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Abstract

In this paper, we empirically investigate whether multilateral adjustment to large U.S. external imbalances can help explain movements in the bilateral exchange rates of three commodity currencies---the Australian, Canadian and New Zealand (ACNZ) dollars. To examine the relationship between exchange rates and multilateral adjustment, we develop a new regime-switching model that augments a standard Markov-switching framework with a threshold variable. This enables us to model the exchange rate dynamics of our commodity currencies in the context of two regimes: one in which multilateral adjustment to large U.S. external imbalances is an important factor driving the commodity currencies and the second in which there are no significant U.S. external imbalances and hence multilateral adjustment is not a factor. We compare the performance of this model, both in and out-of-sample, to several other alternative models. In addition to developing this new model, another distinguishing feature of our paper is that we estimate all of our models using a Bayesian approach. We opt for a Bayesian approach in this context because it provides a simpler and more intuitive means of evaluating and comparing our different non-nested models. Moreover, it is relatively straightforward using a Bayesian approach to evaluate the importance of nonlinearities in the relationship between exchange rates and multilateral adjustment. Our findings suggest that during periods of large U.S. imbalances, fiscal and external, an exchange rate model for the ACNZ dollars should allow for multilateral adjustment effects. Moreover, we also find evidence to suggest that the adjustment of exchange rates to multilateral adjustment factors is best modelled as a non-linear process.

JEL classification: F31, F32, C11, C22
Bank classification: Exchange rates; Econometric and statistical methods

Résumé

Dans cette étude, les auteurs cherchent à établir empiriquement si l'ajustement multilatéral aux importants déséquilibres de la balance extérieure des États-Unis peuvent aider à expliquer les mouvements des taux de change bilatéraux de trois monnaies dont le cours est lié aux matières premières, soit les dollars australien, canadien et néo-zélandais (ACNZ). Afin d'examiner la relation entre taux de change et ajustement multilatéral, ils élaborent un nouveau modèle à changement de régime en intégrant une variable de seuil à un modèle standard de Markov. Ceci leur permet de modéliser la dynamique des taux de change des monnaies en question dans le contexte de deux régimes. Dans le premier, l'ajustement multilatéral aux déséquilibres considérables de la balance extérieure des États-Unis influe fortement sur l'évolution des devises, alors que dans le second, cette balance extérieure ne présente pas de déséquilibres notables, si bien que le facteur de l'ajustement multilatéral n'intervient pas. Les auteurs comparent la performance de leur modèle à plusieurs autres, tant à l'intérieur qu'à l'extérieur de l'échantillon. Outre la mise au point de ce nouveau modèle, une particularité de cette étude est que tous les modèles utilisés sont estimés à l'aide d'une approche bayésienne. Cette méthode est retenue parce qu'elle offre, dans ce contexte, une façon plus simple et plus intuitive d'évaluer et de comparer les différents modèles non imbriqués. En outre, il est relativement simple d'utiliser une approche bayésienne pour évaluer l'importance des éléments non linéaires dans la relation entre taux de change et ajustement multilatéral. Les résultats de l'étude donnent à penser que durant les périodes caractérisées par d'importants déficits extérieurs et budgétaires aux États-Unis, un modèle de taux de change applicable aux dollars ACNZ devrait tenir compte des effets de l'ajustement multilatéral. De plus, les fait observés tendent à indiquer que l'ajustement des taux de change aux facteurs reliés à l'ajustement multilatéral est le mieux modélisé s'il est considéré comme un processus non linéaire.

Classification JEL : F31, F32, C11, C22
Classification de la Banque : Taux de change; Méthodes économétriques et statistiques

1 Introduction

In recent years, the U.S. has been running a growing current account deficit which now stands at about 6 per cent of gross domestic product (GDP). Many observers, such as Obstfeld and Rogoff (2000a, 2005), contend that the U.S. current account deficit is on an unsustainable trajectory and that its inevitable unwinding will be accompanied by a significant depreciation of the U.S. dollar. As shown in Table 1 below, some of this adjustment in the U.S. dollar may have already occurred. Indeed, the U.S. dollar (on an effective basis) has depreciated by about 19 per cent since the beginning of 2002. At that time, the United States was running a current account deficit of over 4 per cent of GDP that many observers felt was unsustainable at existing exchange rate levels. This perceived unsustainability is generally believed to have been an important factor in triggering the depreciation of the U.S. dollar over the period 2002-2004. And although the U.S. dollar has not weakened by much since the end of 2004, many observers interpret this as a pause and expect it to resume its depreciation in the not-so-distant future. For example, Obstfeld and Rogoff (2005) and Blanchard et al. (2005) predict that the resolution of global imbalances will involve sizable depreciations of the U.S. dollar (i.e., 30 per cent or more).

Table 1: Nominal appreciation vs. U.S. dollar (percentage)

From January 2, 2002

	To Dec 31, 2004	**To Dec 30, 2005**	**To July 5, 2007**
Canadian Dollar	32.76	37.05	51.23
Euro	49.83	31.06	50.44
British Pound	32.58	18.91	39.17
Japanese Yen	28.54	11.95	7.48
Australian Dollar	51.62	42.62	66.43
New Zealand Dollar	70.14	61.49	84.97
Broad US Dollar Index	−15.34	−12.58	−19.00

Notes:
(i) Based on daily recorded values at 12:00 pm E.S.T. by the Bank of Canada.
(ii) The index for the U.S. dollar was obtained from the Federal Reserve Board.

The purpose of this paper is to examine the hypothesis that because the U.S. economy occupies a predominant position in the world economy, the bilateral exchange rates of U.S. trading partners would appreciate or depreciate relative to the U.S. dollar in order to facilitate global adjustment to large U.S. external imbalances, especially deficits that are potentially unsustainable. We describe this exchange rate adjustment process as "multilateral adjustment" because it involves a sizable movement of the U.S. dollar against most, if not all, of its trading partners.

As shown in Table 1, the values of all of the major floating currencies did indeed appreciate significantly relative to the U.S. dollar over the 2002-2004 period. A similar pattern of exchange rate adjustment also occurred in the mid- to late 1980s, during another episode when the U.S. current account deficit was considered to be large. This stylized evidence thus suggests that during periods when U.S. external imbalances are significant and potentially unsustainable, multilateral exchange rate adjustment may play an important role in the determination of the bilateral exchange rates between the United States and each of its trading partners. Moreover, this observed pattern of global exchange rate adjustment in episodes when U.S. external imbalances are large is consistent with simulations produced by multi-region dynamic general equilibrium (DGE) models based on the new open economy macroeconomics (NOEM) paradigm.[1]

Understanding the implications of the emergence and unwinding of large U.S. external imbalances for the behaviour of the bilateral exchange rates in the N-1 other countries is important because disorderly adjustment to these imbalances could trigger large and abrupt movements in the currencies of America's trading partners. These resulting exchange rate movements could pose a challenge for monetary policy in these economies because they would imply a dramatic change in the relative price of domestic goods and thus could have a substantial impact on aggregate demand. In addition, these exchange rate fluctuations could put pressure on inflation, to varying degrees depending on the extent of exchange rate pass-through. Understanding the causes of these exchange rate movements is, therefore, critical for determining the appropriate monetary policy response.

More generally, understanding the role of multilateral adjustment to U.S. external imbalances in driving bilateral exchange rate movements may contribute to a better understanding of exchange rate determination. As is well known, empirical models of exchange rate determination based on macroeconomic fundamentals have not had much success in either explaining or forecasting exchange rate movements (Meese and Rogoff (1983); Obstfeld and Rogoff (2000b); Cheung et al. (2005)). These standard models, however, typically ignore multilateral influences and this omission may contribute to their poor performance—particularly during episodes when exchange rate movements in many non-U.S. economies are driven largely by multilateral adjustment to U.S. imbalances. Our paper seeks to address this shortcoming in the literature by incorporating multilateral exchange rate adjustment to U.S. external imbalances into a standard empirical exchange rate model based on country-specific macroeconomic fundamentals and testing whether such a feature helps to improve explanatory power and forecasting performance.

[1]For example, see the simulations in Faruqee et al. (2005) using the International Monetary Fund's (IMF) Global Economic Model (GEM).

In modelling the link between exchange rates and multilateral adjustment, it would seem important to account for the non-linear nature of this relationship. Indeed, as we discuss in more detail in Section 2, the stylized evidence for the post-Bretton Woods period suggests that the U.S. current account balance and the U.S. effective exchange rate are correlated but only during episodes when the U.S. current account deficit is large and coincides with the occurrence of a fiscal deficit. Not surprisingly, in other periods, movements in the U.S. current account do not appear to influence exchange rates. This is consistent with evidence that suggests the presence of threshold effects in current account adjustment.[2]

The link between the U.S. twin deficits and multilateral exchange rate adjustment is intuitively appealing because during episodes in the post-Bretton Woods period when the U.S. fiscal deficit was large, the United States often had a substantial current account deficit. Yet the reverse was, in general, less often true, because current account deficits also occurred during investment booms when there was a fiscal surplus. Hence, U.S. current account deficits generated by increases in government spending or tax cuts appear to have been viewed by the market as less sustainable (perhaps because the foreign borrowing was not used to finance investments that would have generated sufficient returns to service the debt) and thus warranted a substantial multilateral adjustment.

In this paper, we empirically investigate whether multilateral adjustment to large U.S. external imbalances can help explain movements in the bilateral exchange rates of Australia, Canada and New Zealand—often referred to as "commodity currencies" because of the importance of commodities in their exports and in the determination of their exchange rates. Given this commonality in the set of fundamentals underlying their exchange rates, these countries seemed like ideal candidates for a comparative analysis. In addition, these currencies have all appreciated significantly in recent years. Part of this appreciation is undoubtedly due to the strenght in commodity prices, but some may also be due to multilateral adjustment to the large U.S. current account deficit. The response of monetary policy to these different sources of the appreciation would likely not be the same since these shocks have different implications for the domestic output gap and expected inflation in these inflation-targeting countries. Thus understanding the magnitude of the impact of multilateral adjustment is critical in determining the central bank's optimal response to an exchange rate appreciation.

To examine the relationship between exchange rates and multilateral adjustment, we develop a new regime-switching model that generalizes a standard Markov-switching model with a time-varying transition matrix that depends on a threshold variable. This model is thus characterized by an unobservable state of the economy which stochastically shifts between

[2]For example, Clarida et al. (2006) find significant evidence of threshold effects in current account adjustment for the G7 countries.

two regimes based on a time-varying transition probability matrix. This enables us to model the exchange rate dynamics of our commodity currencies in the context of two regimes: one in which multilateral adjustment to large U.S. external imbalances is an important factor driving the commodity currencies (in addition to standard country-specific macroeconomic fundamentals) and the second in which there are no significant U.S. external imbalances and hence multilateral adjustment is not a factor. Our threshold variable is the U.S. fiscal balance-to-GDP ratio. Our choice of threshold variable is motivated by the stylized evidence that suggests that multilateral exchange rate adjustment is more likely to occur when U.S. external imbalances are caused in part by fiscal imbalances, most likely because they are perceived as being less sustainable. The time-varying transition matrix is constructed such that if the U.S. fiscal balance-to-GDP ratio is below the threshold value, the probability of the first regime occurring is larger than the probability of the second regime and vice-versa if the threshold variable is above the threshold value.

This new Markov-switching model with a threshold variable is a generalization of the traditional threshold model because movements between regimes are assumed to be stochastic rather than deterministic. We compare the performance of this new Markov-switching model with a threshold variable, both in- and out-of-sample, to several other alternative models, notably a model based on standard country-specific macroeconomic fundamentals, a model that incorporates multilateral adjustment but in a linear manner, a deterministic threshold model and a random walk.

This paper follows the work of Engel and Hamilton (1990) and others in modelling exchange rates in the context of a regime-switching model. It also builds on earlier work by Chen and Rogoff (2003) and Djoudad et al. (2001) that emphasized the role of commodity prices as a key determinant of the Australian, Canadian and New Zealand (ACNZ) dollars. Moreover, this study extends the work of Bailliu et al. (2005) who estimated a threshold model for the Canadian dollar and found evidence that factors related to multilateral adjustment are an important determinant of movements in this currency in periods where U.S. imbalances are significant.

