence, our proxy for financial stability is the AAA credit spread taken from the FRED database. At the same time, because of overall high credit quality of AAA companies, the spread will, to a large degree, measure the effect of liquidity.[16] As an alternative measure of liquidity, we use month-to-month growth in MZM in order to directly measure the money supply. Finally, we relate the shock f_1 to the year-to-year growth of the public government debt to gauge its association with fiscal shocks. In this regard, our analysis is related to Dai and Philippon (2004), who use the budget deficit as one of the macro variables directly affecting the short interest rate. As we have seen earlier, our shock f_1 operates on longer yields only.

[15] Rudebusch (2002) discusses shocks of a similar nature, however, he asserts that the "... rule deviations are not 'exogenous policy shocks,' that is, actions undertaken by central bankers that are independent of the economy ... Instead, these deviations are endogenous responses to a variety of influences that cannot be captured by some easily observable variable such as output or inflation." Our interpretation is consistent with Rudebusch, and we use the word "exogenous" (without the qualifier "policy") to emphasize the response to variables that are totally unrelated to inflation and real activity.

[16] The Treasury bonds are often perceived to be trading at "liquidity premium" – higher prices that reflect high demand for money-like instruments.

The proposed macro variables are, most likely, correlated with our measures of real activity and inflation. These relationships might obscure the degree of association with the factors f. Therefore, we prewhiten the above-mentioned macro variables by regressing them on twelve lags of real activity and inflation. As a result, we will be relating the factors f to the innovations in measures of liquidity or fiscal policy, which is consistent with our view of f as exogenous shocks.

In order to identify the factors f with the discussed variables clearly, we rotate them so that they are orthogonal to each other and the correlation between f_1 and the public debt growth is maximized. This rotation resolves the usual indeterminacy of latent factors. Appendix B discusses the rotation details. In the context of our model and our dataset, the proposed rotations lead to a mild change in the factors. The new versions are strongly correlated with the old ones. Factor f_1 becomes more volatile after the transformation, and factor f_2 is largely unaffected.

We compute impulse response functions in Figures 4 and 5 to aid our interpretations of factors. We compare the impulse responses from our model to the ones obtained from the regular VAR. We estimate a VAR(12) specification using three yields (three months, two and ten years) and four macro variables (real activity, g, inflation, π, growth in public debt, a proxy for f_1, and AAA credit spread, a proxy for f_2). The responses are based on the recursive identification scheme using the order (g, π, f_1, f_2).

4.2.2 Responses of the State Variables

Figure 4 shows the impulse responses of the state variables. It appears that, after taking into account the statistical uncertainty, the impulse responses implied by our model are not, in most cases, qualitatively different from the ones implied by VAR. This outcome gives us additional confidence in our model and the choice of macro variables related to f_1 and f_2 – it is restricted relative to VAR, but captures the same features of the state variables dynamics.

4.2.3 Fiscal Shock and Factor f_1

Figure 6 graphs factor f_1 with the public government debt annual growth. We find a very strong association between the two series, as the correlation between the two is 59%. The impulse response functions in Figure 5 show that the factor f_1 has a big (40 basis points) impact on the slope, which dies out in about 2 years. It also has a modest (15 basis points), but apparently permanent, impact on the ten-year yield. These observations are related to a number of studies of fiscal and monetary policies.

Dai and Philippon (2004) use the budget deficit, which is equivalent to the growth, as one of the factors in their model. They find an even stronger relationship between the fiscal policy and the long-term debt.[17] Engen and Hubbard (2004) argue that studying the relationship between the level of the long rate and the changes in debt is not appropriate because it is unrelated to the "crowding out" theory, which makes predictions about either levels of debt and interest rates, or changes in both debt and interest rates. These authors nonetheless estimate whether the changes in debt predict the level of the long interest rate and find that, after controlling for other sources of variation, there is no significant relationship between the two. We do not use as many macro variables as Engen and Hubbard do, yet our measures of fit indicate that our model describes the yield curve sufficiently well to take its implications seriously. It may be different observation frequency (they use quarterly data), reliance on only one (long) yield, and lack of no-arbitrage restrictions that account for the differences in our findings.

There is a strong association between the fiscal shock, inflation and the ten-year yield; both inflation and the long yield strongly respond to shocks in f_1 in Figures 4 and 5, respectively. Therefore, it is natural to ask whether it is the fiscal shock that really matters for the ten-year yield, or rather, its contribution via expected inflation. We compute the contribution of f_1 to the variation in expected average inflation over the ten-year horizon to address this question.[18] It turns out that the fiscal shock explains only 0.45% of the variation in the expected inflation. We conclude that a strong reaction of the ten-year yield is driven by its direct response to f_1.

[17] These authors find a different pattern in the ten-year yield response. We will revisit this issue when we discuss the risk premia. The magnitudes of the responses are different, perhaps, because of the differences in shock identification.

[18] Our model implies, see, e.g., (2.3):

$$E_t\left(\frac{1}{\tau}\sum_{i=1}^{\tau}\pi_i\right) = \frac{1}{\tau}e_2'\left(\sum_{i=1}^{\tau}(I-\Phi)^{-1}(I-\Phi^i)\mu + (I-\Phi)^{-1}(I-\Phi^\tau)\Phi z_t\right),$$

where e_2 is a vector of zeros with a one in the second position. In other words, in our affine model the expectation of the average inflation is a linear function of the state variables. Therefore, we can use our projection-based decomposition (2.6) into macro lags M and exogenous shocks f. As a result, it is easy to compute the contribution of the fiscal shock f_1 to the overall variability of the expected inflation, because it is orthogonal to all other factors.

4.2.4 Monetary Shock and Factor f_2

Figure 7, panel (a) shows the orthogonalized latent factor f_2 (with a minus sign) against the pre-whitened spread between AAA Moody's corporate index and ten-year Treasury bond yield. The two series have many common spikes, which could be interpreted as "flight to quality," or, more generally, liquidity events. A correlation between f_2 and the credit spread of is -38%. This indicates that occasionally open market operations are strongly related to liquidity by either reacting to or causing it, and this is reflected in a separate market of quality corporate securities.

An alternative measure of this effect comes from correlating f_2 with the monetary aggregate MZM. Figure 7, panel (b) shows the two variables. While the correlation is weak at -12%, it is clear that there are periods of very strong association between the two (especially after the monetary experiment).

The impulse response functions in Figure 5 show that increases in f_2 have a transient impact along the whole curve and are associated with the increases in the interest rates, with the short rate being the largest. Also, as indicated by Figures 4 and 5, all the yields move in the opposite direction of inflation, in response to the shock in f_2. Thus, the response of the yield curve is driven by a liquidity effect rather than an expected inflation effect (this conclusion is consistent with Evans and Marshall, 1998). Taken together, the evidence and the observation that the fiscal factor f_1 has a minimal impact on the short end of the curve indicate that f_2 could be interpreted as a monetary policy shock.

4.3 Risk premia

Our analysis has direct implications for the role of risk premia because the state variables that we have identified through our projection-based decomposition affect the dynamics of the stochastic discount factor (2.13). There are at least three interesting questions that we can now explore. First, do shocks affect yields primarily through expectations about the future short yields, or do they directly affect the risk premium? Second, what is the contribution of the various macro risk factors to the term premia, and how do they change over time? Third, which particular macro factors are responsible for the deviations from the expectations hypothesis?

4.3.1 Impulse Responses of the Term Premia

To answer the first question, we decompose the yields into the expectations of the short rate and term premia parts. The expectations could be computed via the yield formula (2.15) by setting the risk parameters to zero. The difference between the yields and the expectations delivers the term premium part.

Figure 8 shows the impulse response functions for the expectations (depicted by circles) and term premia (depicted by asterisks) benchmarked against the impulse responses of the full yields (depicted by solid lines) in Figure 5. We see that, not surprisingly, the term premia have virtually no impact at the short end of the curve. The responses of the ten-year yield to inflation and liquidity shocks are primarily driven by the responses of the term premia. The responses of both expectations and term premia to real activity and fiscal shocks are large and of opposite directions.[19]

4.3.2 Decomposition of the Term Premia

To answer the second question, we further decompose the term premia into the contributions of the various macro variables using the projection-based representation. The yield equation (2.15) implies that the term premium of maturity τ can be expressed as:

$$
\begin{aligned}
TP(\tau) &= a^{TP}(\tau) + b^{TP}(\tau)' z_t \qquad (4.3)\\
&= a^{TP}(\tau) + \underbrace{b_g^{TP}(\tau) g_t + b_x^{TP}(\tau)' \hat{x}(M_t)|_{G_t}}_{\equiv B_g(\tau) G_t} + \underbrace{b_\pi^{TP}(\tau) \pi_t + b_x^{TP}(\tau)' \hat{x}(M_t)|_{\Pi_t}}_{\equiv B_\pi(\tau) \Pi_t} \\
&+ b_{x_1}^{TP}(\tau) f_{1t} + b_{x_2}^{TP}(\tau) f_{2t}.
\end{aligned}
$$

The term $b_x^{TP}(\tau)' \hat{x}(M_t)|_{G_t}$ denotes the part of the projection of the latent factors x, which depends on G_t, the current and lagged values of real activity only. We use a similar notation for the inflation-only component. The decomposition allows us to characterize how term premia vary together with changes in the macro variables and shocks.

Figure 9 shows the one-year and ten-year term premia as examples. The top panels provide the time series of the respective yields and the corresponding term premia and the short rate expectations

[19] Note that the response pattern of the short rate expectations over the ten-year horizon to the fiscal shock is similar to the one reported in Dai and Philippon (2004). Therefore, the differences in the yield responses highlighted earlier are driven by the differences in the risk premia. This is not surprising as we estimate different model specifications.

parts. We note that the term premia are generally countercyclical and have a reasonable magnitude. At the one-year horizon, the average absolute premium is 0.73% with a standard deviation of 0.63%; for the ten-year horizon, the numbers are 2.29% and 1.59%, respectively.

