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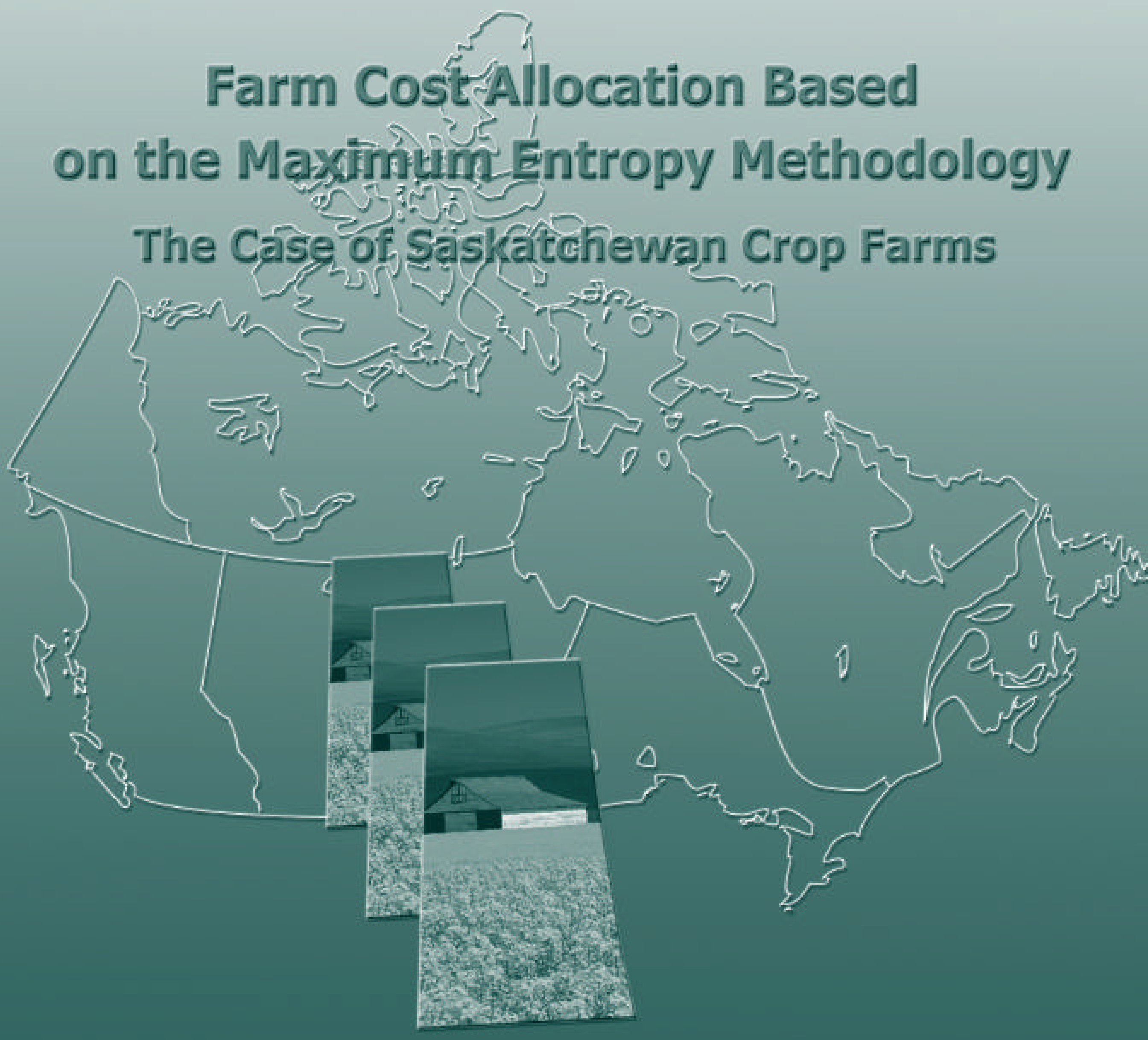
Farm Cost Allocation Based on the Maximum Entropy Methodology

The Case of Saskatchewan Crop Farms



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Foreword

The Government of Canada and the provincial and territorial governments are working with the agriculture and agri-food industry and interested Canadians to develop an architecture for agricultural policy for the 21st century. The objective of the Agricultural Policy Framework (APF) is for Canada to become the world leader in food safety and food quality, innovation and environmentally-responsible production. To contribute to these goals, Agriculture and Agri-Food Canada (AAFC) has an ongoing research program to provide information on the effects of agricultural policy and technology scenarios on the environment and on the economic performance of the agriculture sector.

Included in this work program is a project to improve our farm level data on the cost of production and the farm management practices for economic and environmental analysis. As part of this effort to improve our data, this report evaluates an analytical method, called Maximum Entropy (ME), for its effectiveness in extracting detailed, enterprise-level cost of production information from whole-farm data sets. The ME method has been shown to be a promising and cost-effective option for obtaining these enterprise-level estimates from whole farm data sets already available. Traditionally, direct collection of these estimates has been difficult and costly. Further research on the ME methodology is underway. This study will improve our capacity to carry out analysis in support of the APF by expanding our understanding of the linkages between farm operations and environmental indicators.

Any policy views, whether explicitly stated, inferred or interpreted from the contents of this report do not necessarily reflect the views or policies of AAFC.

Executive summary

Governments have gone to considerable lengths in the past to obtain economic data on the individual enterprises of multi-output farms. For policy purposes, governments need information with respect to the costs per unit of output, measured by so-called input-output coefficients, or (preferably) cost-allocation coefficients. However, collecting this information is costly, as considerable resources have been devoted to apportion the whole-farm accounting data among the individual enterprises. Consequently, there is a need for policy analysts and practitioners to access a methodology that can offer reliable estimates at a significantly lower cost.

The Maximum Entropy (ME) methodology is a valid option for obtaining these enterprise-level estimates from whole-farm data. The ME estimates have the desirable property of being consistent with the available data and known constraints while, at the same time, not requiring a host of assumptions by the analyst. As well, since the estimates of the underlying commodity cost (and hence, production) structures are obtained from data sets already available, such as the Taxfiler Database or the Net Income Stabilization Account (NISA) Database, the results can be used to improve the Canadian Regional Agricultural Model (CRAM) and other farm-level models without undertaking more costly data collection.

The purpose of this study is to demonstrate the relevance of the ME methodology in estimating cost-allocation coefficients and to implement the ME methodology using a data set from a sample of Saskatchewan crop farms. Furthermore, the study assesses the "quality" of the ME methodology by looking at the precision with which the enterprise-level cost structure can be extracted from the whole-farm data. Specifically, the estimation results for a sample of farms are compared to the actual enterprise-level data that are available for the same sample of farms.

Two applications of the ME methodology are conducted. The first application estimates cost allocation coefficients assuming that each cost item is a linear function of several output-specific revenues. The ME methodology is then applied to a complete set of linear relations, linking each cost item to several output variables. In addition, the empirical model is adjusted to take into account several "zero observations" where, for some farms, the sample expenditures on certain input categories, including net operating income, are equal to zero or are negative. The second application includes the first application but also takes into account the particular nature of Saskatchewan agriculture, in which a significant proportion of land is put aside each year due to adverse agro-climatic conditions. Maintaining this so-called "fallowed land" requires certain farm operations, including tillage and weed control. Since these operations have significant costs, the costs need to be considered in the analysis by adjusting the ME methodology accordingly.

Empirical implementation of the ME methodology is conducted using the General Algebraic Modelling System (GAMS) computer program. Cost-allocation coefficients are estimated for five crop enterprises: wheat, other grains, canola, other oilseeds and other crops. From the empirical results, two main findings emerge:

The ME methodology is a valid approach to estimate cost-allocation coefficients, in the sense that the estimated coefficients fall within the range of their observed counterparts.

Some minor qualifications are needed, depending on the category of crops under consideration. The estimated and observed cost-allocation coefficients are quite comparable for "Wheat," "Other Grains," and to a lesser extent "Canola." However, for the two remaining crop categories, "Other Oilseeds" and "Other Crops," some caution is in order when comparing the estimated and observed cost allocations. The discrepancies found are not really surprising given the heterogeneity in those categories. Nevertheless, further investigation may be necessary by, for example, creating more homogenous crop categories.

Given the exploratory nature of the present study, further work is still needed to obtain more efficient and realistic estimates–for example, by using larger samples, exploiting the "richness" of panel data and/or employing more reliable prior information on the support values of the cost-allocation coefficients. Some other practical and methodological problems remain. In particular, one issue that needs further investigation is how to model, in a more satisfactory way, the decision process of fallow land maintenance. It would require the use of sample data for several years to capture the delayed effect of fallow on the returns of crop farmers, or the use of more homogeneous crop farms–for example, crop farms that are located in the same region or that are using similar technologies. Another issue that deserves more emphasis is the necessity that crop and livestock enterprises be described and defined with greater accuracy. Finally, it is necessary to raise the issue of heteroscedasticity, because input structures are likely to vary according to the volume of production on each farm. This issue could be examined in future research using appropriate model specifications.

Section 1: Introduction

In conducting an economic analysis into farm level response to changes in markets, technology or policy, a clear understanding of the production processes and the associated cost structures is required. Unfortunately, this type of detailed data is rarely available, as farmers do not usually keep this type of management or financial records or if they do, it is not available to those conducting the economic analysis. Normally, the source of data that is readily available is whole-farm data, in which the revenue obtained from various enterprises may be specified, but costs for the various inputs are lumped together for all enterprises.

Governments have gone to considerable lengths in the past to obtain economic data on the individual enterprises of multi-output farms. For policy purposes, governments need information with respect to the *costs per unit of output*, measured by so-called *input-output coefficients* or (preferably) *cost-allocation coefficients*. However, collecting this information is very costly, as considerable effort has to be expended to apportion the whole-farm accounting data among the individual enterprises. As a result, there is a need for policy analysts and practitioners to access a methodology that can provide reliable estimates at a significantly lower cost.

The Maximum Entropy (ME) methodology is a valid option for obtaining these enterprise-level estimates from whole-farm data. The ME estimates have the desirable property of being consistent with the available data and known constraints while, at the same time, not requiring a host of assumptions by the analyst. As well, since the estimates of the underlying commodity cost (and hence, production) structures are obtained from data sets already available, such as the Taxfiler Database or the Net Income Stabilization Account (NISA) Database, the results can be used to improve the Canadian Regional Agricultural Model (CRAM) and other farm-level models without undertaking more costly data collection.

The objective of this study is to use data that currently exist within the Research and Analysis Directorate (RAD) of Agriculture and Agri-Food Canada (AAFC) to demonstrate and to test the proposed ME methodology. Furthermore, the study will assess the "quality" of the ME methodology by looking at the precision with which the enterprise-level cost structure can be extracted from whole-farm data. Specifically, the estimation results for a sample of farms will be compared to the actual enterprise-level data that are available for the same sample of farms.

This study describes two applications of the ME methodology to a data set from a sample of Saskatchewan crop farms.[1] The first is a direct application of the ME methodology, developed by Léon et al. (1999) for a group of farms in the Brittany Region of France, where each cost item is assumed to be a linear function of several output-specific revenues. The ME methodology is then applied to a complete set of linear relations, linking each cost item to several output variables. In addition, the empirical model is adjusted to take into account several "zero observations" where for some farms, the sample expenditures on certain input categories, including net operating income, are equal to zero or are negative. The second application includes the first application but also takes into account the particular nature of Saskatchewan agriculture, in which a significant proportion of land is put aside each year due to adverse agro-climatic conditions. Maintaining this so-called "fallowed land" requires certain farm operations, including tillage and weed control. Since these operations have significant costs, the costs need to be considered in the analysis and the ME methodology has to be adjusted accordingly.

The present study is based on 1994 accounting data from a sample of 38 Saskatchewan crop farms, as supplied by the management of the Farm-Level Database System of the Strategic Policy Branch, AAFC. The data are part of the Top Management Model[2] developed in the early 1980s by Richard Schoney at the University of Saskatchewan (Schoney 1991).

All the farms in the sample produce several crops, including grains (wheat, barley, oats and rye), oilseeds (canola, linseed and mustard seed), specialty crops (canary seeds, caraway seeds, lentils, etc.) and forage crops. From a total of 38 farms in the sample, eight farms were excluded from the analysis because they were involved either in single-crop production (wheat) or in non-crop-related activities, such as seed cleaning and custom work. In other words, only multi-crop farms producing grains, oilseeds, specialty crops and forage crops were considered in the analysis. In addition, the farms in the sample are not concentrated in one specific region of Saskatchewan but are rather evenly distributed, thus providing a balanced representation of its agricultural sector.