The main contribution of our paper relative to existing work is that we investigate the role of multilateral adjustment in the determination of exchange rate dynamics using a new Markov-switching model with a threshold variable and do so for the Australian and New Zealand dollars, in addition to the Canadian dollar. Another distinguishing feature of our paper is that we estimate all of our models using a Bayesian approach. Although Markov-switching models have been estimated using both the classical and Bayesian approaches, we believe the latter is better for evaluating our new model and comparing its performance to that of the alternative models. There are two main advantages to using a Bayesian approach in this context. First, it provides a simpler and more intuitive means of evaluating and comparing our different non-nested models. And second, with the Bayesian method, it is relatively straightforward to evaluate the importance of nonlinearities in the relationship between exchange rates and multilateral adjustment. Our findings suggest that during periods of large U.S. imbalances, fiscal and external, an exchange rate model for the ACNZ dollars should allow for multilateral adjustment effects. Moreover, we also find evidence to suggest that the adjustment of exchange rates to multilateral adjustment factors is best modelled as a non-linear process.

The paper is organized into five sections. The next section examines large U.S. external imbalances in the post-Bretton Woods period and their implications for the adjustment of exchange rates, including the ACNZ dollars. The empirical exchange rate models that we estimate and compare are presented in Section 3. The methodology used to estimate, evaluate and compare the models is presented in Section 4, along with the empirical results. Concluding remarks are made in the final section.

2 U.S. External Imbalances and Exchange Rates

Since 1960, the United States has run current account deficits on several occasions (as shown in Figure 1). And while the first two episodes in the 1970s (i.e., 1971-72 and 1977-80) were relatively small and short in duration, the two most recent episodes (i.e., 1984-89 and the ongoing episode, which started in 1992) have been much larger (exceeding 2 per cent of GDP for most of the period) and much more persistent. Indeed, over the four episodes, the size and persistence of the deficits have been increasing monotonically; the current deficit has lasted for over 13 years, with no clear signs of abating, and is now in excess of 6 per cent of GDP. The increasing size and persistence of these imbalances is consistent with the argument presented by Greenspan (2004), among others, that the increasing globalization of financial markets has made it easier for countries to run large current account deficits (or surpluses) by reducing the cost and increasing the reach of international financial intermediation, resulting in an increase in the pool of international savings available to finance external deficits.

As shown in Figure 2, there appears to be a strong, albeit slightly out-of-phase, correlation between the U.S. current account deficit and the U.S. nominal effective exchange rate during the latter two episodes when the deficits were the largest.[3] In particular, the figure shows that during the period of the large U.S. current account deficits of the mid-1980s, the U.S. dollar started to appreciate slightly before the U.S. current account deficit widened and then

[3] The U.S. nominal effective exchange rate in Figure 2 is expressed as the U.S.-dollar price of a trade-weighted basket of currencies.

began its depreciation phase, again slightly before, the narrowing of the U.S. current account deficit. During the most recent episode, the U.S. dollar again began to appreciate just before the U.S. deficit widened and since 2002, it has entered a depreciation phase.[4] Moreover, it is worth noting (as shown in Figure 2) that since the mid-1970s, all of the episodes in which the U.S. dollar has undergone large "swings" have coincided with periods characterized by the build-up and unwinding of large current account deficits.[5]

Interestingly, these two periods of large swings in the U.S. dollar and current account balance were also characterized by large U.S. fiscal deficits. This is depicted in Figure 3 which plots the U.S. current account deficit and the U.S. federal government fiscal deficit. Also shown is the difference between private savings and investment, which is calculated as the residual. This breakdown helps to illustrate, consistent with the national income accounting identity, that a U.S. current account deficit can occur when there is either a fiscal deficit and/or a deficit of private savings relative to domestic investment. As shown, over the periods 1984-88, and 2001-06, the large U.S. external imbalances coincided with large fiscal deficits.[6] When this simultaneous occurrence was observed in the 1980s, it was labeled the "twin-deficits" phenomenon, and the argument was made that the significant reductions in taxes and the concomitant increase in military spending during the Reagan administration caused the deficits in both the fiscal and current accounts over this period. There was much public debate over this causal argument at the time, and this debate has been revived in recent years with the re-emergence of the twin deficits under the Bush administration.

There is theoretical support for the view that fiscal deficits can cause current account deficits.[7] Indeed, twin deficits can arise in a non-Ricardian, open-economy model based on the NOEM paradigm—now the standard workhorse in the international macroeconomics literature. Although Ricardian equivalence holds in the original NOEM model based on Obstfeld and Rogoff (1995) (i.e., the *Redux* model), subsequent work has extended this framework to situations where Ricardian equivalence is not expected to hold. For example, Ganelli (2005)

[4]While the currencies of most of the major trading partners of the United States have appreciated over the 2002-2006 period by a relatively large amount, two exceptions stand out: China and Mexico. China's exchange rate had remained almost fixed vis–a-vis the U.S. dollar until July 2005 when there was a small 2.1% revaluation; since then it has appreciated by about another 6%.

[5]Engel and Hamilton (1990) noted that the U.S. dollar appeared to follow long swings and showed that movements in the dollar over the period they studied (i.e., 1973 to 1988) could be described well by Hamilton's (1989) Markov switching model.

[6]It is also noteworthy that in the three recessionary periods (i.e., 1974-75, 1981-82, and 1991-92), there was a slowdown in economic activity, and consequently, the fiscal position went into deficit because of lower tax revenues and increased transfers, and the current account deficit declined as imports fell. In these situations, higher fiscal deficits did not coincide with current account deficits, because aggregate investment fell below savings as economic prospects turned negative.

[7]There is also empirical evidence to support the twin-deficit hypothesis. For example, Bartolini and Lahiri (2006) find that increases in fiscal deficits are associated with declines in the current account in a sample of Organisation for Economic Cooperation and Development (OECD) countries over the 1972 to 2003 period.

combines the *Redux* model with an overlapping generations (OLG) structure of the Blanchard (1985) type. In his set-up, a debt-financed tax cut leads to an increase in domestic consumption (due to the wealth effect of the tax cut) and an appreciation of the domestic currency in the short run.[8] The subsequent expenditure switching effect generates a trade (and hence current account) deficit and a corresponding long-run deterioration in the net foreign asset position. The latter, in turn, generates permanent wealth effects that imply that the effects on relative consumption, the current account and the exchange rate are reversed in the long run. Botman et al. (2006) extend the framework in Ganelli (2005) by introducing two additional channels, in addition to the OLG structure, through which government debt can affect private activity: distortionary taxes and rule-of-thumb consumers.[9]

The predictions of this type of model are therefore consistent with the emergence of the twin deficits as well as with the exchange rate and current account dynamics in the U.S. during the two recent periods when U.S. external imbalances were large and persistent. For instance, this type of model predicts that a debt-financed tax cut will lead to an appreciation (depreciation) of the domestic exchange rate and a deterioration (improvement) of the current account balance in the short run (long run). This was clearly the case in the U.S. during the mid-1980s in the period following the large tax cuts of the Reagan administration. However, the second period of significant external imbalances (from 1998 to the present) is somewhat different because the current account deficit emerged several years before the string of fiscal deficits began in 2001; indeed, the U.S. current account initially went into deficit when there was a large fiscal surplus. The critical difference is that over the period 1998-2000, the current account deficit was caused by an investment boom and relatively low domestic savings and not by fiscal policy.[10] This situation changed starting in 2001, however, when the Bush administration announced large tax cuts and a fiscal deficit emerged; concomitantly, the U.S. dollar continued to appreciate and the current account balance further deteriorated. Thus, it can be argued that the current situation is also consistent with the predictions of Ganelli's (2005) model.

[8] Since domestic consumption increases by more than foreign consumption, the pressure on relative money demand appreciates the domestic currency.

[9] Either one of these three features on their own (i.e., OLG structure, distortionary taxes or rule-of-thumb consumers) would be sufficient to ensure that Ricardian equivalence does not hold. For example, Erceg et al. (2005) rely only on the introduction of rule-of-thumb consumers as a means of introducing non-Ricardian behaviour into their model—the Fed's open economy DGE model (SIGMA)—in their analysis of the effects of U.S. fiscal shocks on the trade balance. Although twin deficits can arise in their model, they find that a fiscal deficit in the U.S. has a relatively small effect on the trade balance.

[10] As a result of this investment-savings gap, foreign capital flowed into the United States in expectation of higher returns owing to the rapid increases in productivity, which were anticipated to continue for the foreseeable future. This expectation of higher productivity growth also increased domestic consumption and reduced savings as U.S. residents intertemporally shifted higher expected future outputs and incomes to the present.

Because the U.S. economy occupies a predominant position in the world economy, it is reasonable to expect that large movements in its exchange rate that occur as a result of the emergence and unwinding of its large external imbalances would also entail significant changes in the values of the currencies of its trading partners, including the three commodity currencies that are the focus of this paper. Figure 4 shows that the strong correlation identified between the U.S. current account deficit and the U.S.-dollar effective exchange rate during the two periods where there were large U.S. twin deficits also applies to the bilateral exchange rates between the U.S. dollar and the ACNZ dollars. In particular, the figure shows that during the period of the large U.S. twin deficits of the mid-1980s, the ACNZ dollars started to depreciate against the U.S. dollar slightly before the U.S. current account deficit widened and then began an appreciation phase, again slightly before, the narrowing of the U.S. current account deficit. During the most recent episode, the commodity currencies again began to depreciate against the U.S. dollar just before the U.S. deficit widened and since 2002, they have all appreciated against the greenback.

This pattern of global exchange rate adjustment in response to a build-up and unwinding of large U.S. twin deficits is consistent with simulations produced by multi-region DGE models based on the NOEM paradigm. For example, Faruqee et al. (2005) use the IMF's GEM to produce a baseline scenario that accounts well for the current episode of global imbalances.[11] Under this scenario, a steady rebalancing of the U.S. current account with an orderly unwinding of financial positions and currency realignments is achieved,[12] including a gradual and generalized real depreciation of the U.S. dollar against the currencies in the other three regions.[13]

In summary, we have tried to demonstrate in this section that multilateral exchange rate adjustment to large U.S. external imbalances may have played a substantial role in explaining bilateral exchange rate movements between the United States and its trading partners over the past few decades. Moreover, this multilateral exchange rate adjustment is more likely to occur when these external imbalances are caused in part by fiscal imbalances, rather than by private investment-savings imbalances. Fiscal deficits often result from an increase in government spending or tax cuts that fuel domestic consumption, neither of which directly expands the productive capacity of the domestic economy. In constrast, investment booms

[11]In their baseline scenario, the global macroeconomic imbalances of the early 2000s can be attributed to a combination of the following factors: expansionary U.S. fiscal policy, declining rates of U.S. private savings, increased foreign demand for U.S. assets, strong productivity growth in Asia, lagging productivity growth in Japan and the euro area, and gaining export competitiveness in emerging Asia.

[12]As the authors emphasize, far less benign adjustment scenarios are also possible. Notably, if mounting concerns over imbalances trigger a sudden loss in appetite for U.S. dollar assets, then this could result in more swift and sizeable changes in interest and exchange rates with negative implications for global growth.

[13]The analysis in Faruqee et al. (2005) is based on a version of GEM with four regions: the United States, Japan and the euro area countries, Emerging Asia and the remaining countries.

have the opposite effect and thus can produce returns to service increased foreign indebtedness. Consequently, current account deficits that are driven primarily by endogenous increases in fiscal deficits are less likely to be sustained at prevailing exchange rates. We also showed that this pattern of global exchange rate adjustment in response to a build-up and unwinding of large U.S. twin deficits is consistent with theory. Thus, incorporating multilateral adjustment to U.S. external imbalances into empirical bilateral exchange rate models may improve their explanatory and forecasting power relative to standard models based only on macroeconomic fundamentals in the two countries.

3 Empirical Exchange Rate Models

In this section, we develop the Markov-switching model with a threshold variable that we use to examine the effects of multilateral adjustment on exchange rates. We first present two other models that we use as benchmarks to assess the performance, both in- and out-of-sample, of our new model. The first is an empirical exchange rate model based on standard country-specific macroeconomic fundamentals that ignores the effects of multilateral adjustment—we refer to this model as our benchmark model. The second is an extension of this first model where we incorporate multilateral adjustment factors but assume that any such effects on the exchange rate are linear—thus there is only one regime in this framework. We refer to this second model as the linear multilateral effects model. We also compare the performance of our new model to two other models: a random walk and a non-linear threshold model. The latter is a special case of our Markov-switching model and will thus be discussed in Section 3.3 along with the presentation of our new model.

