Methods for the Evaluation and Design of Light-Pipe Systems for Commercial Buildings

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Abstract

This report describes a study to investigate the potential of using current measurement and modeling techniques to characterize the performance of a Fusion-lamp light-pipe system in a commercial building application. To accomplish this task, a commercially available light-pipe was installed in a laboratory setting and measurements were conducted to determine its light output and distribution. The experimental data was then used to generate a series of standard data files that were used as input to an industry-standard lighting-design software program. Experimental measurements subsequently taken on a grid pattern, illuminated by the light-pipe, were found to closely match those predicted by the software program. The results of this study demonstrate the feasibility of evaluating light-pipe systems and illustrates the use of standard-lighting design software for the design of light-pipe installations.

Bref résumé

Le présent rapport contient une étude visant à évaluer les possibilités d'utilisation des techniques actuelles de mesure et de modélisation afin de caractériser le rendement d'un système de conducteur de lumière pour lampe à fusion dans une application liée aux bâtiments commerciaux. Pour ce faire, on a installé dans un laboratoire un système de conducteur de lumière vendu en magasin et on a procédé à des mesures en vue d'en déterminer la luminosité et les résultats obtenus en la matière. Les données expérimentales obtenues ont servi à produire une série de fichiers de données standard qui ont été introduits dans un logiciel de conception d'éclairage selon des normes approuvées dans le secteur industriel. Les mesures expérimentales prises par quadrillage de la surface illuminée par le conducteur de lumière se sont révélées très semblables à celles prévues par le logiciel. Les conclusions de l'étude démontrent clairement la possibilité d'évaluer les systèmes de conducteur de lumière, en plus de mettre en évidence l'utilité d'un logiciel de conception d'éclairage standard pour l'élaboration d'une installation de ce genre d'appareil.

Methods for the Evaluation and Design of Light-pipe Systems

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Executive Summary

This report describes a study to investigate the potential of using current measurement and modeling techniques to characterize the performance of a Fusion-lamp light-pipe system in a commercial building application. To accomplish this task, a commercially available light-pipe was installed in a laboratory setting and measurements were conducted to determine its light output and distribution.

To conduct the experimental evaluation, the light-pipe was divided into 20 sections. Light levels were measured, at a series of complex angles, on each section while the other sections were masked. In effect, the light-pipe was evaluated as if it was composed of twenty point-sources, each with its own characteristics.

The experimental data was then used to generate a series of standard data files that were used as input to an industry-standard lighting-design software program. The program was subsequently used to predict the light levels on a horizontal plane located approximately two meters below the light-pipe axis. Experimental measurements taken on a grid pattern, illuminated by the light-pipe, were found to closely match those predicted by the software program.

Finally, due to the complexity involved in the experimental characterization, a simplified model that represented the light-pipe as a planar "lambertian" surface was proposed and compared to the experimental measurements. This simplified model was found to produce comparable results to the two previous methods when experimental errors are considered.

The results of this study demonstrate the feasibility of evaluating light-pipe systems and illustrates the use of standard-lighting design software for the design of light-pipe installations.

Résumé

Le présent rapport contient une étude visant à évaluer les possibilités d'utilisation des techniques actuelles de mesure et de modélisation afin de caractériser le rendement d'un système de conducteur de lumière pour lampe à fusion dans une application liée aux bâtiments commerciaux. Pour ce faire, on a installé dans un laboratoire un système de conducteur de lumière vendu en magasin et on a procédé à des mesures en vue d'en déterminer la luminosité et les résultats obtenus en la matière.

Afin de réaliser cette évaluation expérimentale, on a divisé le conducteur de lumière en 20 sections. Les niveaux de luminosité ont été déterminés en fonction d'une série d'angles complexes dans une section donnée, alors que les autres parties du conducteur étaient voilées. Dans les faits, le conducteur de lumière a été évalué comme s'il était composé de vingt points de source de luminosité, chacun présentant ses propres caractéristiques.

Les données expérimentales obtenues ont servi à produire une série de fichiers de données standard qui ont été introduits dans un logiciel de conception d'éclairage selon des normes approuvées dans le secteur industriel. Par la suite, ce programme a permis de déterminer à l'avance les niveaux de luminosité correspondant à un plan horizontal situé à environ deux mètres sous l'axe du conducteur de lumière. Les mesures expérimentales prises par quadrillage de la surface illuminée par le conducteur de lumière se sont révélées très semblables à celles prévues par le logiciel.

Finalement, compte tenu de la complexité entourant la caractérisation expérimentale, un modèle simplifié, représentant le conducteur de lumière comme étant une surface de plans conforme à la Loi de Lambert, a été proposé et comparé aux mesures obtenues par expérimentation. On a alors constaté que ce modèle simplifié donnait des résultats comparables à ceux obtenus avec les deux méthodes précédentes où étaient prises en compte les erreurs d'expérimentation.

Les conclusions de l'étude démontrent clairement la possibilité d'évaluer les systèmes de conducteur de lumière, en plus de mettre en évidence l'utilité d'un logiciel de conception d'éclairage standard pour l'élaboration d'une installation de ce genre d'appareil.

Methods for the Evaluation and Design of Light-pipe Systems for Commercial Buildings

1.0 Introduction

The sulfur fusion lamp light-pipe is a high efficiency, high efficacy luminaire that shows much promise for industrial lighting applications. It offers many attractive features including low maintenance and excellent spectral characteristics and is expected to generate significant cost savings for companies that employ them. In order to design buildings incorporating the use light-pipes, an accurate computer model is needed that will allow architects to determine floor-level illuminance values in rooms where light-pipes have been installed. This report outlines the work of the Solar Calorimetry Laboratory at Queen's University towards creating a point-source array model for performing accurate illuminance calculations with the light-modeling program Micro Lumen 7. The correlation between the illuminance values calculated with this method and those measured in the laboratory were typically better than 99%. A simplified mathematical model of the light-pipe was also developed which allows first order approximations of the floor-level illuminance values to be quickly calculated. This model yielded correlations of 95%.

1.1 IES Files

The Illuminating Engineering Society of North America has developed a standard file format for creating computer models of point-source luminaires. These .IES files contain a set of intensity readings for different horizontal and vertical angles around the luminaire as well as other information such as dimensions, ballast factor, efficacy etc. The set of intensity values contained in an .IES file is all a computer program needs to be able to model the luminaire as a point source. An extended source can also be modeled with .IES files if it is treated as an array of point sources, each with its own .IES file. This technique was used to model the light-pipe.[1] Appendix A gives an example of an .IES file.

2.0 Apparatus and Measurements

In order to model the light-pipe as a series of point sources it was necessary to measure light intensity from different portions of the pipe separately. To this end, opaque, black paper was used to mask the light-pipe so that only one 20 cm wide section was showing at a time as in Figure 1. The four-meter long light-pipe was thus broken down into twenty sections, each of which could be modeled as a separate point source.

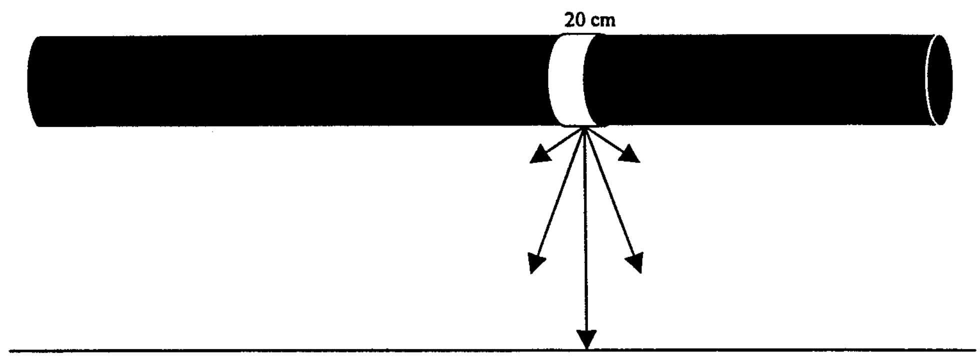

Fig. 1. Light-pipe showing masking technique used for conducting illuminance tests.

Once the light-pipe was masked, illuminance readings were taken at vertical angles of 0, 15, 30, 45, 60, 75 and 90 degrees for each horizontal angle of 0, 45, 90, 135 and 180 degrees. The measurements were taken with a Licor Li-21052 photometer attached to a Fluke 8500A digital multimeter and mounted on a scanning goniometer. The goniometer was built in 1997 as part of a mechanical engineering project[2], and was originally intended to be completely automated, allowing a PC to collect data as a stepping motor controlled the motion of a photometer along a quarter circle track. For this experiment, though, it proved simpler to take the measurements manually. The photometer was therefore placed by hand at the correct angle along the track and the track was moved into the correct position by the experimenter.

Each section of the light-pipe displayed a similar illumination pattern. Ideally, it was expected that each section would behave like a Lambertian source, or a source for which the intensity of emitted radiation is proportional to the cosine of the incident angle. It was found that in the "point-source" sections of the pipe, this ideal light distribution was somewhat distorted. This distortion was likely due to an experimental error in determining the location of the effective point-source centre and to the insufficient distance from the point-source to the measuring point. Both these sources of error will be explained in a later section. Polar graphs of some measured illuminance readings from different sections of the luminaire along with a polar graph of an ideal Lambert source can be found in Appendix B.

Once a set of illuminance measurements was taken for each of the twenty sections along the light-pipe, it was necessary to convert the data into a set of intensity values as required by the .IES format.

For a point-source at a distance D from a surface, and incident on a surface at angle ξ, the illuminance E of the surface is related to the luminous intensity I emitted by the source by the equation:

$$I = \frac{ED^2}{\cos\xi}$$

This equation, often called the square-cosine rule, is derived for point sources[3], but is reasonably accurate for a luminaire that is small compared to its distance from the observer. A general rule of thumb is that if the distance between the luminaire and the observer is five times the largest dimension of the luminaire, then the square-cosine rule should be accurate to within five percent[4].

In order to use the equation, however, it was first necessary to determine the location of the *effective* point-source that would be used to model each section of the pipe. An attempt was made to calculate the location of the effective source by taking two measurements of intensity at different distances from the luminaire and relating them by the cosine-square rule. It was found, however, that this method was too sensitive to measurement error to produce accurate results. Changes in the illuminance values of less than 3% at either of the two points led to changes of 100% or more in the calculated distance to the source. Since in general, illuminance measurements are only valid to within 5%, it was apparent that this method would not be sufficiently accurate to calculate the effective point-source location. It was therefore decided to take the location of the diffusing strip at the top of the tube to be the effective source as most of the light reaching the floor diffuses off this strip.

In the laboratory, the light-pipe was hung so that the diffusing strip was 1.98 m from the floor. Since the largest dimension of each section was half the circumference of the pipe or 40 cm, the distance between the observation point and the luminaire was very nearly 5 times the largest luminaire dimension. Intensity values calculated using this equation were therefore good to within approximately 5% of the actual intensities.

