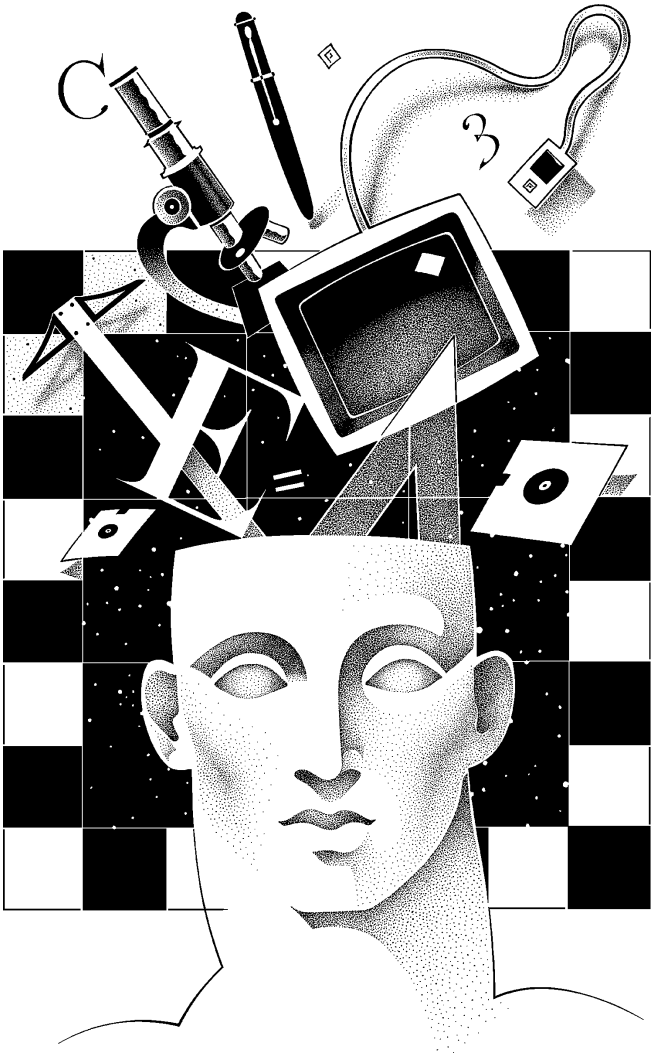
School Achievement Indicators Program

Mathematics Assessment

Criteria and Framework



2001

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MATHEMATICS ASSESSMENT CRITERIA 2001

Introduction

The provincial and territorial ministers of education agreed to undertake the School Achievement Indicators Program (SAIP) in 1989. Objectives for the SAIP adopted by the ministers included the collection of information about the achievement levels of 13- and 16-year-olds in mathematics content and problem solving, reading and writing, and science.

The first stage in the creation of assessment instruments to provide this information involved the development of criteria describing student performance across five broad levels of achievement in each of the four assessment areas. The criteria for the assessment of mathematics content and problem solving are provided in this document. They were originally drafted in the fall of 1990 and have gone through successive revisions in response to feedback across Canada. The criteria were used as the basis for the development of the 1993 and 1997 SAIP mathematics assessment materials. For this third round of SAIP Mathematics, the criteria have been modified to reflect the current trends in mathematics teaching.

The purpose of the mathematics assessment is to describe students' abilities to acquire a variety of mathematics concepts, to carry out a variety of mathematics procedures and to use them to solve problems in both familiar and unfamiliar situations.

This mathematics assessment is divided into two major components, one dealing with the skills related to mathematical content and the other dealing with the skills related to problem solving.

The *strands* chosen to measure students' skills in *mathematical content* are designed to evaluate levels attained on:

- numbers and operations;
- algebra and functions;
- measurement and geometry; and
- data management and statistics.

The *strands* chosen to measure students' skills in *problem solving* are designed to evaluate levels attained on:

- a range of problems and solutions;
- the use of numbers and symbols;
- the ability to reason and to construct proofs;
- providing information and making inferences from databases;
- pursuing evaluation strategies; and
- demonstrating communication skills.