3.1 The Benchmark Model

To assess whether multilateral effects are an important determinant of the exchange rates of our ACNZ currencies, it is useful to compare the performance of our new model to an empirical exchange rate model based on macroeconomic fundamentals that ignores the effects of multilateral adjustment. Given that Australia, Canada and New Zealand are all large net exporters of commodities, we adopt as a benchmark model a framework that emphasizes the role of commodities in exchange rate determination. This is consistent with previous work on the determinants of these currencies by Chen and Rogoff (2003) and Djoudad et al. (2001).[14]

[14] Djoudad et al. (2001) find that commodity prices play an economically and statistically significant role in explaining the behaviour of all three currencies. Chen and Rogoff (2003), on the other hand, find that although commodity prices were statistically significant in explaining exchange rate movements for Australia and New Zealand, they were not significant for Canada. Differences in the methodology used most likely explain the

More specifically, we use an exchange rate equation that was initially developed for Canada by Amano and van Norden (1993). This single-equation error-correction model is built around a long-run relationship between the real exchange rate, real energy commodity prices and real non-energy commodity prices.[15] Short-run dynamics are captured by an interest rate differential.

The commodity price indices are intended to be proxies for the terms of trade and should play a role in the determination of the long-run value of our three commodity currencies. Since all three countries are major exporters of non-energy commodities, one would expect that an increase in their price would lead to an appreciation of all three currencies. As for energy commodities, only Australia and Canada are net exporters. Therefore, one would expect that an increase in the price of energy would cause an appreciation of both the Australian and Canadian dollars. As discussed by Amano and van Norden (1993), the interest rate differential may be thought of as a proxy for the difference in the stance of monetary policy—thus the inclusion of this term should capture the usual short-run dynamics implied by the standard monetary models of the exchange rate. Thus, one would expect that an increase in the interest rate differential (i.e., relatively higher interest rates in the domestic economy) would lead to an appreciation of the commodity currencies.

For convenience, we focus on bilateral exchange rates in our paper. This is justified by the fact that the bilateral and effective series for our three countries are highly correlated (as shown in Figures 5 to 7). This result is hardly surprising for a country like Canada, where roughly 87% of exports go to the United States, but it is a little more surprising for Australia and New Zealand, where the U.S. share of exports—at 9% and 15%, respectively—is much smaller.[16] As pointed out by Djoudad et al. (2001), this could be due to the number of their trading partners in Asia that peg their currencies to the U.S. dollar, either explicitly or implicitly.

The specifications for the benchmark model for the Australian, Canadian, and New Zealand dollars, respectively, are as follows:

$$\begin{aligned} \Delta \ln(rfx_t^a) &= \alpha^a(\ln(rfx_{t-1}^a) - \beta^a - \phi^a \ln(comtot_{t-1}^a) - \pi^a \ln(enetot_{t-1}^a)) \\ &+ \delta^a Rintdif_{t-1}^a + \varepsilon_t \end{aligned} \tag{1}$$

mixed results for Canada. Notably, Chen and Rogoff (2003) linearly detrend the unit root variables in their equations.

[15]Under certain circumstances, a single-equation–as opposed to estimating the entire vector error-correction model–can be justified. Indeed, as discussed by Johansen (1992), estimation and inference based on the single-equation system will be equivalent to that of the full system if there is only one cointegrating vector and all the other cointegrating variables are weakly exogenous with respect to the first variable under consideration (in this case, the real exchange rate). As shown in Tables 7–9 and Table 10 in Appendix C, cointegration and weak exogeneity tests generally support this approach for our sample countries.

[16]These figures are averages for the 2000-04 period.

$$\begin{aligned}\Delta \ln(rfx_t^b) &= \alpha^b(\ln(rfx_{t-1}^b) - \beta^b - \phi^b \ln(comtot_{t-1}^b) - \pi^b \ln(enetot_{t-1}^b) \\ &- \eta I(t>\tau)\ln(enetot_{t-1}^b) - \gamma I(t>\tau)) + \delta^b intdif_{t-1}^b + \varepsilon_t \end{aligned} \quad (2)$$

$$\Delta \ln(rfx_t^c) = \alpha^c(\ln(rfx_{t-1}^c) - \beta^c - \phi^c \ln(comtot_{t-1}^c)) + \delta^c Rintdif_{t-1}^c + \varepsilon_t, \quad (3)$$

where rfx^a, rfx^b, and rfx^c are the real dollar exchange rates for the Australian, Canadian and New Zealand dollars, respectively;[17] $comtot^a$, $comtot^b$, $comtot^c$ are the real non-energy price indices for Australia, Canada, and New Zealand;[18] $enetot^a$ and $enetot^b$ are the real energy price indices for Australia and Canada; $Rintdif^a$ and $Rintdif^c$ are the real short-term interest rate differentials with the U.S. for Australia and New Zealand; and $intdif^b$ is the Canada-U.S. short-term interest rate differential. Appendix B provides more details on the data.

Unit-root tests were conducted on all the series in equations (1)-(3) using the DF-GLS test developed by Elliot et al. (1996). The results, as well as a description of this test, are provided in Tables 4–6 in Appendix C. They suggest (for all three countries) that *rfx*, *comtot*, and *enetot* are non-stationary, as assumed.[19] Initial results (which are reported) for the interest rate differential were mixed, and suggested that this variable may be non-nonstationary for Australia and Canada. By increasing the sample size, we were able to find support for our priors that these variables should indeed be stationary.

There are some differences in the equations across the three countries. First, the commodity price variables (i.e., *comtot* and *enetot*) are constructed using different products/weights for the three countries, to reflect the different basket of commodities that each country produces and exports. The composition of each of these indices is shown in Tables 2 and 3 and they are all displayed in Figures 8 and 9. Second, the energy price index is not included as a variable in the equation for New Zealand given that New Zealand does not export energy products. Third, we use the nominal interest rate differential as an explanatory variable for Canada but the real interest rate differential for Australia and New Zealand. Finally, following the work of Issa et al. (2006), we include a structural break in the long-run relationship between the Canadian dollar and energy prices.[20]

[17] In each case, the nominal exchange rate (which is expressed in local currency units) is deflated by the ratio of the GDP deflators for the two countries.

[18] The energy and non-energy price indices are each deflated by the U.S. GDP deflator to convert them into real terms.

[19] Johansen cointegration test results, shown in Tables 7–9 , support the presence of one cointegrating vector between the real exchange rate, real non-energy commodity prices and real energy commodity prices for Australia and Canada, and the presence of one cointegrating vector between the real exchange rate and real non-energy commodity prices for New Zealand.

[20] Issa et al. (2006) find a structural break in the relationship between energy prices and the Canadian dollar. In particular, the sign of the relationship changes from negative to positive in the early 1990s. The authors find this break to be consistent with the large increase in Canadian net exports of energy prices since

3.2 The Linear Multilateral Effects Model

In this section, we extend this benchmark framework by incorporating variables to capture effects on the ACNZ dollars stemming from multilateral exchange rate adjustment to large U.S. external imbalances. In contrast to our Markov-switching model, in this set-up we assume that the relationship between these variables and the exchange rate is linear (i.e., there is only one "regime" in this framework). As discussed in Section 2, the two key variables that reflect U.S. imbalances and that are likely to instigate a multilateral adjustment of the U.S. dollar are the U.S. fiscal and current account balances. Thus, we include these two variables in the regression model.

Unit-root tests were also conducted on these two variables and are reported in Table 4. As shown, the DF-GLS unit-root test suggests that both the fiscal balance-to-GDP ratio and the current account-to-GDP ratio contain a unit root. By increasing the time span used in the tests, we found evidence that the fiscal balance-to-GDP ratio follows a stationary process but that the current account ratio does not. The latter is contrary to what one would expect and suggests that the intertemporal budget constraint is violated and that the current account is on an explosive path. Christopoulos and Leon-Ledesma (2004) also find that traditional tests for the U.S. current account-to-GDP ratio suggest that the series is non-stationary, even when the sample is extended back to 1960. However, they argue that these tests suffer from an important loss of power if the dynamics of the series being tested exhibit non-linearities, which they show is the case for the U.S current account. They address this issue by analyzing the stationarity of the U.S. current account using new econometric tests based on a non-linear adjustment, and find evidence that the U.S. current account-to-GDP ratio is stationary when this non-linearity is taken into account. Given these results and our priors based on theoretical considerations, we decide to treat the U.S. current account-to-GDP ratio as a stationary variable in our analysis.

By making these modifications, we obtain the following specifications for the linear multilateral effects model for our three commodity currencies:

$$\begin{aligned}\Delta\ln(rfx_t^a) &= \alpha^a(\ln(rfx_{t-1}^a) - \beta^a - \phi^a\ln(comtot_{t-1}^a) - \pi^a\ln(enetot_{t-1}^a)) \\ &+ \delta^a Rintdif_{t-1}^a + \chi^a US\_cabal_{t-1} + \lambda^a US\_fisbal_{t-1} + \varepsilon_t\end{aligned} \quad (4)$$

$$\begin{aligned}\Delta\ln(rfx_t^b) &= \alpha^b(\ln(rfx_{t-1}^b) - \beta^b - \phi^b\ln(comtot_{t-1}^b) - \pi^b\ln(enetot_{t-1}^b) \\ &- \eta I(t>\tau)\ln(enetot_{t-1}^b) - \gamma I(t>\tau)) + \delta^b intdif_{t-1}^b \\ &+ \chi^b US\_cabal_{t-1} + \lambda^b US\_fisbal_{t-1} + \varepsilon_t\end{aligned} \quad (5)$$

the early 1990s.

$$\Delta \ln(rfx_t^c) = \alpha^c(\ln(rfx_{t-1}^c) - \beta^c - \phi^c \ln(comtot_{t-1}^c)) + \delta^c Rintdif_{t-1}^c + \chi^c US\_cabal_{t-1} + \lambda^c US\_fisbal_{t-1} + \varepsilon_t, \tag{6}$$

where *US_cabal* is the U.S. current account balance as a proportion of GDP and *US_fisbal* is the U.S. fiscal balance as a proportion of GDP.

3.3 A Markov-Switching Model with a Threshold Variable

In this paper, we generalize the threshold (auto)regressive (THR) model developed by Tong (1977, 1990), Tong and Lim (1980), and Hansen (1996, 2000) by incorporating a threshold variable into the Markov regime-switching (MS) model introduced by Hamilton (1989). Our Markov regime-switching model augmented with a threshold variable (henceforth, MSTV model) is therefore a generalization of the THR model because it allows for some positive probability (where the probability is ≤ 1) of switching regimes based on the position of the threshold variable relative to its threshold value. The MSTV model is thus less restrictive because switches between regimes are assumed to be stochastic rather than deterministic as in the THR model.[21] Our MSTV model is characterized by an unobservable state of the economy which shifts between two regimes based on a time-varying transition probability matrix. This enables us to model the exchange rate dynamics of our commodity currencies in the context of two regimes: one in which multilateral adjustment to large U.S. external imbalances is an important factor driving the commodity currencies (in addition to standard country-specific macroeconomic fundamentals) and the second in which there are no significant U.S. external imbalances and hence multilateral adjustment is not a factor.

The innovative feature of the model is that the transition probability matrix depends on a time-varying threshold variable. Although Hamilton (1989) assumed that the probability of switching from one regime to another is constant, others have used time-varying transition probability matrices in MS models.[22] However, in our MSTV model, the transition probability matrix is not only time-varying but it changes depending on the position of the threshold variable relative to its threshold value. Our threshold variable is the U.S. fiscal balance to GDP ratio. Our choice of threshold variable is motivated by the stylized evidence that suggests that multilateral exchange rate adjustment is more likely to occur when U.S. external

[21] Threshold models split the sample into regimes based on the threshold value of an observed variable, the so-called threshold variable. Given that the threshold value of this variable is typically unknown, it needs to be estimated along with the other parameters of the model.

[22] For example, Ghysels et al. (1998) developed a MS model where the transition probability matrix of the state of the economy changes based on the seasonal characteristics of the data. Moreover, Filardo (1994) also developed an MS model to examine the properties of U.S. business cycles in which the transition probability matrix is time varying and given as a logistic function of an exogenous information variable.

imbalances are caused in part by fiscal imbalances, most likely because they are perceived as being less sustainable. The time-varying transition matrix is constructed such that if the U.S. fiscal balance to GDP ratio is below the threshold value, the probability of state 0 occurring is larger than the probability of the state 1 occurring and vice-versa if the threshold variable is above the threshold value. As far as we know, this paper is the first to develop and use a MS model that incorporates a threshold variable.