By treating segments of the diffusing strip as the effective point sources, the directional intensity of each section could then be found from the illuminance readings. Since on the goniometer the photometer always directly faced the light-pipe, the angle ξ was always zero, so the intensity at each point was simply the illuminance at the point divided by the square of its distance from the point-source.

The intensity distribution for each of the 20 point-sources making up the light-pipe model were then used to construct a set of IES files. The code for one of these files along with a brief explanation can be found in Appendix A. If the intensity distributions are stored in Microsoft Excel format, then a simple macro can automatically produce all the IES files for the light-pipe array.

The twenty IES files modeling the light-pipe are all that is needed to represent it in a computer light modeling program. The model was tested in Micro Lumen 7[5], but the same results should be obtainable from any light simulation software that accepts .IES formatted files.

3.0 Evaluation of the model

When all the directional illumination measurements had been taken, the light-pipe was moved to the laboratory. This provided a setting in which it would be possible to measure the floor-level illumination due to the light-pipe and to compare it to values predicted by the computer simulation.

A one-meter by one-meter grid was laid down on the floor of the laboratory, with one line of the grid lying directly below the centre line of the light-pipe. Illuminance measurements were taken with a Hioki 3422 Digital Photometer for illuminances under 2000 lux (the maximum range of the photometer) and a Licor Li-21052 photometer for illuminances over 2000 lux. As the graph in Appendix C shows, the Licor Li-21052 measured illuminance to be 8.4% higher at locations where both meters were used. To compensate for this, all readings with the Licor photometer were reduced to make them consistent with the readings taken with the Hioki photometer. Appendix D shows contour and surface plots of floor-level illumination levels.

Illumination measurements on the grid were taken twice using a slightly different room configuration each time. Measurements were taken on the grid intersection points as well as halfway between grid intersection points. More measurements were taken in the second room configuration than in the first one.

Computer models of the laboratory were generated on Micro Lumen v.7.5. Furniture was modeled as rectangular boxes with uniform reflectance of 0.5, except for a desk on the left side of the room which was modeled as having a reflectance of 0.2, since it had a matte finish and was dark brown in colour. Walls were modeled as having a standard reflectance of 0.5, the ceiling a reflectance of 0.8 and the floor a reflectance of 0.2. The room plans and the calculations performed by Micro Lumen 7 can be found in Appendix E. When the simulation was run, a strong correlation was found between the calculated and the measured illuminance values. For the first and second room configurations, correlations[1] of 99.7% and 99.8% were found between measured and computer generated illuminances. Graphs showing the correlation plots for each room configuration can be found in Appendix F. Both sets of calculations with Micro Lumen 7 showed floor-level illumination to be on average 16% lower than it was measured to be.

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[1] Correlation is a statistical measure of the likelihood that two sets of data are from the same parent distribution. Correlation is defined as the covariance between the two sets divided by the standard deviation of each set, or:

$$cor(x,y) = \frac{\frac{1}{n}\sum(x_i - \mu_x)(y_i - \mu_y)}{\sigma_x \sigma_y}$$

4.0 Measurement Limitations

There are a number of factors accounting for the 16% discrepancy between calculated and measured illuminances. The most obvious source of error was the photometers that typically have an accuracy of 5-10%. Another major factor is the simple model of room reflectance used. The use of more accurate reflectance values could significantly improve agreement between the calculated and measured results. There is also some error accountable to the assumption that the location of the effective point-source is on the diffusing strip. As well, transmissive diffusion also occurs on the exterior surface of the light-pipe, making the location of the effective point-source difficult. Given the high errors associated with photometer measurements, however, the assumption that the effective point-source lies on the diffusing strip seems an adequate approximation for most applications.

Another, easily corrected, error occurred due to the experimental procedure followed and related to the placement of the goniometer. During testing, the goniometer was not centred on the diffusing strip but rather on the centre of the tube. Since the point-source model treats the diffusing strip as the point source, the centre of each strip section should have also been the centre of the spherical coordinate system used in measurements. This error should be corrected when subsequent illuminance measurements are taken. Finally, the point-source model could be significantly improved by breaking the light-pipe into smaller sections. The 20 cm sections used in the preceding tests have dimensions just barely within scope of the "five times rule," and the error could likely be reduced by 4-5% by increasing the number of sections in the model from twenty to forty.

The Illuminating Engineering Society recommends illumination levels for various occupations within a 10% tolerance range. It is likely that a more refined computer model using smaller sections as point sources and more accurate reflectance values could produce values that agree with measurements to within this 10% range.

5.0 Simplified Mathematical Model

The following mathematical model can be used as a first-order approximation in the calculation of floor-level illuminance values. Rather than modeling the light-pipe as an array of point sources, it models the pipe as a single diffusing strip of uniform exitance. Ideally each section of the light-pipe outputs an equal amount of luminous flux from a thin Lambert strip located at the top of the pipe. A reasonably good approximation to the light-pipe's illuminance at a grid point (x,y) models the pipe as a Lambert strip of width w, length L, and uniform exitance M suspended above the x-axis at height h with the one end at the origin. In this case, the illuminance can be calculated by evaluating the integral[6]

$$E(x,y) = \frac{M}{\pi}\iint \frac{\cos\theta\cos\xi}{D^2}\,dA = \frac{Mw}{\pi}\int_0^L \frac{h^2}{\left[(x-l)^2 + y^2 + h^2\right]^2}\,dl$$

An analytic solution to this integral exists but it is rather long. The Maple V release 4.0 worksheet containing the evaluated integral along with surface and contour plots can be found in Appendix G

The equation above can also be written as E=MC where the configuration factor C is given by

$$C = \frac{1}{\pi}\iint \frac{\cos\theta \cos\xi}{D^2} dA = \frac{w}{\pi}\int_0^L \frac{h^2}{\left[(x-l)^2 + y^2 + h^2\right]^2} dl$$

The configuration factor is a purely geometric quantity that determines how light will be delivered from a diffuse source. The length and height of the diffusing strip can all be measured easily, but the exitance and the strip width are more difficult to find. In fact these numbers themselves aren't needed to find the floor-level illuminance. It is sufficient to find the product of exitance and strip width, and this can be done empirically. If a number of illuminance readings are taken, and their average divided by the ratio of the configuration factor to the strip width at the point where the readings were taken, then this value is a good estimate of the product of exitance and strip width. This need only be done once, as the exitance is independent of where and how the luminaire is positioned. By using this technique on the data from the two room configurations, the product of exitance and width was found to be 3855 lux·m in the first room and 3892 lux·m in the second room. More detailed measurements taken in an open room could produce a constant value of the exitance-strip width product which could be used in all future lighting analysis.

Data calculated using the diffusing strip model was compared to measured illuminance data, and the model was found to be quite good. Correlation between illuminances calculated with this method and measured values was 94.4% and 96.6% for room configurations one and two respectively. Furthermore, as the graphs in Appendix H show, a trendline fit of the data reveal that calculated values match the measured values quite closely and produce fits nearly as good as the computer model fits. It would seem likely that in a larger, emptier room where wall reflectance is less of a factor, the model would be even better.

The diffusing strip model offers a good first-order illuminance calculation method during the early design stages, and can be supplemented later on by more accurate computer models taking room reflections into account.

6.0 Model Limitations

This model does not take into account the effect of the prismatic film in reducing transmission of light rays with high angles of incidence. As a result, it will only work at points where, rays from most of the light-pipe strike the point at incident angles less than 60 degrees. The place where this limitation is most evident is along the axis of the light-pipe where because of the effect of the prismatic film, illuminance falls off more rapidly than along other lines at points equally distant from the pipe. It may be possible to correct this problem if the equation in Appendix G is used as the basis of a computer model in conjunction with information about the characteristics of the prismatic film.

Another problem is that the intensity of the actual light-pipe tends to be higher closer to the sulfur fusion-lamp. Whereas the diffusing strip model predicts a maximum floor level illuminance directly below the centre of the strip, the actual illuminance is at a maximum closer to the fusion lamp end of the light-pipe than the far end. This discrepancy could be corrected in the same way that the point-source model is corrected, by breaking the tube down into sections, and modeling each one as a diffusing strip of constant exitance. The more sections the light-pipe is divided into, the more accurate the analysis will be.

7.0 Conclusions

Using interior lighting design software with a point-source array model of the fusion lamp light-pipe, it is possible to calculate the illuminance from the pipe to within 16% of measured illuminance values. The Illuminating Engineering Society recommends that calculated illuminances be accurate to within 10%. By making a few changes to the point-source model employed in this experiment there seems no reason why this 10% accuracy cannot be achieved. For early design stages, the less accurate, but much simpler diffuse source model can be used to calculate illuminances. It has a reasonable accuracy and will be most effective in large industrial settings with high fixtures and few room objects capable of causing reflections.

8.0 References

[1] Illuminating Engineering Society of North America, IESNA Standard File Format for Electronic Transfer of Photometric Data LM-63-1995

[2] Stetic, John Design of a Prototype PC Controlled Moving-Detector Photometer

[3] Illuminating Engineering Society of North America Lighting Handbook 8th Ed. pp. 384

[4] For further details see Lighting Handbook 8th Ed., pp. 389.

[5] Micro Lumen 7 is an interior illumination modeling program published by Lighting Technologies Inc.

[6] Lighting Handbook, 8th Ed., pp.385

APPENDIX A: SAMPLE IES FILE

Appendix A: .IES Files

This is an example of an .IES file according to the LM-63-1995 standard of the Illuminating Engineers Society of North America. .IES files are to be save in ASCII text format.

TILT=NONE

1 135000 1 7 5 1 1 0 .65625 -.833

1 1 1425

0 15 30 45 60 75 90

0 45 90 135 180

810	692	376	91	38	11	0
810	735	537	295	231	193	59
810	741	623	472	408	322	242
810	671	472	268	145	113	64
810	607	306	81	32	11	0

The TILT=NONE line indicates to the modeling software that values for the present illuminaire follow directly.

The first line of numbers give number of lamps, lumens per lamp, a candela multiplier (if intensity values are in another unit system), number of vertical angles, number of horizontal angles, photometric type (type 3), units type for dimension measurements (feet), and dimensions of the luminaire.