DESCRIPTION OF ASSESSMENT DOMAINS

1. Mathematical Content Criteria

The various strands for each level will include the ability to estimate within a given range, the ability to evaluate a solution and the ability to justify the validity of any results obtained.

Students will be expected to use the mathematical symbolism and terminology implicit in the criteria for each strand and level. As an example, students at level one in the numbers and operations strand will be expected to understand and use the terms sum, difference, product, equal to, greater than and less than and to understand and use the symbols +, –, ×, =, > and <.

NUMBERS AND OPERATIONS

Level One

The student…

1. recognizes the natural number as a property of a group of objects or a set of elements.
2. uses the place value property of the decimal system of numeration for natural numbers smaller than 1 000.
3. writes, in numerical form, numbers smaller than 1 000.
4. orders a set of natural numbers, using the symbols for less than, greater than and equal to.
5.* performs in writing additions or subtractions of natural numbers smaller than 1 000 with or without regrouping.
6.* multiplies two natural numbers smaller than 10.
7. solves real-life problems that require, for their solution, a single addition or subtraction of natural numbers less than 1 000, or a single multiplication of two natural numbers smaller than 10.

Level Two

The student…

1. converts a natural number smaller than 1 000 000 into base ten extended notation and vice versa.
2. orders a set of natural numbers whose elements are each smaller than 1 000 000.
3. demonstrates an understanding of the sense of the four basic arithmetic operations with natural numbers.
4.* performs single calculations or simple sequences of the same operation with natural numbers.
5. solves real-life problems that require several operations with natural numbers less than 1 000 or with decimals with one or two figures beyond the decimal point.
6. estimates relative size of natural numbers less than 1 000 000.
7. rounds a decimal to the nearest whole number.
8. uses concrete examples that illustrate the idea of negative numbers.
9. demonstrates an understanding of the principle of place value in numbers with decimal points.

*Items noted with an asterisk will be influenced by the use of calculators.

10.* performs the four basic arithmetic operations on positive numbers with no more than two figures beyond the decimal point.
11. uses concrete examples that illustrate the concept of fraction.
12. with the aid of concrete materials, performs and represents additions, subtractions and multiplications on fractions having denominators that are simple multiples of each other.
13. determines the intersection and union of two given sets.

Level Three

The student…

1. solves problems related to the numeration system for natural numbers.
2. determines the lowest common multiple and the greatest common factor of a set of natural numbers.
3. orders numbers in the following sets: positive rational numbers (fractional and decimal representations); integers.
4. finds several alternative expressions for the same number, using both fractional and decimal representations (limited to thousandths).
5. performs the four basic arithmetic operations with all natural numbers, all integers and all positive rationals (both fractional and decimal).
6. rounds a number to any given degree that may vary from hundredths to millions.
7. translates and solves real-life problems that require in their solution the use of integers to 1 000 000, positive fractions or positive decimal numbers to thousandths.
8. demonstrates an understanding of the concepts of ratio, proportion and percentage.
9. uses percentages to solve simple problems of buying and selling.
10. translates and solves simple real-life problems involving ratios, proportions and percentages.
11. solves simple combinatorial real-life problems.

Level Four

The student…

1. performs chained calculations on natural numbers, integers and rationals.
2. translates and solves real-life problems that require, in their solution, the use of chained calculations and the full range of rational numbers.
3. demonstrates an understanding of the concept of factor in a numerical context.
4. determines the value of numerical expressions containing integer exponents.
5. demonstrates an understanding of the meaning of irrational numbers.

*Items noted with an asterisk will be influenced by the use of calculators.

6. rounds any rational or irrational number to any given degree.
7. uses scientific notation to write very small and very large numbers.
8. describes the relationships between subsets of the real number system.
9. solves complex combinatorial real-life problems.

Level Five

The student…

1. illustrates the use of irrational numbers through examples such as right-angled triangles.
2. uses expressions and performs calculations containing irrational numbers.
3. performs calculations on numbers expressed in scientific notation.
4. translates and solves real-life problems requiring the use of the full range of the real number system.