We propose the following MSTV model:

$$y_t = \begin{cases} X_t\theta_0 + e_{0,t} & \text{if } S_t = 0, \\ X_t\theta_1 + e_{1,t} & \text{if } S_t = 1, \end{cases} \tag{7}$$

$$e_{0,t} \sim \text{i.i.d.}N(0, \sigma^2_{e0}), e_{1,t} \sim \text{i.i.d.}N(0, \sigma^2_{e1}),$$

where y_t is the dependent variable, X_t is a vector of explanatory variables, and $e_{0,t}$ and $e_{1,t}$ are Gaussian heteroscedastic errors, respectively. This is a regime-switching model with two states (i.e., $S_t = \{0, 1\}$). The specification of the model in each state follows that for the linear multilateral effects model (as shown in equations (4) – (6)), except that the coefficients on all the explanatory variables are allowed to vary in each state. Thus, when the state of the economy S_t is equal to 0, the coefficients are given by θ_0 whereas when the state S_t is equal to 1, they take on the values in θ_1 (where $\theta_1 \neq \theta_0$). $S_t = 0$ is the regime in which multilateral adjustment to U.S. imbalances is an important factor driving the ACNZ dollars and $S_t = 1$ is the other regime.

In this model, the transition probability matrix is time-varying and depends on a threshold variable q_{t-1}:

$$\Gamma_t = \begin{bmatrix} m_0 & 1-n_0 \\ 1-m_0 & n_0 \end{bmatrix} \quad \text{if } q_{t-1} \leq \gamma \quad \text{and} \quad \begin{bmatrix} m_1 & 1-n_1 \\ 1-m_1 & n_1 \end{bmatrix} \quad \text{if } q_{t-1} > \gamma \tag{8}$$

where the (i, j) element of the transition probability matrix represents the probability of $S_t = i$ conditional on $S_{t-1} = j$ and either $q_{t-1} \leq \gamma$ or $q_{t-1} > \gamma$ (e.g., $m_0 = P\left[S_t = 0 | S_{t-1} = 0, q_{t-1} \leq \gamma\right]$). The threshold variable in this case is the U.S. fiscal balance as a proportion of GDP. Following Hansen (1996, 2000), we further assume that the threshold variable q_{t-1} is distributed such that:

$$q_{t-1} \sim \text{i.i.d. with the following cdf:} \quad P[q_{t-1} \leq \gamma] = F^q(\gamma), \tag{9}$$

where $F^q(\gamma)$ is the probability that the threshold variable q_{t-1} takes on a value of less than or equal to γ.

An important characteristic of the MSTV model consisting of equations (7), (8) and (9) is that it nests the THR model as a special case with the following transition probabilities: $m_0 = n_1 = 1$ and $m_1 = n_0 = 0$. The transition probability matrix for the THR model is thus given by:

$$\Gamma_t = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \quad \text{if } q_{t-1} \leq \gamma \quad \text{and} \quad \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \quad \text{if } q_{t-1} > \gamma. \tag{10}$$

The transition probability matrix in (10) implies that, if $q_{t-1} \leq \gamma$, the current state of the economy (S_t) is 0 with probability 1, independent of the past state of the economy (S_{t-1}). Thus, if the U.S. fiscal balance is below the estimated threshold value, then the economy is in the regime characterized by multilateral adjustment. Similarly, if $q_{t-1} > \gamma$, the current state of the economy (S_t) is 1 with probability 1, independent of the past state of the economy (S_{t-1}). Thus, if the U.S. fiscal deficit is above the estimated threshold value (or is in surplus), then the economy is in the other regime. We can thus rewrite the MSTV model in (7) for this special case where it becomes a THR model:

$$y_t = \begin{cases} X_t\theta_0 + e_{0,t} & \text{if } q_{t-1} \leq \gamma, \\ X_t\theta_1 + e_{1,t} & \text{if } q_{t-1} > \gamma. \end{cases} \tag{11}$$

Therefore, the MSTV model nests the THR model as a special deterministic case with the more restrictive transition probability matrix given by (10).

The time-varying transition matrix (8) has another useful non-time-varying representation. To see this, we introduce a new state variable S_t* which can take on four discrete values as follows:

$$S_t^* = \begin{cases} 0 & \text{if} \quad S_t = 0 \text{ and } q_t \leq \gamma \\ 1 & \text{if} \quad S_t = 0 \text{ and } q_t > \gamma \\ 2 & \text{if} \quad S_t = 1 \text{ and } q_t \leq \gamma \\ 3 & \text{if} \quad S_t = 1 \text{ and } q_t > \gamma \end{cases}.$$

From the transition probability matrix (8) and the cumulative density function of the threshold variable q_{t-1}, $F^q(\gamma)$, we can show that the new state variable S_t^* has the following non-time-varying transition matrix Γ^*:

$$\Gamma^* = \Gamma^*(m_0, n_0, m_1, n_1, \gamma) = \{P[S_t^* = i|S_{t-1}^* = j]\}$$
$$\begin{bmatrix} F^q(\gamma)m_0 & F^q(\gamma)m_1 & F^q(\gamma)(1-n_0) & F^q(\gamma)(1-n_1) \\ [1-F^q(\gamma)]m_0 & [1-F^q(\gamma)]m_1 & [1-F^q(\gamma)](1-n_0) & [1-F^q(\gamma)](1-n_1) \\ F^q(\gamma)(1-m_0) & F^q(\gamma)(1-m_1) & F^q(\gamma)n_0 & F^q(\gamma)n_1 \\ [1-F^q(\gamma)](1-m_0) & [1-F^q(\gamma)](1-m_1) & [1-F^q(\gamma)]n_0 & [1-F^q(\gamma)]n_1 \end{bmatrix}. \tag{12}$$

This can be shown by writing out the probability of $S_t^* = 0$ conditional on $S_{t-1}^* = 0$ as follows:

$$\begin{aligned} P[S_t^* = 0|S_{t-1}^* = 0] &= P[S_t = 0, q_t \leq \gamma|S_{t-1} = 0, q_{t-1} \leq \gamma] \\ &= P[q_t \leq \gamma|S_{t-1} = 0, q_{t-1} \leq \gamma]P[S_t = 0|q_t \leq \gamma, S_{t-1} = 0, q_{t-1} \leq \gamma] \\ &= F^q(\gamma)P[S_t = 0|S_{t-1} = 0, q_{t-1} \leq \gamma] \\ &= F^q(\gamma)m_0. \end{aligned}$$

Similar expressions can be written out for all the other sets of S_t^* and S_{t-1}^* to derive the conditional probabilities $P[S_t^* = i|S_{t-1}^* = j]$ for $i, j = \{0, 1, 2, 3\}$. In the MSTV model, therefore, the probability of being in the current state of the economy depends on the past state of the economy through the first-order Markov process (12). Furthermore, once the transition probability matrix (12) is constructed, the threshold variable q_t has no additional information for the model because q_t affects the model only through its cumulative density $F^q(\gamma)$.[23]

In order to identify our MSTV model , we need to impose two sets of restrictions. First, we impose the following inequality restrictions on the transition probability matrices in equations (8) and (12): $m_0 > m_1$ and $n_1 > n_0$. To illustrate the economic intuition behind these two inequalities, it is useful to point out that $P[S_t = 0|S_{t-1} = 0, q_{t-1} \leq \gamma] > P[S_t = 0|S_{t-1} = 0, q_{t-1} > \gamma]$ and $P[S_t = 0|S_{t-1} = 1, q_{t-1} \leq \gamma] > P[S_t = 0|S_{t-1} = 1, q_{t-1} > \gamma]$ both hold. These inequalities with respect to the conditional probabilities imply that, if the U.S. fiscal balance is below or equal to the estimated threshold value (i.e., $q_{t-1} \leq \gamma$), then the state associated with multilateral adjustment (i.e., State 0) has a higher probability of occurring in the current period than it would if it was above the estimated threshold value (i.e., $q_{t-1} > \gamma$)—regardless of the state of the economy in the previous period. Similarly, it is worth noting that $P[S_t = 1|S_{t-1} = 1, q_{t-1} > \gamma] > P[S_t = 1|S_{t-1} = 1, q_{t-1} \leq \gamma]$ and

[23]This property comes from our assumption that q_t is identically and independently distributed with the cumulative density $F^q(\gamma)$.

$P[S_t = 1|S_{t-1} = 0, q_{t-1} > \gamma] > P[S_t = 1|S_{t-1} = 0, q_{t-1} \leq \gamma]$ also both hold. Therefore, if the U.S. fiscal balance as a proportion of GDP is above the estimated threshold value (i.e., $q_{t-1} > \gamma$), then the state that is not associated with multilateral adjustment (i.e., State 1) has a higher probability of occurring in the current period than it would if it was below or equal to the estimated threshold value (i.e., $q_{t-1} \leq \gamma$)—regardless of the state of the economy in the previous period. Hence, in this model, State 0 is identified as the state which has a strictly higher probability of occurring when the threshold variable (q_{t-1}) is less than or equal to the threshold value (γ). Similarly, State 1 is identified as the state which has a strictly higher probability of occurring when the threshold variable (q_{t-1}) is greater than the threshold value (γ). And second, we allow the prior distributions of the parameters vectors (θ_0 and θ_1) to differ in the two states in our Bayesian estimation of our MSTV model, which is explained in the next section. This inequality also helps to identify our model.

4 Estimation and Model Evaluation

4.1 Bayesian Estimation

We estimate our MSTV model, as well as our three alternative models, using a Bayesian approach. Although MS models have been estimated using both the classical and Bayesian approaches, we believe the latter is a better approach for evaluating our MSTV model and comparing its performance to that of the alternative models. There are two main advantages to using a Bayesian approach in this context. First, it provides a simpler and more intuitive means of evaluating and comparing different non-nested models. In our paper, we compare our MSTV model to three other models. Although the THR model is nested within the MSTV model, the benchmark and linear multilateral effects models are not.[24] And comparing these non-nested specifications using the classical approach is challenging because there is no obvious classical test statistic that we can use to compare the models in terms of their overall ability to fit the data.[25] These non-nested models, however, can easily be compared under the Bayesian approach by comparing the marginal likelihood of each model (which is simply the probability of the data given the model). Thus, the best model will be the one with the highest marginal likelihood. Second, with the Bayesian method, it is relatively straightforward to evaluate the importance of nonlinearities in the relationship between exchange rates

[24] Even though the THR model is nested within the MSTV model, we cannot apply the standard LR statistic to compare the two models because the THR model imposes restrictions on the boundaries of the admissible ranges of parameters of the MSTV model.

[25] One cannot simply compare the likelihoods across models because the likelihood functions depend on parameters that are not shared across all the models.

and multilateral adjustment. This is simply accomplished by comparing the marginal likelihoods of the linear multilateral effects model and the two nonlinear models with multilateral adjustment (i.e., the THR and MSTV models). On the other hand, it is not clear how in the context of the MSTV model one would test for the threshold effect using the classical approach given that the threshold parameter is not identifiable under the null hypothesis of "no threshold effect"; the threshold parameter is only identifiable as a nuisance parameter under the alternative.

In order to estimate our MSTV model in equation (7) using the Bayesian approach, we first need to specify our joint prior distribution of the parameters of our model, $P(\theta_0, \theta_1, m_0, m_1, n_0, n_1, \gamma, \sigma_{e0}, \sigma_{e1})$. We then draw posterior inferences on the parameters, $P(\theta_0, \theta_1, m_0, m_1, n_0, n_1, \gamma, \sigma_{e0}, \sigma_{e1}|y^T, X^T)$. Letting Θ denote the parameter set for our model, this is done using Bayes' theorem:

$$P(\Theta|y^T, X^T) \propto P(\Theta)L(y^T|\Theta, X^T), \tag{13}$$

where $L(y^T|\Theta, X^T)$ is the likelihood of the data y^T conditional on the parameters of the model θ and the set of explanatory variables X^T.

It is relatively straightforward to construct the likelihood function for the MSTV model given the data y^T and X^T. Using the Gaussian property of the error terms $e_{0,t}$ and $e_{1,t}$, we can construct the log-likelihood function $\ln L$ as follows:

$$\begin{aligned} \ln L(y^T|\Theta, X^T) &= \Sigma_{t=1}^T \ln f(y_t|\Psi_{t-1}, X_t) \\ &= \Sigma_{t=1}^T \ln\{\Sigma_{S_t^*=0}^3 f(y_t|S_t^*, \Psi_{t-1}, X_t)P[S_t^*|\Psi_{t-1}, X_t]\}, \end{aligned} \tag{14}$$

where

$$f(y_t|S_t^*, \Psi_{t-1}, X_t) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma_{e0}^2} \exp\left\{-\frac{(y_t - X_t\theta_0)^2}{2\sigma_{e0}^2}\right\} & \text{if } S_t^* = 0 \text{ or } S_t^* = 1, \\ \frac{1}{\sqrt{2\pi}\sigma_{e1}^2} \exp\left\{-\frac{(y_t - X_t\theta_1)^2}{2\sigma_{e1}^2}\right\} & \text{if } S_t^* = 2 \text{ or } S_t^* = 3, \end{cases}$$

and the second equality comes from the probabilistic fact that $f(y_t) = \sum_{S_t^*=0}^3 f(y_t, S_t^*)$ and $f(y_t, S_t^*) = P[S_t^*]f(y_t|S_t^*)$.