The remaining panels show the decomposition of the term premium into the real activity, $B_g(\tau)G_t$, and inflation, $B_\pi(\tau)\Pi_t$, components of the premia in the panels of the intermediate row, and the fiscal, $b_{x_1}^{TP}(\tau)f_{1t}$, and liquidity, $b_{x_2}^{TP}(\tau)f_{2t}$, shock components of the premia in the panels of the bottom row. We see that, for the ten-year horizon, the real activity and inflation components of the premia are larger in magnitude than the fiscal and liquidity components, and tend to increase during recessions. The real activity component is less variable than the inflation component, and is smaller in magnitude except for the period beginning in December 1998. At that time, the Fed started the tightening streak that led to the eventual collapse of the stock market. Figure 9 indicates that concerns regarding the impact of these events on real activity dominated the inflation fears. The fiscal and liquidity components of the premia often move in opposite directions of each other. The liquidity component was the largest during the monetary experiment. The fiscal premium was large during the Bush presidency from 1989-1993 and went down as a result of the Clinton budget agreement of late 1993.

Subsequently, we compute the population variances of the various ingredients of the term premia and evaluate their relative contribution to the term premia variances.[20] Because the factors f_1 and f_2 are orthogonal to the macro factors, the decomposition of the term premia variance for a generic maturity τ simplifies to

$$\begin{aligned} \operatorname{var}(TP(\tau)) &= \operatorname{var}(B_g(\tau)G_t) + 2\cdot\operatorname{cov}(B_g(\tau)G_t, B_\pi(\tau)\Pi_t) + \operatorname{var}(B_\pi(\tau)\Pi_t) \\ &+ \operatorname{var}\left(b_{x_1}^{TP}(\tau)f_{1t}\right) + \operatorname{var}\left(b_{x_2}^{TP}(\tau)f_{2t}\right). \end{aligned} \tag{4.4}$$

When computing the percentage contribution of the various factors to the term premia variance, we attribute one half of the covariance term to the contribution of real activity and the other half to the contribution of inflation. We compute the variance of the macro components by simulating a long path (500,000 observations) from the estimated model and computing the sample variance based on

[20]Because risk premia are not observable, the term premia are computed based on our model. Therefore these relative contributions cannot be interpreted as R^2 that we reported for the principal components.

it. We also compute the 95% confidence bounds for each factor's contribution to the variance via the parametric bootstrap.

Panel (a) of Table 6 shows that our initial macro variables, real activity and inflation, contribute 53% to 68% to variation in the term premia. In independent work, Ludvigson and Ng (2005) explore the bond risk premia and their relation to the macro variables in the context of excess returns predictability regressions. These authors find, similar to us, that inflation and real activity are the most important factors in explaining variation in the risk premia. Overall, inflation and liquidity risk premia have the largest and statistically significant effect; the combined impact of the two premia is about 65% to 85%, depending on a bond's maturity. Naturally, the liquidity premium is more prominent at the short horizon. It explains 42% and 22% of the variation in the one-year and ten-year premiums, respectively. The contribution of real activity is most pronounced at the intermediate maturities; it explains 28% of the five-year term premium.[21]

4.3.3 The Expectation Hypothesis

As highlighted in DS2 and Duffee (2002), the essentially affine specifications of risk in (2.14) are important for replication of the expectation hypothesis' failure observed in the data. We have verified that this claim holds in our macro-based model. Panel (a) of Figure 10 replicates the DS2 results by plotting the slope coefficients ϕ_τ from the regression

$$y_{t+1}(\tau-1) - y_t(\tau) = \text{constant} + \phi_\tau (y_t(\tau) - y_t(1))/(\tau - 1) + \text{residual}, \tag{4.5}$$

as implemented in our sample and implied by our model.

In the context of our model we can also characterize the contribution of the different macro factors to the explanation of the expectation hypothesis violation by relying on the risk premia decomposition. To this end we rely on DS2 who show that

$$y_{t+1}(\tau-1) - y_t(\tau) = (y_t(\tau) - y_t(1) - D^*_{t+1}(\tau))/(\tau - 1) + \text{residual}, \tag{4.6}$$

[21]As is the case with the principal components, the variance decompositions of risk premia reported in ADP speak to a different aspect of the model, i.e. the contribution of the various shocks to the variance of the forecast error. We report these for one (infinite) horizon for completeness in panel (b) of Table 6.

where the "pure premium term" $D^*_{t+1}(\tau)$ is provided in appendix C for completeness. Therefore,

$$\phi_\tau = \frac{\operatorname{cov}(y_{t+1}(\tau-1) - y_t(\tau), (y_t(\tau) - y_t(1))/(\tau-1))}{\operatorname{var}((y_t(\tau) - y_t(1))/(\tau-1))} = 1 + \frac{\operatorname{cov}(-D^*_{t+1}(\tau), y_t(\tau) - y_t(1))}{\operatorname{var}(y_t(\tau) - y_t(1))}. \quad (4.7)$$

Because D^*_{t+1} can be computed from our model and then decomposed into the contributions of the factors similar to the term premium in (4.3), the covariance part in the last expression could be split into four elements. We plot them in panel (b) of Figure 10.

We see that real activity and the monetary policy factor f_2 have an upward effect on the regressions at long horizons. Therefore, inflation and the fiscal factor f_2 contribute most to the violations of the expectations hypothesis. This outcome is intuitive as inflation and fiscal shocks are the major drivers of long yields.

5 Conclusion

We propose an approach that allows us to establish, in the no-arbitrage affine framework, both which and how macroeconomic variables contribute to the evolution of the yield curve. We rely on two ingredients. First, we allow for a rich model specification involving both preselected macro variables, such as inflation, real activity, and latent factors. Second, in order to identify the novel information in the latent factors, we dynamically project them onto the macro variables, and study the projection residuals. This new interpretation gives maximal flexibility to the measures of inflation and real activity to explain the yield curve. The residuals could be compared to other macro variables in order to identify additional macro factors and shocks affecting monetary policy.

In contrast to previous studies, in the context of a four-factor model we find that real activity, inflation, and their optimally weighted lags explain 80% of the variation in the short interest rate. We find that the unexplained part (the projection residual) is correlated with measures of the budget deficit and money supply (liquidity). These residuals, which we interpret as exogenous monetary and fiscal shocks, have a prominent impact on the short and long end of the yield curve, respectively. Jointly, they are as important as inflation and real activity in explaining the long part of the term structure. The residual factors explain 50% of the slope variation.

We explore the impact of the macro variables on the term premia in our model. We find that the ten-year yield responses to inflation and liquidity shocks are primarily driven by the responses to the term premia. We also decompose the term premia into contributions of the four macro risk factors. Inflation and liquidity shock jointly provide the strongest explanatory power at any maturity (65% to 85%). Inflation and fiscal shock have the largest contributions to the violations of the expectations hypothesis.

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A Projection

In this appendix we first provide the projection formulas, and then use them to show how our model is related to the traditional VAR analysis.

A.1 Recursive formulas

The model controlling the evolution of state z in (2.4) does not represent a state-space system. Nonetheless, Liptser (1997) derives the projection of one element of the VAR(1) on the other using the same ideas as in a standard Kalman filter. In particular, they derive the following expression for the conditional mean, $\hat{x}(M_t)$, often referred to as "forecast," and variance, P_t, of the forecast error,

$$\begin{aligned}
\hat{x}(M_t) &= \mu^x + \Phi^{xx}\hat{x}(M_{t-1}) + \Phi^{xm}m_{t-1} \\
&+ \left(\Sigma^{xx}\Sigma^{mx\prime} + \Sigma^{xm}\Sigma^{mm\prime} + \Phi^{xx}P_{t-1}\Phi^{mx\prime}\right)\left(\Sigma^{mx}\Sigma^{mx\prime} + \Sigma^{mm}\Sigma^{mm\prime} + \Phi^{mx}P_{t-1}\Phi^{mx\prime}\right)^{-1} \\
&\times \left(m_t - \mu_m - \Phi^{mx}\hat{x}(M_{t-1}) - \Phi^{mm}m_{t-1}\right) \qquad \text{(A.1)} \\
P_t &= \Phi^{xx}P_{t-1}\Phi^{xx\prime} + \left(\Sigma^{xx}\Sigma^{xx\prime} + \Sigma^{xm}\Sigma^{xm\prime}\right) \\
&- \left(\Sigma^{xx}\Sigma^{mx\prime} + \Sigma^{xm}\Sigma^{mm\prime} + \Phi^{xx}P_{t-1}\Phi^{mx\prime}\right)\left(\Sigma^{mx}\Sigma^{mx\prime} + \Sigma^{mm}\Sigma^{mm\prime} + \Phi^{mx}P_{t-1}\Phi^{mx\prime}\right)^{-1} \\
&\times \left(\Sigma^{xx}\Sigma^{mx\prime} + \Sigma^{xm}\Sigma^{mm\prime} + \Phi^{xx}P_{t-1}\Phi^{mx\prime}\right)'. \qquad \text{(A.2)}
\end{aligned}$$

We introduce additional notations to describe the projection initialization. The long run mean z is:

$$(I-\Phi)^{-1}\mu = \begin{bmatrix} \Theta^m \\ \Theta^x \end{bmatrix} \qquad \text{(A.3)}$$

(A.4)

The steady-state matrix P satisfies

$$\begin{aligned}
P &= \Phi^{xx}P\Phi^{xx\prime} + \left(\Sigma^{xx}\Sigma^{xx\prime} + \Sigma^{xm}\Sigma^{xm\prime}\right) \\
&- \left(\Sigma^{xx}\Sigma^{mx\prime} + \Sigma^{xm}\Sigma^{mm\prime} + \Phi^{xx}P\Phi^{mx\prime}\right)\left(\Sigma^{mx}\Sigma^{mx\prime} + \Sigma^{mm}\Sigma^{mm\prime} + \Phi^{mx}P\Phi^{mx\prime}\right)^{-1} \\
&\times \left(\Sigma^{xx}\Sigma^{mx\prime} + \Sigma^{xm}\Sigma^{mm\prime} + \Phi^{xx}P\Phi^{mx\prime}\right)' \qquad \text{(A.5)}
\end{aligned}$$

Then the projection is initialized as follows:

$$\hat{x}(m_0) = \Theta^x + V^{xm}(V^{mm})^{-1}(m_0 - \Theta^m), \;\; P_0 = P \tag{A.6}$$

In this case $P_t = P$, and the projection is time-stationary. An alternative strategy is to initialize P_0 at the unconditional variance of z. In this case, the sequence P_t will converge to P. In our model it happens in twelve steps.