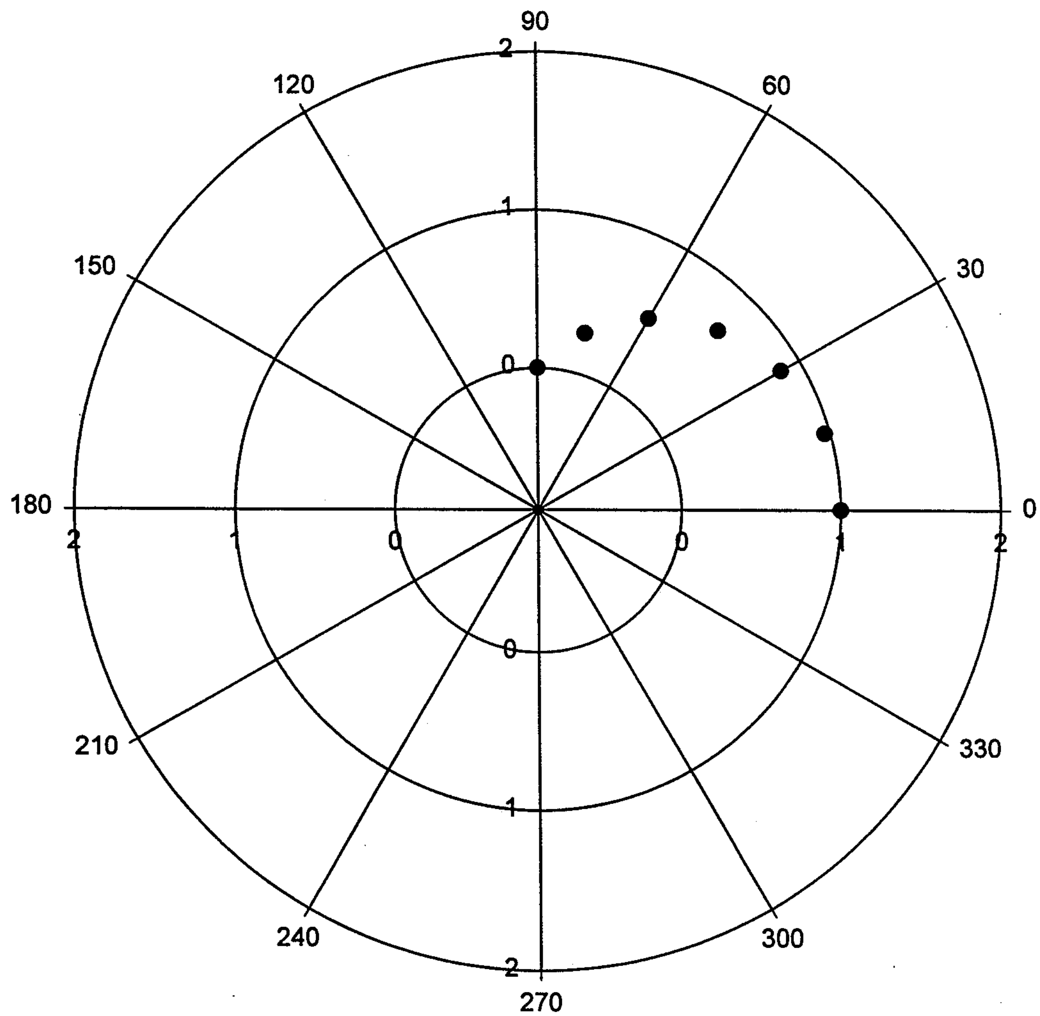
The second line gives a ballast factor of 1 indicating no ballasting, a second number set to one and reserved for future use by the IES and the power input into the light in watts.

After this is a list of vertical angles, then horizontal angles at which intensity was determined. The last five lines each contain seven numbers corresponding to intensity at each of the seven vertical angles for each of the five horizontal angles. The intensity measurements are in foot-candles.