The main difficulty in constructing the likelihood function lies in drawing an inference on S_t^* conditional on the past information set Ψ_{t-1} and the current X_t, i.e., $P[S_t^*|\Psi_{t-1}, X_t]$. In this paper, we exploit Hamilton's (1989) filter to construct $P[S_t^*|\Psi_{t-1}, X_t]$ for $t = 1, 2, ..., T$.

Note that:

$$
\begin{aligned}
P[S_t^*|\Psi_{t-1}, X_t] &= \Sigma_{S_{t-1}^*=0}^3 P[S_t^*, S_{t-1}^*|\Psi_{t-1}, X_t] \\
&= \Sigma_{S_{t-1}^*=0}^3 P[S_{t-1}^*|\Psi_{t-1}, X_t] P[S_t^*|S_{t-1}^*, \Psi_{t-1}, X_t] \\
&= \Sigma_{S_{t-1}^*=0}^3 P[S_{t-1}^*|\Psi_{t-1}] P[S_t^*|S_{t-1}^*], \qquad (15)
\end{aligned}
$$

where the third equality is the direct result of the transition probabilities (12). Given $P[S_{t-1}^*|\Psi_{t-1}]$, equations (12) and (15) therefore yield $P[S_t^*|\Psi_{t-1}, X_t]$. Then, given the data y_t, we update our inference on the current state of the economy following the updating formula:

$$
\begin{aligned}
P[S_t^*|\Psi_t] &= P[S_t^*|\Psi_{t-1}, y_t, X_t] \\
&= \frac{f(y_t, S_t^*|\Psi_{t-1}, X_t)}{f(y_t|\Psi_{t-1}, X_t)} \\
&= \frac{P[S_t^*|\Psi_{t-1}, X_t] f(y_t|S_t^*, \Psi_{t-1}, X_t)}{\Sigma_{S_t=0}^3 P[S_t^*|\Psi_{t-1}, X_t] f(y_t|S_t^*, \Psi_{t-1}, X_t)}. \qquad (16)
\end{aligned}
$$

Using $P[S_t^*|\Psi_{t-1}, X_t]$, $f[y_t|S_t^*, \Psi_{t-1}, X_t]$, and y_t as inputs, formula (16) updates our inference on the current state of the economy, $P[S_t^*|\Psi_t]$. Iterating equations (15) and (16) from $t = 1$ to T generates the sequence of $P[S_t^*|\Psi_t]$ for $t = 1, 2, ..., T$.

We exploit a random-walk Metropolis-Hastings algorithm to simulate the posterior joint distribution of the MSTV parameters in (13). We use normal, beta, and inverse-gamma distributions as the prior distributions of the regression parameters, the transition probabilities, and the standard deviations of the Gaussian error terms, respectively. We assume that all parameters are distributed independently. To identify the MSTV model, the inequality restrictions $m_0 > m_1$ and $n_1 > n_0$ are imposed with probability 1 during the Markov-chain Monte-Carlo (MCMC) samplings. Following the recommendation of Gelman et al. (2004, chap. 11), we choose the approximated mode of the posterior joint distribution as the initial value of Θ for the MCMC samplings, and use the inverse Hessian matrix evaluated at the mode as the variance-covariance matrix of the multivariate normal proposal distribution of the random-walk Metropolis-Hastings algorithm. All the posterior inferences in this paper are based on 100,000 MCMC samplings out of which the first 10,000 draws are discarded. We set the rejection rate around 20 per cent.[26]

[26]Before implementing the MCMC samplings, we need to have a consistent estimate of the cdf of threshold variable q_t, $F^q(\gamma)$. In this paper, $F^q(\gamma)$ is approximated by the cumulative normal distribution with the mean and standard deviation of q_{t-1}. Another way to estimate F^q would be to use a nonparametric kernel density estimation. Or alternatively, one could characterize the parametric stochastic process of q_t as an AR process (i.e., $q_t = \rho q_{t-1} + \epsilon_t$ where ϵ_t is white noise). In the latter case, the transition probability matrix (12) would also be time-varying depending on q_{t-1}. We leave these alternative approaches for future research.

4.2 Estimation Results

In this section, we present the estimation results for our MSTV model as well as for the alternative models for our three sample countries (all the relevant tables are found in Appendix D). We present and discuss our results for Canada first, followed by those for Australia and New Zealand.

4.2.1 Estimation results for Canada

The estimation results for Canada are shown in Tables 11–14 in Appendix D. All models for Canada are estimated using data for the period 1973Q1 to 2005Q4. Table 11 depicts the estimates for the benchmark model. The prior distribution for the benchmark model was constructed using the ordinary least squares (OLS) point estimates and standard errors. As shown by the results for the posterior distribution, the parameters are estimated fairly precisely and are of the expected sign. The estimated long-run effects suggest that an increase in the non-energy price index leads to an appreciation of the Canadian dollar (in real terms), as does an increase in the energy price index after 1993.[27] And the coefficient on the variable that captures short-run dynamics suggests that an increase in the Canada-U.S. short-term interest rate spread will induce an appreciation of the Canadian dollar.

Table 12 depicts the estimates for the linear multilateral effects model. The prior distribution for this model was also constructed using the OLS point estimates and standard errors. The results for the posterior distribution suggest that the coefficients for all of the explanatory variables, except for the U.S. current account-to-GDP ratio, are estimated fairly precisely. Moreover, the coefficients on all the variables, including those that are intended to capture multilateral adjustment effects, are of the expected sign. Indeed, the coefficients on the multilateral adjustment variables suggest that a deterioration of both the U.S. fiscal and current account balances (as a proportion of GDP) will lead to an appreciation of the Canadian dollar.

The estimates for the prior and posterior distributions for our MSTV model for Canada are shown in Table 13. Our prior densities for all the coefficients are equal to those used for the linear multilateral effects model, with one important exception: the prior means for the coefficients on our two multilateral variables are set equal to zero in State 1. The latter reflects our prior that multilateral adjustment should only be an important determinant of the Canadian dollar in the regime where U.S. external imbalances are large, and are caused in part by a fiscal deficit. We use the sample median and standard deviation of the U.S. fiscal balance-to-GDP ratio to construct our prior distribution for the threshold parameter

[27]Prior to 1993, an increase in energy prices led to a depreciation of the Canadian dollar.

(γ). The construction of our priors for the two non-linear models is intended to enable us to test easily for the imporance of non-linearities in the link between multilateral adjustment and exchange rate dynamics (i.e., the priors are the same as those for the linear models with the exception of the parameters on the non-linear effects).

The results for our posterior distributions for the MSTV model suggest that the estimated value of the threshold parameter is -2.39%. Thus, in the MSTV model, State 0 has a higher probability of occurring when the U.S. is running a fiscal deficit that is larger or equal to 2.39% of GDP. State 1 thus has a higher probability of occurring when the U.S. fiscal deficit is smaller than 2.39% of GDP. Figure 10 plots the evolution of the posterior mean of the probability of State 0 occuring along with 95% credible intervals.[28] As shown, the probability of State 0 occuring has increased since 2001 and is now about 70%. Thus, our results suggest that there is a fairly high probability that we have entered the multilateral adjustment regime in recent years where we can expect the Canadian dollar to adjust in response to a global adjustment in the U.S. dollar. Figure 11 plots the evolution of the threshold variable – the U.S. fiscal balance as a proportion of GDP – along with the posterior mean of the probability of State 0 occuring. As shown, in periods when the threshold variable falls below its estimated threshold value, the probability of State 0 occuring increases. Or in other words, when the U.S. fiscal deficit becomes large enough, our results suggest that the probability of being in State 0 – the state in which a multilateral adjustment of the U.S. dollar is more likely to occur – rises.

The coefficients on all the variables in the MSTV model, including those that are intended to capture the multilateral adjustment effects, are of the expected sign. In contrast to our results for the linear multilateral effects model, the results for the posterior distribution suggest that the coefficients of all the explanatory variables, including that for the U.S. current account-to-GDP ratio, are estimated fairly precisely. Moreover, the magnitude of the coefficient on the U.S. current account-to-GDP ratio is much larger (more than 3 times larger) compared to its size in the linear multilateral effects model. The results from the posterior distribution are thus consistent with our prior that multilateral adjustment factors are an important determinant of the Canadian dollar, but only in periods where the U.S. is running large twin deficits. In the other regime, multilateral adjustment factors do not influence the exchange rate.

Finally, the results for the THR model (shown in Table 14) are very similar to those for the MSTV model both in terms of the magnitude of the coefficients and the precision with which they are estimated.[29] Moreover, the estimated value of the threshold parameter is roughly

[28] Appendix E provides a description of how we draw inference on the probabilities of the states of the economy.

[29] The priors for the THR model as the same as those for the MSTV model.

the same. Thus, in the THR model, State 0 is characterized by a situation in which the U.S. is running a fiscal deficit that is larger than 2.39% of GDP. State 1 is then characterized by periods in which the U.S. fiscal deficit is smaller than 2.39% of GDP or in which there is a fiscal surplus. Figure 12 plots the evolution of the two multilateral variables –the U.S. current and fiscal account balances as a proportion of GDP – across the two regimes identified by the THR model for Canada, where the shaded area represents State 0. As expected, in each case, State 0 contains most of the observations for the periods when there were large twin deficits in the United States. Consistent with our results for the MSTV model, our results for the THR model suggest that in most quarters since 2001, the probability of being in State 0 has been 1.

4.2.2 Estimation results for Australia

The estimation results for Australia are shown in Tables 15–18 in Appendix D. All models for Australia are estimated using data for the period 1985Q1 to 2005Q4. The prior distributions for all four models for Australia were constructed in the same way as they were for Canada. Table 15 depicts the results for the benchmark model. As shown, the parameters from the posterior distribution are estimated fairly precisely, except for the coefficient on the energy price index. Moreover, the coefficients are of the expected sign, again with the exception of the energy price index. Thus, the estimated long-run effects suggest that an increase in the non-energy price index leads to an appreciation of the Australian dollar (in real terms), but that changes in the energy price index do not appear to influence this currency. An increase in the Australian-U.S. short-term interest rate spread seems to induce an appreciation of the Australian dollar, as expected, although the magnitude of this coefficient is relatively small.

Table 16 depicts the estimates for the linear multilateral effects model for Australia. The results for the posterior distribution suggest that the coefficients for all of the explanatory variables, except for the energy price index, are estimated fairly precisely. Moreover, the coefficients on all the variables, except the energy price index, are of the expected sign. This includes both variables that are intended to capture multilateral adjustment effects, in contrast to the results of the linear multilateral effects model for Canada where only the fiscal variable was estimated precisely. Moreover, the magnitude of the coefficients on the two multilateral variables is much larger than those estimated for Canada. Thus, the coefficients on the multilateral adjustment variables suggest that a deterioration of both the U.S. fiscal and current account balances (as a proportion of GDP) will lead to an appreciation of the Australian dollar.

The estimates for the prior and posterior distributions for our MSTV model for Australia are shown in Table 17. The estimated value of the threshold parameter from the posterior distribution is -1.29%. Thus, in the MSTV model, State 0 has a higher probability of occurring when the U.S. is running a fiscal deficit that is larger or equal to 1.29% of GDP. State 1 thus has a higher probability of occurring when the U.S. fiscal deficit is smaller than 1.29% of GDP. Figure 13 plots the evolution of the posterior mean of the probability of State 0 occuring along with 95% credible intervals. As is the case for Canada, the probability of State 0 occuring has increased in recent years – for Australia it is now above 70%. Thus, our results suggest that there is a fairly high probability that we have entered the multilateral adjustment regime in recent years where we can expect the Australian dollar to continue to adjust in response to a global adjustment in the U.S. dollar. Figure 14 plots the evolution of the threshold variable – the U.S. fiscal balance as a proportion of GDP – along with the posterior mean of the probability of State 0 occuring. As shown, in periods when the threshold variable falls below its estimated threshold value, the probability of State 0 occuring increases.

Although the coefficients on the interest rate spread and the energy price index are not estimated very precisely, the coefficients on all the other variables are, including those that are intended to capture the multilateral adjustment effects. The latter are also of the expected sign. Finally, the results for the THR model (shown in Table 18) are very similar to those for the MSTV model both in terms of the magnitude of the coefficients and the precision with which they are estimated, with one important exception. The estimated value of the threshold parameter from the posterior distribution is larger; in fact it is positive (at 0.387%). It is also not estimated very precisely. Thus, in the THR model, State 0 is characterized by a situation in which the U.S. is running a fiscal surplus that is smaller than 0.387% of GDP (or is running a deficit). State 1 is then characterized by periods in which the U.S. fiscal surplus is larger than 0.387% of GDP. Figure 15 plots the evolution of the two multilateral variables –the U.S. current and fiscal account balances as a proportion of GDP – across the two regimes identified by the THR model for Canada, where the shaded area represents State 0. Most of the observations for the THR model for Australia fall into State 0, most likely a reflection of the fact that the threshold estimate in not estimated very precisely in this case.