A.2 Relation to VAR

Recall from (2.7) that

$$f_t = x_t - \hat{x}(M_t). \tag{A.7}$$

By construction, its variance is equal to P_t. Equations (2.4) and (A.1) imply that

$$\begin{aligned} f_t &= \Phi^{xx} f_{t-1} + \Sigma^{xx}\epsilon_t^x + \Sigma_{xm}\epsilon_t^m \\ &- \underbrace{\left(\Sigma^{xx}\Sigma^{mx\prime} + \Sigma^{xm}\Sigma^{mm\prime} + \Phi^{xx}P_{t-1}\Phi^{mx\prime}\right)\left(\Sigma^{mx}\Sigma^{mx\prime} + \Sigma^{mm}\Sigma^{mm\prime} + \Phi^{mx}P_{t-1}\Phi^{mx\prime}\right)^{-1}}_{\mathcal{P}_{t-1}} \\ &\times \left(\Phi^{mx} f_{t-1} + \Sigma^{mx}\epsilon_t^x + \Sigma^{mm}\epsilon_t^m\right) \\ &= \left(\Phi^{xx} - \mathcal{P}_{t-1}\Phi^{mx}\right) f_{t-1} + \left(\Sigma^{xx} - \mathcal{P}_{t-1}\Sigma^{mx}\right)\epsilon_t^x + \left(\Sigma^{xm} - \mathcal{P}_{t-1}\Sigma^{mm}\right)\epsilon_t^m \end{aligned} \tag{A.8}$$

Therefore, the "residual" factors f follow the VAR(1) process. However, in contrast to the dynamics of m and x the conditional mean and variance of f are state dependent. The steady-state Kalman filter theory implies that $\mathcal{P}_t$ converges to a fixed matrix together with P_t.

Denote $\mathcal{P} = \lim_{t\to\infty} \mathcal{P}_t$. We introduce the following explicit notations for simplified referencing:[22]

$$\Phi^{ff} \equiv \Phi^{xx} - \mathcal{P}\Phi^{mx} \tag{A.9}$$

$$\Sigma^{ff} \equiv \Sigma^{xx} - \mathcal{P}\Sigma^{mx} \tag{A.10}$$

$$\Sigma^{fm} \equiv \Sigma^{xm} - \mathcal{P}\Sigma^{mm} \tag{A.11}$$

[22] The identification assumptions regarding the matrix Σ (3.3) simplify many projection equations. One notable simplification is that $\Sigma^{ff} = \Sigma^{xx}$.

Therefore,

$$\begin{bmatrix} m_t \\ f_t \end{bmatrix} = \begin{bmatrix} \mu^m \\ 0 \end{bmatrix} + \begin{bmatrix} \Phi^{mm} & \Phi^{mx} \\ 0 & \Phi^{ff} \end{bmatrix} \begin{bmatrix} m_{t-1} \\ f_{t-1} \end{bmatrix} + \begin{bmatrix} \Phi^{mx}\hat{x}(M_{t-1}) \\ 0 \end{bmatrix} + \begin{bmatrix} \Sigma^{mm} & \Sigma^{mx} \\ \Sigma^{fm} & \Sigma^{ff} \end{bmatrix} \begin{bmatrix} \epsilon_t^m \\ \epsilon_t^x \end{bmatrix} \tag{A.12}$$

Recursive substitution of $\hat{x}$ from (A.1) and bond valuation based on (2.3) yield the VAR system:

$$\begin{bmatrix} m_t \\ f_t \\ y_t \end{bmatrix} = \begin{bmatrix} \mu^m \\ 0 \\ \mu^y \end{bmatrix} + \begin{bmatrix} \Phi^{mm}(L) & \Phi^{mx} & 0 \\ 0 & \Phi^{ff} & 0 \\ \Phi^{ym}(L) & \Phi^{yf} & 0 \end{bmatrix} \begin{bmatrix} m_{t-1} \\ f_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} \Sigma^{mm} & \Sigma^{mx} & 0 \\ \Sigma^{fm} & \Sigma^{ff} & 0 \\ \Sigma^{ym} & \Sigma^{yf} & 0 \end{bmatrix} \begin{bmatrix} \epsilon_t^m \\ \epsilon_t^x \\ \omega_t \end{bmatrix}. \tag{A.13}$$

This is a stylized representation; in order to focus on the most important issues, we do not reproduce full expressions for some matrices (e.g., $\Phi^{ym}(L)$).

B Latent Factor Indeterminacy

One of our objectives is to establish whether the new variables f_1 and f_2 are related to additional observables. The two latent factors could span the space generated by some macro variables, i.e., regressing f_1, f_2 on the proposed macro variables would generate high R^2, but, in principle, we cannot easily interpret the latent factors. We can, however, exploit the indeterminant nature of the latent factors to our advantage.

While the previous section imposes identifying restrictions, Dai and Singleton (2000) point out that such restrictions are not necessarily unique. There are many sets of restrictions, or invariant transformations of the model, such that the yields are left unchanged. Naturally, when a parameter configuration changes, the respective latent variables change as well by "rotating." It is sensible to rotate the factors to identify f with macro variables. We will use the invariant affine transformation, which scales factors by a matrix. Appendix A of Dai and Singleton (2000) describes how such a transformation affects model parameters.

We examine two types of rotations. The first rotation, $\mathcal{O}$, ensures that the two factors are orthogonal to each other; i.e., the variance-covariance matrix of f, P (see the expression in Appendix A), becomes diagonal. We define $\mathcal{O} = Rf_t$, where the matrix R is such that RPR' is diagonal.

The matrix R is not unique; i.e., the rotation of type $\mathcal{O}$ can generate many pairs of orthogonal factors f. Our second proposed rotation, $\mathcal{M}$, can be applied after any of the rotations from the class $\mathcal{O}$, resolves this type of indeterminacy. Define $\mathcal{M} = Uf_t$, where the matrix U is the orthogonal matrix; i.e., $UU' = I$, that preserves the correlation structure between the factors. In our two-dimensional case, the matrix U is determined by a single parameter, which is established by maximizing the correlation between one of the latent factors and one of the observable macro factors, which we choose to be the public debt growth.

C Pure Premium Term in Campbell-Schiller regressions

In this section we briefly review, based on the work of DS2, the expressions allowing to compute $D^*_{t+1}(\tau)$ in (4.6). The forward rate is equal to

$$f_t(\tau) = (\tau + 1)y_t(\tau + 1) - \tau y_t(\tau). \tag{C.1}$$

Then the forward term premium is defined as:

$$p_t(\tau) = f_t(\tau) - E^P_t(r_{t+\tau}). \tag{C.2}$$

The yield term premium is defined as:

$$c_t(\tau) = y_t(\tau) - \frac{1}{\tau}\sum_{i=0}^{\tau-1} E^P_t(r_{t+i}) \equiv a^{TP}(\tau) + b^{TP}(\tau)' z_t. \tag{C.3}$$

Because

$$c_t(\tau) = \frac{1}{\tau}\sum_{i=0}^{\tau-1} p_t(i), \tag{C.4}$$

both premia are introduced for the convenience of notation and interpretation. Finally, the "pure premium term" is equal to:

$$D^*_{t+1}(\tau) = -(\tau - 1)(c_{t+1}(\tau - 1) - c_t(\tau)) + p_t(\tau - 1). \tag{C.5}$$

Table 1 : Descriptive Analysis

We provide the preliminary data analysis by reporting the adjusted R^2 from univariate regressions of level, slope and curvature on contemporaneous inflation and real activity in the first column, on 12 lags of inflation and real activity in the second column and from the VAR(12) analysis of the vector comprised of level, slope, curvature, inflation, and real activity in the third column. Level is defined as the three-month yield, $y_t(3)$; slope is equal to $y_t(120) - y_t(3)$; and curvature is $y_t(3) + y_t(120) - 2 \cdot y_t(24)$.

	m_t only	m_t+lags	VAR
level	54.89	66.73	96.35
slope	40.83	43.90	88.21
curvature	9.57	14.23	70.20

Table 2 : Estimated Parameters

The table lists the parameter values for our model

$$
\begin{aligned}
z_t &= (g_t, \pi_t, x_{1t}, x_{2t}) \\
r_t &= \delta_0 + \delta_z' z_t \\
z_t &= \mu + \Phi z_{t-1} + \Sigma \epsilon_t \\
\Lambda_t &= \Lambda_0 + \Lambda_z z_t \\
\log \xi_t &= -r_{t-1} - \frac{1}{2}\Lambda_{t-1}'\Lambda_{t-1} - \Lambda_{t-1}'\epsilon_t
\end{aligned}
$$

The bootstrapped 95% confidence intervals are reported in parentheses. The parameters restricted by the identification requirements are highlighted by the letter combination 'id' in place of the confidence bounds. The dash '-' indicates that a parameter was restricted in the course of estimation. Note that we report the long-run mean, $(I-\Phi)^{-1}\mu$, of the state variables.