The remainder of this study is organized into four sections. Section 2 provides a preliminary overview of the Saskatchewan farm. Section 3 presents the ME methodology. Section 4 reports the two applications and we discuss the corresponding results. Section 5 is a summary of the main findings, along with recommendations for further research. The list of references and five appendices on the data and selected crop categories appear at the end of the study.

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1. The initial purpose of this study was to apply the ME methodology to two groups: a group of crop farms in Saskatchewan and a group of livestock farms in Ontario. Eventually however, only the Saskatchewan crop farms were retained, because the sample of Ontario livestock farms was not suitable for the present analysis. Most of the farms in the Ontario sample were characterized by an overly strong specialization toward hogs (no multi-enterprises farms).
2. The Top Management Model was initially designed as a farm management extension tool, which provided highly sophisticated enterprise-level and farm-level financial analysis of the current and projected periods. Later, the model was adapted to serve as a data-collection tool. The Saskatchewan data were collected through the Top Management Workshops, a series of extension workshops held annually from 1987 to 1994. The workshops were conducted by the University of Saskatchewan and received joint funding from AAFC and the Saskatchewan Department of Agriculture and Food.

Section 2: Preliminary data analysis

A preliminary data analysis was necessary for three reasons. It enabled us to organize the data according to specific cost and revenue categories, to correct several data inconsistencies, and to ensure the accounting balance between the revenues and costs for each farm.

In Section 2, we discuss several aspects of the data sample of Saskatchewan farms. We define the various cost and revenue items retained for the ME estimation of cost-allocation coefficients in Section 2.1. Then we present the main characteristics of the selected Saskatchewan crop farms in Section 2.2, followed in Section 2.3 by a discussion of agro-climatic conditions prevailing in Saskatchewan. We review the inconsistencies appearing in the data sample of Saskatchewan crop farms and provide ways (adjustments) adopted to overcome them.

2.1 Aggregate cost and revenue categories

To make the estimation of the cost-allocation coefficients feasible, the original farm data were aggregated according to a limited number of categories (see Box 1). Specifically, five aggregate revenue (output) and nine aggregate cost (input) categories were considered. The selection of these categories was carried out in consultation with experts from AAFC. The selected categories reflect the major characteristics of Saskatchewan farms in terms of crop orientation and production systems.

Each of the aggregated output categories comprises a wide range of crop enterprises, which are listed in Appendix A.

In addition to the nine input categories, the analysis considers "fallow costs" (see Footnote 24 for details on the definition of fallow costs). "Fallow land" was considered here which included not only fallow land, but also the land planted with crops aimed at improving soil fertility. The crops of concern are mainly green manure and green feed. Appendix B contains the figures on net revenues and costs for each farm included in the sample.

Box 1: Overview of revenue and cost categories

Revenues (output)	Costs (input)
Wheat–excluding wheat grown for seed production[a]	Application 1
Other Grains–including (feed and malt) barley, oats and rye[b]	Seeds
Canola	Fertilizers
Other Oilseeds–including flax, sunola and mustard seeds[c]	Pesticides
Other crops[d]	Other direct material inputs
	Fuel, power and grease
	Repairs
	Paid salaries
	Other fixed cash expenses
	Net operating income
	Application 2 also includes fallow costs

a. Spring wheat and durum wheat grown for seed production had to be excluded from the analysis, because these two types require special treatment. Wheat grown for seed production corresponds to crop enterprises, such as “certified and pedigree wheat,” which are viewed as producing specialty crops and hence, are part of “Other crops.”
b. Although viewed as a hybrid grain, triticale was categorized as a specialty crop, and was not included in “Other grains.” This crop plays a minor role in the present analysis, as it is grown only by a small number of farms.
c. Similar to wheat, some oilseeds are grown for seed production. In this case, they are considered a specialty crop and hence, are included in “Other crops.”
d. This category includes all the crops excluded from the other categories. It is made up, mainly, of specialty crops and forage crops.

2.2 Farm characteristics

The “average” farm in the sample earns a gross operating income of $258,000 on 2,186 hectares (see Table 1). Wheat and canola are the two major crops in the sample representing 33 percent and 31 percent of total revenues, and 33 percent and 19 percent of total hectares, respectively. The remaining crops represent about one third of gross farm income. The net operating income of the average farm is about $98,000, whereas total costs represent 62 percent of total gross revenue. A closer look at the various costs reveals that, except for other fixed cash expenses at 16 percent, each represents less than 10 percent of total costs.

The farms in the sample show considerable heterogeneity in acreages, revenues and costs as shown by the statistical indicators presented in the last three columns of Table 1. Specifically, the coefficients of variation are quite large (larger than 100 percent). This finding is confirmed by the magnitudes of the range (minimum-maximum) of the revenue and cost categories. However, the farms show considerable variation in net operating income. While some farms experienced a negative net operating income, the most profitable farms have a net operating income that is four times greater than the average value.

2.3 Summerfallow and crop rotations

The agro-climatic conditions prevailing in Saskatchewan require summerfallow and specific crop rotations. The crop rotations vary according to three soil zones that run through the province: southern brown soil (SBS), central dark brown soil (CDBS) and black soil (BS). Summerfallow plays an integral part of the crop rotation system in the SBS zone and in the drier parts of the CDBS zone.[3] Farmers in the BS zone and parts of the CDBS zone, on the other hand, have more or less discontinued the use of summerfallow. Given that the BS and CDBS zones receive greater precipitation and have higher soil fertility, the adoption of new production techniques, tillage equipment and crop varieties has gradually allowed for continuous cropping.[4] The sample data clearly reflect these characteristics of the province's agriculture and the underlying agro-climatic conditions. Six farms in the sample do not use summerfallow which, for the other farms, represents on average about one fifth of the total acreage.

2.4 Data inconsistencies and appropriate adjustments

Preliminary inspection of the original data set revealed inconsistencies that required appropriate adjustments. These inconsistencies stem partly from the fact that the original data were recorded according to two different methods. Specifically, revenues (and costs) were recorded on either a *cash* basis or an *accrual* basis. The two methods are different only in how cash costs and sales are recorded: the former distinguishes cash costs from farm input and crop inventory changes, while the latter records costs of the inputs effectively used by the farms and crop outputs produced by the farms.

Given the purpose of the present study to estimate *cost-allocation* coefficients associated with the inputs *effectively used* by the Saskatchewan farmers, the accrual cost data approach was adopted. This choice is more in line with the economic notion of costs (and revenues). Also, the database reporting costs and revenues on an accrual basis was organized for each crop enterprise, which allowed for assessing the performance of the ME methodology. By contrast, the data file presenting costs and revenues on a cash basis provides only totals, with no breakdown by categories of crops.

3. Typical rotations in these regions are #1 Fallow-wheat-wheat, #2 Fallow-wheat-feed grain (barley, oats and rye), #3 Fallow-flaxseed or mustard seed-wheat-feed grain, #4 Fallow-wheat-lentils-wheat, and #5 Fallow-wheat-feed grain-forage crop (three–five years). The length of the rotation depends on the agro-climatic conditions of the local area. Dry regions with low soil fertility have short rotations (e.g. #1 and #2) while areas with more moisture and greater fertility have longer rotations (e.g. #3 and #4).
4. Typical rotations in these regions include #1 Wheat-wheat-feed grain, #2 Canola-peas-wheat-wheat, #3 Canola-feed grain-wheat-peas-wheat, #4 Wheat-feed grain-forage crop (for two–four years). Summerfallow may still be used periodically to treat disease, insect and weed problems or during drought years, but is no longer a regular component of the rotation.

Table 1: Saskatchewan farm data

		Mean			C.V.	Minimum		Maximum	
		(ha)	($)	(percent)	(percent)	(ha)	($)	(ha)	($)
Acreages	Fallow land	362		16.6	92.9	0		1,229	
	Wheat	723		33.2	115.9	0		4,640	
	Other grains	201		9.3	107.4	0		805	
	Canola	421		18.9	121.4	0		2,320	
	Other oilseeds	102		4.7	240.1	0		1,300	
	Other crops	377		17.3	84.1	0		981	
	Total	2,186		100.0	82.0	0		10,040	
Revenues	Wheat		85,937	33.3	94.4		0		387,450
	Other grains		26,603	10.3	136.3		0		131,800
	Canola		79,628	30.9	125.7		0		372,938
	Other oilseeds		11,001	4.3	233.0		0		132,600
	Other crops		54,538	21.2	137.3		0		367,590
	Total		257,707	100.0	89.9		65,969		1,025,226
Costs	Seeds		14,727	5.7	86.7		981		59,645
	Fertilizers		22,194	8.6	97.8		0		95,770
	Pesticides		24,346	9.5	116.7		4,325		145,437
	Other direct material inputs		11,584	4.5	70.2		1,450		32,309
	Fuel, power and grease		14,584	5.7	84.4		0		62,670
	Repairs		14,491	5.6	86.2		0		66,200
	Paid salaries		15,759	6.1	173.1		0		138,264
	Other fixed cash expenses		42,034	16.3	82.3		2,510		172,018
	Total		159,719	62.0	82.8		29,940		670,550
Net operating Income			97,988	38.0	116.8		-12,031		406,514

Notes: C.V. = Standard deviation*100/Mean

The last two columns present the minimum and maximum of each item obtained from the data sample of Saskatchewan farms and for this reason all revenue, cost and net operating income items do not add up.

Before applying the ME methodology, one has to ensure that each cost item be measured similarly by using the accrual and cash cost methods. To do so required the reconciliation and harmonization of the two databases (supplying costs on either a cash or an accrual basis, respectively). Some cost categories, which were recorded for some farms as custom work in the cash cost database, were recorded differently in the accrual cost database. This method of recording is certainly valid for expenses associated with seed, fertilizer and pesticide operations. It was straightforward to reconcile all the accrual and cash cost data associated with direct material inputs (i.e. seeds, fertilizers, pesticides, other direct material inputs); fuel, power and grease; and repairs. On the other hand, for paid salaries and other fixed cash expenses, it was impossible to reconcile the accrual and cash cost data. In fact, too many discrepancies existed, and there was no way that a thorough investigation, aimed at reconciling cost data for these two categories, could be conducted. For this reason, these two cost categories were measured by using their cash cost proxies. Finally, the net operating income was obtained residually by taking the difference between total accrued sales, as measured by total net returns in the crop enterprises database and the total cost.

Another inconsistency, which is linked to crop activities, stems from the fact that many farms in the sample are also involved in some non-crop activities, such as seed cleaning, custom work, and non-agricultural activities (e.g. bird habitat conservation). It is difficult to consider these farms, because there is no way to deal with non-crop activities using the ME methodology. As a result, all farms involved in such non-crop activities are excluded from the analysis. Along the same line, some Saskatchewan farms were single-enterprise farms and were therefore also excluded.

Section 3: The Maximum Entropy methodology

A common approach to estimate cost-allocation coefficients[5] from whole-farm accounting data has been to consider a system of linear equations, representing the derived demand for each farm input as a function of several farm outputs. In other words, inputs and outputs, both expressed as costs and revenues, are treated as the dependent and independent variables, respectively.[6]

Given I inputs used by T farms to produce K outputs, the system of linear input-demand equations can be written as follows (see also Box 2 for the adopted notation):

$$x_i^t = \sum_{k=1}^{K} a_{ik}\, y_k^t + u_i^t \quad \text{for } i = 1,2,\ldots,N \;\text{ and }\; t = 1,2,\ldots T \qquad (1)$$

where

x_i^t = the total cost associated with input *i (i = 1, 2,..., I)*, paid by farm *t*,

a_{ik} = the unknown cost-allocation coefficients, which are defined here as the average (i.e. for all farms) expenditure on input *i* required per unit of output value *k*,

y_k^t = the total revenue corresponding to output *k (k = 1, 2,.., K)* produced by farm *t*,

u_i^t = a random disturbance, which is specific to each input and to each farm.

5. This section is taken in part from Léon et al. (1999), where the use of the ME methodology to estimate cost-allocation coefficients is developed at full length.
6. The theoretical foundations of such a linear input demand model rest upon the adoption of a Leontief (fixed proportions) technology or a linear production process framework as described in Ray (1985) and Scandizzo (1990).

Box 2: Notation

Index	Domain	Dimension (applications, see below)
i(I)	inputs - excluding fallow costs (application 1) - including fallow costs (application 2)	 9 10
k(K)	outputs	5
t(T)	farms	30
m(M)	support values for the a_{ik} parameters	11
n(N)	support values for the errors	3

Several "classical" tools can be (and were) used to estimate the a_{ik} parameters of the input-demand system in (1), such as linear regression techniques (LRT), bayesian estimation techniques (BES) and linear programming (LP). However, the use of these techniques creates various practical problems. For example, it is a well-known result that the application of LRT may lead to a_{ik} estimates that are *negative*. Also, due to the overall accounting constraint (which equates total costs to total revenues), the disturbance terms of the various input-demand equations are not independent from each other. As a result, the system of input demand equations is singular, which further invalidates the use of the LRT technique (Bewley 1986).[7]

To ensure the non-negativity of the estimated production coefficients, one may consider using constrained estimation procedures, such as BES or inequality restricted least squares methods (see, for instance, Moxey and Tiffin 1994). These alternative methods, however, are very cumbersome to implement and they do not incorporate the overall accounting balance because all the equations in the system are treated separately.