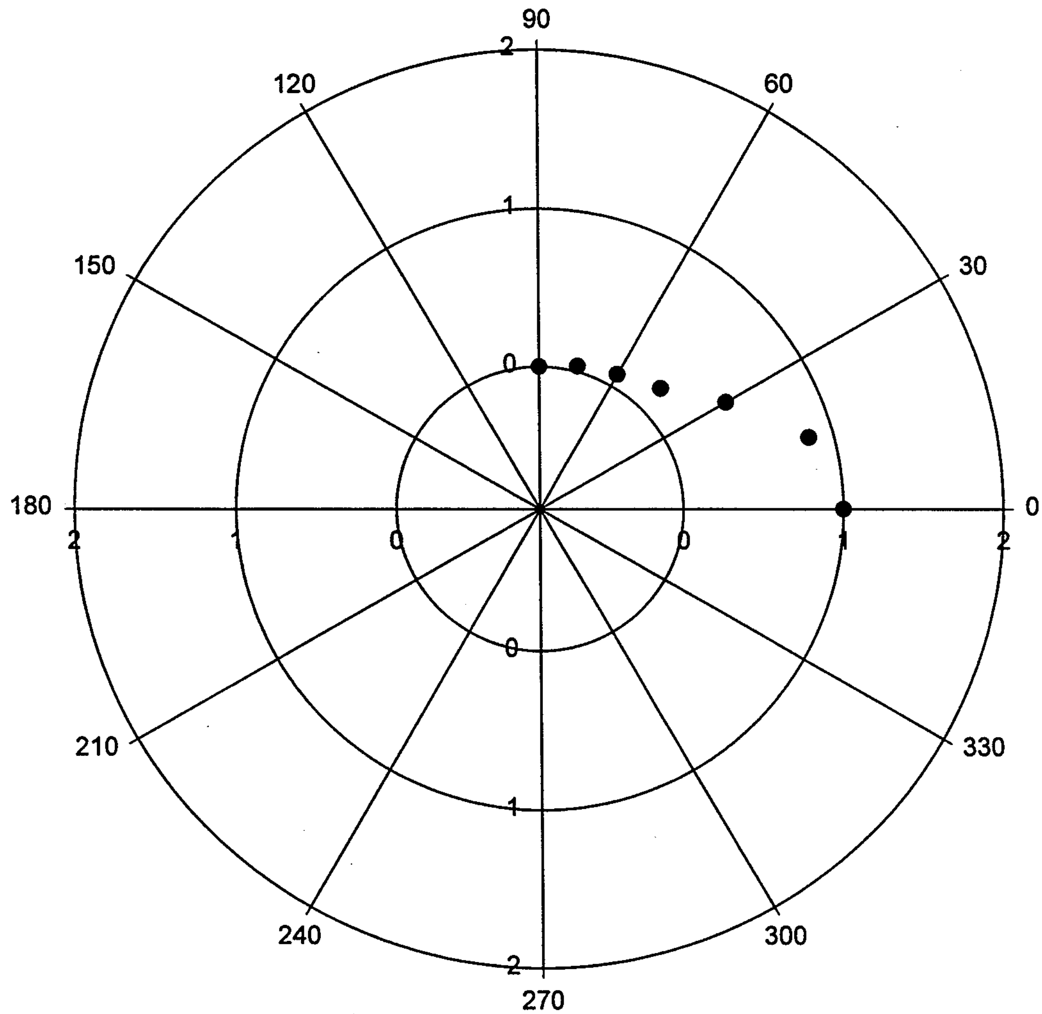
APPENDIX B: RADIAL LIGHT DISTRIBUTIONS

Appendix B: Polar plots of Lambert source and light pipe point sources

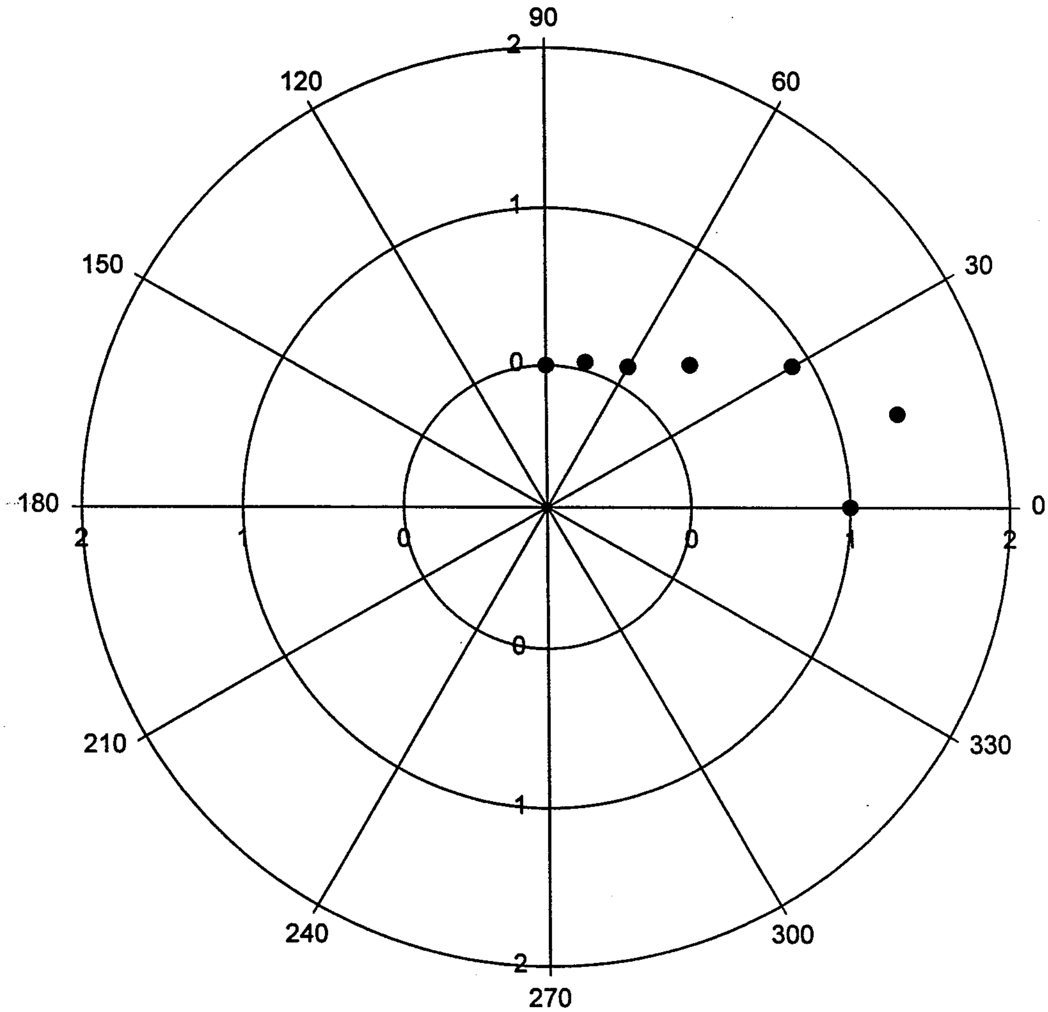
Ideal Lambert Diffuse Source



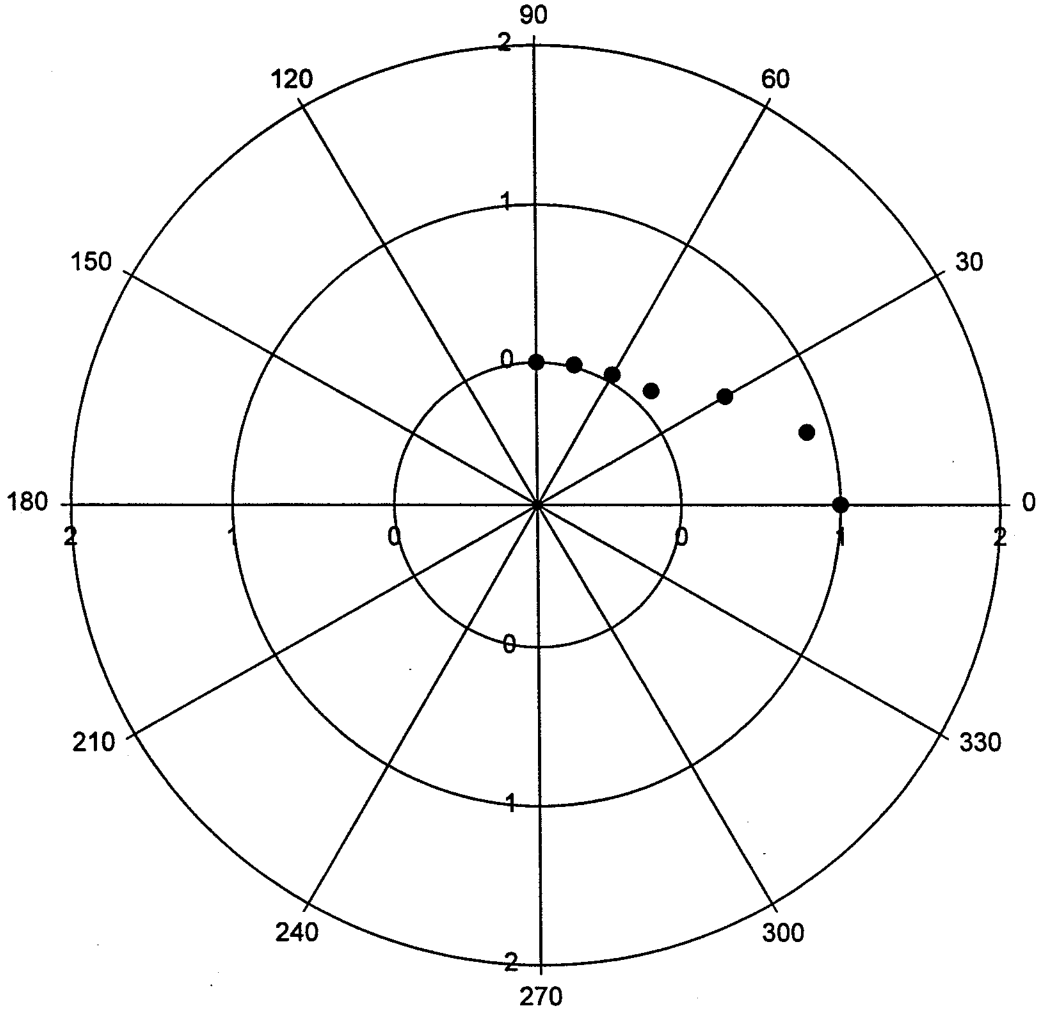
Point Source Close to Centre of Light Pipe