Interest Rate Rule

δ_0	δ_z			
	g	π	x_1	x_2
-13.03	1.62	1.45	1	1
(-33.61, 10.61)	(-1.23,4.60)	(-0.54,4.60)	id	id

State equation

	$(I-\Phi)^{-1}\mu$	Φ				Σ			
		g	π	x_1	x_2	g	π	x_1	x_2
g	7.41	0.96	0	-0.02	0.55	0.84	0	0	0
	(6.22,8.62)	(0.86,1.03)	-	(-0.04,-0.01)	(0.01,5.86)	(0.77,0.89)	id	id	id
π	4.19	0.07	0.95	0.01	-0.80	0.08	1.07	0	0
	(2.25, 5.96)	(0.02,0.18)	(0.86,0.99)	(0.00,0.04)	(-8.83,-0.04)	(-0.02,0.17)	(0.98,1.13)	id	id
x_1	0	-0.10	0.04	0.92	0.98	-0.99	-1.40	1.96	0
	id	(-0.40,0.27)	(-0.15, 0.27)	(0.80,1.02)	(-3.93,9.98)	(-3.46,1.39)	(-4.57,0.78)	(1.72,2.10)	id
x_2	0	-0.01	0	0	1.02	0	-0.07	0	0.06
	id	(-0.04,0.00)	-	-	(0.94,1.09)	-	(-0.67,-0.01)	id	(0.01,0.73)

Risk premia

	Λ_0	Λ_z			
		g	π	x_1	x_2
g	-93.24	10.62	6.31	1.68	0
	(-199.21,-31.83)	(3.51,23.87)	(1.25, 16.56)	(-1.43,6.77)	-
π	57.49	-5.64	-5.40	-0.83	-24.32
	(12.60,135.34)	(-15.24,-0.68)	(-13.28, -1.84)	(-3.76,1.09)	(-247.37,-1.92)
x_1	25.82	-2.91	-1.88	-0.83	0
	(2.44,60.91)	(-7.71,-0.03)	(-5.95,0.10)	(-2.50,-0.08)	-
x_2	0	0	0	0.10	-2.69
	-	-	-	(-0.09,0.43)	(-47.05,-0.08)

Table 3 : Pricing Errors and Moments

We report average absolute pricing errors in yields by maturity in panel (a). Panel (b) reports various moments of the observables computed from the dataset (monthly observations from 1970 to 2002) and implied by the estimated model. The bootstrapped 95% confidence intervals are reported in parentheses. The boldfaced sample statistics are outside the confidence bounds.

Panel (a). Pricing Errors.

Maturity, months	3	6	12	24	36	60	84	120
Error, b.p.	19.88	12.05	6.20	27.15	28.01	8.93	13.12	33.64

Panel (b). Moments.

	Means, %		Std. Dev., %		Skewness		Kurtosis		Autocorr.	
	Data	Model	Data	Model	Data	Model	Data	Model	Data	Model
g	7.72	7.42	1.64	1.58	-0.27	-0.01	2.08	2.62	0.99	0.98
	(6.12, 8.72)		(0.99, 2.29)		(-0.89, 0.83)		(1.87, 3.92)		(0.97, 0.99)	
π	4.82	4.21	2.89	2.87	**1.19**	-0.01	3.64	2.50	0.99	0.99
	(2.34, 6.12)		(1.66, 4.21)		(-0.89, 0.89)		(1.74, 3.83)		(0.98, 1.00)	
level,	6.49	5.53	2.78	2.95	**1.05**	0.00	**4.44**	2.56	0.98	0.97
$y(3)$	(2.33, 8.87)		(1.75, 4.38)		(-0.84, 0.79)		(1.78, 3.84)		(0.94, 0.99)	
$y(24)$	7.20	6.21	2.59	2.71	**0.85**	0.00	3.78	2.51	0.98	0.98
	(2.99, 9.56)		(1.52, 4.11)		(-0.90, 0.83)		(1.72, 3.86)		(0.95, 0.99)	
$y(120)$	7.85	7.01	2.21	2.27	**0.97**	-0.01	3.55	2.50	0.99	0.98
	(4.19, 9.78)		(1.27, 3.45)		(-0.95, 0.88)		(1.70, 3.87)		(0.96, 0.99)	
slope,	1.36	1.48	1.43	1.44	-0.60	0.00	3.19	2.75	0.93	0.94
$y(120) - y(3)$	(0.92, 2.01)		(1.04, 1.92)		(-0.65, 0.66)		(2.08, 3.82)		(0.90, 0.97)	
curvature,	-0.04	0.13	**0.76**	0.38	0.04	0.01	**4.03**	2.52	**0.81**	0.97
$y(3) + y(120) - 2 \cdot y(24)$	(-0.37, 0.63)		(0.21, 0.60)		(-0.79, 0.85)		(1.72, 3.75)		(0.94, 0.99)	

Table 4 : Correlations Between the Traditional Latent Factors and the Residual Latent Factors

We correlate the three latent factors (level, $y(3)$, slope, $y(120) - y(3)$, and curvature, $y(3) + y(120) - 2 \cdot y(24)$) which jointly capture 99% of the yield curve variation with two sets of latent factors that feature in our model. The factors x enter our model directly, joint with macro factors. The factors f are residuals of projection of x on to the history of the macro variables. We also report correlations between the respective xs and fs. The low correlations imply that inferring the impact of macro variables with x as latent factors is very different from using f.

	x_1	x_2	f_1	f_2
level	-0.38	-0.26	-0.04	0.60
slope	0.64	0.38	0.62	-0.46
curvature	-0.02	0.08	-0.36	-0.35
x_1			0.22	
x_2				0.31

Table 5 : Principle Components

In panel (a) we establish which fraction of the yield curve variation is explained by various versions of the interest rate rule. We represent the yield curve by its first three principle components: level, $y(3)$, slope, $y(120) - y(3)$, and curvature, $y(3)+y(120)-2\cdot y(24)$. We consider two versions of the interest rule, which depend on the difference in how the factors are used and on how the rules is estimated. The different rules are derived from from the no-arbitrage model. Given the estimated coefficients, we compute the cumulative variance decomposition, i.e., fraction of the unconditional variance explained. First, we evaluate the rule using the lagged macro variables

$$r_t = \delta_0 + \delta_m' m_t + \delta_x' \hat{x}(M_t).$$

Finally, then we use the full set of state variables:

$$r_t = \delta_0 + \delta_m' m_t + \delta_x' \hat{x}(M_t) + \delta_x' f_t \equiv \delta_0 + \delta_m' m_t + \delta_x' x_t$$

Panel (b) reports the contribution of the factors' shocks to the variance of the error in forecasting the principal components at the infinite horizon. The bootstrapped 95% confidence intervals are reported in parentheses in both panels.

Panel (a). Theoretical R^2.

PC	m_t+lags	full model
level	79.59	99.75
	(43.86, 90.21)	(99.21, 99.86)
slope	52.36	97.45
	(22.36, 75.77)	(95.17, 98.43)
curvature	42.74	55.83
	(8.43, 64.04)	(24.87, 72.70)

Panel (b). Variance Decomposition of the Mean Squared Forecasting Error, Infinite Horizon.

PC	g	π	f_1	f_2
level	46.72	4.91	31.79	16.55
	(24.35, 63.19)	(1.86, 22.06)	(2.60, 45.18)	(10.87, 48.91)
slope	31.58	5.32	49.62	13.47
	(16.11, 48.60)	(1.09, 16.88)	(20.34, 60.30)	(7.55, 37.72)
curvature	40.25	1.46	38.73	19.54
	(14.18, 60.25)	(0.46, 17.69)	(4.59, 57.51)	(9.79, 58.43)

Table 6 : Term Premia

In panel (a) we report the percentage contribution of macro risk factors to the overall unconditional variation in the term premia. The contribution of the covariance between the inflation and real activity components is split equally. We consider three maturities: one, five, and ten years. Panel (b) reports the contribution of the risk factors' shocks to the variance of the error in forecasting the term premia at the infinite horizon. The bootstrapped 95% confidence intervals are reported in parentheses in both panels.

Panel (a). Decomposition of the Unconditional Variance of the Term Premia.

	g_t	π_t	f_{1t}	f_{2t}
1-year	11.43	42.27	4.44	41.84
	(0.00, 39.91)	(12.00, 61.97)	(0.00, 15.76)	(21.86, 66.74)
5-year	28.47	39.39	6.39	25.74
	(2.24,49.18)	(17.23, 61.86)	(0.00, 34.26)	(10.01, 46.19)
10-year	17.24	51.25	8.84	22.65
	(0.00, 39.40)	(14.57, 80.47)	(0.00, 44.34)	(7.17, 34.95)

Panel (b). Variance Decomposition of the Mean Squared Forecasting Error, Infinite Horizon.