Our results for Australia suggest that the Australian dollar responds more strongly to multilateral adjustment to the U.S. dollar than does the Canadian dollar. This may seem counterintuitive given the stronger economic ties between Canada and the United States. However, this result is consistent with simulations conducted in the context of the Global Economic Model which show that the size of an exchange rate adjustment in response to a U.S. fiscal shock will be smaller if the trade links between the two countries are stronger.[30]

[30] See, for example, Faruqee et al. (2005). The intuition is that the fiscal shock will create an external imbalance. If the trade linkages are substantial then a smaller exchange rate adjustment would be required to restore the external balance.

4.2.3 Estimation results for New Zealand

The estimation results for New Zealand are shown in Tables 19–22 in Appendix D. All models for New Zealand are estimated using data for the period 1986Q1 to 2005Q4. The prior distributions for all four models for New Zealand were constructed in the same way as they were for Canada and Australia. Table 19 depicts the results for the benchmark model. As shown, the parameters from the posterior distribution are estimated fairly precisely and are of the expected sign. Thus, the estimated long-run effect suggests that an increase in the non-energy price index leads to an appreciation of the New Zealand dollar (in real terms). An increase in the New Zealand-U.S. short-term interest rate spread seems to induce an appreciation of the New Zealand dollar, as expected, although the magnitude of this coefficient is relatively small (as is the case for Australia).

Table 20 depicts the estimates for the linear multilateral effects model for New Zealand. The results for the posterior distribution suggest that the coefficients for all of the explanatory variables, except for the interest rate spread, are estimated fairly precisely. Moreover, the coefficients on all the variables, except the interest rate spread, are of the expected sign. This includes both variables that are intended to capture multilateral adjustment effects, in contrast to the results of the linear multilateral effects model for Canada where only the fiscal variable was estimated precisely. Moreover, as is the case for Australia, the magnitude of the coefficients on the multilateral variables is much larger than for Canada. Thus, the coefficients on the multilateral adjustment variables suggest that a deterioration of both the U.S. fiscal and current account balances (as a proportion of GDP) will lead to an appreciation of the New Zealand dollar.

The estimates for the prior and posterior distributions for our MSTV model for New Zealand are shown in Table 21. The estimated value of the threshold parameter from the posterior distribution is -1.53%. Thus, in the MSTV model, State 0 has a higher probability of occurring when the U.S. is running a fiscal deficit that is larger or equal to 1.53% of GDP. State 1 thus has a higher probability of occurring when the U.S. fiscal deficit is smaller than 1.53% of GDP. Figure 16 plots the evolution of the posterior mean of the probability of State 0 occuring along with 95% credible intervals. As is the case for Canada and Australia, the probability of State 0 occuring has increased in recent years. Thus, our results suggest that there is a fairly high probability that we have entered the multilateral adjustment regime in recent years where we can expect the New Zealand dollar to continue to adjust in response to a global adjustment in the U.S. dollar. Figure 17 plots the evolution of the threshold variable – the U.S. fiscal balance as a proportion of GDP – along with the posterior mean of the probability of State 0 occuring. As shown, in periods when the threshold variable falls below its estimated threshold value, the probability of State 0 occuring increases.

Although the coefficient on the interest rate spread is not estimated very precisely, the coefficients on all the other variables are, including those that are intended to capture the multilateral adjustment effects. The latter are also of the expected sign. Finally, as is the case for Australia, the results for the THR model (shown in Table 22) are very similar to those for the MSTV model except that the estimated value of the threshold parameter from the posterior distribution is larger and positive (at 0.584%). It is also not estimated very precisely.

4.3 Model Evaluation and Comparison

In this paper, we evaluate and compare our MSTV model and our three alternative models with respect to their (i) overall statistical fit; (ii) performance in in-sample dynamic simulations; and (iii) performance in out-of-sample forecasting exercises.

We compare the different models using three measures of overall statistical fit. First, we use the marginal likelihood which is simply a measure of the probability of the data given the model; so the better model will have the higher marginal likelihood. And as pointed out by Geweke (1999), the marginal likelihood also summarizes the out-of-sample prediction record of the model. Thus, the marginal likelihood is both a measure of a model's adequacy and of its out-of-sample prediction record. Second, we compare models using the posterior odds ratio—more technical details on how this measure is constructed is provided in Appendix E. The posterior odds ratio of model i versus model j provides an indication of whether model j fits the data better than model j, which will be the case if it is larger than 1. In order to construct the posterior odds ratio, we assign the same prior model probability across all models. And finally, we also compare competing models using the deviance information criterion (DIC). One of the advantage of the DIC is that it is a measure of fit that takes into account model complexity.[31] There are thus two components to the DIC: one that measures goodness of fit and the second that can be thought of as a penalty term for increasing model complexity (as measured by the effective number of parameters in the model). The smaller the DIC of model i, the better the model fits the data. Technical details on the construction of the DIC are provided in Appendix E.

So based on these three measures of goodness of fit, we can rank the models in terms of their overall statistical fit for each one of our sample countries. As shown in Table 23, the best model of the Canadian dollar is the MSTV model, followed by the THR model, then the linear multilateral effects model and finally the benchmark model. For Australia, as shown in Table 25, the best model is the THR model, followed by the MSTV model, the linear

[31]It should be noted that the marginal likelihood also takes into account model complexity.

multilateral effects model and finally the benchmark model. And as depicted in Table 27, the results for New Zealand are mixed. The posterior odds ratio and the marginal likelihood select the linear multilateral effects model as the best model, whereas the DIC suggests that the MSTV model is the best performer.

As discussed earlier, one of the advantages of using a Bayesian approach in this context is that it is relatively straightforward to evaluate the importance of nonlinearities in the relationship between exchange rates and multilateral adjustment.This is accomplished by comparing the marginal likelihood of the linear multilateral effects model and the two nonlinear models with multilateral effects (i.e., the THR and MSTV models). As shown in Tables 23 and 25, the marginal likelihood of the two nonlinear models is higher than that of the linear multilateral effects model for Canada and Australia. Thus, the results for these two countries suggest that the adjustment of exchange rates to multilateral factors are best modelled as a nonlinear process. The results for New Zealand, however, do not support this view. Indeed, as shown in Table 27, in the case of New Zealand the marginal likelihood is higher for the linear multilateral effects model than it is for the two nonlinear models.

Next, we compare the different models for our sample countries using in-sample, dynamic simulations. Technical details on how we construct these dynamic simulations is provided in Appendix E. Figures 19, 20, and 21 depict the dynamic simulations for our three sample countries along with credible intervals and the posterior mean of the Theil U-statistic.[32] As shown in Figure 19, all of the models for Canada are fairly successful at accounting for broad movements in the Canada-U.S. dollar over the sample period. As shown the correspondence between the actual and simulated values is quite close. There are, however, differences across the models. Indeed, the simulated values from the MSTV and THR models appear to match more closely the actual values than the other two models. The Theil U-statistic suggests that the MSTV model is the best model for Canada. In contrast to the simulations for Canada, the models are not as successful at tracking the broad movements in the Australian and New Zealand dollars (as shown in Figures 20 and 21). Nonetheless, the MSTV and THR models appear to perform better than the other two models, as is the case for Canada. And the Theil U-statistic once again suggests that the MSTV model outperforms the other models.

Finally, we also compare the competing models across our three sample countries using their out-of-sample forecasting performance, shown in Figures 22 – 24. In each case, the forecasting period selected is 2000Q4 to 2005Q4. We use two statistics to measure the forecasting performance of each model: Theil's U-statistic and the DIC (where the DIC is measuring the fit of the predicted values). Technical details on our approach are provided in Appendix E. For Canada, the U-statistic suggests that the MSTV model has the best out-of-sample

[32]Credible intervals are similar to the confidence intervals used in the classical approach.

forecasting performance whereas the DIC suggests that the THR model is the superior model along this dimension. For Australia, the U-statistic ranks the THR model first whereas the DIC suggests that the MSTV model is the best performer. And finally, for New Zealand, both measures suggest that the THR model has the best out-of-sample forecasting performance.

5 Conclusion

Understanding the implications of the emergence and unwinding of large U.S. external imbalances for the behaviour of the bilateral exchange rates of its trading partners is important for determining the optimal response of monetary policy. In particular, these imbalances could entail large movements in these exchange rates, which would pose a challenge for monetary policy in these economies because they would imply a significant change in the relative price of domestic goods and thus could have a substantial impact on aggregate demand and inflation. In addition, understanding the role of multilateral adjustment to U.S. external imbalances in driving exchange rate dynamics may contribute to a better understanding of exchange rate determination.

In this paper, we empirically investigate whether multilateral adjustment to large U.S. external imbalances can help explain movements in the bilateral exchange rates of Australia, Canada and New Zealand. Although the results are generally stronger for the Canadian dollar than for the other two currencies, largely because of the longer sample period, our findings suggest that during periods of large U.S. imbalances, fiscal and external, an exchange rate model for the ACNZ dollars should account for multilateral adjustment effects. Moreover, we also find evidence to suggest that the adjustment of exchange rates to multilateral factors is best modelled as a nonlinear process.

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Appendix A: Figures and Tables