ALGEBRA AND FUNCTIONS

Level One

The student…

1. establishes classifications, patterns and relations from concrete materials and representations.
2. starting from concrete materials and representations, draws patterns in situations related to his/her domains of interest.
3. translates and solves single-step problems that are expressed in the form of open sentences and that have solutions in the form of natural numbers.
4. when presented with pictorial material, uses diagrams to represent and describe simple relations in areas associated with his/her domains of interest.

Level Two

The student…

1. establishes and uses classifications, patterns and relations in real-life situations as well as in sets of numbers or sets of operations.
2. translates and solves two-step problems that are presented in the form of open sentences and that have solutions that are in the form of natural numbers or of decimals limited to hundredths.
3. plots points with whole number coordinates either on number lines or in the first quadrant of Cartesian grids.

Level Three

The student…

1. demonstrates an understanding of the concepts of variable, algebraic expression and algebraic equation.

2. calculates the numerical value of a first-degree or second-degree algebraic expression using integers for the replacement of the variables.
3. carries out the four basic arithmetic operations with monomial algebraic expressions that have integers as coefficients.
4. translates and solves problems involving familiar formulas or simple first-degree equations in one real variable.
5. chooses an appropriate continuation of a simple sequence.
6. describes relations using other representations such as tree, arrow diagrams and networks.
7. identifies and plots points in all four quadrants of the Cartesian grid.

Level Four

The student…

1. demonstrates an understanding of the concept of factor in an algebraic context.
2. carries out the four basic arithmetic operations with algebraic expressions containing several terms and rational number coefficients.
3. calculates the numerical value of an algebraic expression using rational number substitutions for the replacement of the variables.
4. translates and solves real-life problems that reduce either to a first-degree algebraic expression (equation or inequation) in one real variable or to a first-degree equation system in two real variables.
5. graphs first-degree function statements in one real variable and their solution sets, as well as first-degree equation systems, in two real variables and their solution sets.
6. finds the continuation of a simple sequence.
7. demonstrates an understanding of concepts of relation and function.
8. produces appropriate tables of values for sequences and constant or linear functions.
9. translates between list, table, rule and equation forms for constant and linear functions.
10. translates and solves real-life problems that reduce to a constant or linear function.

Level Five

The student…

1. uses the laws of exponents to evaluate algebraic expressions.
2. factorizes algebraic expressions, using common factors, grouping in pairs and trinomial factor methods.
3. graphs first-degree inequation systems in two real variables and their solution sets.

4. translates and solves real-life problems that reduce either to a first-degree inequation system in two real variables or to a second-degree algebraic expression in one real variable.
5. uses a translation to generate the graph or equation of a function when the graph or equation of a closely related function is given.
6. draws the Cartesian graph, determines the domain and range and applies the properties of linear functions, quadratic functions, exponential functions, logarithmic functions and the three trigonometric functions of sine, cosine and tangent.
7. translates between table, rule, graph and equation forms for quadratic functions, absolute value function, inverse variation function, square root function and greatest integer function.
8. translates and solves real-life problems that reduce to linear functions, quadratic functions, exponential functions, logarithmic functions and the three trigonometric functions of sine, cosine and tangent.

MEASUREMENT AND GEOMETRY

Level One

The student…

1. estimates and measures lengths and areas in terms of non-conventional units.
2. estimates and measures lengths in metres, decimetres or centimetres.
3. demonstrates an understanding of the concept of magnitude and the concept of area.
4. demonstrates an understanding of the concepts of interior, exterior and boundary.
5. with the aid of concrete materials, derives certain characteristics such as colour, form and size for solid forms and for two-dimensional figures.
6. demonstrates an understanding of the concept of symmetry in simple activities such as folding and the completion of drawings.

Level Two

The student…

1. estimates and measures dimensions of familiar objects, using SI units.
2. establishes relationships between SI length units.
3. demonstrates an understanding of the notion of volume.
4. estimates and calculates areas in both conventional and non-conventional units.
5. uses SI symbols, including the prefixes milli, centi, deci and kilo, in the context of length and area measurements.
6. solves simple real-life problems using SI units of length, area and time.