The approach adopted here implies the reparameterization of the linear statistical model in (1) for the ME methodology, so that all the non-negativity and adding-up constraints are incorporated into the estimation procedure.[8]

To do so, Section 3.1 gives the definition of the entropy measure used to estimate the statistical input-demand model in (1). Then the standard ME and ME-tobit formulations of the cost-allocation problem are presented in Sections 3.2 and 3.3, respectively. Finally, Section 3.4 presents and discusses the statistical properties of the ME and ME-tobit systems of equations.

7. System-of-equations estimation procedures, such as the iterative seemingly-unrelated estimation technique, could have been used to overcome this singularity problem. However, such an estimation procedure does not prevent the possibility that a_{ik} estimates can be negative.
8. The ME methodology is advocated when we are facing "ill-posed" problems in the sense of insufficient sample information as described by Golan et al. (1996b). More specifically, such a situation arises when the number of parameters exceeds the number of observations. We do not experience this problem in the case of our sample of Saskatchewan crop farms.

3.1 Shannon's entropy measure

Given n data points, the entropy measure, introduced by Shannon (1948), is expressed in terms of unobserved probabilities p = [p_1, p_2, ..., p_n]′ as

$$H(p) = -\sum_{j=1}^{n} p_j \ln(p_j) \tag{2}$$

where $p_j \ln p_j = 0$ for $p_j = 0$, and $H(p)$ reaches a maximum when $p_1 = p_2 = ... = p_n = 1/n$; that is, when the probabilities are uniform. In other words, maximizing the entropy measure $H(p)$ amounts to choosing the probability vector p that is closest to the uniform distribution and yet consistent with what we know, that is, with the available data and relevant constraints (Jaynes 1957a, b).[9]

3.2 Standard ME formulation of the cost-allocation problem

Expression (1) can be treated either as a single-equation linear statistical model or as a system-of-equations statistical model.

Single-equation linear statistical model

Given the above entropy measure (2), the estimation of the parameters a_{ik} and the error process terms u_i^t of the system of input-demand equations in (1) can be formulated as the problem of finding the expected value of a probability distribution. This value is defined over a set of known and discrete "support values" within pre-specified intervals (Golan et al. 1996b). Thus, for each input i we get the following equations:

$$a_{ik} = \sum_{m=1}^{M} z_{ik}^m \; p_{ik}^m = z_{ik}' \, p_{ik}, \quad \text{for } i = 1,2,...I \text{ and } k = 1,2,...,K \tag{3}$$

and

$$u_i^t = \sum_{n=1}^{N} v_{it}^n \; w_{it}^n = v_{it}' \, w_{it}, \quad \text{for } i = 1,2,...I \text{ and } t = 1,2,...,T \tag{4}$$

where

z_{ik} = the support vectors (of dimension M) for the K production coefficients a_{ik} associated with input i,

v_{it} = the support vectors (of dimension N) for the T error terms u_i^t associated with input i

$p_{ik}(w_{it})$ = the corresponding unknown probability vectors for a_{ik} [u_i^t].

9. It can easily be shown that the implied probability distribution is the one that "has the lowest information content" or that "can be realized in the greatest number of ways" consistent with what we know.

The elements of the support vectors are centred about the likely value of the parameters and noise components to be recovered. For our purposes, it suffices to consider *common* sets of support values for the unknown production coefficients and error terms, respectively; that is, $z_{ik} = z = [z^1, z^2, ..., z^M]$, for all i, k, and $v_{it} = v = [v^1, v^2, ..., v^N]$ for all i, t. Doing so simplifies our notation considerably, so that the expressions (3) and (4) can be rewritten as follows:

$$a_{ik} = \sum_{m=1}^{M} z^m \; p_{ik}^m = z' \, p_{ik}, \quad \text{for } i = 1, 2, \ldots .K \tag{5}$$

and

$$u_i^t = \sum_{n=1}^{N} v^n \; w_{it}^n = v' w_{it}, \text{ for } i = 1, 2, \ldots ..I \text{ and } k = 1, 2, \ldots \ldots K \tag{6}$$

System-of-equations statistical model

Alternatively, expression (1) can be treated as a system of interdependent (seemingly-unrelated) equations. Doing so means that all the inputs are taken into account *simultaneously*.[10] Hence, an additional accounting restriction should be imposed for each type of output k:

$$\sum_{i=1}^{I} a_{ik} = 1, \quad \text{for all } k \tag{7}$$

This restriction ensures that the overall adding-up consistency or accounting balance between total revenue and total cost is always satisfied–as explained below.

Given the accounting balance between total revenue and total cost for each farm, a singularity problem occurs. Specifically, given the I equations in (1) for each farm, we can write:

$$\sum_{i=1}^{I} x_i^t = \sum_{i=1}^{I} \left(\sum_{k=1}^{K} a_{ik} y_k^t + u_i^t \right) \tag{8}$$

Introducing the cross-equation restriction $\Sigma_i a_{ik} = 1$ implies that

$$\sum_{i=1}^{I} x_i^t = \sum_{k=1}^{K} \left(\sum_{i=1}^{I} a_{ik} \right) y_k^t + \sum_{i=1}^{I} u_i^t = \sum_{k=1}^{K} y_k^t + \sum_{i=1}^{I} u_i^t \tag{9}$$

Since, by construction,

$$\sum_{i=1}^{I} x_i^t = \sum_{k=1}^{K} y_k^t \tag{10}$$

10. Considering the I linear models in (1) simultaneously, for each farm t, implies that we assume that the errors u_i^t are "contemporaneously" correlated (i.e. for each individual farm), but uncorrelated across the farms.

this implies that

$$\sum_{i=1}^{I} u_i^t = 0 \tag{11}$$

Hence, the system is singular and the inverse of the variance-covariance matrix does not exist. Consequently, ordinary least squares (OLS) or maximum likelihood (ML) procedures cannot be used properly to estimate the unknown parameters of the system of equations.

With the above specifications, the following ME problem can now be formulated:
given the support vectors [z, v] along with the sample data on inputs x_i^t and outputs y_k^t, find the probability vectors $p >> 0$ and $w >> 0$ that maximize the following entropy measure:

$$\mathrm{H(p, w) = -p' \ln p - w' \ln w} \tag{12-1}$$

subject to the *data-consistency* constraints,

$$\mathrm{x = Ya + u = Y(z'p) + v'w} \tag{12-2}$$

the *adding-up* constraints,

$$\sum_{m=1}^{M} p_{ik}^m = 1, \quad \text{for all } i, k \tag{12-3}$$

$$\sum_{n=1}^{N} w_{it}^n = 1, \quad \text{for all } i, t \tag{12-4}$$

and the additional *cross-equation* or *accounting* restriction

$$\sum_{i=1}^{I} a_{ik} = \sum_{i=1}^{I} z' p_{ik} = 1, \quad \text{for all } k \tag{12-5}$$

The solution to the above ME optimization problem is unique and the unknown probability vectors $\hat{p}$ and $\hat{w}$ can be derived by using non-linear programming software, such as the computer program GAMS (General Algebraic Modelling System).[11]

3.3 ME-tobit formulation of the cost-allocation problem

If some cost items are equal to zero, as could be observed for a number of farms, the use of a censored-data or tobit variant of the linear statistical model is recommended, where the observations are ordered as follows:

$$\mathrm{x = YA + u} = \begin{bmatrix} \mathrm{x_1 > 0} \\ \mathrm{x_2 = 0} \end{bmatrix} = \begin{bmatrix} \mathrm{Y_1} \\ \mathrm{Y_2} \end{bmatrix} \mathrm{A} + \begin{bmatrix} \mathrm{u_1} \\ \mathrm{u_2} \end{bmatrix} \tag{13}$$

11. For more details on this computer program, see Brooke et al. (1988).

where x_1 is observed and x_2 is unobserved, and Y_1 and Y_2 are the corresponding output matrices. Using the ME formalism yields the following reparameterized model (Golan et al. 1996b, p. 270):

$$\begin{bmatrix} x_1 > 0 \\ x_2 = 0 \end{bmatrix} = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} z'p + \begin{bmatrix} v_1'w_1 \\ v_2'w_2 \end{bmatrix} \tag{14}$$

which leads to the following ME-tobit problem:

given the support vectors [z, v_1, v_2] along with the sample data on inputs x_i^t and outputs y_k^t, find the probability vectors p >> 0, w_1 >> 0, and w_2 >> 0 that maximize the following entropy measure:

$$H(p, w_1, w_2) = -p'\ln p - w_1'\ln w_1 - w_2'\ln w_2 \tag{15-1}$$

subject to the *data-consistency* constraints,

$$x_1 = Y_1 z'p + v_1'w_1 \tag{15-2}$$

$$0 \geq Y_2 z'p + v_2'w_2 \tag{15-3}$$

and the *adding-up* constraints,

$$\sum_{m=1}^{M} p_{ik}^{m} = 1, \quad \text{for all } i, k \tag{15-4}$$

$$\sum_{n=1}^{N} w_{1it}^{n} = 1, \quad \text{for all } i \text{ and } t = 1, 2, \ldots T \tag{15-5}$$

$$\sum_{n=1}^{I} w_{2it}^{n} = 1, \quad \text{for all } i, \text{ and } t = 1, 2, \ldots T_2, \text{ where } T_1 + T_2 = T \tag{15-6}$$

and the *accounting* constraint

$$\sum_{i=1}^{I} a_{ik} = \sum_{i=1}^{I} z'\, p_{ik} = 1 \tag{15-7}$$

Given the above specification, there are T_1 (farms with) positive observations and T_2 (farms with) zero observations for input *i*.

Again, the above ME optimization problem can be solved along similar lines as the problems previously defined, using standard non-linear programming software, such as GAMS.

3.4 Statistical properties of the ME and ME-tobit systems of equations

The statistical properties of ME and ME-tobit systems of equations are assessed by three indicators: the entropy ratio statistic, the normalized entropy indicator and asymptotic standard errors.

Entropy ratio statistic

The statistical properties of the ME estimators of the (constrained or unconstrained) system of equations were recently investigated by Golan and Judge (1996a), Golan et al. (1996b, 2001), Marsh et al. (1998) and Mittelhammer and Cardell (1997). Assuming mild conditions on the error terms, on the support values z and v, and on the matrix of explanatory variables Y,[12] it can be shown that the ME estimators of the a_{ik} parameters are consistent and asymptotically normal. Given this result, it is possible to show that the entropy ratio statistic for the different parameters of the unknown distribution generating the data distributions has a limiting distribution. Applied to the system of linear input-demand equations, the entropy ratio (ER) statistic is defined as follows.

Under the null-hypothesis "H_0 = the sum of the output coefficients a_{ik}, for a given output k, is equal to one $(\Sigma_k a_{ik} = 1)$", the entropy ratio[13] is defined as:

$$\mathrm{ER}\left\{\sum_k a_{ik} = 1\right\} = 2S_{UN}\left(\sum_k a_{ik} \neq 1\right) - 2S_R\left(\sum_k a_{ik} = 1\right) \tag{16}$$

where S_{UN} and S_R are the values of the unrestricted and restricted ME functions, respectively, as given by the expressions (12-1) and (15-1), respectively. Under the null-hypothesis H_0, the entropy ratio (ER) statistic follows a x_2 distribution, with K degrees of freedom, where K is the number of constraints imposed on the a_{ik} parameters.

Normalized entropy indicators

The relative performance of the ME estimation of the a_{ik} coefficients can be measured by the normalized entropy indicator, defined as the proportion of the total remaining uncertainty. The normalized entropy measure, for $I \times K$ parameters (I inputs and K outputs) and M possible outcomes (M support values for the a_{ik} parameters), is defined by Golan et al. (1996b, p. 93) as follows:

$$S(\hat{p}) = \frac{-\hat{p}' \ln \hat{p}}{IK \ln(M)} \tag{17}$$

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12. Specifically, there are four conditions (Golan et al. 2001, p. 24): the error support values v are symmetric around zero, the support values z associated with the a_{ik} parameters span the true value for each of the unknown parameters and have finite lower and upper bounds, the errors are independently and identically distributed with mean zero and with a contemporaneous N x N variance-covariance matrix Σ; and the matrix Y'Y exists and is non-singular.