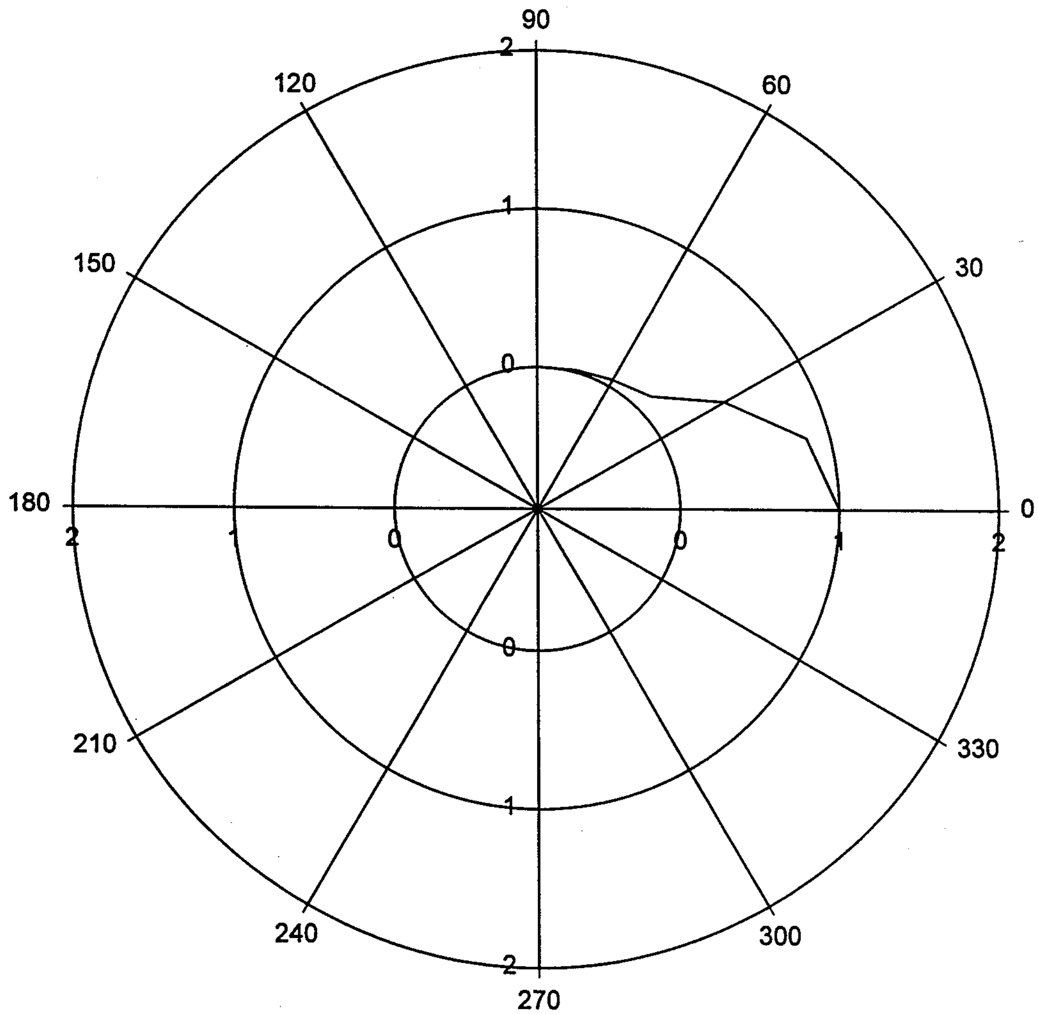
Point Source Furthest from Fusion Lamp



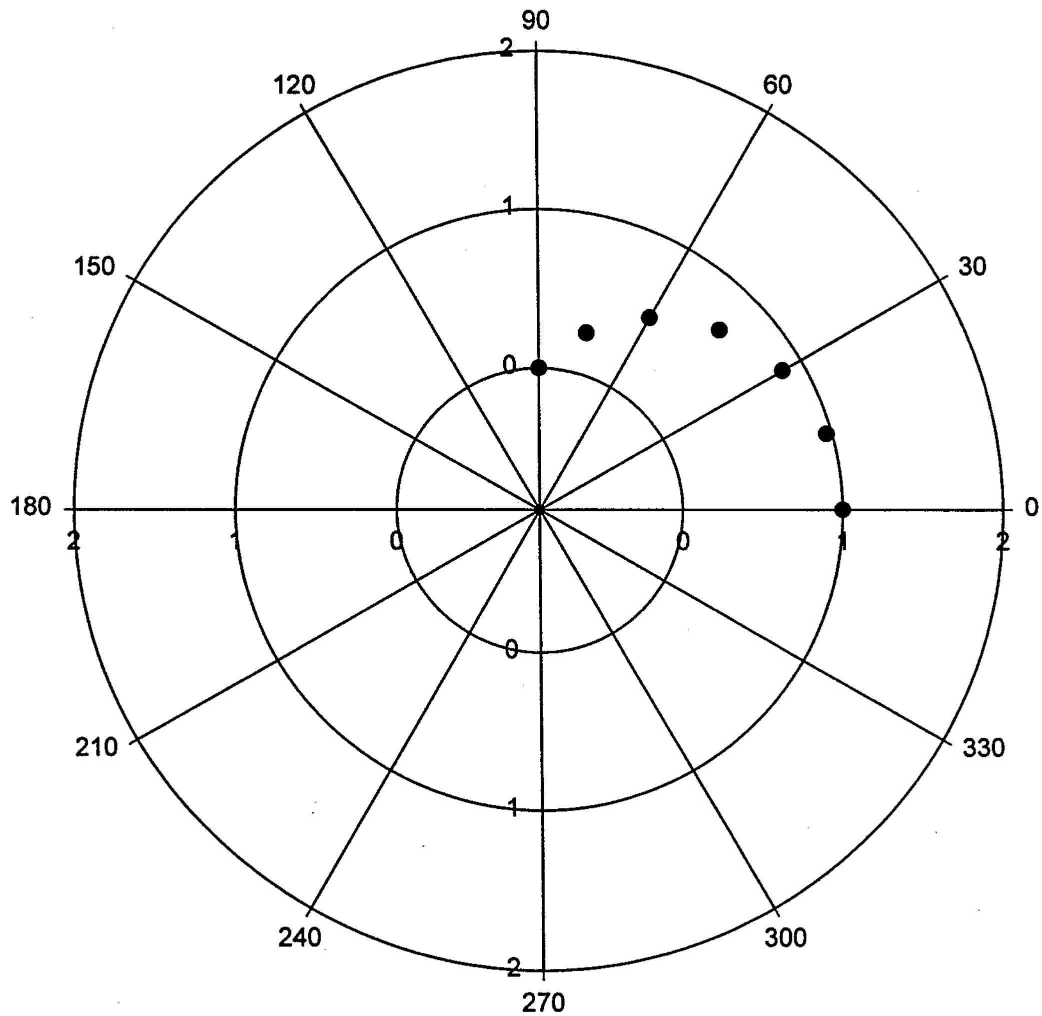
Point Source Closest to Fusion Lamp



Point Source Close to Fusion Lamp

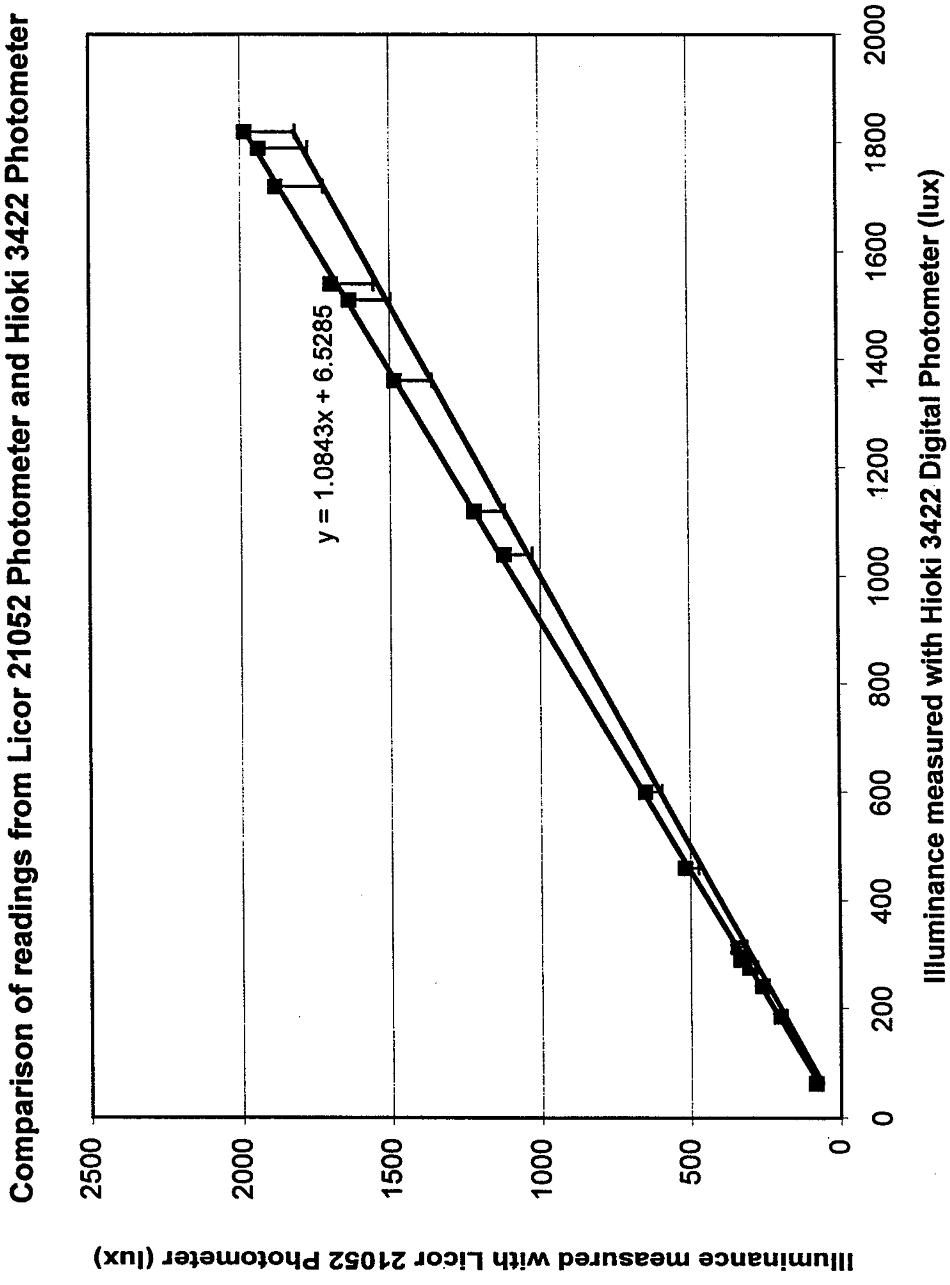


Ideal Lambert Diffuse Emitter



APPENDIX C: PHOTOMETER CALIBRATION PLOT

Appendix C

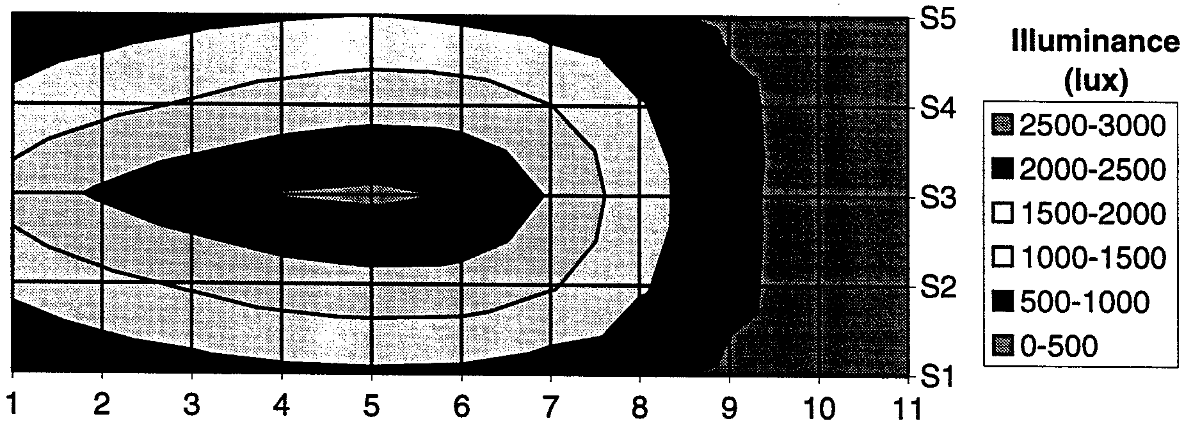


Comparison of readings from Licor 21052 Photometer and Hioki 3422 Photometer
y = 1.0843x + 6.5285
Illuminance measured with Licor 21052 Photometer (lux)
Illuminance measured with Hioki 3422 Digital Photometer (lux)
0
500
1000
1500
2000
2500
0
200
400
600
800
1000
1200
1400
1600
1800
2000