7. demonstrates an understanding of the concepts of angle and measure of an angle.
8. develops and applies problem-solving strategies related to spatial relations.
9. knows and uses characteristics of various solid forms for the purpose of classifying the solid forms.
10. determines, starting from concrete materials, certain characteristics of solid forms and of the two-dimensional figures that form their boundaries.
11. constructs solid forms from simpler forms.
12. describes, draws and classifies polygons and polyhedra according to certain of their properties.
13. describes and performs single geometric transformations of translation, rotation or reflection.

Level Three

The student…

1. using concrete examples, demonstrates an understanding of the concept of the measure of an angle.
2. solves real-life problems relating to angle measure, segment length, area, volume, mass and time.
3. reproduces plane figures by using the repetition of one of the plane geometric transformations of translation, rotation, reflection or similarity.
4. constructs various plane geometric figures.
5. identifies and uses various characteristics relating to triangles, quadrilaterals and circles.
6. calculates area, circumference and diameter of a circle.
7. establishes relationships between SI units (including squared and cubic units).

Level Four

The student…

1. solves problems derived from real-life situations that use the characteristics of solid forms.
2. finds the unknown elements in right-angled triangles and isosceles triangles using scale drawing or calculation methods, provided that the student is given the option of which method he/she can use.
3. solves real-life problems using the notion and units of capacity.
4. solves problems derived from real-life situations involving area, circumference and diameter of a circle.
5. reproduces plane figures using ordered sequences of the following plane transformations: translation, reflection, rotation and similarity.
6. demonstrates an understanding of the concepts of isometry (congruence) and similitude (similarity).
7. solves real-life problems using the concepts of isometry and similitude in triangles and polygons.

Level Five
The student…

1. calculates arc and chord lengths, sector and segment areas, associated with a circle.
2. solves real-life problems involving length and angle relationships within the circle or the right-angled triangle.
3. finds unknown elements in general triangles using scale drawing or calculation methods, provided that the student is given the option of the method that he/she can use.
4. constructs a rigorous proof (not necessarily in two-column form) by stating properties, theorems, or corollaries involved in the solution.

DATA MANAGEMENT AND STATISTICS

Level One
The student…

1. extracts data stated in tables or lists illustrating familiar situations.
2. answers direct questions on data included in tables or lists that illustrate familiar situations.
3. performs a count or a tally as information is gathered.
4. conducts trials of a simple experiment and lists the experimental outcomes (not necessarily all possible outcomes).

Level Two
The student…

1. extracts data stated in diagrams or charts illustrating familiar situations.
2. draws simple conclusions from data stated in diagrams and charts.
3. represents organized sets of data in table or diagram form.
4. estimates and calculates the arithmetic mean of a set of data.
5. answers questions about basic relationships between two or more sets of data.
6. demonstrates a qualitative understanding of probability scales (impossible, improbable, probable, certain, 0–100 %).

Level Three
The student…

1. combines information from data extracted from different diagrams or charts illustrating a single situation to answer a simple question.
2. represents familiar situations in a data table or in various diagrams (pie graphs, bar graphs, stem-and-leaf plots, histograms, etc.).

3. identifies applications of the mean in real life.
4. calculates and demonstrates an understanding of the mean in real-life situations.
5. demonstrates an understanding of the concept of probability in real-life activities such as decision-making.
6. calculates the experimental probability of a result in real-life activities and the results of random experiments.
7. identifies trends and patterns as represented by graphs or diagrams.
8. recognizes types of data (categorical, count, continuous).

Level Four

The student…

1. constructs a data table (such as frequency distribution table) or other suitable diagram from unorganized data.
2. represents and interprets a data distribution by an appropriate graphic.
3. estimates and calculates the median or mode of a set of data.
4. identifies applications of the median and mode in real life.
5. determines the measure of central tendency that is most useful in describing a distribution.
6. calculates the theoretical probability of any single event.
7. calculates the number of possible results of a random experiment by using a tree diagram or other methods of enumerating the sample space.
8. draws and compares frequency distribution graphs.
9. describes sources of potential bias in design and data collection.