13. It is worth noting the similarity between this entropy ratio statistic and the likelihood ratio statistic.

where $S(\hat{p})\varepsilon[0, 1]$, and where *IKln(M)* represents the maximum uncertainty—that is, the entropy level of the uniform distribution. When $S(\hat{p}) = 0$, there is no uncertainty; when $S(\hat{p}) = 1$, the distribution of the *K*×*I* parameters is identical to the uniform prior distribution. Along similar lines, the informational content of the noise or error component can be assessed through the normalized entropy measure, for *I*×*T* errors (*I* inputs and *T* farms) and *N* possible outcomes (*N* support values for the error terms), defined by Golan et al. (1996b, p. 93) as follows:

$$S(\hat{w}) = \frac{-\hat{w}'\ln\hat{w}}{IT\ln(N)} \tag{18}$$

where $S(\hat{w})\varepsilon[0, 1]$, and where *ITln(N)* represents the maximum uncertainty–that is, the entropy level of the uniform distribution.

Asymptotic standard errors

The asymptotic standard errors of the estimated parameters a_{ik} are calculated by defining, in the first stage, the variance-covariance matrix of the estimated residuals, denoted by Σ. The elements of Σ are defined as follows:[14]

$$\hat{\sigma}_{ij} = \frac{1}{T}\sum_t \hat{u}_{it}\,\hat{u}_{ijt} \tag{19}$$

where

$$\hat{u}_{it} = \sum_n w_{it}^n\, v_{it}^n \tag{20}$$

Given the estimated variance-covariance matrix Σ, the asymptotic variance-covariance matrix of the constrained parameters a_{ik} is:

$$\Omega_{R-GME} = \Omega - \Omega R'(R\Omega R')^{-1} R\Omega \tag{21}$$

where

$$\Omega = p\lim T^{-1}\left[Y'(\sum{}^{-1} \otimes I)\,Y\right]^{-1} \tag{22}$$

The matrix *R*, of dimension *K*×*I*, is defined in such a way (containing “1’s” in the appropriate positions) that *Ra* = *e*, where *a* is the vector of a_{ik} parameters and *e* is a unit vector. As a result, the complete set of *K* equality conditions that the sum of the a_{ik} parameters equal one, for each output *k* (*k* = *1*,…,*K*), are satisfied.

14. It is a well-known result that for small samples the use of *1/T* in expression (19) yields a biased estimate of σ_{ij}. To remedy this problem, Zellner and Huang (1962) suggested the replacement of *T* by $(T\text{-}K_i)^{0.5}(T\text{-}K_j)^{0.5}$ in expression (19), where K_i and K_j are the number of explanatory variables.

Next, it can be established that the restricted ME estimator of the a_{ik} parameters is consistent and asymptotically normally distributed (Golan et al. 2001):

$$\sqrt{T}(\hat{a}-a) \xrightarrow{d} N(0, \Omega_{R-GME}) \tag{23}$$

All these statistical results are used in applying the ME methodology to Saskatchewan farms and are reported in Section 4.

Section 4: Maximum entropy applied to the Saskatchewan data sample

The ME methodology is applied to the accounting data for 30 crop farms in Saskatchewan.[6] As some farms in the sample exhibit zero and/or negative values for certain input categories,[7] the standard ME methodology had to be adjusted. Specifically, the tobit formulation of the ME methodology (ME-tobit), as given by expression (13), is used and applied to the data sample of farms.[8]

The data are for nine input categories *($I = 9$)* and five output categories *($K = 5$)*. All input expenditures (accounting costs) and output values are expressed in Canadian dollars ($).

This section has six sub-sections. Section 4.1 has three alternative ME-tobit model specifications with varying support sets. In Section 4.2, these alternative model specifications are then applied to two variants of the Saskatchewan data sample—one excluding fallow costs as a separate cost item, the other including the fallow costs as a separate cost item. We present the estimation results of the various ME-tobit model specifications in Section 4.3. We discuss and validate the performance of these alternative ME-tobit model specifications in the last three sub-sections.

4.1 Three alternative model specifications with varying support sets

Three alternative model specifications are used with varying designs of the support sets for the cost-allocation coefficients. The three sets of support values (intervals) are different only with regard to the *width* of the interval and/or the type of *spacing* of the support values within the chosen interval.[9] An overview of the three model specifications, designated here as Models A, B and C and their associated support sets are given in Table 2. For each model, eleven values *($M = 11$)* were chosen for the common parameter support vector z (for all i, k),

6. Given the small size of the sample (only 30 observations), the σ_{ij} 's are corrected for the number of degrees of freedom using Zellner and Huang's suggestion (see Footnote 14).
7. This situation occurs for the four input categories: fertilizers (one farm), fuel, power and grease (one farm), paid salaries (six farms) and net operating income (three farms).
8. The standard ME applications were also conducted but the results which are not reported here are available from the authors upon request. For other recent applications of the ME-tobit approach to systems of equations in the context of agricultural economics, see Arndt (1999) and Golan et al. (2001)

along with three values *(N = 3)* for the common error support vector v (for all *i, t)*. The selected values of the common support vector z are motivated by "prior information" about the range of magnitudes for the cost-allocation coefficients.[10]

In Model A, the interval is defined in such a way that all the support values are within the admissible [0.0, 1.0] range, while being equally spaced over the interval. Model B is grounded on the plausible assumption that the coefficients should be closer to zero than to one. Hence, the size of the interval is reduced, spanning the range [0, 0.5]. In addition, the support values are equally spaced over the reduced interval. In Model C, a so-called "asymmetric" and "left-skewed" type of spacing over the reduced interval [0, 0.5] is considered, with a larger number of support values that are close to zero and fewer support values that are close to 0.5.

The error support vector v is treated differently, depending on whether all the observed values of the input demand are positive or whether some of them are negative or zero. In the first case (all positive), the selected support values are based on the results of prior OLS estimations. The support values are always symmetrically defined around zero. The endpoints $[-3\hat{\sigma}_u 0, +3\hat{\sigma}_u]$ of the intervals are based on the so-called "three-sigma rule", where the range was derived from prior OLS standard-error-of-regression results.[11] In the second case (some negative or zero), the OLS estimate of the standard-error-of-regression is biased and hence, cannot be used to generate the support range in the ME-tobit model. Therefore, an alternative estimate proposed by Golan et al. (1997) is used, which is based on an assumed uniform distribution of the data.[12, 13]

4.2 Two applications excluding and including fallow costs

*APPLICATION 1: The ME-tobit model **excluding** fallow costs as a separate cost item*

The first application implies a tobit version of the ME methodology (ME-tobit), where input expenditures are linked to five crop outputs, expressed in monetary terms. This application leads to the Models A1, B1 and C1. Fallow costs as such are included in the estimation (but not as a separate cost item). In all three model variants, it is implicitly assumed that all the

9. For a more thorough analysis on the sensitivity of the ME estimates to the width of the support interval (end points) and/or the type of spacing of the support values within the pre-specified intervals, see Léon et al. (1999, p. 435-437).
10. The "prior information" relates to the fact that all the cost-allocation coefficients must lie within the range [0, 1]. Furthermore, common sense and the existence of a large number of cost items (nine or ten depending upon the applications) lead us to conclude that the estimated a_{ik}'s should be closer to zero than one. This fact is confirmed by OLS estimation experiments that all estimated coefficients were close to zero and smaller than 0.5. All this factual evidence also suggests that using left-skewed distribution for the values of the common support vector z is well justified.
11. Concerning the use of this "three-sigma rule," we followed the recommendations of Golan et al. (1996b, p. 88) who used it in their econometric experiments.
12. This alternative estimate of σ_u is obtained by assuming a uniform and censored distribution of the data. Let x_{max} denote the largest value of x_i^t. Then, knowing the range in the sample and the proportions of the censored observations (that is, where $x_i^t = 0$) in the data sample, the unobserved x_{min} can be derived. The uniform variance $s_x^2 = (x_{max} - x_{min})^2 / 12$ is used as an estimator of the variance $\sigma_u 2$.
13. In terms of software used, the system of input-demand equations was estimated using the GAMS computer program, while the variance-covariance matrix of the restricted ME parameters was obtained using the econometric Time Sharing Processor (TSP) computer package (Hall and Cummins 1998). Examples of these two computer programs are provided in Appendices C and D, respectively.

inputs (costs) associated with fallowed land operations are imputed to each crop output, and are invariant with respect to time—that is, there is no year-to-year variation. Given this assumption, there is no need to include the number of hectares of fallowed land as an additional explanatory variable in the problem formulation. Ignoring fallow costs as a separate cost item is not entirely satisfactory because the resulting *cost-allocation coefficients* are considerably overestimating their true values.

Table 2: Model specifications and corresponding support sets for the unknown cost-allocation parameters a_{ik}

Model specification	Number of support values	Type of spacing	Selected support values
Model A	11	Symmetric	0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1.0
Model B	11	Symmetric	0,0.05,0,10,0.15,0.20,0.25,0.30,0.35,0.40,0.45,0.50
Model C	11	Non-symmetric left-skewed	0,0.025,0.050,0.075, 0.100, 0.125, 0,150, 0.200, 0.300, 0.400,0.500

*APPLICATION 2 : The ME-tobit model **including** fallow costs as a separate cost item*

The second application implies a tobit version of the ME methodology (ME-tobit) where, in contrast with the first application, fallow costs are included as a separate cost item. Specifically, an additional input category, called "fallow costs," is incorporated into the model.[14] This input category depends on the five categories of crop outputs.[15] As a result, the model contains ten input-demand equations and the three estimated model specifications are now designated as Models A2, B2 and C2. For purposes of comparison, the ME-tobit model was also estimated for the three different sets of support values defined earlier.

4.3 Results of the ME-tobit estimation

The estimation results are presented in Tables 3 and 4 for the ME-tobit Applications 1 and 2, respectively. Additional results on the statistical tests and the performance of the models are reported in Tables 5, 6, 7 and 8. The empirical results are quite reasonable in all respects, regardless of the model specification used. The only issue of concern is the relatively weak statistical significance of the estimated parameters (see Table 5). Looking at Model A1, it can be seen that 26 out of 45 estimated coefficients (nine inputs times five outputs) are statistically significant.

14. Six farms out of 30 do not use summerfallow and hence are characterized by fallow costs equal to zero.
15. The costs associated with fallow operations were obtained using the cropping enterprise database which records all the expenses imputed to the maintenance of fallowed land. These operations concern mainly pesticides, some other direct material inputs, fuel, power and grease and repairs. In the category of paid salaries, the imputation of the cost attributable to fallowed land was undertaken by taking the proportion of management fees devoted to fallow land appearing in the cropping enterprise database. Finally, in other fixed cash expenses, cash cost associated with fallowed land was measured as follows: interest paid, property taxes, overhead materials, and cash land lease that can be imputed to fallow maintenance operations were computed for each farm using the cropping enterprise database. These expenses were then converted into percentages which were applied to total fixed cash expenses. Finally, to ensure that the accounting identity between total revenues and costs is satisfied for each farm, all expenses other than fallow costs were adjusted downward by an amount imputed to fallow costs.

Specifically, 14 coefficients are significant at the 5 percent level of significance, while 12 coefficients are significant at the 10 percent and 15 percent levels of significance, respectively. Examining each input-demand equation separately, it can be seen that three to four out of five coefficients are significant in the equations for seeds, pesticides, fuel, power and grease and other fixed cash expenses, whereas only one coefficient is significant in the equations for fertilizers, other direct material inputs and repairs. In the equation for net operating income, only one coefficient is significant at the 10 percent level of significance. A similar pattern is discernible for the other model specifications.

Consistent estimation of the coefficients was possible by imposing the adding-up constraint, which also ensures that the sum of the coefficients, for each output separately, is equal to one. These restrictions are tested by comparing the restricted and the unrestricted models, using the entropy ratio (ER) statistic. The null hypothesis, stating that the sum over all inputs of the a_{ik} coefficients is equal to one, cannot be rejected if the calculated statistic ER is smaller than 11.09—which is the critical χ^2 value at the 5 percent level of significance. An inspection of these test results, provided in Table 6, reveals that the null-hypothesis can be rejected only for Models A1 and A2.

The normalized entropy (NE) indicators are important criteria for assessing the informational content of each model specification. Specifically, a model is viewed as "superior" if its associated NE measure $S(\hat{p})$ is greater than the values of the same indicator obtained for alternative model specifications. This means that the "superior" model would yield a solution for the recovered cost-allocation coefficients that is more "uniform" or closer to the prior distribution. Using this criterion, $S(\hat{p})$ increases from 0.55 in Model A1 to 0.77 in Model C1 (see Table 7). Hence, we can conclude that a "left-skewed" spacing of the support values over a reduced interval [0, 0.5] yields more satisfactory results. This finding is not surprising however, and is in line with our prior judgement that the coefficients are more likely to be closer to zero than to one. A computation of the normalized entropy indicator[16] for all the recovered cost allocation coefficients (the a_{ik}'s) confirms this point (see Appendix E for more details on results). A similar pattern holds in the case of Application 2 where Model C2 is preferred to Model A2.