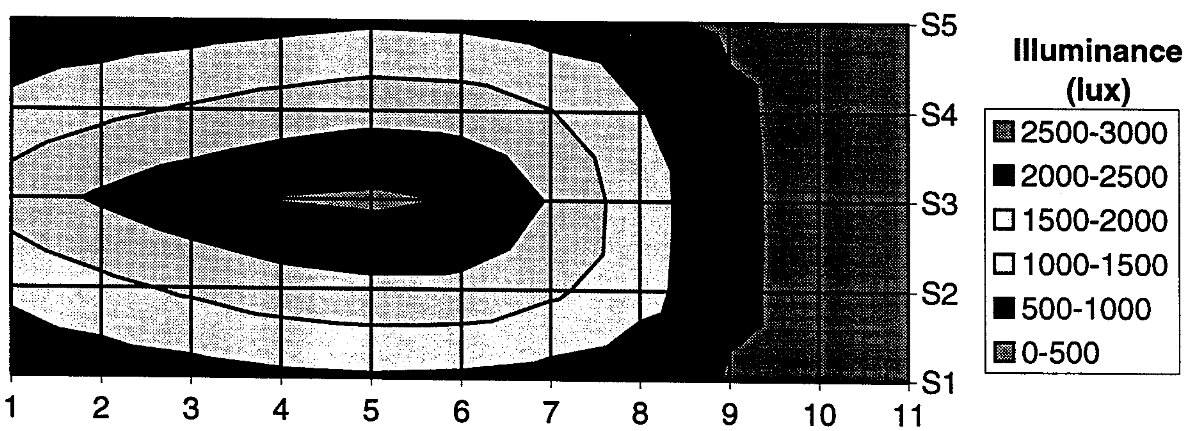
APPENDIX D: CONTOUR AND SURFACE PLOTS OF FLOOR-LEVEL ILLUMINANCES

Appendix D

Contour Plot of Floor-level Illuminance in Second Workshop Configuration

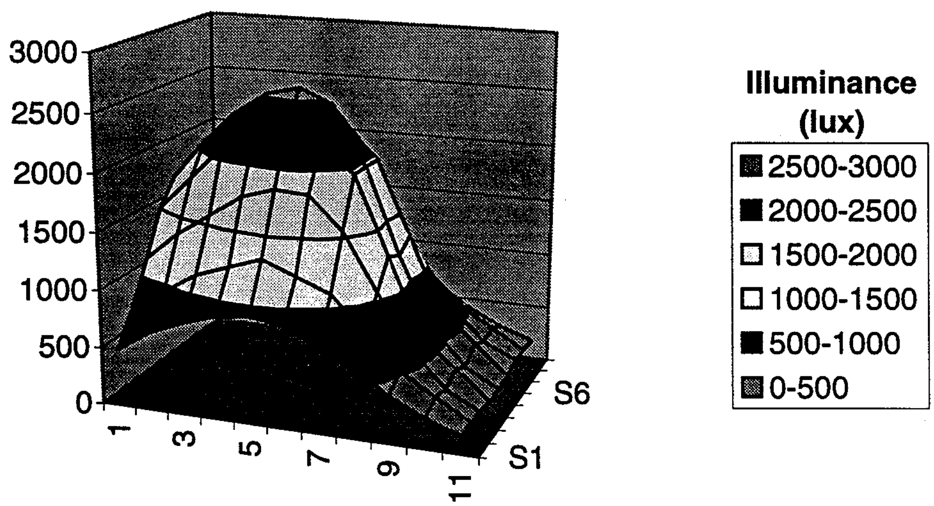


Contour Plot of Floor-level Illuminance in First Workshop Configuration

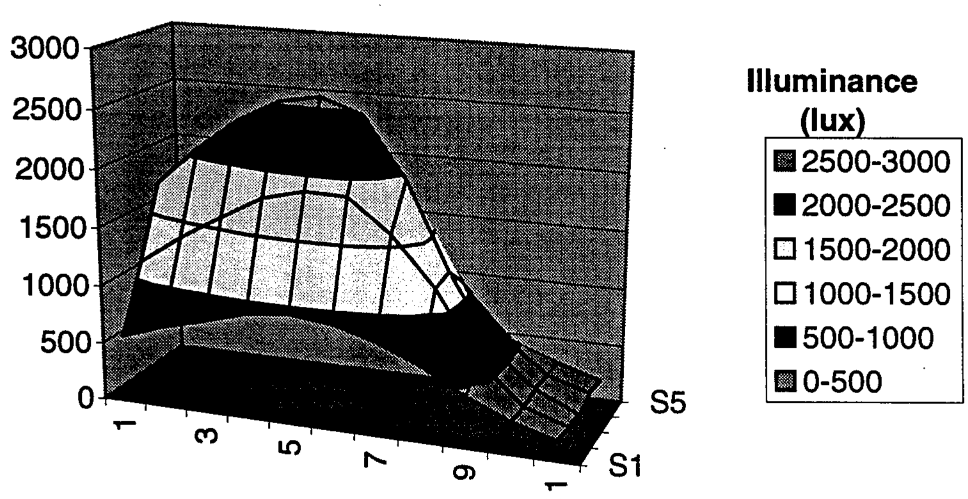


Appendix D

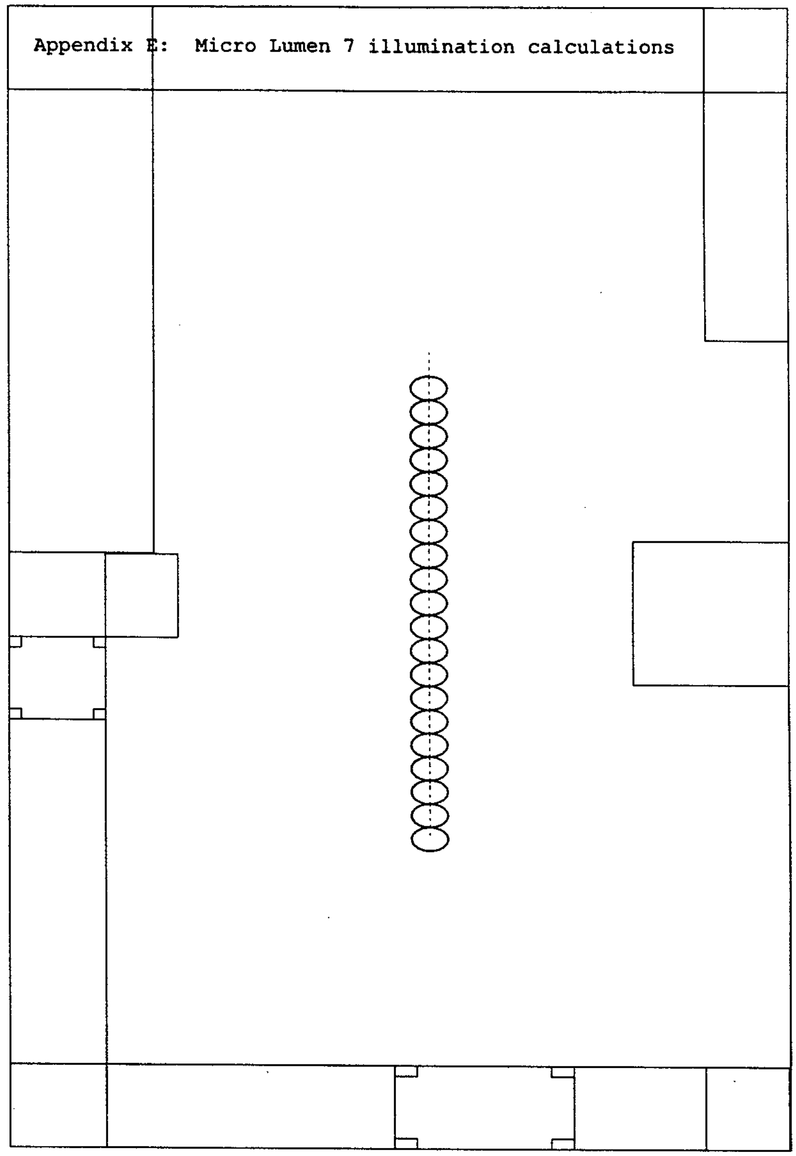
Surface Plot of Floor-level Illuminance in Second Workshop Configuration



Surface Plot of Floor-level Illuminance in First Workshop Configuration



APPENDIX E: MICRO LUMEN 7 CAD DRAWINGS AND ILLUMINATION RESULTS

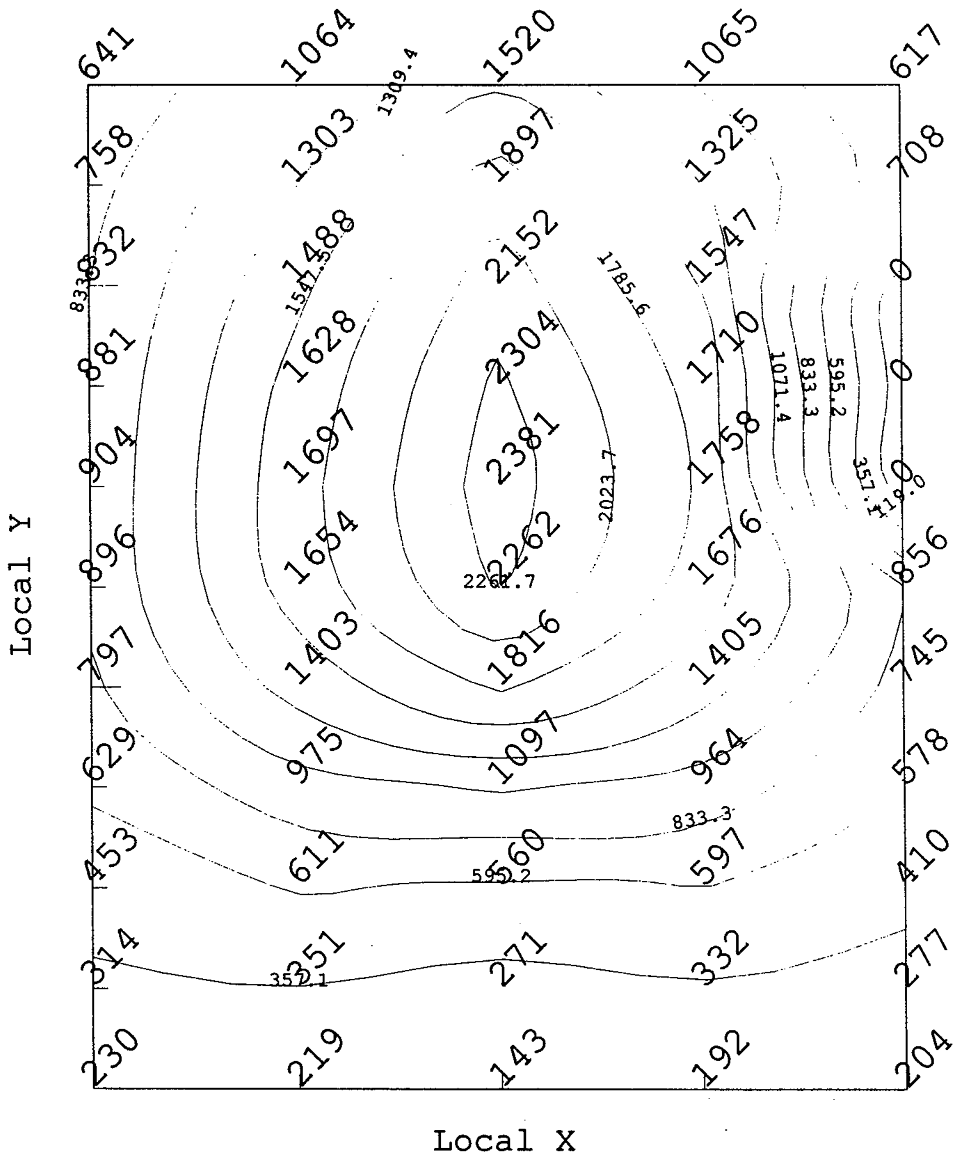


First Laboratory Configuration Plan View

Scale: Scale to Fit

Queen's University

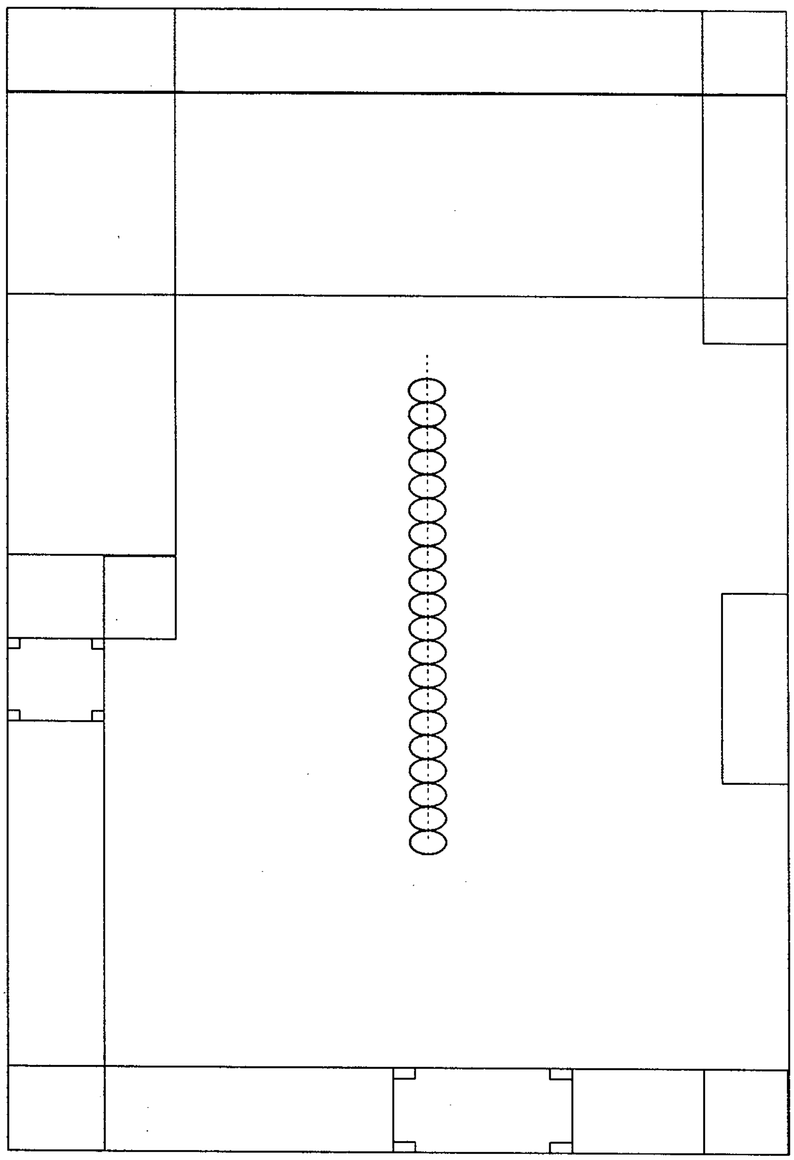
7/15/99 Prepared for:



First Laboratory Configuration, Horizontal Grid, Horizontal Illuminance

Scale: Scale to Fit

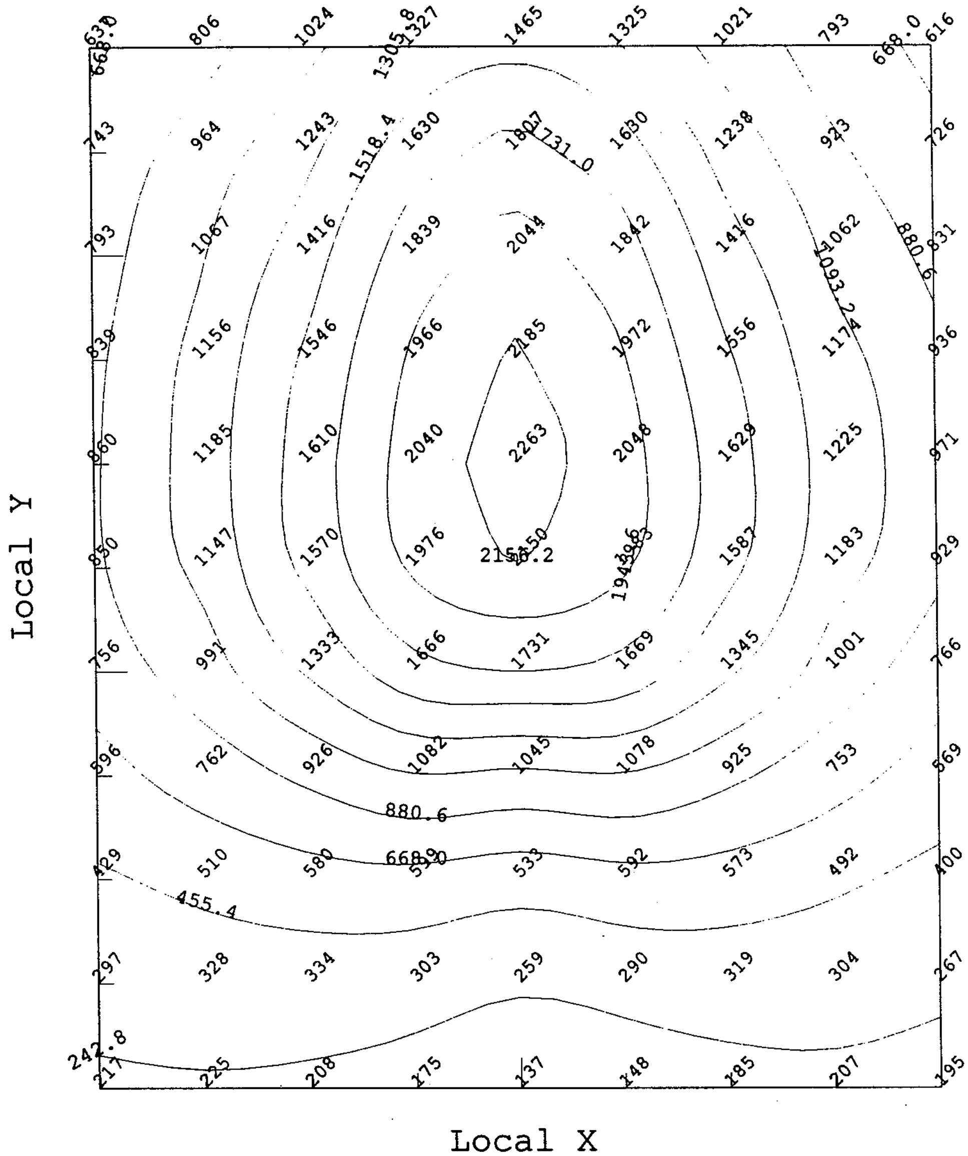
LM	Queen's University	
7/14/99	Prepared for:	