Level Five

The student…

1. calculates the weighted average of a data set.
2. estimates and calculates measures of deviation (range, standard deviation, interquartile range) in order to describe the dispersion of data.
3. makes qualitative inferences from data distributions using frequency distributions and the relative placement of the measures of central tendency.
4. determines the probability of a composite event, restricting such problems to mutually exclusive or independent events.
5. describes the assumptions to be considered when drawing a sample.
6. calculates and interprets z-scores and probabilities related to sample data.
7. identifies issues to the generalizability of a data set.
8. identifies mutually exclusive or non-mutually exclusive, independent or dependent events.

Content Component Framework

Domain		*Level 1*	*Level 2*	*Level 3*	*Level 4*	*Level 5*
Numbers and operations	(32 items)	7	6	7	6	6
Algebra and functions	(31 items)	6	6	6	6	7
Measurement and geometry	(31 items)	6	6	6	7	6
Data analysis and statistics	(31 items)	6	7	6	6	6
Totals	(125 items)	25	25	25	25	25

Note: Numbers refer to quantity and not to particular question numbers.

2. Problem-Solving Criteria

For the purpose of this assessment instrument, the following four strands form the basis for the holistic assessment by levels of achievement of problem solving:

- range of problems and solutions
- use of numbers and symbols
- ability to reason and construct proofs
- providing information and making inferences from databases

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Two additional strands within problem solving are:

- pursuing evaluation strategies
- demonstrating communication skills

These strands, evaluation and communication, are more appropriately evaluated through interviews, observations and small-group interactions. This assessment will provide limited information for these strands and only on criteria that can be accomplished by paper-and-pencil assessment.

PROBLEM SOLVING

Level 1

The student can find a single solution to a one-step problem, using obvious sets of procedures and a limited range of whole numbers; use one case to establish a proof.

The student…

1. translates and solves short problems requiring the use of a simple and obvious algorithm.
2. considers that similar problems are different and uses different methods of solution for similar problems.
3. solves problems requiring the use of whole numbers less than 1 000 and solves single-step problems using basic arithmetic operations on these numbers.
4. uses concrete materials and concrete representations to develop single-solution procedures.
5. uses trial-and-error methods to find a single solution.
6. uses a single particular case as verification and considers that as an adequate proof for an assessment.
7. solves problems by answering direct questions requiring a reading of data presented in the form of tables or diagrams.
8. draws elementary conclusions from data tables when guided by a series of direct questions.

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Evaluates a solution when given specific direction as how to approach the evaluation.

The student verbally explains a sense of the problem when asked to do so.

Level 2
The student can make a choice of sets of procedures to find a solution to multi-step problems, using a limited range of whole numbers, and one-step problems, using rational numbers; use more than one particular case to establish a proof; use common vocabulary to present solutions.

The student…

1. translates and solves problems requiring the selecting of an algorithm from algorithms familiar to the student.
2. finds one solution to a problem and provides a solution to a closely related problem if the solution procedure requires only a minor modification.
3. solves problems requiring the use of whole numbers less than 1 000 or positive rational numbers (in decimal form limited to hundredths or in fractional forms with denominators multiples of each other).
4. uses figurative and schematic representations to generate solutions to problems.
5. solves multi-step problems using basic arithmetic operations on whole numbers less than 1 000 and solves single-step problems using a basic arithmetic operation on positive rational numbers.
6. uses random trial-and-error methods to generate more than one solution.
7. uses an inductive proof based on an accumulation of particular cases, with these verifications being considered as adequate proof for an assertion.
8. solves problems requiring the extraction of information from data directly stated in table or chart form, or as an organized list.
9. without guidance, draws conclusions from data stated in tables with simple data treatment required to produce the conclusion.

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Confines evaluation to making numerical checks of the final answer.

When told an answer is wrong, will redo the calculation using the same procedure.

Will attempt an alternative methodology for evaluation only if instructed to do so.

The student's solution is not organized and minimal work is done.

Level 3
The student can choose from two sets of procedures to find a solution to multi-step problems, using a limited range of rational numbers; use necessary and sufficient cases to establish a proof; use mathematical vocabulary, although not with great precision, to present solutions.