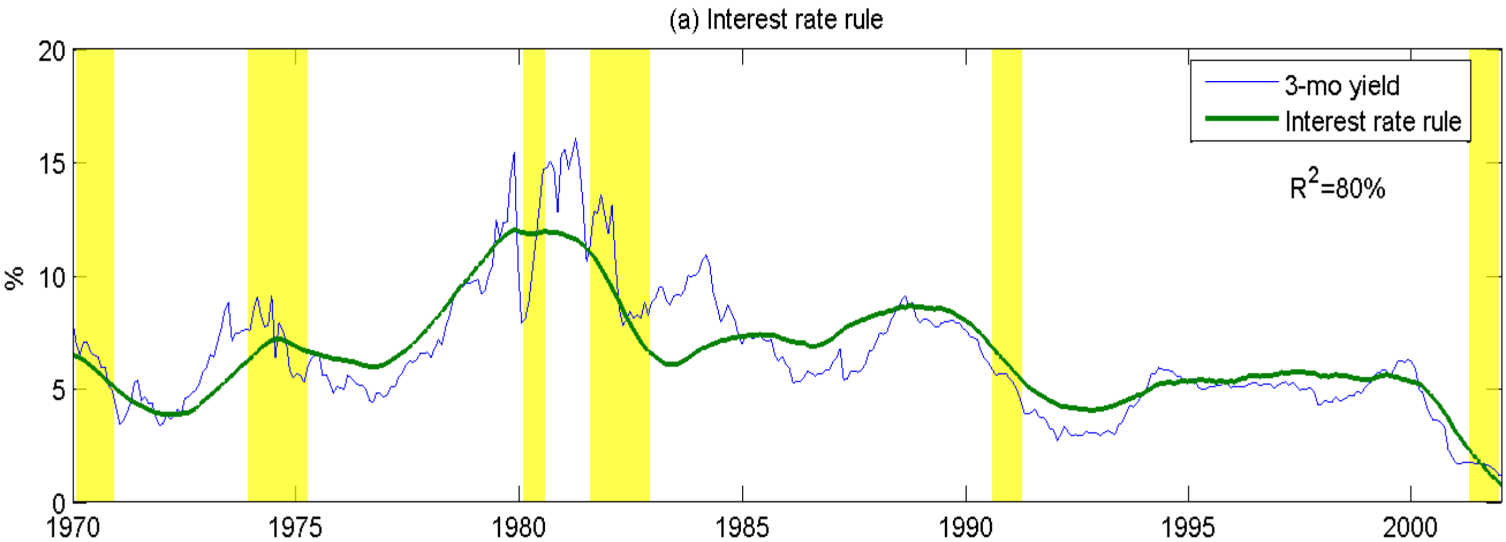
	g_t	π_t	f_{1t}	f_{2t}
1-year	30.10	13.50	9.81	46.58
	(8.15, 46.13)	(5.32, 25.74)	(1.83, 27.01)	(20.97, 72.54)
5-year	33.81	16.25	12.95	36.96
	(10.29, 49.81)	(6.69, 31.54)	(3.51, 32.46)	(13.10, 65.30)
10-year	37.61	18.12	16.43	27.81
	(11.91, 51.38)	(7.84, 31.69)	(5.84, 40.37)	(11.48, 54.57)

Figure 1. The Interest Rate Rule.

We plot the time series of the three-month zero yield and the estimate of r_t based on the projection

$$\hat{r}_t = \delta_0 + \delta_m' m_t + \delta_x' \hat{x}(M_t)$$

in panel (a). Panel (b) shows the first 24 loadings on the macro variables and their lags that generate the interest rate rule.



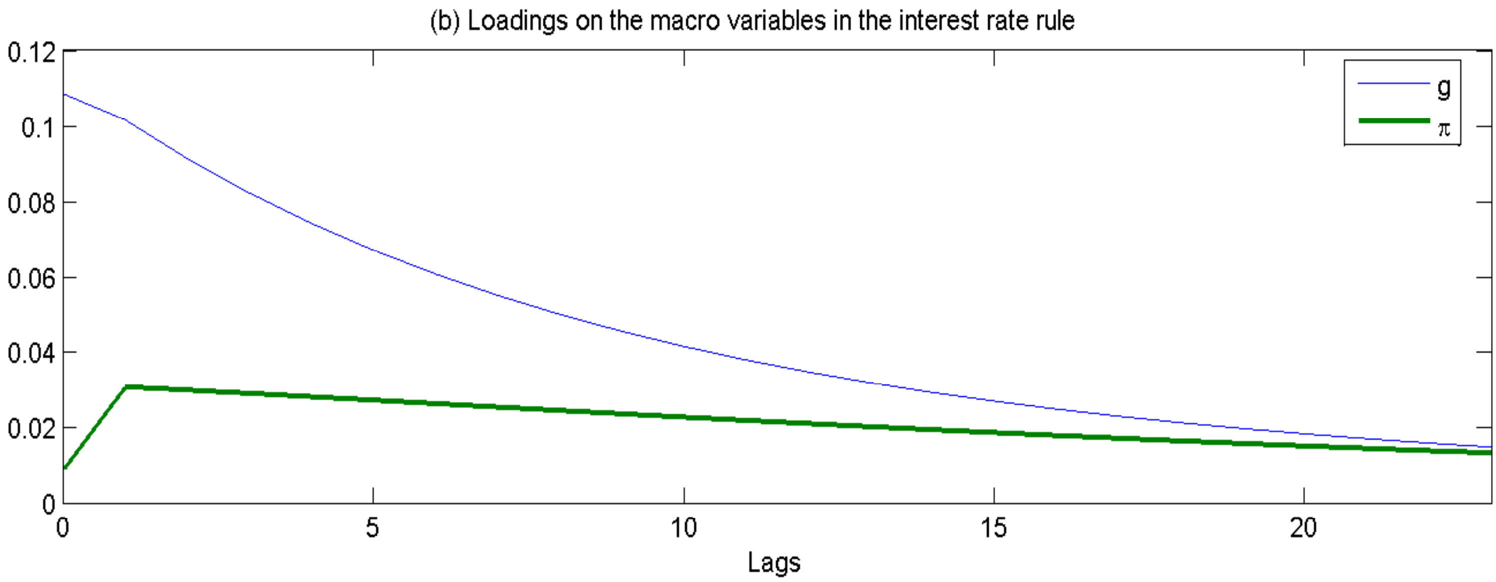


Figure 2. Term Structure Response to Shocks in the Latent State Variables.

We plot how term structure changes, in basis points, in response to one standard deviation change in one of the two residual factors f. The thin line corresponds to f_1 (left scale) and the thick line corresponds to f_2 (right scale).

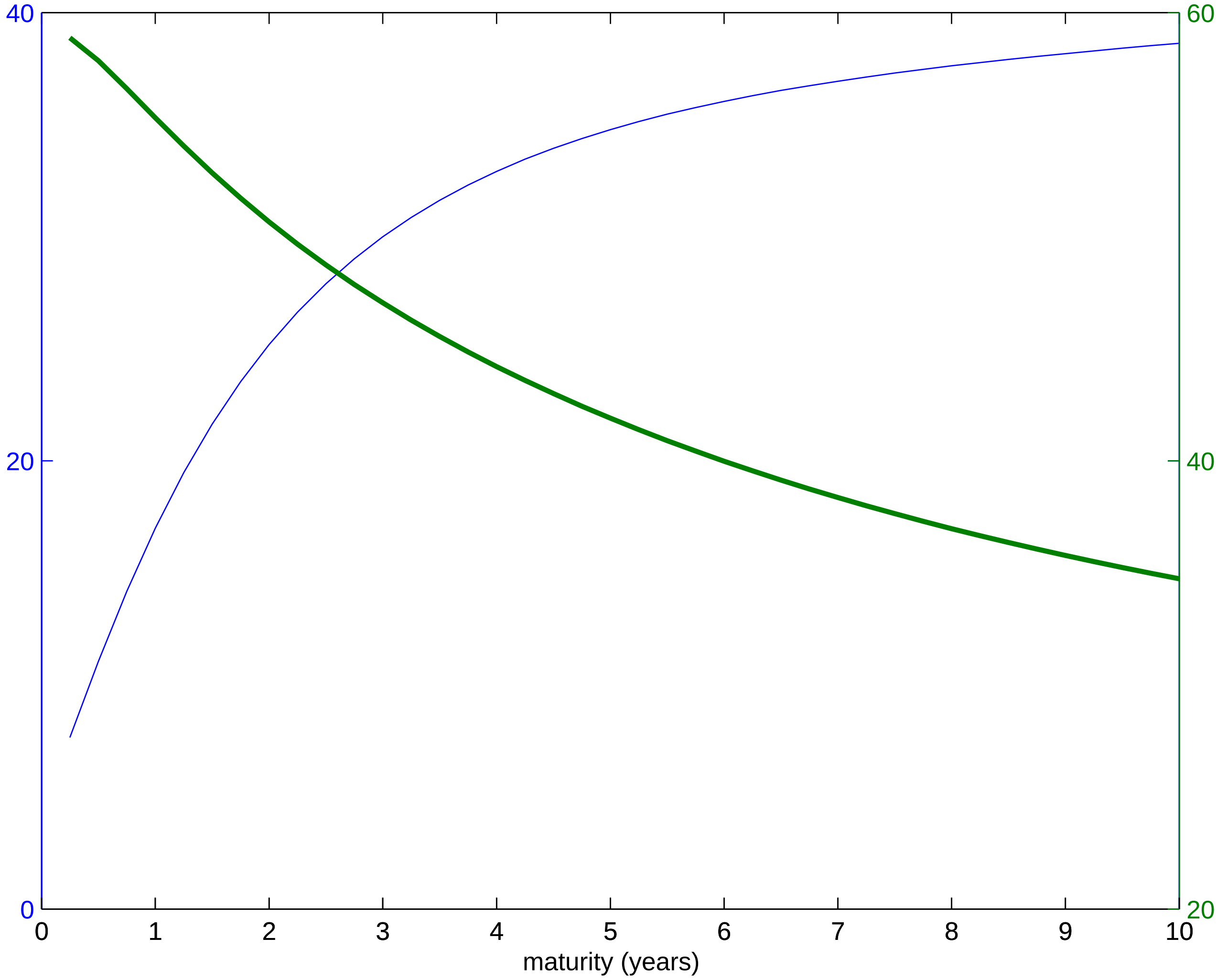


Figure 3. The Slope.

We plot the time series of the slope and its estimate computed using the projection-based interest rate rule

$$\hat{r}_t = \delta_0 + \delta_m' m_t + \delta_x' \hat{x}(M_t)$$

in panel (a). Panel (b) shows the first 24 loadings on the macro variables and their lags that generate the slope.