Second Workshop Configuration Plan View

Scale: Scale to Fit

7 LM		
	Queen's University	
7/14/99	Prepared for:	



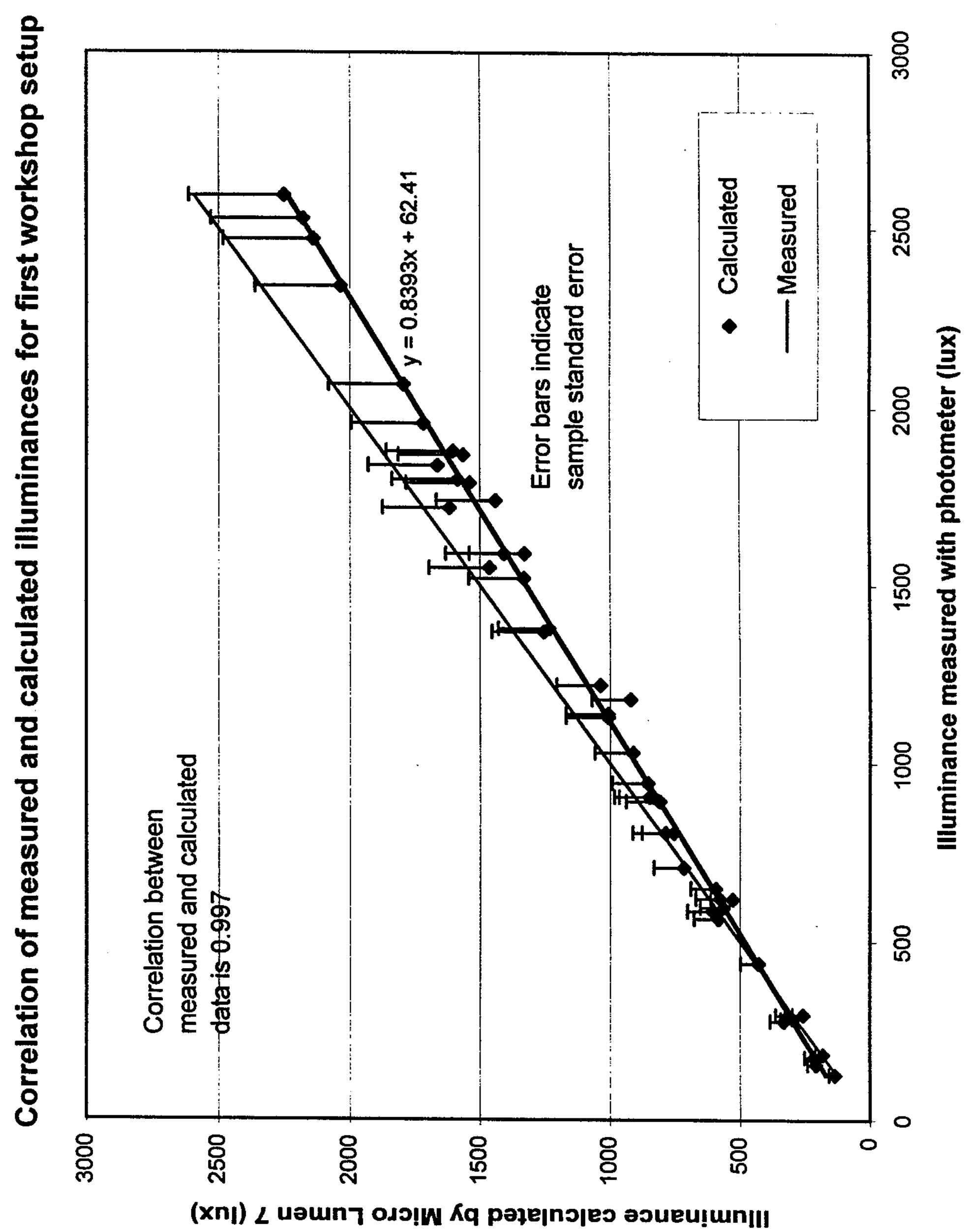
Second Workshop Configuration, Horizontal Grid, Horizontal Illuminance

Scale: Scale to Fit

LM	Queen's University	
7/14/99	Prepared for:	

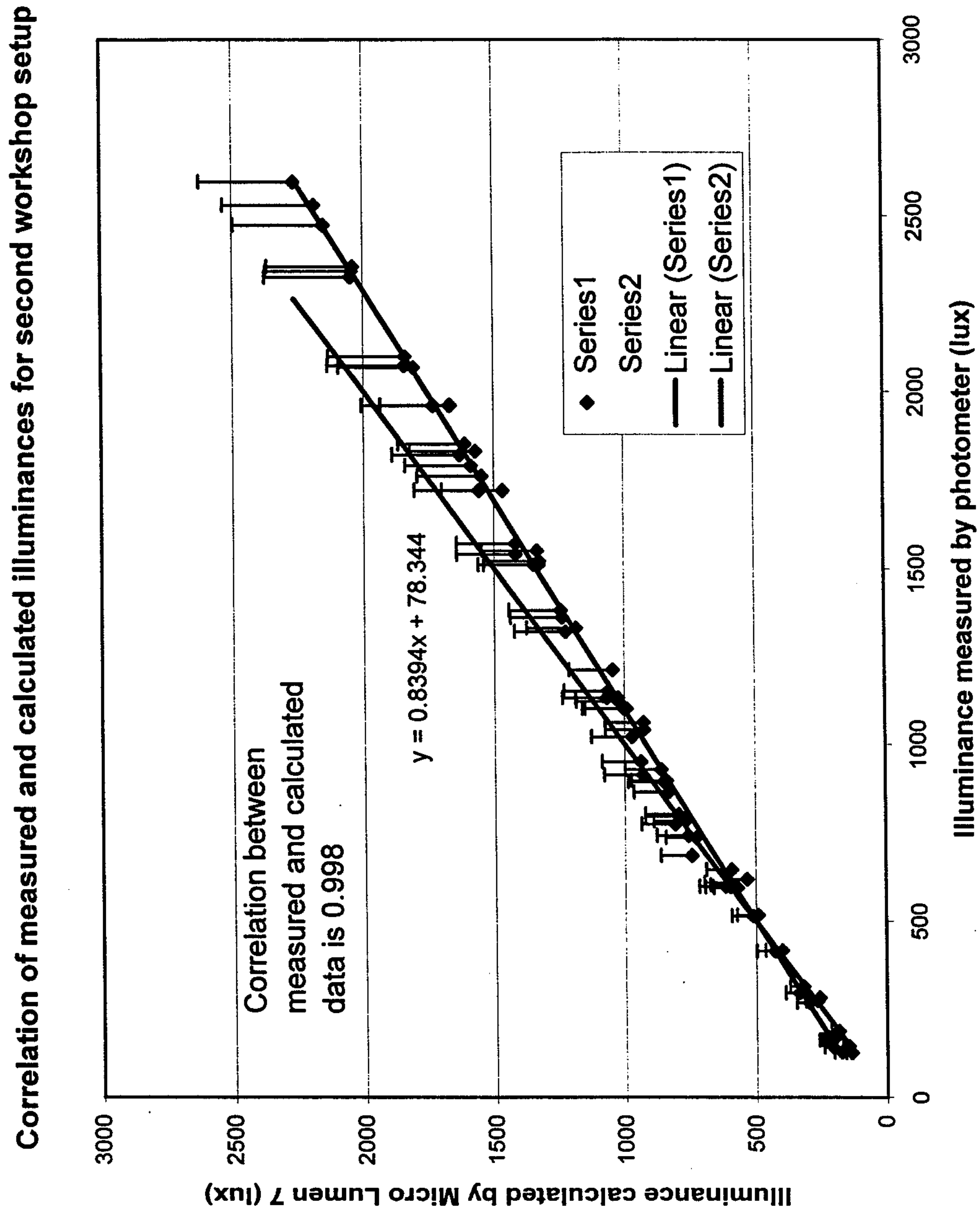
APPENDIX F: CORRELATION PLOTS OF MEASURED AND CALCULATED ILLUMINANCE

Appendix F



Correlation of measured and calculated illuminances for first workshop setup
Illuminance calculated by Micro Lumen 7 (lux)
Illuminance measured with photometer (lux)
0
500
1000
1500
2000
2500
3000
Correlation between measured and calculated data is 0.997
y = 0.8393x + 62.41
Error bars indicate sample standard error
Calculated
Measured

Appendix F



Correlation of measured and calculated illuminances for second workshop setup
Correlation between measured and calculated data is 0.998
y = 0.8394x + 78.344
Series1
Series2
Linear (Series1)
Linear (Series2)
Illuminance calculated by Micro Lumen 7 (lux)
Illuminance measured by photometer (lux)
0
500
1000
1500
2000
2500
3000