Figure 1: The U.S. current account balance

Figure 2: The U.S. dollar and the U.S. current account balance

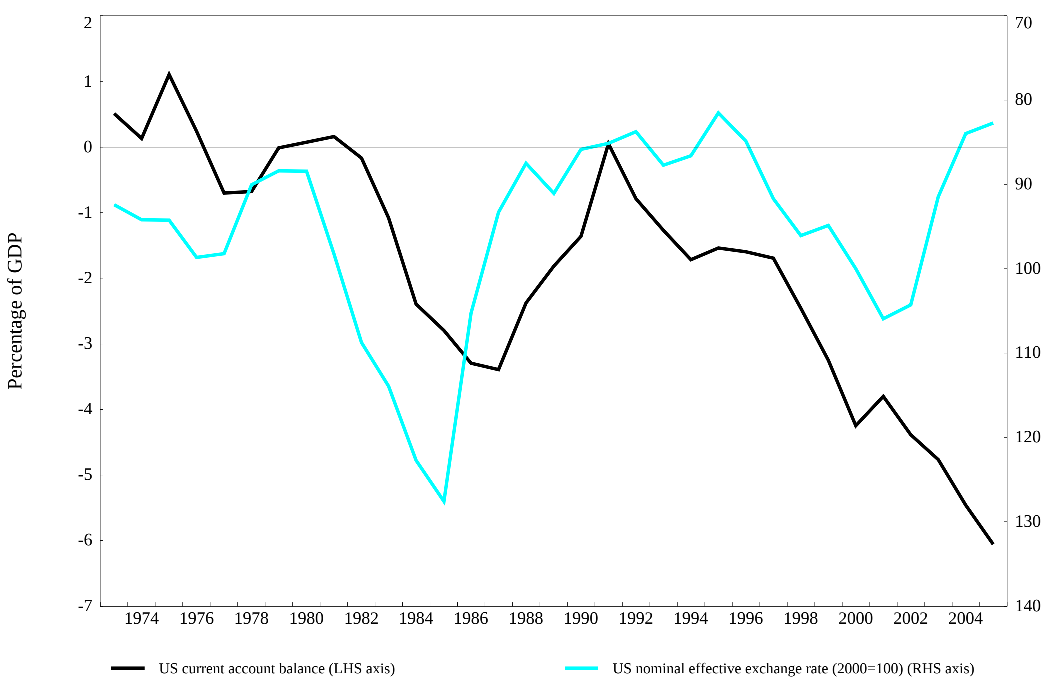


Figure 3: The U.S. current and fiscal account balances

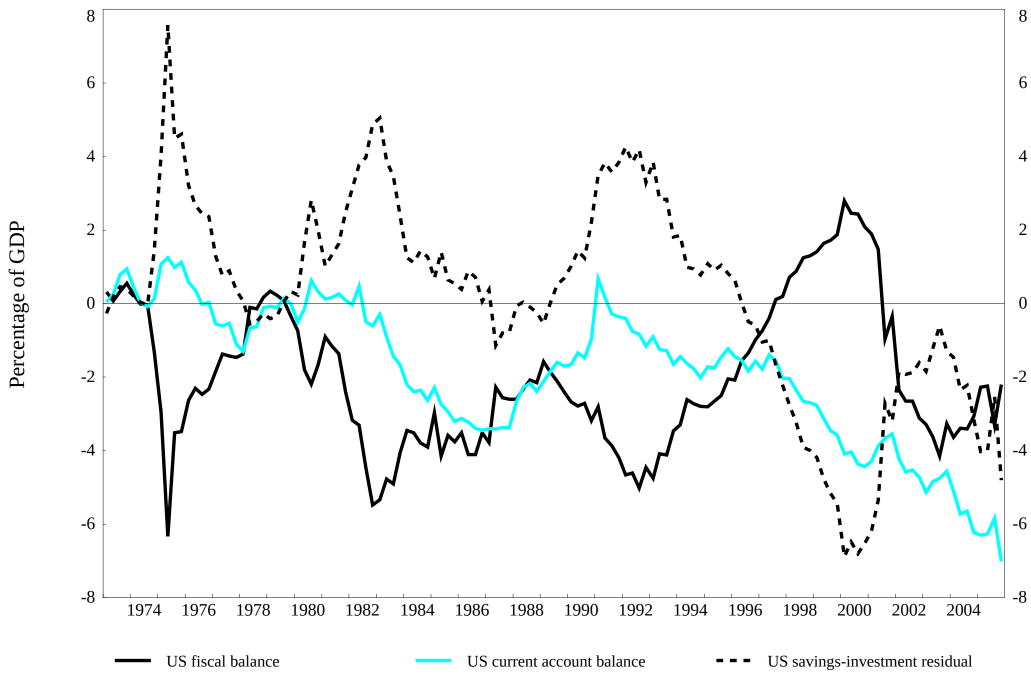


Figure 4: The commodity currencies and the U.S. current account balance

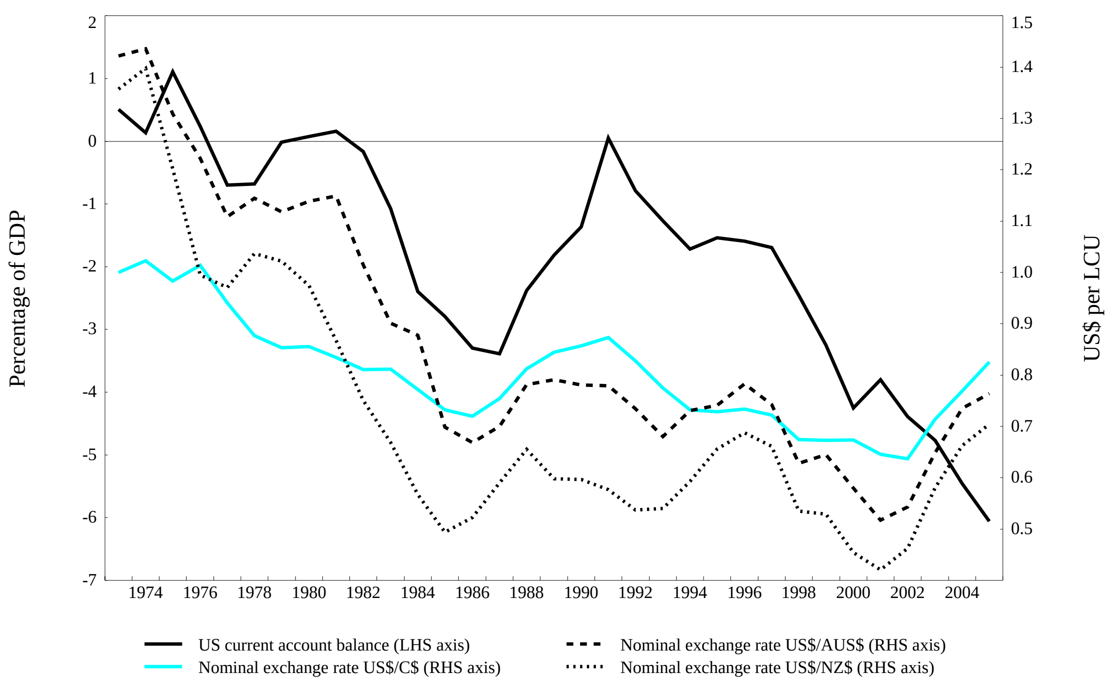


Figure 5: Canadian real effective and bilateral exchange rate (2000=100)

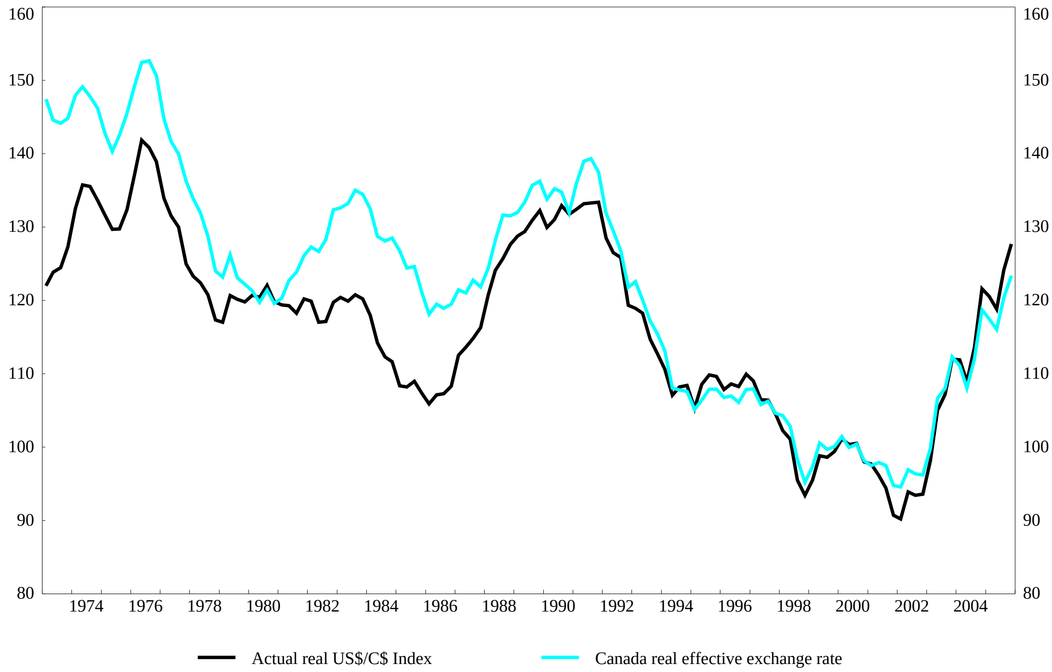


Figure 6: Australian real effective and bilateral exchange rate (2000=100)

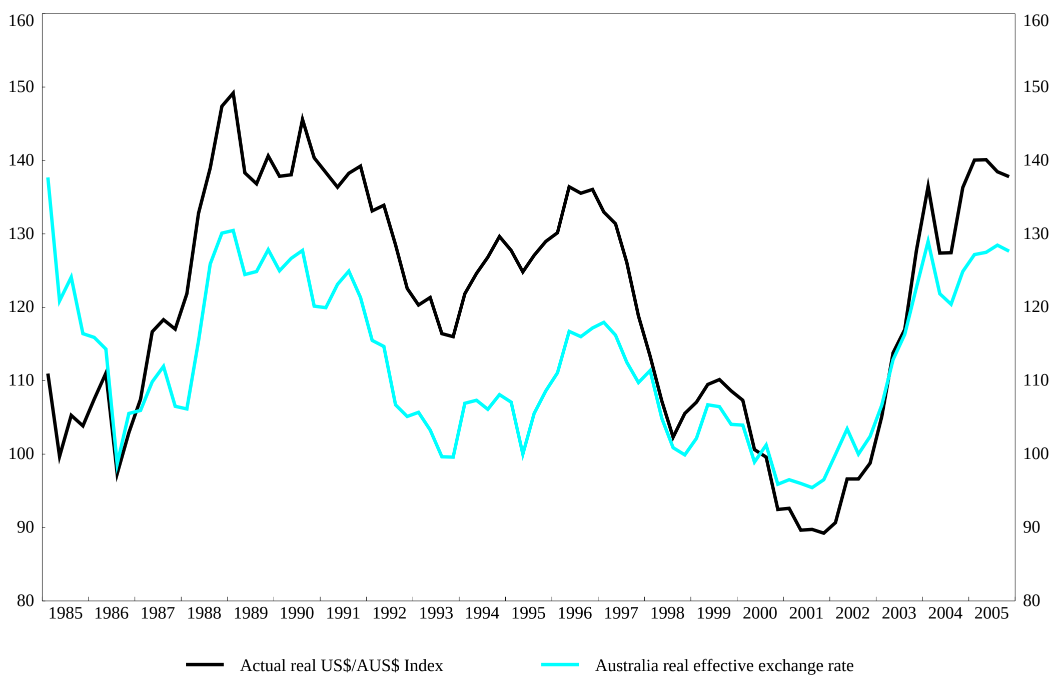


Figure 7: New Zealand real effective and bilateral exchange rate (2000=100)