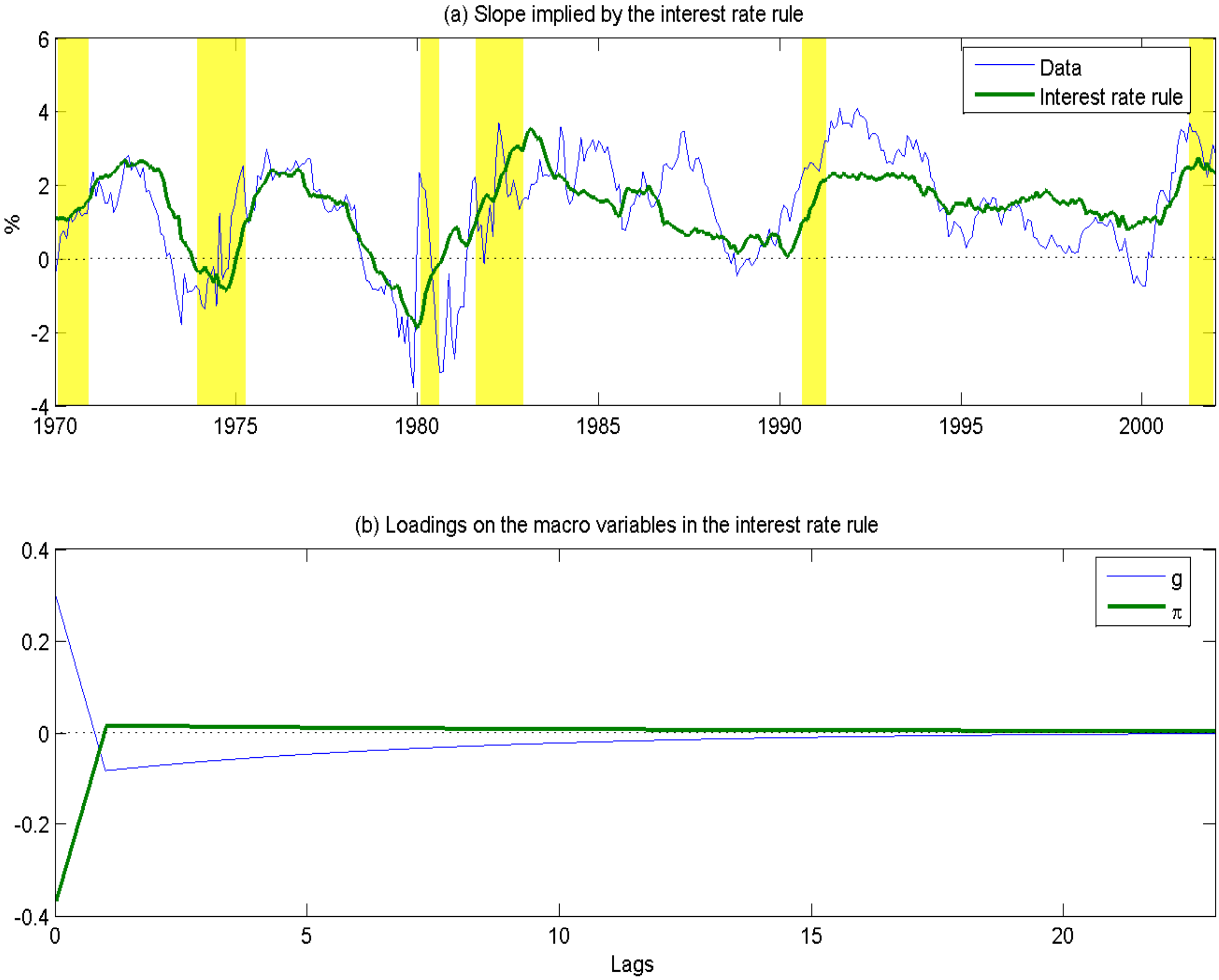


Figure 4. Impulse Response Functions: State Variables.

The figure shows the impulse responses to one standard deviation shock to the factors in our model (dark thick line) and in a regular VAR (thin line). The bootstrapped 95% confidence intervals for the IR in our model are represented by dashed lines. We estimate a VAR(12) using three yields (3 months, 2 and 10 years) and four macro variables (real activity, g; inflation, π; growth in public debt, f_1; and AAA credit spread multiplied by negative one, f_2. We report confidence bounds only for VAR from our model to avoid clutter.

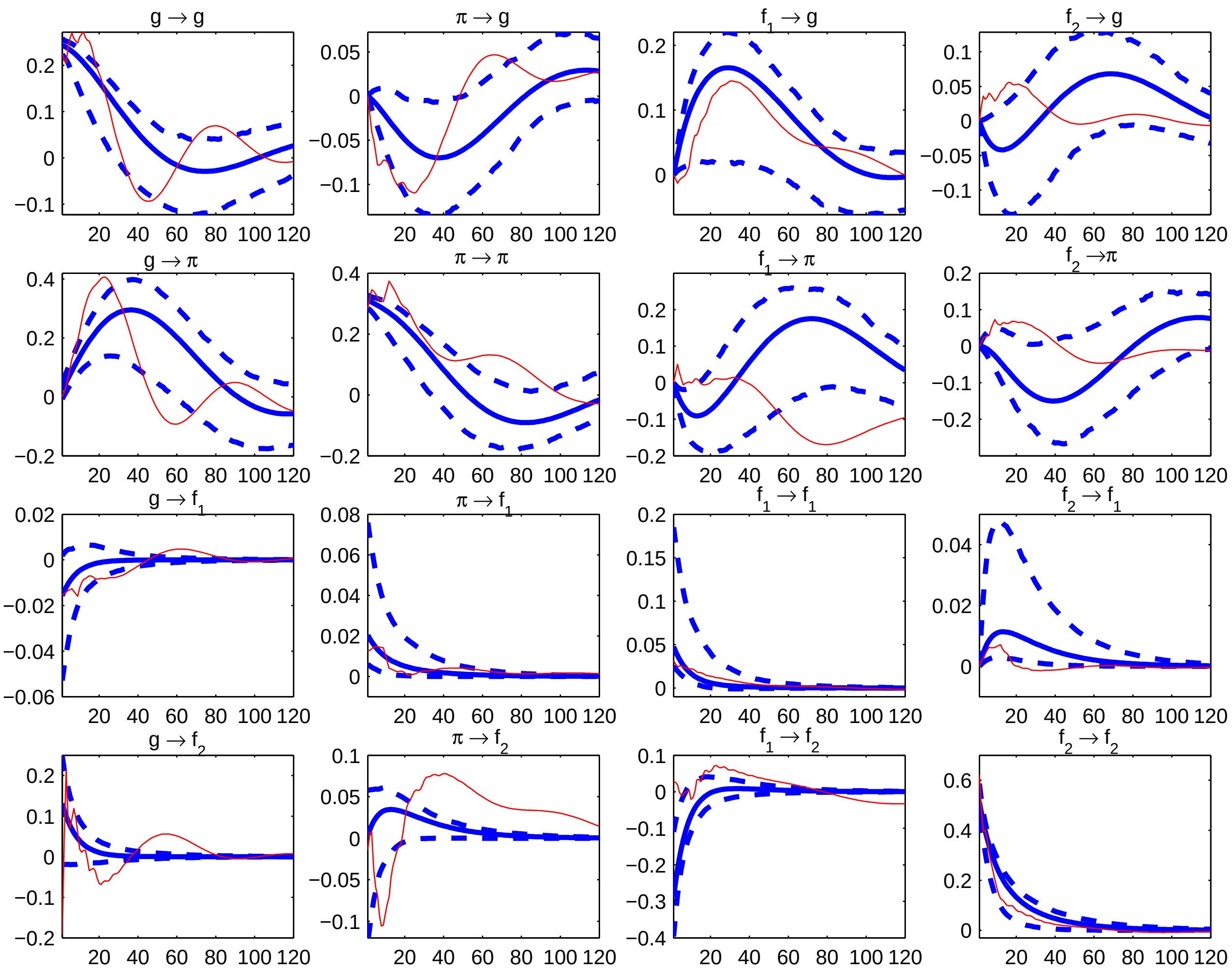


Figure 5. Impulse Response Functions: Yields and Slope.

The figure shows the impulse responses of the three-month and ten-year yields and the slope to one standard deviation shocks in the state variables in our model (dark thick line) and in a regular VAR (thin line). The bootstrapped 95% confidence intervals for the IR in our model are represented by dashed lines. We estimate a VAR(12) using three yields (3 months, 2 and 10 years) and four macro variables (real activity, g; inflation, π; growth in public debt, f_1; and AAA credit spread multiplied by negative one, f_2. We report confidence bounds only for VAR from our model to avoid clutter.