Comparing the two "best performing" model specifications, that is, Models C1 and C2, then Model C2 is "superior" because it shows a higher value (0.7706 against 0.8363) (see Table 7) for the NE measure $S(\hat{p})$. With regard to the informational content of the noise ratio, the NE measure $S(\hat{w})$ is invariant to the choice of the support set (see Table 7). This result is not surprising, as the initial support values for w have not been changed across the various model specifications.

16. The normalized entropy ratio associated to each a_{ik} is defined as follows: $S_{ik}(\hat{p}) = \dfrac{-\sum_{m=1}^{M} p_{ik}^{m} \ln\left(p_{ik}^{m}\right)}{\ln(M)}$

where M is the number of support values equal to eleven.
The interpretation of $S_{ik}(\hat{p})$ is the same as for $S(\hat{p})$: if it is equal to one, it means that the distribution of the coefficient a_{ik} is identical to the uniform prior distribution. An examination of the $S_{ik}(\hat{p})$'s in Appendix E shows that all of them increase significantly when we switch from Models A to B. This finding confirms the fact that shrinking the range of the support values leads to cost allocation coefficients which are more in line with the prior belief that the a_{ik}'s are closer to zero than one. This pattern is also reinforced, but to a lesser extent, when we move from Model specifications B to C. However, this is not valid for all the estimated coefficients a_{ik}. In fact, few of them have their normalized entropy indicator declining in value. This latter pattern indicates that the symmetric type of spacing of support values is more appropriate for those cost allocation coefficients.

Table 3: Application 1—ME-tobit estimates of cost-allocation coefficients (*excluding* fallow costs as a separate cost item)

	Model A1		Model B1		Model C1	
	Coefficients	Standard errors	Coefficients	Standard errors	Coefficients	Standard errors
1. Seeds						
Wheat	0.067	0.0218	0.062	0.0219	0.063	0.0222
Other grains	0.013	0.0307	0.020	0.0297	0.020	0.0313
Canola	0.034	0.0183	0.034	0.0177	0.034	0.0187
Other oilseeds	0.069	0.0645	0.075	0.0629	0.072	0.0663
Other crops	0.066	0.0157	0.069	0.0152	0.067	0.0160
2. Fertilizers						
Wheat	0.194	0.1237	0.192	0.0481	0.193	0.1276
Other grains	0.158	0.1791	0.153	0.0676	0.156	0.1795
Canola	0.204	0.1072	0.206	0.0404	0.206	0.1074
Other oilseeds	0.090	0.3796	0.094	0.1433	0.090	0.3806
Other crops	0.112	0.0917	0.117	0.0346	0.114	0.0920
3. Pesticides						
Wheat	0.088	0.0430	0.101	0.0381	0.090	0.0439
Other grains	0.005	0.0604	0.006	0.0535	0.010	0.0617
Canola	0.057	0.0362	0.053	0.0320	0.056	0.0369
Other oilseeds	0.400	0.1281	0.356	0.1135	0.386	0.1309
Other crops	0.096	0.0307	0.091	0.0274	0.095	0.0316
4. Other direct material inputs						
Wheat	0.034	0.0215	0.032	0.0189	0.033	0.0217
Other grains	0.047	0.0302	0.051	0.0266	0.050	0.0306
Canola	0.029	0.0180	0.029	0.0159	0.028	0.0183
Other oilseeds	0.032	0.0641	0.041	0.0563	0.040	0.0648
Other crops	0.061	0.0155	0.063	0.0136	0.062	0.0157
5. Fuel, power and grease						
Wheat	0.055	0.0305	0.057	0.0315	0.059	0.0343
Other grains	0.114	0.0428	0.124	0.0443	0.117	0.0482
Canola	0.062	0.0256	0.070	0.0265	0.065	0.0289
Other oilseeds	0.071	0.0908	0.077	0.0939	0.073	0.1023
Other crops	0.063	0.0219	0.075	0.0227	0.070	0.0247
6. Repairs						
Wheat	0.089	0.0312	0.080	0.0285	0.081	0.0321
Other grains	0.071	0.0438	0.080	0.0400	0.076	0.0451
Canola	0.017	0.0262	0.023	0.0240	0.023	0.0270
Other oilseeds	0.076	0.0929	0.080	0.0849	0.076	0.0957
Other crops	0.034	0.0225	0.040	0.0205	0.039	0.0231
7. Paid salaries						
Wheat	0.077	0.1080	0.091	0.0909	0.088	0.1149
Other grains	0.170	0.1519	0.169	0.1278	0.166	0.1617
Canola	0.116	0.0909	0.143	0.0765	0.125	0.0967
Other oilseeds	0.064	0.3220	0.074	0.2709	0.071	0.3427
Other crops	0.135	0.0778	0.157	0.0655	0.143	0.0828
8. Other fixed cash expenses						
Wheat	0.196	0.0610	0.180	0.0520	0.190	0.0618
Other grains	0.211	0.0857	0.199	0.0731	0.204	0.0870
Canola	0.125	0.0513	0.141	0.0438	0.131	0.0521
Other oilseeds	0.120	0.1818	0.115	0.1551	0.110	0.1844
Other crops	0.101	0.0439	0.114	0.0375	0.107	0.0446
9. Net operating income						
Wheat	0.199	0.2503	0.204	0.1854	0.202	0.2660
Other grains	0.211	0.3520	0.198	0.2608	0.200	0.3741
Canola	0.356	0.2106	0.301	0.1561	0.332	0.2239
Other oilseeds	0.077	0.7462	0.088	0.5529	0.081	0.7931
Other crops	0.332	0.1803	0.273	0.1336	0.303	0.1917

Table 4: Application 2—ME-tobit estimates of cost-allocation coefficients (*including* fallow costs as a separate cost item)

	Model A2		Model B2		Model C2	
	Coefficients	Standard errors	Coefficients	Standard errors	Coefficients	Standard errors
1. Seeds						
Wheat	0.065	0.0222	0.061	0.0226	0.058	0.0230
Other grains	0.015	0.0313	0.022	0.0318	0.028	0.0324
Canola	0.035	0.0187	0.035	0.0190	0.035	0.0194
Other oilseeds	0.069	0.0663	0.073	0.0674	0.071	0.0688
Other crops	0.063	0.0160	0.066	0.0163	0.065	0.0166
2. Fertilizers						
Wheat	0.086	0.0492	0.097	0.0599	0.099	0.0579
Other grains	0.125	0.0693	0.130	0.0843	0.129	0.0815
Canola	0.1270	.0414	0.141	0.0504	0.135	0.0487
Other oilseeds	0.069	0.1468	0.075	0.1787	0.073	0.1728
Other crops	0.102	0.0355	0.115	0.0432	0.112	0.0418
3. Pesticides						
Wheat	0.069	0.0425	0.070	0.0436	0.064	0.0434
Other grains	0.002	0.0597	0.002	0.0614	0.006	0.0616
Canola	0.063	0.0357	0.065	0.0367	0.065	0.0368
Other oilseeds	0.385	0.1266	0.352	0.1301	0.365	0.1305
Other crops	0.104	0.0306	0.105	0.0314	0.108	0.0315
4. Other direct material inputs						
Wheat	0.036	0.0213	0.035	0.0216	0.035	0.0218
Other grains	0.045	0.0300	0.049	0.0304	0.050	0.0307
Canola	0.028	0.0180	0.029	0.0182	0.029	0.0184
Other oilseeds	0.030	0.0636	0.038	0.0645	0.044	0.0650
Other crops	0.061	0.0154	0.062	0.0156	0.061	0.1572
5. Fuel, power and grease						
Wheat	0.060	0.0301	0.064	0.0384	0.067	0.0389
Other grains	0.106	0.0424	0.112	0.0540	0.110	0.0453
Canola	0.060	0.0254	0.070	0.0323	0.069	0.0271
Other oilseeds	0.068	0.0899	0.071	0.1144	0.070	0.0960
Other crops	0.059	0.0217	0.070	0.0277	0.071	0.0232
6. Repairs						
Wheat	0.089	0.0304	0.082	0.0314	0.076	0.0322
Other grains	0.064	0.0428	0.072	0.0442	0.073	0.0453
Canola	0.015	0.0256	0.021	0.0265	0.026	0.0271
Other oilseeds	0.076	0.0907	0.077	0.0938	0.073	0.0960
Other crops	0.032	0.0219	0.037	0.0227	0.041	0.0232
7. Paid salaries						
Wheat	0.090	0.1085	0.110	0.1282	0.110	0.1241
Other grains	0.162	0.0428	0.157	0.1804	0.155	0.1746
Canola	0.119	0.0256	0.150	0.1079	0.142	0.1045
Other oilseeds	0.061	0.0907	0.069	0.3823	0.069	0.3701
Other crops	0.127	0.0219	0.143	0.0924	0.138	0.0895
8. Other fixed cash expenses						
Wheat	0.193	0.1085	0.182	0.0520	0.185	0.0520
Other grains	0.176	0.1526	0.169	0.0732	0.168	0.0731
Canola	0.116	0.0913	0.130	0.0438	0.128	0.0437
Other oilseeds	0.117	0.3235	0.108	0.1551	0.099	0.1550
Other crops	0.094	0.0782	0.103	0.0375	0.102	0.0375
9. Fallow cost						
Wheat	0.023	0.0507	0.025	0.0274	0.028	0.0294
Other grains	0.096	0.0713	0.098	0.0386	0.092	0.0414
Canola	0.018	0.0426	0.022	0.0231	0.025	0.0248
Other oilseeds	0.049	0.1511	0.053	0.0817	0.054	0.0877
Other crops	0.024	0.0365	0.030	0.0197	0.033	0.0212
10. Net operating income						
Wheat	0.300	0.2069	0.273	0.2532	0.278	0.2494
Other grains	0.215	0.2910	0.190	0.3562	0.188	0.3508
Canola	0.446	0.1741	0.337	0.2131	0.346	0.2010
Other oilseeds	0.076	0.6168	0.084	0.7551	0.080	0.7437
Other crops	0.367	0.1491	0.268	0.1825	0.269	0.1797

Table 5: Number of statistically significant parameters

Application	Model specification	Number of statistically significant parameters				
		5% level	10% level	15% level	20% level	Total
Application 1	A1	14	5	7	0	26
	B1	19	8	0	1	28
	C1	14	5	6	1	26
Application 2	A2	17	6	7	0	30
	B2	15	3	12	0	30
	C2	14	6	10	0	30

Note: The critical values are 1.960, 1.645, 1.439 and 1.281 for a 5%, 10%, 15% and 20% level of significance, respectively.

Table 6: Entropy ratio tests of model performance

Application	Model specification	Entropy ratio statistic
Application 1	A1	15.284
	B1	7.688
	C1	7.170
Application 2	A2	16.624
	B2	9.718
	C2	5.522

Note: $\chi^2(5) = 11.07$ for a level of significance of 5%.

Table 7: Normalized entropy indicators of the ME-tobit models

Application	Model specification	$S(\hat{p})$	$S(\hat{w})$
Application 1	A1	0.5463	0.9033
	B1	0.7375	0.8983
	C1	0.7706	0.9003
Application 2	A2	0.5110	0.9529
	B2	0.6999	0.9469
	C2	0.8363	0.9458

4.4 Predictive power of the ME-tobit estimates

The various model specifications can also be assessed in terms of their predictive power. Many different indicators can be used for this purpose. Here, we decided to use the "pseudo-R^2" which is defined as the square of the correlation coefficient between the predicted and the observed values for each cost item. This indicator is positive but smaller than one. The predictive power of the model improves as the value approaches one.

The "pseudo-R^2" indicator was computed for each input, according to the six model specifications. A closer inspection of the results presented in Table 8 reveals that the values are higher than 0.800 for all the inputs, except two. For paid salaries, the "pseudo-R^2" takes a value of about 0.501 for all model specifications. For fallow costs, it gravitates around the value of 0.714.

Table 8: Predictive power of the ME-tobit models in terms of the "pseudo-R^2" statistic

Input cost	Model specification					
	Application 1			Application 2		
	A1	B1	C1	A2	B2	C2
Seeds	0.911	0.908	0.908	0.910	0.907	0.904
Fertilizers	0.906	0.906	0.906	0.894	0.895	0.895
Pesticides	0.924	0.920	0.922	0.919	0.915	0.915
Other direct material input	0.854	0.849	0.850	0.854	0.852	0.850
Fuel, power and grease	0.885	0.879	0.882	0.885	0.882	0.882
Repairs	0.828	0.818	0.819	0.828	0.818	0.811
Paid salaries	0.496	0.497	0.493	0.507	0.504	0.504
Other fixed cash expenses	0.918	0.914	0.916	0.923	0.920	0.920
Fallow costs	N/A	N/A	N/A	0.723	0.714	0.705
Net operating income	0.936	0.932	0.934	0.926	0.924	0.923

Note: "N/A" means not applicable.