The student…

1. translates and solves problems requiring the use of at least two familiar algorithms or the application of the algorithm to a new situation.
2. generates a single solution to a problem and succeeds in generating solutions to families of closely related problems.
3. solves multi-step problems requiring the use of simple operations on positive and negative rational numbers (in decimal form to thousandths and most common fractional forms).
4. uses schematic representations and symbolic representations in one real variable to generate solutions.
5. methodically uses trial and error to find several solutions that have clearly established links connecting them.
6. uses a structured inductive form of proof, including in the proof only those examples essential to the justification of the assertion.
7. extracts information from data stated in table, chart, or organized list form, making interpolations if necessary.
8. makes conclusions and predictions that require treatment of a data set.

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Checks problem solutions for errors in numerical computations or for procedural errors.

Extends the scope of the evaluation using estimates, if given appropriate instructions.

The student's work is not always ordered. There are errors in notation and only some steps are explained.

Level 4
The student can adapt one or more sets of procedures to find solutions to multi-step problems, using the full range of rational numbers; construct structured proofs that may lack some detail; use mathematical and common vocabulary correctly, but solutions may lack clarity for the external reader.

The student…

1. translates and solves problems requiring the adapting of one or more than one algorithm in a multi-step problem.
2. recognizes the possibility of developing general solutions for certain types of problems but succeeds in developing them only in simple cases.
3. solves problems requiring the use of the full range of rational numbers, including very large and very small numbers, and can use them in long sequences of chained operations.
4. uses symbolic representations in two real variables to generate solutions to particular problems.
5. uses precise reasoning to find the essential elements of a problem solution.
6. constructs formal structured proofs of general results but the proposed proofs lack some details or some justifications.
7. solves problems requiring the extraction of information, by interpolation if necessary, from data presented in a disorganized fashion; organizes the data as required.
8. draws conclusions requiring data treatment, makes simple inferences and makes basic intuitive predictions from data stated in table or diagram form.

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Checks problem solutions for errors in numerical or algebraic computations, or for procedural errors.

Evaluates by analyzing more than one solution, but shows no clear evidence of having checked assumptions behind the solution.

The student's work is well organized and consistent. Mathematical and language errors do not detract. Most steps are explained.

Level 5
The student can create original sets of procedures to find solutions to multi-step problems, using the full range of real numbers; construct structured proofs that provide full justification of each step; use mathematical and common vocabulary correctly and provide clear and precise solutions.

The student…

1. translates and solves problems requiring the creating or developing of an algorithm that is original for the student and essential for solving the problem.
2. recognizes that some solutions can be generalized and can generate such general solutions to complete families of problems.
3. solves problems requiring the use of the full range of real numbers, both rational and irrational, and solves both single-step and multi-step problems using these numbers.
4. uses symbolic representations in two real variables to generate solutions to families of problems.
5. uses a problem-solving methodology based on deductive reasoning and previous experience to find the essential elements of a problem solution.
6. creates structured proofs based on formal logic that contain all the steps necessary and that provide full justification of each step in the proof.
7. solves problems requiring the structuring of data presented in unorganized form and requiring the extraction of information, by interpolation or extrapolation if necessary.
8. identifies data that are missing and provides links between related data sets in order to draw conclusions and to make simple inferences from the data sets.

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Evaluates the solution by using numerical checks of the solution procedure and by using more than one solution procedure.

Checks validity of assumptions and approximations, using data included in the problem statement and data external to the problem statement; reworks the solution in the light of information gained in the evaluation process.

The student's work is very elegant. It is obvious to all what is occurring.

Problem-Solving Component Framework

Problem	*Level 1*	*Level 2*	*Level 3*	*Level 4*	*Level 5*
The Boxcars (Measurement and Geometry and Algebra)	Part 1	Part 2	Part 3	Part 4	Part 5
Kelly and the Cat (Data Management)	Part 1	Part 2	Part 3	Part 4	Part 5
The Trip to Europe (Data Management and Algebra)	Part 1	Part 2	Part 3	Part 4	Part 5
The Cubes (Algebra)	Part 1	Part 2	Part 3	Part 4	Part 5
The Whole Number Machine (Numbers and Operations and Algebra)	Part 1	Part 2	Part 3	Part 4	Part 5
Nancy's Restaurant (Numbers and Operations and Algebra)	Part 1	Part 2	Part 3	Part 4	Part 5
Total number of questions per level	6	6	6	6	6