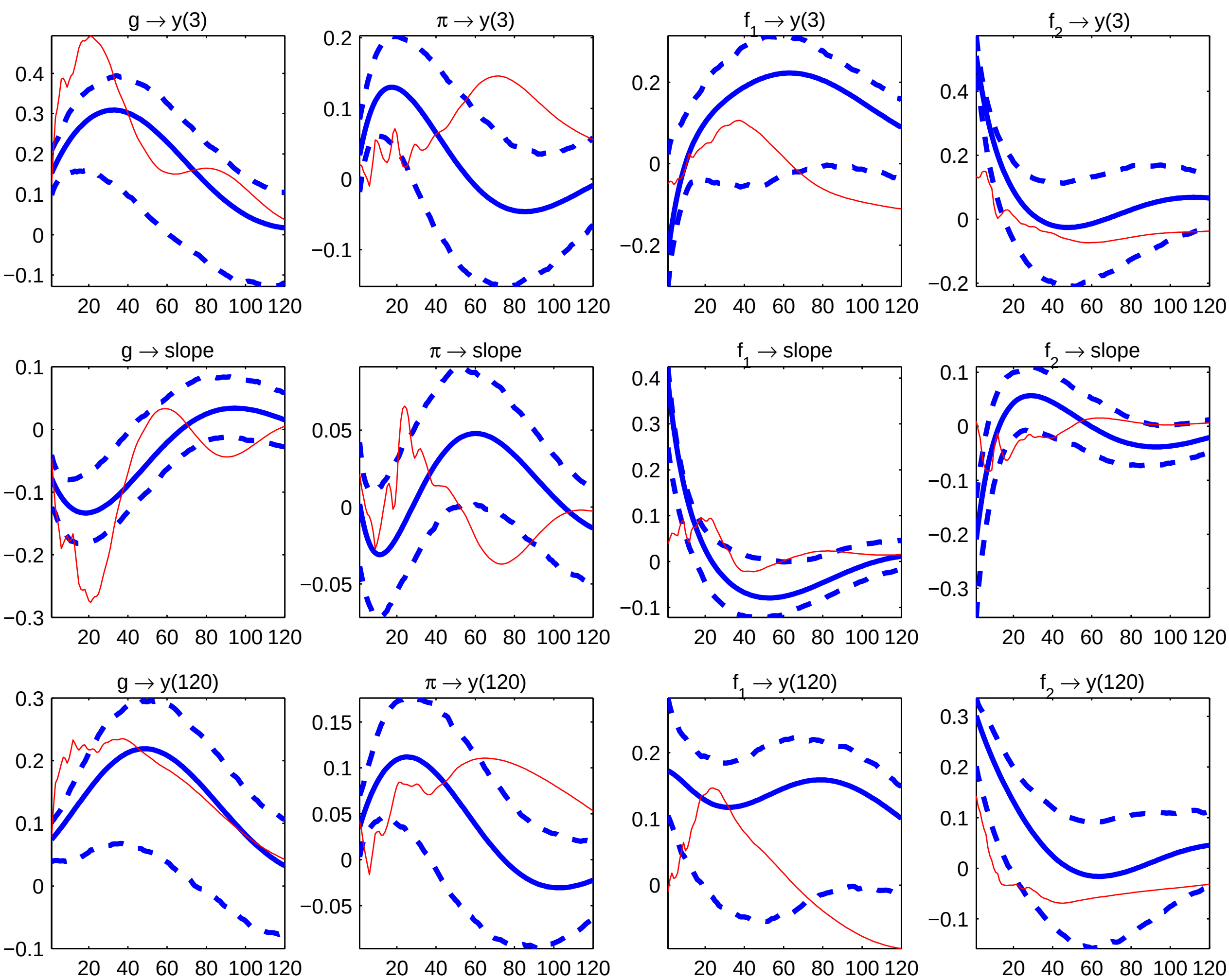


Figure 6. Orthogonalized Factor f_1 and the Annual Public Debt Growth Rate.

The plot shows the monthly series of the estimated factor f_1 (thin line, left scale) against the annual government public debt growth rate (thick line, right scale). The latter series are residuals from regressing the debt growth rate on twelve lags of inflation and real activity. Both series are standardized to facilitate comparison.

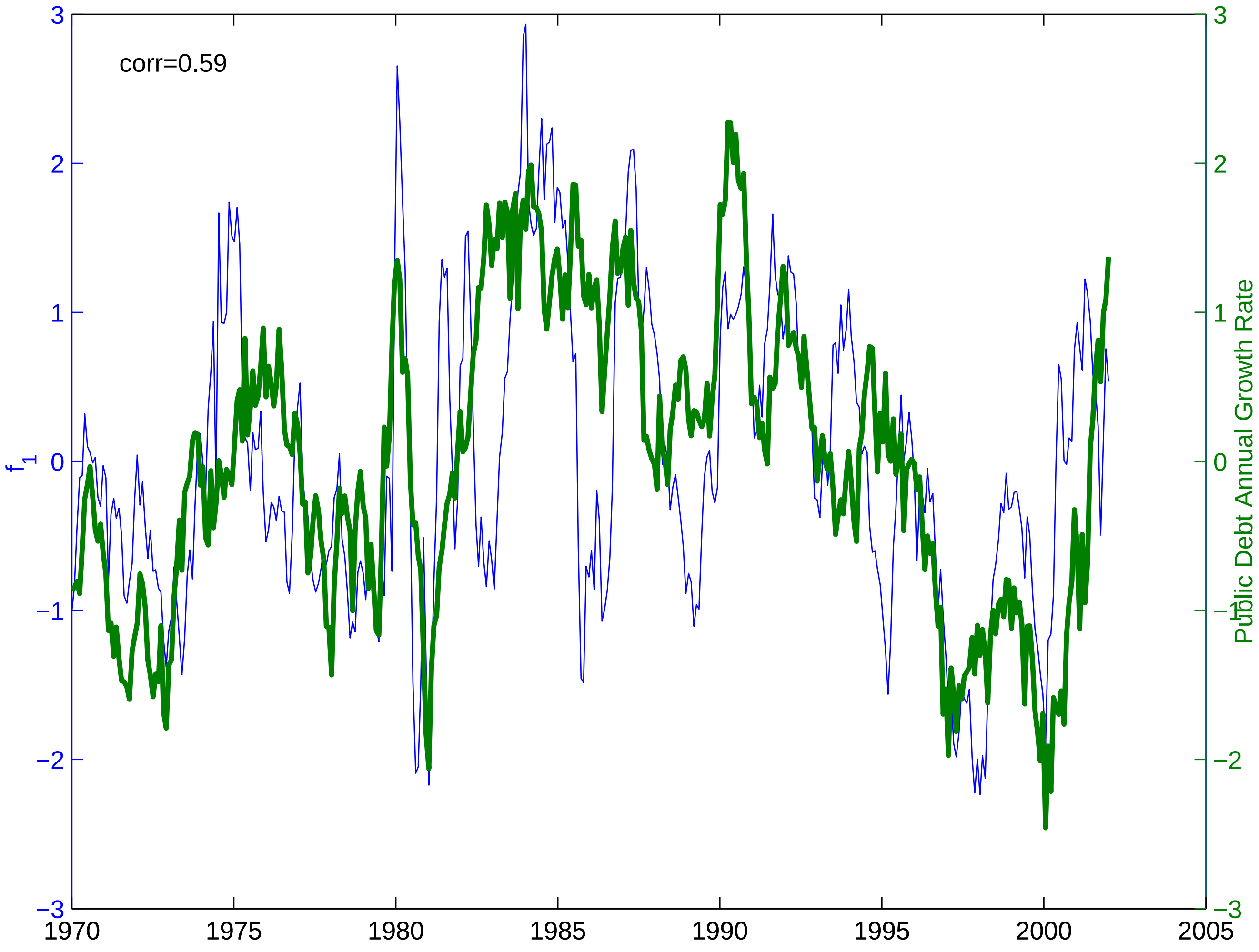


Figure 7. Orthogonalized Factor f_2 and the AAA Credit Spread and MZM growth rate

The plot shows the monthly series of the estimated latent factor f_2 (with minus sign) (thin line, left scale) against the spread between AAA Moody's corporate index and ten-year Treasury bond yield on panel (a) and MZM monthly growth rate on panel (b) (thick line, right scale). Both macro series are residuals from regressing the AAA spread (or MZM rate) on inflation and real activity. All three series are standardized to facilitate comparison. The MZM series are available from 1975.

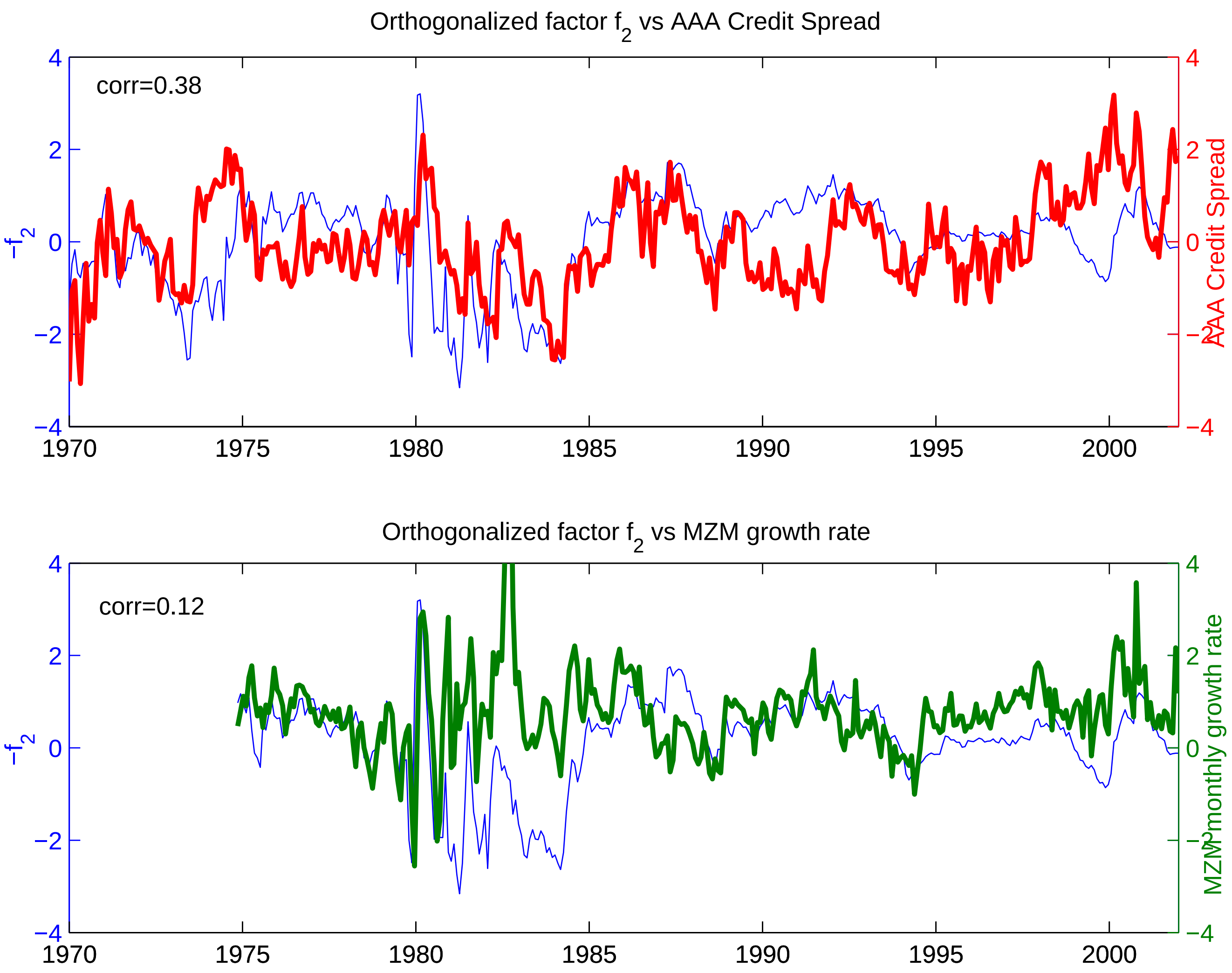


Figure 8. Impulse Response Functions: Expectations and Risk Premia.