4.5 General assessment of the ME estimates

What conclusions can be drawn from these estimation results? Is it justifiable to say, by looking at these various indicators and statistical tests, that a particular model specification is superior to the others? If the assessment is conducted in terms of statistical performance and predictive power, it is rather difficult to rank the models in terms of their performance. On the other hand, if one uses the NE criterion, more conclusive statements can be made. In fact, one could say without any doubt that Model C2, which is characterized by an "asymmetric" and "left-skewed" spacing and a reduced interval of the support values z, is decisively the best model.

4.6 Validation of the estimated cost allocation coefficients

So far, the ME results have been studied only from econometric and statistical points of view, without providing any comments on (the plausibility of) their magnitudes. Evaluating the magnitudes of the coefficients is conducted in two stages, based on the results of the two model specifications selected, Models C1 and C2. After a first evaluation in general terms we conducted a comparison of the ME estimates with the available observed coefficients using the cost data appearing in the crop enterprise database.

In general, most of the ME estimates obtained for the direct inputs, like seeds, fertilizers, pesticides, fuel, power and grease and repairs take values smaller than 0.20. There are a few exceptions to this rule, however. For example, in the case of pesticides, the coefficient associated with the other oilseeds output is 0.386 in Model C1 and 0.365 in Model C2. In addition, a surprising result was obtained for the coefficients in the equation for fertilizers in Application 1 (Models A1, B1 and C1). The Application 1 coefficients in the equations for wheat, other grains and canola are much higher than the corresponding coefficients in Application 2 (Models A2, B2 and C2). The Application 2 coefficient values, however, are more in line with prior expectations. Also the values of the coefficients in the equation for net operating income changed drastically from Model C1 to Model C2 (the two best model specifications). Specifically, the wheat coefficient increased from 0.202 in Model C1 to 0.278 in Model C2. Also, the magnitudes of the coefficients associated with other grains, canola and other oilseeds decreased significantly for these two model specifications. Overall, the ME estimates obtained with Model C2 seem to be more realistic. In addition, they are in line with similar estimates obtained by the authors of this study for crops in Brittany (see Léon et al. 1999).

Furthermore, it is instructive to investigate whether the addition of fallow costs to the list of inputs leads to a change in some coefficients. A cross-checking of the coefficients (reported in Tables 3 and 4) in the equations for pesticides, fuel, power and grease and repairs (inputs used in fallow-maintenance operations) reveals that the coefficients associated with wheat, canola and other oilseeds changed drastically from Model C1 to Model C2.[17] A similar pattern was also observable for the coefficients in the net operating income equation. In most other cases, the values of coefficients remained fairly stable or they increased only marginally.

Finally, the validity of the ME-tobit results is tested by comparing the estimated cost-allocation coefficients a_{ik} with the actual or observed cost allocations aik that are available from the data set at hand (cost enterprise database). Although the actual cost allocations are available at the level of the individual crop enterprises (indexed *j = 1,…, J*), the comparison is possible only at the level of the *aggregated* crop outputs (indexed *k = 1,…,K*), given the nature of the ME estimates. The comparison is carried out for the five direct input categories (seeds, fertilizers, pesticides, fuel, power and grease and repairs).

Before proceeding with this comparison however, weighted averages of the actual cost allocations $a_{ij\{k\}}$ are also calculated (for information purposes) at the level of the individual crop enterprises $j\{k\}$ belonging to each aggregated crop category *k*. The weights are the shares of each individual farm's enterprise acreage $(ac^t_{j(k)})$ in the corresponding total enterprise acreage of the sample $(ac_{j(k)})$:

$$w^t_{j(k)} = \frac{ac^t_{j(k)}}{SUM_t(ac^t_{j(k)})} = \frac{ac^t_{j(k)}}{ac_{j(k)}} \tag{24}$$

and

$$a_{i,j\{k\}} = SUM_t(w^t_j c^t_{i,j\{k\}}) \tag{25}$$

where $c^t_{i,j(k)}$ is the actual or observed cost for input *i* allocated to crop enterprise *j* belonging to aggregated crop *k*.

17. For wheat, the pesticide coefficient decreased from 0.090 in Model C1 to 0.064 in Model C2; for canola, the corresponding coefficient increased from 0.056 in Model C1 to 0.065 in Model C2; for other oilseeds, the corresponding coefficients decreased from 0.386 in Model C1 to 0.365 in Model C2.

Next, weighted averages of the actual cost allocations at the level of the *aggregated* crops k (for k = $1,\ldots,K$) are calculated, where the weights are the shares of each enterprise's crop acreage (acj_k) in the corresponding total crop acreage of the sample (ac_k):

$$W_{jk} = \frac{ac_{j(k)}}{ac_k} = \frac{SUM_t(ac^t_{j(k)})}{SUM_j(ac_{j(k)})} \tag{26}$$

and

$$a_{ik} = SUM_j(W_{jk}a_{i,j(k)}) \tag{27}$$

The calculated, actual cost allocations, both at the level of the individual crop enterprises and the aggregated crops (appearing under the heading "aggregated other crops"), are given in Table 9, along with the corresponding ME estimates at the aggregated level. Overall, the ME estimates for the five direct inputs under consideration are clearly within the range of the observed coefficients.

A closer inspection of the actual (calculated) and estimated (ME) results in Table 9 leads us to make four comments:

- For wheat, which is the main crop grown in Saskatchewan, the average observed cost allocations are smaller that the corresponding ME cost-allocation estimates for seeds, fuel, power and grease and repairs. For fertilizers, the average observed cost allocation (which is equal to 0.175) is much higher than the ME estimate generated by Model C2 (0.099), but smaller than the value obtained in Model C1 (0.193). For pesticides, the observed and estimated cost allocations are very close. With regard to the sum, *per output*, of the cost-allocation coefficients for the five direct inputs, the discrepancy between the observed and estimated totals is smaller than 10 percent for Model C2.

- For other grains, Model C2 is reproducing the average observed cost allocations reasonably well for three direct inputs, namely seeds, pesticides and repairs. By contrast, the cost-allocation estimates for fuel, power and grease are completely out of line with their observed counterparts. For fertilizers, the estimate generated by Model C2 is very close to the observed cost allocation. Finally, (as for wheat) the sum, *per output*, of the estimated cost-allocation coefficients for the five inputs is again very close to the observed total.

- The cost-allocation estimates for canola are quite reasonable for three inputs, seeds, pesticides and repairs. The cost-allocation coefficients in the equations for the other inputs were overestimated by a significant amount. Consequently, it is not surprising to find a wide discrepancy between the sum of the estimated and observed cost allocations for the five direct inputs.

- For the remaining two crop categories, other oilseeds and other crops, one can discern a wide discrepancy between the average observed and estimated coefficients. This finding is not really surprising, however, given the heterogeneous composition of these two crop aggregates.

Given the empirical evidence above, two main findings emerge:

- Using the ME methodology to estimate cost-allocation coefficients is a valid approach in the sense that the estimated coefficients fall within the range of their observed counterparts.

- Some minor qualifications are needed, depending on the category of crops under consideration.

Hence, for wheat, other grains and to a lesser extent also canola, the estimated and observed cost-allocation coefficients are quite comparable. For the remaining crop categories, other oilseeds and other crops, some caution is in order when comparing the estimated and observed cost allocations. The discrepancies found are not really surprising, however, though they may require further investigation by creating more homogenous crop categories.

Table 9: Comparison of observed and estimated cost-allocation coefficients

Observed values									ME-to bit estimates						
Crop Enterprise	Crop Code	Acres	Seeds	Fertiliz-ers	Pesti-cides	Fuel, power and grease	Repairs	Total		Seeds	Fertiliz-ers	Pesti-cides	Fuel, power and grease	Repairs	Total
Wheat on fallow	3	3,778	0.0497	0.0390	0.0875	0.0398	0.0742	0.2902	Wheat						
Wheat on stubble	103	11,108	0.0499	0.1553	0.0997	0.0398	0.0733	0.4180	Model C1	0.063	0.193	0.090	0.059	0.081	0.486
Durum on fallow	43	4,861	0.0386	0.0272	0.0458	0.0260	0.517	0.1893	Model C2	0.058	0.099	0.064	0.065	0.067	0.353
Durum on stubble	143	4,130	0.0405	0.1183	0.0567	0.0387	0.0894	0.3436							
Aggregated wheat	–	23,877	0.045	0.104	0.079	0.037	0.072	0.3370							
Barley on fallow	5	2,006	0.0314	0.0696	0.0731	0.0401	0.0649	0.2791	Other grains						
Barley on stubble	105	5,113	0.0364	0.1279	0.0623	0.0465	0.0739	0.3470	Model C1	0.020	0.1560	0.010	0.117	0.081	0.364
Oats on stubble	108	585	0.0766	0.1906	0.0559	0.0579	0.0743	0.4553	Model C2	0.028	0.129	0.050	0.110	0.070	0.387
Aggregated other grains	–	7,704	0.0382	0.1175	0.0646	0.0457	0.0716	0.3376							
Canola on fallow	6	7,576	0.0360	0.0439	0.0733	0.0178	0.0276	0.1986	Canola						
Canola on stubble	106	5,490	0.0406	0.1227	0.1026	0.0245	0.0349	0.3253	Model C1	0.034	0.206	0.056	0.065	0.023	0.350
Aggregated canola	–	13,067	0.0379	0.0770	0.0856	0.0207	0.0307	0.2519	Model C2	0.035	0.135	0.065	0.069	0.026	0.330
Flax on stubble	107	2,154	0.0433	0.1449	0.1250	0.0237	0.0503	0.3872	Other oilseeds						
Mustard on fallow	14	531	0.0598	0.0341	0.0896	0.0377	0.0471	0.2683	Model C1	0.072	0.090	0.386	0.073	0.076	0.697
Aggregated other oilseeds		13,067	0.0379	0.0770	0.0856	0.0207	0.0307	0.3638	Model C2	0.071	0.073	0.365	0.070	0.073	0.652
Lentils on stubble	113	3,560	0.1408	0.0314	0.1711	0.0228	0.0468	0.4129	Other crops						
Peas on stubble	115	3,309	0.1654	0.0495	0.1661	0.0288	0.0505	0.4603	Model C1	0.067	0.114	0.095	0.070	0.039	0.385
Canary seeds on stubble	112	1,002	0.0693	0.1122	0.0649	0.0215	0.0294	0.2973	Model C2	0.065	0.112	0.108	0.071	0.041	0.397
Aggregated other crops	–	7,871	0.1420	0.0493	0.1555	0.0252	0.0461	0.4181							

Section 5: Concluding remarks

This study described the results of the application of the Maximum Entropy (ME) methodology to estimate the enterprise-level cost allocations, using a sample of data from 30 Saskatchewan crop farms. Although many different methods could have been used (and have been used by several analysts), this study opted for the ME methodology. Apart from the fact that the ME methodology offers a less expensive alternative to conducting a special survey—a property which the ME methodology has in common with other estimation techniques, such as OLS or LP—the choice for the ME methodology was primarily motivated by its properties of flexibility, transparency and relative ease of implementation. These properties make it valuable to both practitioners and policy analysts. Finally, using the ME methodology allowed us to overcome the usual problems of implausible outcomes, such as negative signs for the coefficients (OLS) and corner-point or zero solutions (LP).[8]

Empirical results indicate that the estimated cost-allocation coefficients are "realistic", falling within the range of observed values. This finding holds true especially for well defined crop enterprises (wheat, other grains and canola) but to a lesser extent for the two heterogeneous crop categories, other oilseeds and other crops. Given the exploratory nature of the present study, further work is needed to obtain more efficient and realistic estimates—for example, by using larger samples, exploiting the "richness" of panel data and/or employing more reliable prior information on the support values of the cost-allocation coefficients.

Some other practical and methodological problems remain. Specifically, one issue that needs further investigation is how to model, in a more satisfactory way, the decision process of fallow land maintenance. This would require the use of sample data for several years to capture the delayed effect of fallow on crop farmers' returns, or the use of more homogeneous crop farms—for example, crop farms that are located in the same region or that are using similar technologies. Another issue that deserves more emphasis has to do with the necessity that crop and livestock enterprises be described and defined with great accuracy.

8. To show the merits of the ME methodology, we would need to conduct a more in-depth comparative analysis of the different estimation procedures using Monte Carlo simulation experiments.