APPENDIX G: MAPLE WORKSHEET OF SIMPLIFIED MATHEMATICAL MODEL

Light Pipe Modeling Project
Appendix G
Diffusing strip model of light pipe

```
>
> f:=eval(int((h^2/((x-l)^2+y^2+h^2)^2),l=0..L));
```

$$f := -\frac{1}{2} h^2 \left(-L\,(y^2+h^2)^{3/2} + x\,(y^2+h^2)^{3/2} + \%2\,x^2 y^2 + h^2\,\%2\,x^2 - 2\,\%2\,x\,L\,y^2 - 2\,h^2\,\%2\,x\,L + 2\,h^2\,\%2\,y^2 + h^4\,\%2 + \%2\,L^2 y^2 + h^2\,\%2\,L^2 + \%2\,y^4\right) \Big/ \left((y^2+h^2)^{5/2}\,(x^2 - 2\,x\,L + h^2 + L^2 + y^2)\right) + \frac{1}{2}\,\frac{h^2\,(x\,(y^2+h^2)^{3/2} + \%1\,x^2 y^2 + h^2\,\%1\,x^2 + \%1\,y^4 + 2\,h^2\,\%1\,y^2 + h^4\,\%1)}{(y^2+h^2)^{5/2}\,(x^2+y^2+h^2)}$$

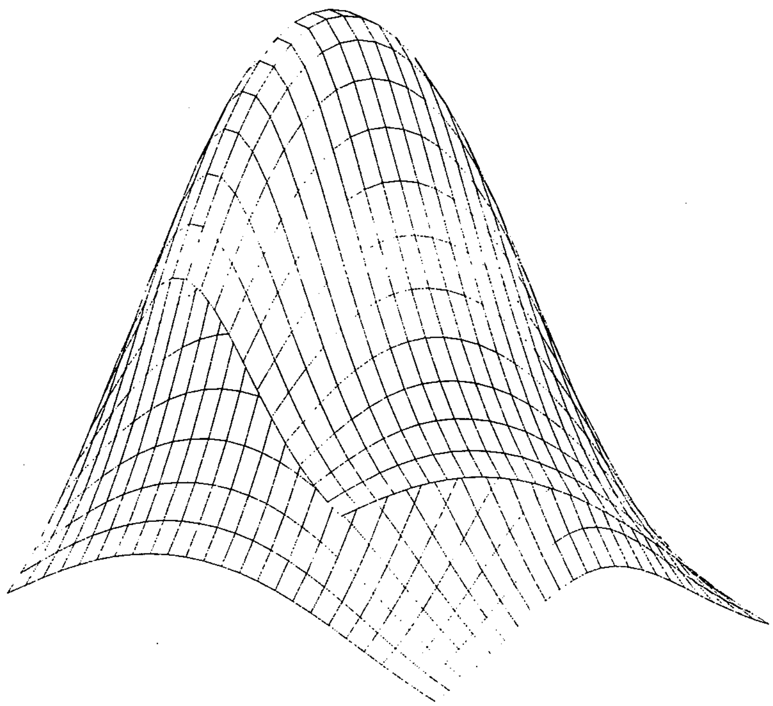
$$\%1 := \arctan\left(\frac{x}{\sqrt{y^2+h^2}}\right)$$

$$\%2 := \arctan\left(\frac{-L+x}{\sqrt{y^2+h^2}}\right)$$

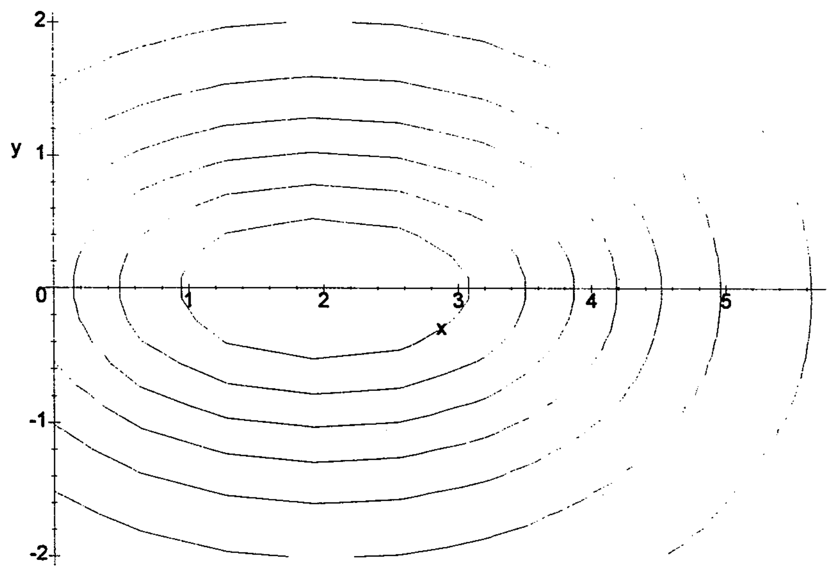
```
> with(plots);
```

[*animate, animate3d, changecoords, complexplot, complexplot3d, conformal, contourplot, contourplot3d, coordplot, coordplot3d, cylinderplot, densityplot, display, display3d, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, odeplot, pareto, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedraplot, replot, rootlocus, semilogplot, setoptions, setoptions3d, spacecurve, sparsematrixplot, sphereplot, surfdata, textplot, textplot3d, tubeplot*]

```
> plot3d(subs({L=4,h=1.81},f),x=0..5,y=-2..2);
```



```
> contourplot(subs({L=4,h=1.81},f),x=0..16,y=-2..2,scaling=constrain
  ed);
```




```
> grid:=proc(A) option remember;
> local i,j;
>
> for i from 3.5 by -.5 to -1.5 do
> for j from -2 by .5 to 2 do
> print(evalf(subs({h=1.99,L=4,x=i,y=j},f)));
>
>
>
> od;
> od;
> end;
```

grid := **proc**(*A*)
local *i*, *j*;
option *remember*;
  for *i* **from** 3.5 **by** -.5 **to** -1.5 **do**
    for *j* **from** -2 **by** .5 **to** 2 **do** print(evalf(subs({ $h = 1.99$, $L = 4$, $x = i$, $y = j$ }, *f*))) **od**

```
    od
end
> grid();
```

.1523649806
.2322777849
.3390546770
.4460230556
.4939633732
.4460230556
.3390546770
.2322777849
.1523649806
.1737421817
.2682254881
.3954982137
.5235678539
.5810569819
.5235678539
.3954982137
.2682254881
.1737421817
.1873157642
.2905241903
.4294287583
.5688184310
.6312410004
.5688184310
.4294287583
.2905241903
.1873157642
.1919282094
.2979742650
.4405202072
.5833162834
.6471849964

.5833162834
.4405202072
.2979742650
.1919282094
.1873157642
.2905241903
.4294287583
.5688184310
.6312410005
.5688184310
.4294287583
.2905241903
.1873157642
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.2682254881
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.5235678539
.5810569819
.5235678539
.3954982137
.2682254881
.1737421817
.1523649806
.2322777849
.3390546770
.4460230556
.4939633732
.4460230556
.3390546770
.2322777849
.1523649806
.1258566195
.1871141878
.2666737405
.3444912210

.3788809835

.3444912210

.2666737405

.1871141878

.1258566195

.09818256258

.1404821585

.1925404225

.2410004515

.2617635730

.2410004515

.1925404225

.1404821585

.09818256258

.07313797132

.09985199011

.1304405514

.1570778599

.1680236709

.1570778599

.1304405514

.09985199011

.07313797132

.05288026263

.06878649974

.0856547923

.0993681284

.1047712206

.0993681284

.0856547923

.06878649974

.05288026263

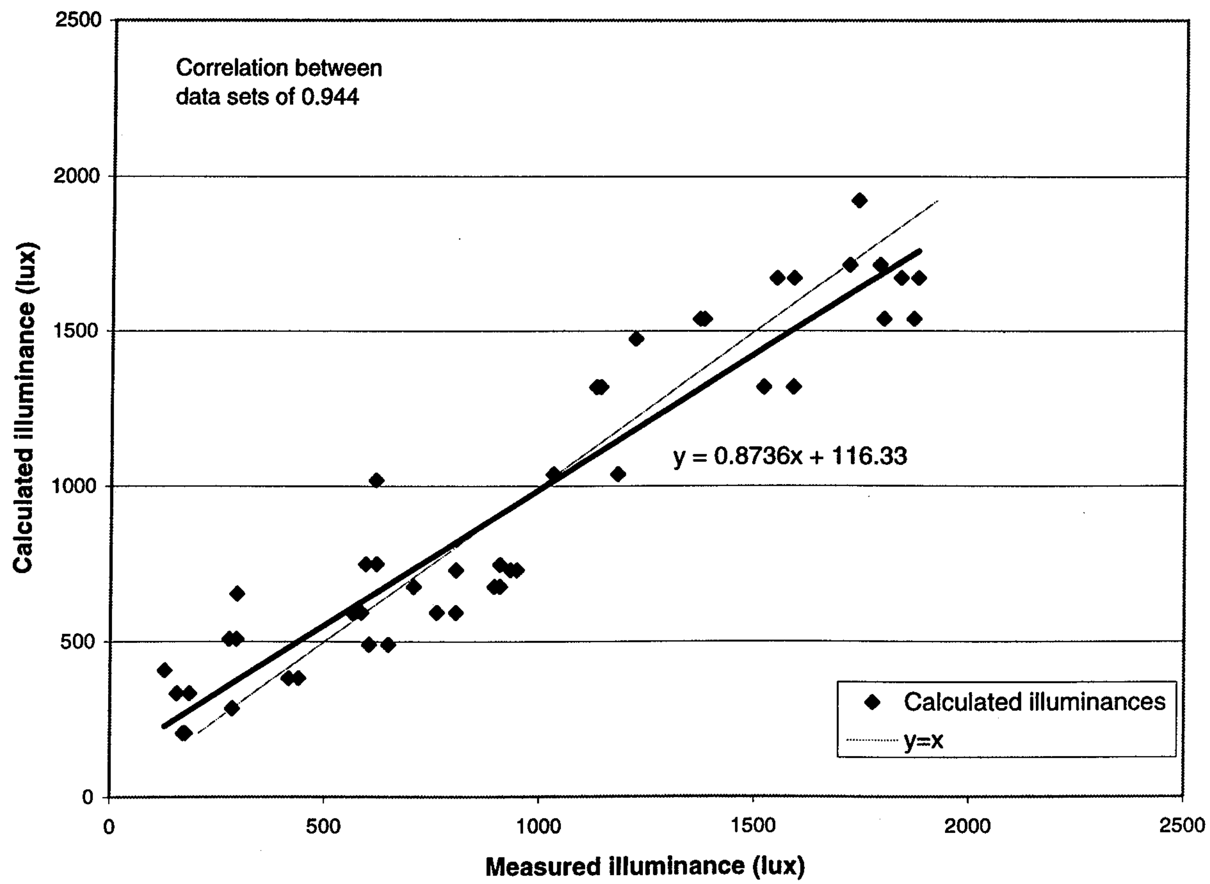
>

>

APPENDIX H: CORRELATION PLOTS OF MEASURED ILLUMINANCE AND ILLUMINANCE CALCULATED WITH THE SIMPLFIED MODEL

Appendix H

Comparison of diffusing strip model and measured values of illuminance for first workshop configuration



Appendix H

Comparison of diffusing strip model and measured values of illuminance for second workshop configuration