The figure decomposes the impulse responses of the three-month and ten-year yields and the slope to one standard deviation shocks in the state variables into the expectations response and the term premia response. The solid line depicts the response of the yield (the sum of the two responses) – the same as in Figure 5.

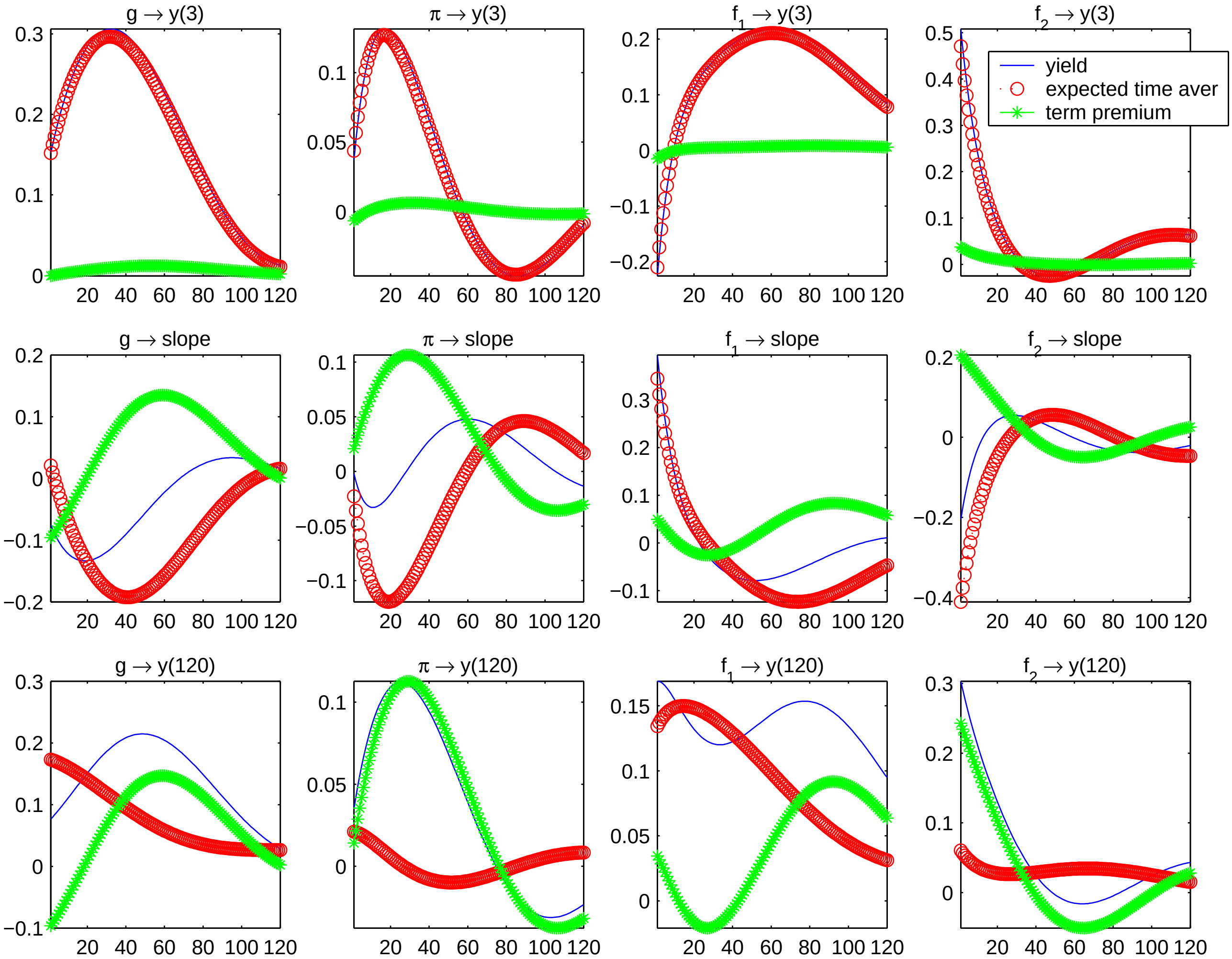


Figure 9. Term Premia Decompositions

The figure shows the time series of the one- and ten-year yields decomposed into the expectations and term premia. The term premia are subsequently decomposed into the contributions of the four determinants of the yield curve: real activity inflation, fiscal and liquidity shocks. The shaded regions show the NBER recessions.

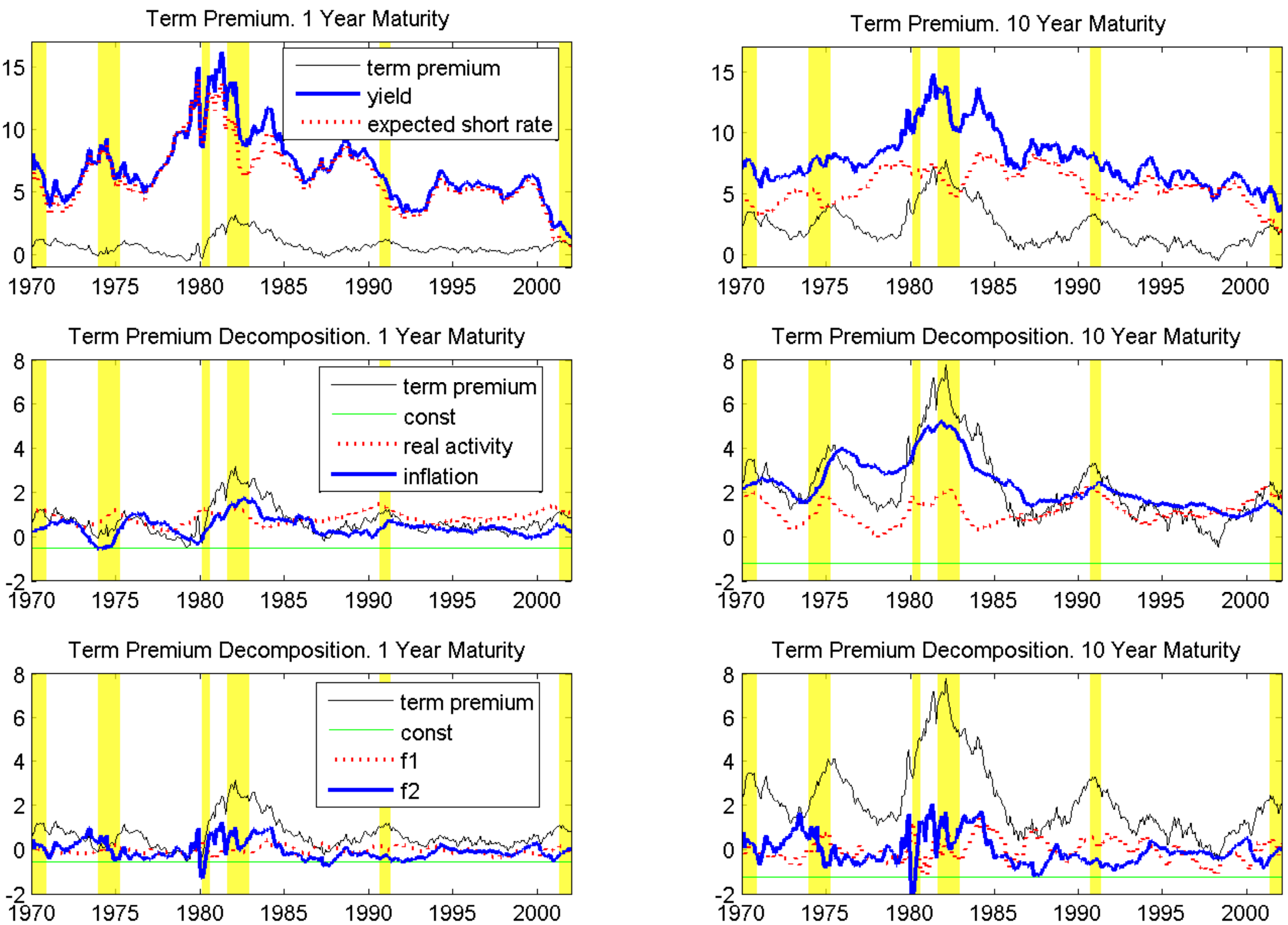


Figure 10. The Expectations Hypothesis

We show how well our model replicates the coefficients of the yield-predicting regressions ϕ_τ in panel (a) by plotting the data and model implied coefficients with the bootstrapped 95% confidence bounds. In panel (b) we decompose the model-implied coefficients according to the contributions of macro factors. These factor-based coefficients, together with the unity (zero risk premia) line add up to the model-implied ϕ_τ.