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Appendix A: Definition of selected groups of crops

Aggregated crops: Number	Aggregated crops: Name	Crop code	Enterprise label	Product code
I	Fallow	1	Fallow, Summerfallow (SMF), Rent Summerfallow	1
		2	Chemical fallow, Cons Fallow	1
		201	Green Manure sf	1
		17	Grass, IWG on rented, Intermediate Wheat GR	16
		52	Greenfeed	35
II	Wheat	3	Wheat on fallow, Wheat on C.F.L., CRSH-Wheat-Fallow, CWRS/Fallow Wheat/fallow Rent, Rent/West, Rent East, Wheat/summerfallow, Wheat/stubble, Wheat/STBL/OWNED, Canadian Red Spring (CRSH)/ wheat-Fall,	3
		103	Wheat on Wheat, Wheat on canola/stubble, CSWH-stubble, CS durum-stubble, S-wheat/stubble zero til, Rented wheat/stubble, CWRS/stubble, Tea/Can St, Wheat/Rent stubble, Wheat/wheat stubble, WHT/rent, CSWHJ-SJ, Wheat/stubble/Rent, Canadian Prairie Spring CPS/fallow, CPS owned, CPS/stubble,	3
		10	Genesis on stubble, Canadian Prairie Spring (CPS) on fallow	10
		110	Pr Spring/on stubble, Grandi on Canst, CPS Wheat/ on stubble, CWES/ rent/fallow, CWES/owned/stubble, CWES/rent/stubble, CWES/fallow, Durum summer fallow, Durum/fallow/rent, Durum chemical fallow Durum/ fallow/rent/FJS, Durum/fallow(Tref), Durum fallow South	10
		43	CRSH durum/Fallow	43
		143	A Durum/stubble, Durum/stubble, Com Durum on Lentils, Durum/stubble/ rent, Durum Canadian Prairie spring (CPS) stubble	43
III	Other grains	5	Barley on fallow, Barley/rented, Barley/owned, M/barley/fallow, Barley/ rented/fallow	5
		105	Barley/stubble, Malt barley on stubble, Feed barley, Good barley, Malt barley/canola stubble	5
		108	Oats, Oats on stubble	8
		109	Fall rye on stubble	9
IV	Canola	6	Canola summerfallow, Rent canola/fallow, Canola on W B, Canola/fallow, Canola/fallow/cropshare, Canola/rented/fallow	6
		106	Canola/stubble, Canola on W B, Rape st, W. Canola /stubble, Canola/stub-ble owned	6
V	Other oilseeds	7	Flax	7
		107	Flax on stubble, Flax on wheat/barley	7
		111	Sunola/stubble	11
		14	Mustard fallow, Y. Mustard fallow	14
		114	Mustard on stubble	14

Aggregated crops		Crop code	Enterprise label	Product code
Number	**Name**			
VI	Other crops	153	Certified wheat fallow, Certified wheat/chemical fallow, Pedigree wheat on fallow	3
		157	Pedigree flax summerfallow	11
		12	Canary seeds, Canary seeds/stubble, Canary seeds/fallow	12
		112	Canary seeds/stubble	12
		13	Lentils, Lentils/fallow	13
		113	Lentils (Rape St), Lentils/canola stubble, Lentils/stubble/rented, Lentils on wheat stubble, CRSH Lentils-St, Lentils/stubble, Richlea/stubble, Richlea lentils, Lentils rent, Lentils/stubble Owned, Lentils/Canola stubble, Cert Est Lentils/fallow	13
		163	Pedigree Lentils/wheat stubble	13
		183	Peas	13
		15	Peas/stubble, peas/cereal stubble, Peas/stubble/rent, Peas on W B.	15
		115	Crsh Peas/Stubble	15
		17	Dahurian	16
		17(1)	Alfalfa(rent)	16
		19	Alfalfa hay	16
		20	Seed red clover	23
		16	Hay	33
		50	Pasture	33
		53	Chickpeas/owned/fallow, Triticale, Caraway, Coriander/fallow, Coriander/ stubble,	36
		45	Borage/stubble	45
		193	Certified Durum, Fallow	43
		213	Ped. Durum/dir stubble	43

Appendix B: Data set

Table B-1: Data set - Net returns

Farm Identification No.	Net returns				
	Wheat	Other grains	Canola	Other oilseeds	Other crops
	Y1	Y2	Y3	Y4	Y5
25	13320.00	31065.00	53816.85	5600.00	69766.80
27	387450.00	31968.00	372937.50	132600.00	100270.5
30	89082.75	44200.00	91650.00	0.00	161.00
33	3661.51	12576.61	33690.10	0.00	22180.500
35	129365.60	86269.20	210754.20	0.00	80821.40
42	18620.93	20550.26	92535.85	0.00	32980.57
56	218625.00	131800.00	287087.50	29448.00	05320.00
58	65827.26	0.00	0.00	0.00	75450.37
77	31500.00	28644.00	0.00	11700.00	0.00
98	40262.30	0.00	0.00	34220.00	53722.50
99	59573.60	6864.25	4156.56	0.00	2395.98
109	72168.00	0.00	0.00	0.00	367590.00
111	156172.40	9584.50	0.00	7424.00	77392.00
121	66494.00	18320.00	46800.00	0.00	0.00
123	122774.00	0.00	129906.82	0.00	179622.00
129	10991.59	20784.09	23120.76	0.00	11073.04
136	174489.90	0.00	312841.32	43279.60	138892.600
167	163335.68	8178.19	0.00	0.00	17139.60
182	102376.56	4375.00	50920.00	0.00	43417.80
185	176017.50	21562.50	206009.78	0.00	81366.00
186	57750.00	0.00	0.00	0.00	15120.00
201	79780.00	110656.00	0.00	0.00	64631.13
215	48737.70	0.00	80850.00	15600.00	14175.00
218	95089.24	6210.00	82881.25	0.00	49600.00
220	0.00	32152.50	47190.00	6300.00	13812.50
221	20312.50	107306.50	35640.00	0.00	0.00
259	30624.00	52195.50	65483.31	18172.00	0.00
286	18564.26	0.00	79339.98	0.00	19264.00
289	76238.60	0.00	56880.30	9045.00	0.00
295	48915.90	12818.00	24360.00	16652.34	0.00

Table B-2: Data Set - Costs

Farm Identification No.	Costs								
	Seeds	Fertilizers	Pesticides	Other direct material inputs	Fuel, power and grease	Repairs	Paid salaries	Other fixed cash expenses	Net operating income
	X1	X2	X3	X4	X5	X6	X7	X8	X9
25	12940.45	20784.00	17932.90	6717.00	14298.00	19086.00	3000.00	42199.00	36611.30
27	59645.38	95770.35	145437.220	32309.00	62670.00	66200.00	36500.00	172018.00	354676.05
30	10805.00	22242.69	16957.76	5738.00	21943.00	18762.00	7000.00	40048.00	81597.31
33	6971.93	10205.60	8999.80	5159.00	7862.88	6321.24	0.00	23688.00	2900.27
35	23374.36	32804.00	26712.50	30959.00	20754.00	19577.00	138264.00	80341.00	134424.55
42	13683.99	24425.75	26505.00	6984.00	11772.00	13244.00	4000.00	32221.00	31851.87
56	17599.25	50617.005	28246.20	14918.00	40995.00	28076.00	55670.00	129645.00	406514.06
58	1530.00	0.00	17437.80	5154.00	3185.19	13571.00	1428.00	23120.00	75851.64
77	6545.50	11400.00	19636.00	3970.00	7904.25	7453.00	0.00	10452.00	4483.25
98	18145.51	367.20	21727.53	7505.50	21475.00	6186.00	12500.00	52329.00	-12030.93
99	981.00	4688.33	8753.31	5952.00	4720.65	6938.00	6682.00	34734.00	-458.90
109	23953.75	18826.00	43070.58	22704.50	11536.00	17646.00	12400.00	38288.00	251333.18
111	20933.00	21460.65	27966.00	11763.00	12869.00	30652.00	5640.00	64692.00	54597.25
121	6720.08	12612.20	15822.45	13429.00	14055.00	15394.00	3500.00	49697.00	384.27
123	28519.55	34547.04	26045.07	25870.00	12780.00	8150.00	15000.00	51756.00	229635.16
129	7888.75	2721.95	4324.50	15029.00	0.00	0.01	5400.00	23920.00	6685.27
136	34526.91	83023.10	91816.31	21264.00	24306.00	10075.00	16500.00	69123.00	318869.10
167	13187.49	10809.00	22969.18	10362.00	17658.00	14106.00	38386.00	38810.00	22365.79
182	17880.04	11079.12	12208.98	7701.00	14577.00	26467.00	33500.00	20155.00	57521.22
185	39402.88	28731.00	45108.50	20554.00	22920.00	19500.00	40000.00	38718.00	230021.41
186	5245.00	9807.00	6555.40	3900.00	1368.00	555.01	0.00	2510.00	42929.59
201	10337.60	5674.50	7568.00	7406.00	3911.66	3531.00	380.00	23018.00	189240.37
215	8886.71	9926.25	9608.21	8996.50	12997.00	5547.00	12000.00	29497.00	61904.04
218	18406.25	36251.10	11112.05	7268.00	13692.00	20344.00	7000.00	25752.00	93955.09
220	6216.25	16857.50	15980.13	6516.00	13014.00	11431.00	7000.00	26062.00	-3621.88
221	4993.88	23817.00	7624.20	9871.00	13814.00	11000.00	4500.00	31667.00	55971.93
259	6350.00	18384.00	15793.20	4757.00	8567.35	9700.00	2511.00	20483.00	79929.26
286	7281.00	5932.00	9130.00	8476.00	4288.00	6623.00	0.00	18938.00	56500.24
289	3744.00	27217.50	7455.75	14845.00	11002.00	14681.00	0.00	28351.00	34867.65
295	5115.25	14851.5	11876.0	1450.00	6571.25	3910.00	0.00	18796.00	40176.19

Appendix C: GAMS program to estimate the cost-allocation coefficients

```
* Filename "saskat.gms"


* The objective of this GAMS program is to estimate cost-allocation coefficients
* for Saskatchewan crop farms. The program develops the ME-tobit specifications
* without fallow cost and with 11 support values symmetrically spaced over the
* [0, 1] interval.


SETS
   L     raw index    / 1 * 15 /

   T     farm index   / 1 * 30 /

   I     input index  / 1 seeds cost
                        2 fertilizers cost
                        3 pesticides cost
                        4 other direct material cost
                        5 fuel, power and grease cost
                        6 repairs cost
                        7 paid salaries cost
                        8 other fixed cash expenses cost
                        9 net operating income /

   K     output index / 1 wheat output
                        2 other grains output
                        3 canola output
                        4 other oilseeds output
                        5 other crops output /

   M     index parameter support / 1 * 11 /

   N     index error support / 1 * 3 /
```

```
   R     acres index  / 1 fallow land
                        2 wheat
                        3 other grains
                        4 canola
                        5 other oilseeds
                           6 other crops /
;

$INCLUDE "saskat1.prn";
$INCLUDE "acres.prn"

ALIAS(I,J);
ALIAS(T,S);

*  Display the raw data on input used, output used (Table MN)
*  and acreages (Table AA)

DISPLAY MM;
DISPLAY AA;

VARIABLES
OBJ;

PARAMETERS

* Z(M) is the vector of support values for the input/output coefficients

 Z(M)   / 1    0.0
          2    0.1
          3    0.2
          4    0.3
          5    0.4
          6    0.5
          7    0.6
          8    0.7
          9    0.8
         10    0.9
         11    1.0/

 X(T,I)
 XYZ(T,I)
 Y(T,K)
 XR(T)
 FID(T)
 DIF(T)
 DIFF(T)
 V(N,I)
 SIGMA(I,J)
 RSQUARE(I)
 RSQUARR(I)
 CRITP
 CRITU
```

```
 XHAT(T,I)
 XF(T,I)
 W(T,I,N)
 AHAT(K,I)
 AHATU(K,I)
 SUMAHAT(K)
 SUMAHATU(K)
;

*  Defining input and output variables

FID(T)   = MM(T,"1");
Y(T,"1") = MM(T,"2");
Y(T,"2") = MM(T,"3");
Y(T,"3") = MM(T,"4");
Y(T,"4") = MM(T,"5");
Y(T,"5") = MM(T,"6");
X(T,"1") = MM(T,"7");
X(T,"2") = MM(T,"8");
X(T,"3") = MM(T,"9");
X(T,"4") = MM(T,"10");
X(T,"5") = MM(T,"11");
X(T,"6") = MM(T,"12");
X(T,"7") = MM(T,"13");
X(T,"8") = MM(T,"14");
X(T,"9") = MM(T,"15");

XR(T)=SUM(K,Y(T,K))-X(T,"1")-X(T,"2")-X(T,"3")-X(T,"4")-X(T,"5")-X(T,"6")-X(T,"7")-
X(T,"8");
DIF(T)=XR(T)-X(T,"9");
DISPLAY FID ;
DISPLAY DIF;
X(T,"9")=XR(T);

* Estimate of the standard error of the residuals

TABLE V1(N,I)

       1     2        3        4       5        6       7         8         9
 1  -6087 -28599.7 -10036.9 -5620.5 -18715.1 -8056.2 -49891.7 -15351.7 -156467.3
 2      0        0        0       0        0       0        0       0          0
 3   6087 -28599.7  10036.9  5620.5  18715.1  8056.2  49891.7  15351.7  156467.3
 ;



* Establishing the “three-sigma” rule for the residuals

V(N,I)=3*V1(N,I);
DISPLAY V;
```

```
POSITIVE VARIABLES

* Probabilities associated with support values of the
* cost-allocation coefficients and residuals

P(K,I,M)
W1(T,I,N)
W2(T,I,N);

* Defining variables that are positive or equal to zero

XYZ(T,I) = 0$(X(T,I) LE 0) + X(T,I)$(X(T,I) GT 0) ;

EQUATIONS
 OBJECTIVE
 ADD1(K,I)
 ADD21(T,I)
 ADD22(T,I,N)
 ADD31(T,I)
 ADD32(T,I,N)
 CON1(T,I)
 CON2(T,I)
 SUMRESTR(K)
;

* Objective function of the ME problem

OBJECTIVE.. OBJ =E=-SUM(I, SUM(K, SUM(M, P(K,I,M)*LOG(1.e-4+P(K,I,M)))))
                   -SUM(I, SUM(T$(XYZ(T,I) GT 0),
                       SUM(N, W1(T,I,N)*LOG(1.e-4+W1(T,I,N)))))
                  -SUM(I, SUM(T$(XYZ(T,I) LE 0),
                          SUM(N, W2(T,I,N)*LOG(1.e-4+W2(T,I,N)))));

*  SET OF EQUATIONS

*  Conditions that the sum of probabilities P(.), W1(.) and W(.)
   is equal to one

ADD1(K,I)..   SUM(M, P(K,I,M)) =E= 1.0 ;

ADD21(T,I)$(XYZ(T,I) GT 0)..  SUM(N, W1(T,I,N)) =E= 1.0 ;
ADD22(T,I,N)$(XYZ(T,I) LE 0)..  W1(T,I,N) =E= 0.0 ;

ADD31(T,I)$(XYZ(T,I) LE 0)..  SUM(N, W2(T,I,N)) =E= 1.0 ;
ADD32(T,I,N)$(XYZ(T,I) GT 0)..  W2(T,I,N) =E= 0.0 ;

*  Input demand equations

CON1(T,I)$(XYZ(T,I) GT 0)..  XYZ(T,I) =E= SUM(K, SUM(M,
                                   P(K,I,M)*Z(M))*Y(T,K))
                                      +SUM(N, W1(T,I,N)*V(N,I)) ;
```

```
CON2(T,I)$(XYZ(T,I) LE 0)..  SUM(K, SUM(M, P(K,I,M)*Z(M))*Y(T,K))
                               + SUM(N, W2(T,I,N)*V(N,I)) =L= 0 ;

*   Adding-up condition on the cost-allocation coefficients

SUMRESTR(K)..  SUM(I, SUM(M, P(K,I,M)*Z(M))) =E= 1 ;

*  Resolution of program using MINOS

OPTIONS ITERLIM = 20000 ;
OPTIONS RESLIM=3000;
MODEL BRET /ALL/;
OPTION NLP=MINOS5;
SOLVE BRET USING NLP MAXIMIZING OBJ ;

* Display results

* Cost-allocation coefficients

AHAT(K,I) = SUM(M, P.L(K,I,M)*Z(M)) ;
DISPLAY AHAT ;

SUMAHAT(K) = SUM(I, AHAT(K,I)) ;
DISPLAY SUMAHAT ;

* Normalized entropy ratios

CRITP = -SUM(I, SUM(K, SUM(M, P.L(K,I,M)*LOG(1.e-4+P.L(K,I,M)))));
CRITU = -SUM(I, SUM(T$(XYZ(T,I) GT 0),
           SUM(N, W1.L(T,I,N)*LOG(1.e-4+W1.L(T,I,N)))))
            -SUM(I, SUM(T$(XYZ(T,I) LE 0),
              SUM(N, W2.L(T,I,N)*LOG(1.e-4+W2.L(T,I,N)))));
DISPLAY CRITP;
DISPLAY CRITU;

* Estimated residuals

W(T,I,N)=W1.L(T,I,N)$(XYZ(T,I) GT 0) + W2.L(T,I,N)$(XYZ(T,I) LE 0);

* Estimated variance/covariance of residuals

SIGMA(I,J) = SUM(T,(SUM(N, W(T,I,N)*V(N,I))
              * SUM(N, W(T,J,N)*V(N,J))))/(CARD(T)-5);

DISPLAY SIGMA;

* Predicted values of input demand

XHAT(T,I)$(XYZ(T,I) GT 0) = SUM(K, AHAT(K,I)*Y(T,K)$(XYZ(T,I) GT >0));
```

```
* Compute R-square

RSQUARR(I) = SUM(T$(XYZ(T,I) GT 0), XHAT(T,I)*X(T,I))
                 * SUM(T$(XYZ(T,I) GT 0), XHAT(T,I)*X(T,I))/
                    (SUM(T$(XYZ(T,I) GT 0), XHAT(T,I)*XHAT(T,I))
                     * SUM(T$(XYZ(T,I) GT 0), X(T,I)*X(T,I)));
DISPLAY RSQUARR;

* Save estimated matrix of variance-covariance of the estimated
* residuals

FILE SASKASOL /"saska.sol"/;
PUT SASKASOL;
SASKASOL.PC=5;
SASKASOL.PW=250;
LOOP(I,
 LOOP(J, PUT  SIGMA(I,J) ;
  );
 PUT /;
);
```

Appendix D: TSP program to estimate the variance-covariance matrix of the ME-estimated cost-allocation coefficients

```
OPTIONS MEMORY=12;
SET N=30;
SMPL 1,N;
LOAD(file='SASKAT1.prn')FARMID Y1 Y2 Y3 Y4 Y5 X1 X2 X3 X4 X5 X6 X7 X8 X9;

GENR Y9R=Y1+Y2+Y3+Y4+Y5-X1-X2-X3-X4-X5-X6-X7-X8;

? *************************
? DEFINITION OF VARIABLES
? **************************
?
? Y1= WHEAT OUTPUT
? Y2= OTHER GRAINS OUTPUT
? Y3= CANOLA OUTPUT
? Y4= OTHER OILSEEDS OUTPUT
? Y5= OTHER CROPS OUTPUT
?
? X1= SEEDS COST
? X2= FERTILIZERS COST
? X3= PESTICIDES COST
? X4= OTHER DIRECT MATERIAL INPUTS COSTS
? X5= FUEL, POWER AND GREASE COSTS
? X6= REPAIRS COSTS
? X7= PAID SALARIES COSTS
? X8= OTHER FIXED CASH EXPENSES COSTS
? X9R= NET OPERATING INCOME I
```

```
 SMPL 1,45;

 LOAD RR1; 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0
           1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0;

 LOAD RR2; 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0
           0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0;

 LOAD RR3; 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0
           0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0;


 LOAD RR4; 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0
           0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0;

LOAD  RR5; 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1
           0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1;

MMAKE RT RR1 RR2 RR3 RR4 RR5;
MAT R=RT';

SMPL 1 30;
MMAKE YY Y1 Y2 Y3 Y4 Y5;
MAT YYT=YY';
MAT YYYY=YYT*YY;

SMPL 1 9;

? SIGMA1  variance/covariance matrix of the estimated residuals

READ(FILE='sigma1.prn')SIGI1 SIGI2 SIGI3 SIGI4 SIGI5 SIGI6 SIGI7 SIGI8 SIGI9;

MMAKE SIGMA SIGI1 SIGI2 SIGI3 SIGI4 SIGI5 SIGI6 SIGI7 SIGI8 SIGI9;
MAT SIGINV=SIGMA";
MAT II=IDENT(30);
MAT DDD=SIGINV#YYYY;
MAT OMEGA=DDD";
MAT NNINV=(R*OMEGA*RT)";
MAT OMEGAME=OMEGA-OMEGA*RT*NNINV*R*OMEGA;
MAT VAR=DIAG(OMEGAME);
MAT SE=VAR**0.5;
PRINT SE;
STOP;
END;
```

Appendix E: Normalized entropy indicators associated with the estimated cost-allocation coefficients

	Model A1	Model B1	Model C1	Model A2	Model B2	Model C2
1. Seeds						
Wheat	0.470	0.642	0.704	0.459	0.635	0.813
Other grains	0.171	0.348	0.428	0.189	0.367	0.606
Canola	0.317	0.476	0.546	0.319	0.480	0.674
Other oilseeds	0.476	0.699	0.738	0.476	0.689	0.865
Other crops	0.463	0.672	0.719	0.455	0.659	0.844
2. Fertilizers						
Wheat	0.780	0.971	0.967	0.535	0.782	0.937
Other grains	0.716	0.918	0.927	0.644	0.872	0.979
Canola	0.797	0.983	0.976	0.648	0.895	0.984
Other oilseeds	0.548	0.771	0.797	0.477	0.698	0.872
Other crops	0.612	0.839	0.857	0.584	0.835	0.959
3. Pesticides						
Wheat	0.542	0.794	0.797	0.477	0.677	0.838
Other grains	0.086	0.154	0.307	0.044	0.080	0.265
Canola	0.429	0.597	0.669	0.451	0.654	0.843
Other oilseeds	0.979	0.901	0.954	0.972	0.909	0.730
Other crops	0.567	0.762	0.810	0.588	0.807	0.952
4. Other direct material inputs						
Wheat	0.318	0.460	0.541	0.327	0.478	0.670
Other grains	0.382	0.584	0.643	0.376	0.570	0.775
Canola	0.285	0.430	0.503	0.283	0.431	0.616
Other oilseeds	0.303	0.522	0.587	0.294	0.504	0.736
Other crops	0.447	0.647	0.698	0.444	0.641	0.826
5. Fuel, power and grease						
Wheat	0.421	0.617	0.685	0.440	0.652	0.849
Other grains	0.617	0.857	0.862	0.594	0.825	0.955
Canola	0.448	0.679	0.713	0.440	0.677	0.857
Other oilseeds	0.482	0.706	0.742	0.472	0.685	0.863
Other crops	0.451	0.701	0.729	0.439	0.677	0.863

	Model A1	Model B1	Model C1	Model A2	Model B2	Model C2
6. Repairs						
Wheat	0.545	0.720	0.769	0.546	0.729	0.881
Other grains	0.483	0.721	0.754	0.459	0.686	0.871
Canola	0.202	0.378	0.454	0.188	0.361	0.587
Other oilseeds	0.502	0.720	0.753	0.502	0.706	0.872
Other crops	0.314	0.517	0.579	0.303	0.495	0.715
7. Paid salaries						
Wheat	0.504	0.761	0.790	0.548	0.821	0.956
Other grains	0.738	0.943	0.940	0.725	0.925	0.996
Canola	0.621	0.899	0.878	0.628	0.912	0.989
Other oilseeds	0.458	0.697	0.733	0.446	0.674	0.859
Other crops	0.668	0.925	0.909	0.649	0.900	0.986
8. Other fixed cash expenses						
Wheat	0.785	0.958	0.965	0.779	0.960	0.999
Other grains	0.807	0.978	0.975	0.751	0.943	0.999
Canola	0.642	0.894	0.890	0.621	0.872	0.978
Other oilseeds	0.632	0.833	0.847	0.624	0.815	0.937
Other crops	0.580	0.833	0.841	0.559	0.800	0.942
9. Fallow cost						
Wheat	N/A	N/A	N/A	0.249	0.402	0.609
Other grains	N/A	N/A	N/A	0.565	0.783	0.923
Canola	N/A	N/A	N/A	0.209	0.371	0.574
Other oilseeds	N/A	N/A	N/A	0.392	0.594	0.793
Other crops	N/A	N/A	N/A	0.255	0.440	0.659
10. Net operating income						
Wheat	0.788	0.982	0.974	0.902	0.995	0.918
Other grains	0.807	0.977	0.973	0.802	0.969	0.998
Canola	0.955	0.978	0.988	0.986	0.934	0.782
Other oilseeds	0.506	0.751	0.770	0.499	0.736	0.892
Other crops	0.939	0.995	0.997	0.940	0.997	0.930