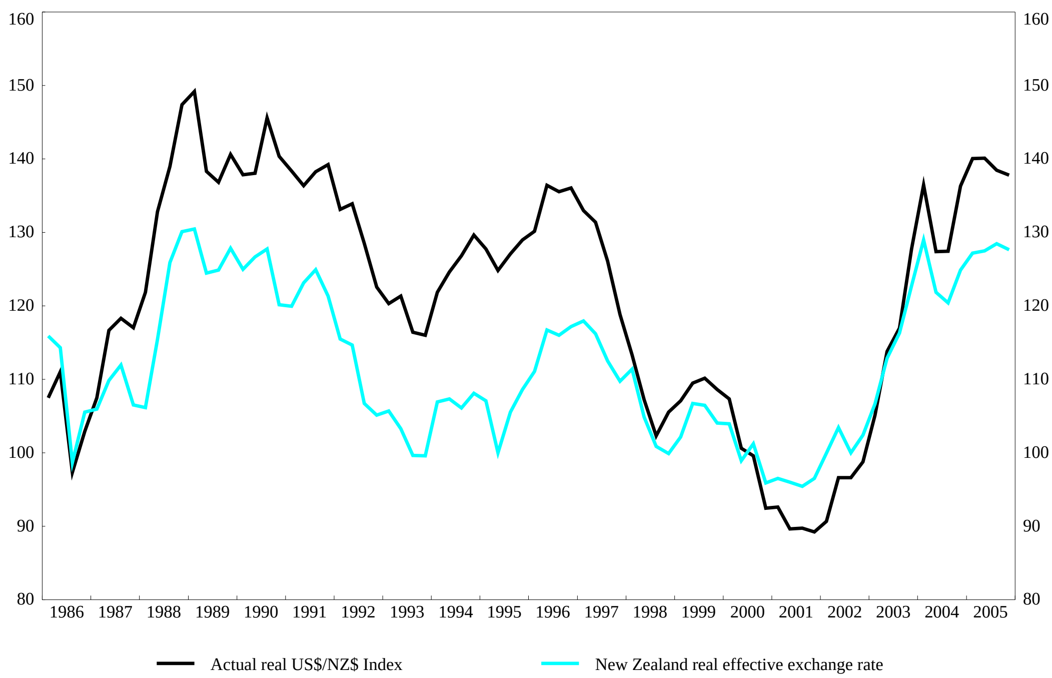


Figure 8: Real non-energy commodity prices

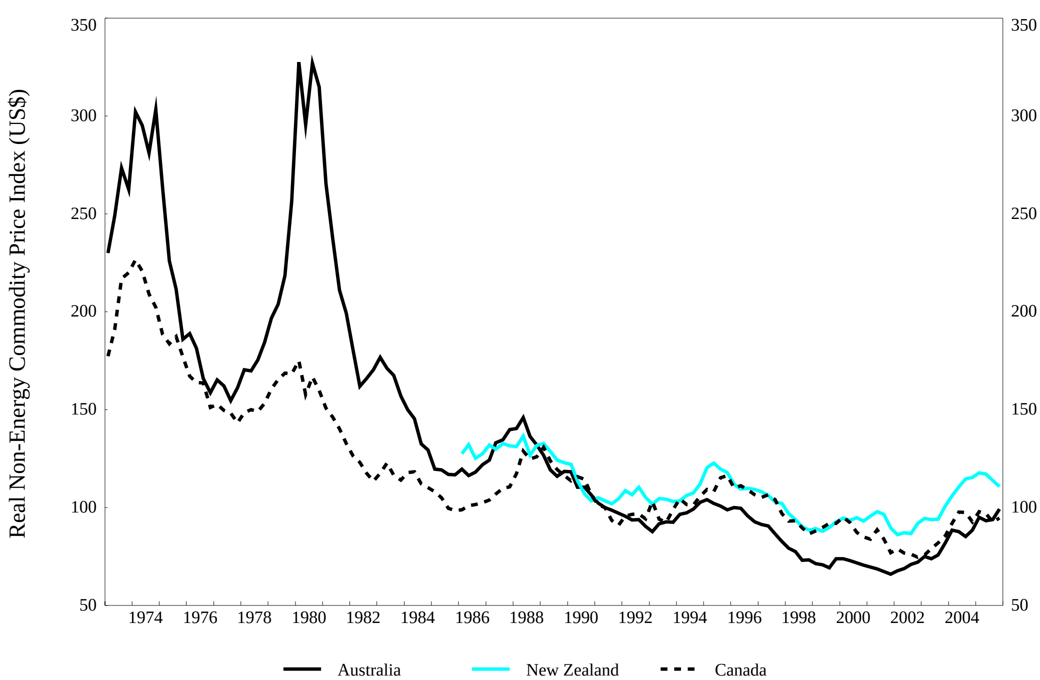


Figure 9: Real energy commodity prices

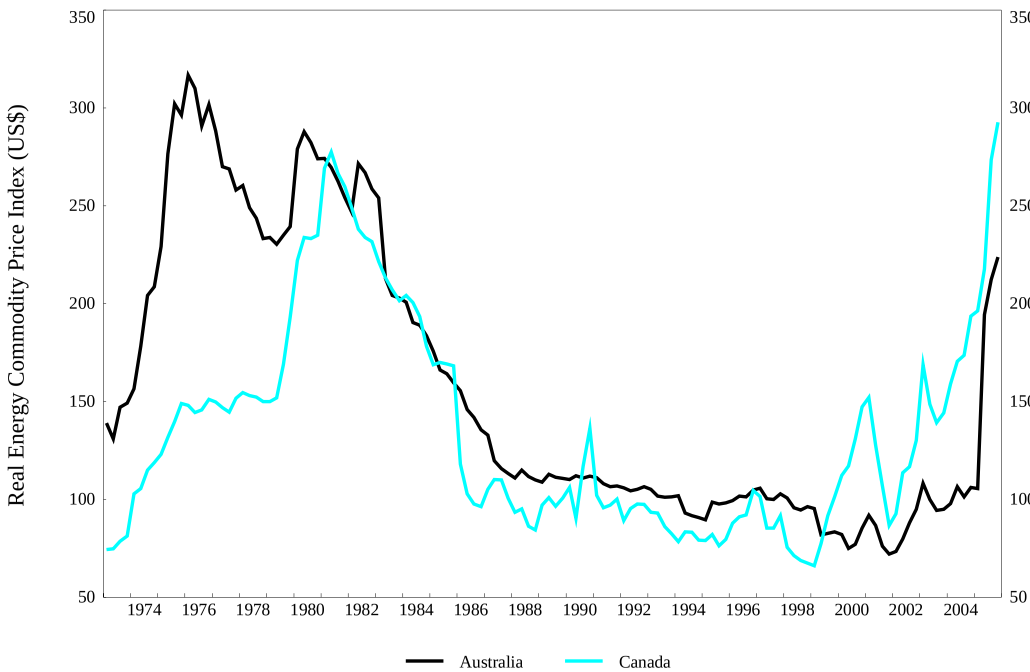


Table 2: Composition of non-energy commodity price indices

Canada	%	Australia	%	New Zealand	%
Aluminium	7.6	Alumina	10.4	Aluminium	6.5
Barley	1.0	Aluminium	11.4	Apples	2.9
Canola	1.9	Barley	2.7	Beef	12.0
Cattle	11.9	Beef and veal	11.1	Dairy	30.8
Copper	3.1	Canola	1.4	Kiwifruit	4.7
Corn	0.8	Copper	3.9	Lamb	13.8
Gold	3.5	Cotton	3.9	Logs	3.3
Hogs	2.7	Gold	13.3	Sawn Timber	6.0
Lumber	20.6	Iron Ore	13.1	Seafood	7.0
Newsprint	11.7	Lead	1.0	Skins	3.3
Nickel	3.6	Nickel	3.7	Venison	1.2
Potash	2.5	Rice	0.7	Wood Pulp	3.1
Pulp	18.3	Sugar	3.5	Wool	5.4
Seafood	1.9	Wheat	11.7		
Silver	0.5	Wool	5.8		
Wheat	5.2	Zinc	2.1		
Zinc	3.4				
Total	100		100		100

Notes:
(i) The Bank of Canada commodity price index is a fixed-weight index based on production values.
(ii) The Reserve Bank of Australia's Index of Commodity Prices are based on the 2001/02 export value weights.
(iii) The New Zealand commodity price index is re-weighted annually and is based on export values.

Table 3: Composition of energy commodity price indices

Canada	%	Australia	%
Crude Oil	63.1	Crude Oil	
Natural gas	31.5	Natural gas	16.4
Coal	5.4	Coal	83.6
Total	100		100

Notes: See notes for Table 2.

Appendix B: Sources and Definitions of Variables

Dependent variable

1. $\triangle \ln(rfx)$:

   - log difference in the real quarterly (Can/Aus/Nzl)-U.S. bilateral exchange rate constructed using the nominal exchange rate, deflated by the ratio of the (Can/Aus/Nzl) and U.S. GDP deflators. Both deflators are indexed to 1997=1.0.

     (a) Canada
       - Nominal exchange rate (Bank of Canada internal database)
       - GDP Deflator (Statistics Canada series *V1997756*)

     (b) Australia
       - Nominal exchange rate (International Financial Statistics (IFS) series *Q.193..RF.ZF...H*)
       - GDP Deflator (OECD Main Economic Indicators series *Q.AUS.EXPGDP.DNBSA*)

     (c) New Zealand
       - Nominal exchange rate (IFS series *Q.196..RF.ZF...H*)
       - GDP Deflator (OECD Economic Outlook series *Q.NZL.PGDP*)

     (d) United States
       - GDP Deflator (U.S. Department of Commerce - Bureau of Economic Analysis series *pdigdp*)

Explanatory variables

1. $\ln(comtot)$:

   - log of the real non-energy commodity price index constructed as the nominal non-energy commodity price index in U.S. dollar terms, deflated by the U.S. GDP deflator.

     (a) Canada
       - Nominal non-energy commodity price index (Bank of Canada internal database)

(b) Australia

– Nominal non-energy commodity price index (Used weights from the Reserve Bank of Australia's Index of Commodity Prices and constructed a non-energy index by reweighting. Price data used for commodities was obtained from the IFS, Datastream (alumina), and the Bank of Canada's internal database)

(c) New Zealand

– Nominal non-energy commodity price index (Australia and New Zealand Banking Group Limited (ANZ) Commodity Price Index).

2. ln(*enetot*):

- log of the real energy commodity price index constructed as the nominal energy commodity price index in U.S. dollar terms, deflated by the U.S. GDP deflator.

(a) Canada

– Nominal energy commodity price index (Bank of Canada internal database)

(b) Australia

– Nominal energy commodity price index (Used weights from the Reserve Bank of Australia's Index of Commodity Prices and constructed an energy index by reweighting. Price data for commodities was obtained from the IFS and the Bank of Canada's internal database.

3. *intdif*:

- short-term interest rate spread constructed as the difference between Canadian and U.S. rates.

(a) Canada

– Three-month prime corporate paper rate (Statistics Canada series *V122491*)

(b) United States

– 90-day AA non-financial commercial paper closing rate (Federal Reserve Board)

4. *Rintdif*:

- short-term real interest rate spread constructed as the difference between (Aus/Nzl) and U.S. real rates.

(a) Australia

  - Yield on 90-day bank-accepted bills (OECD Main Economic Indicators series *Q.AUS.IR3TBB01.ST*)

(b) New Zealand

  - 90-day bank bill rate (OECD Main Economic Indicators series *Q.NZL.IR3TBB01.ST*)

5. *US_cabal*:

   - Balance on U.S. current account as a proportion of GDP.
     - Current account balance (U.S. Department of Commerce, Bureau of Economic Analysis series *bopcrnt*)
     - GDP (U.S. Department of Commerce, Bureau of Economic Analysis series *gdp*)

6. *US_fisbal*:

   - U.S. total government fiscal balance as a proportion of GDP.
     - Fiscal balance (U.S. Department of Commerce, Bureau of Economic Analysis series *netsavg*)

Appendix C: Unit-Root, Cointegration, and Weak Exogeneity Test Results

Table 4: DF-GLS Unit-Root Tests
(Canada, Sample period: 1973Q1 to 2005Q4)

Variable	Trend	No Trend
ln(*rfx*)	-1.72	-1.60
ln(*comtot*)	-2.02	-0.48
ln(*enetot*)	-1.33	-0.28
intdif	-2.04	-1.31
US_cabal	-1.34	1.10
US_fisbal	-1.87	-1.48

Notes:
(i) The Dickey-Fuller Generalized Least Squares (DF-GLS) test is based on Elliott, Rothenberg, and Stock's (1996) modification to the Augmented Dickey-Fuller (ADF) test. Under this test, the variable is first locally detrended/demeaned and then tested for the presence of a unit root in the usual ADF manner. The power of the DF-GLS is substantially improved over the original version of ADF, particularly for finite samples. As with the ADF test, the null hypothesis states that the variable contains a unit root.
(ii) The number of lags used in the test was selected based on the modified Schwarz information criterion, developed by Ng and Perron (2001).
(iii) Bolded values exceed the 5 per cent critical value.

Table 5: DF-GLS Unit-Root Tests
(Australia, Sample period: 1985Q1 to 2005Q4)

Variable	Trend	No Trend
ln(*rfx*)	-1.73	-1.60
ln(*comtot*)	-1.01	-1.11
ln(*enetot*)	0.10	-0.58
Rintdif	-2.44	-1.88

Notes: See notes for Table 4.

Table 6: DF-GLS Unit-Root Tests
(New Zealand, Sample period: 1986Q1 to 2005Q4)

Variable	Trend	No Trend
ln(*rfx*)	-1.50	-0.90
ln(*comtot*)	-1.96	-1.79
Rintdif	-2.37	**-2.39**

Notes: See notes for Table 4.

Table 7: Johansen Cointegration Test Results for ln(rfx), ln($comtot$), and ln($enetot$)

(Canada, Sample period as shown in table)

	No Trend			Trend		
Hypothesized No. of CVs	1973Q1-2005Q4	1973Q1-1993Q3	1993Q4-2005Q4	1973Q1-2005Q4	1973Q1-1993Q3	1993Q4-2005Q4
	Trace Statistics					
Fewer than 1	16.50 (0.68)	**31.52** (0.03)	18.37 (0.54)	32.02 (0.39)	**44.10** (0.04)	29.91 (0.51)
Fewer than 2	8.08 (0.46)	12.05 (0.15)	3.35 (0.95)	8.27 (0.98)	16.82 (0.43)	14.80 (0.59)
	λ-max Statistics					
Fewer than 1	8.42 (0.88)	**19.46** (0.08)	15.02 (0.29)	**23.74** (0.09)	**27.28** (0.03)	15.11 (0.62)
Fewer than 2	5.30 (0.70)	11.24 (0.14)	3.34 (0.92)	5.34 (0.98)	11.52 (0.46)	11.49 (0.46)

Notes:
(i) The values reported under the column labeled "No Trend" assume a constant in the cointegration space and a linear deterministic trend in the data.
(ii) The values reported under the column labeled "Trend" assume a constant and a linear deterministic trend in the cointegration space, as well as a linear deterministic trend in the data.
(iii) Bold values denote rejection of the null of no cointegration at the 10 per cent significance level based on critical values calculated by MacKinnon, Haug, and Michelis (1999).
(iv) Lag selections based on sequential modified likelihood ratio test statistic.
(v) P-values are in parentheses.

Table 8: Johansen Cointegration Test Results for ln(rfx), ln($comtot$), and ln($enetot$)

(Australia, Sample period: 1985Q1 to 2005Q4)

Hypothesized No. of CVs	No Trend	Trend
	Trace Statistics	
Fewer than 1	**42.74** (0.00)	**62.80** (0.00)
Fewer than 2	10.04 (0.28)	21.05 (0.18)
	λ-max Statistics	
Fewer than 1	**32.71** (0.00)	**41.75** (0.00)
Fewer than 2	9.36 (0.26)	12.37 (0.38)

Notes: See notes for Table 7.

Table 9: Johansen Cointegration Test Results for ln(rfx) and ln($comtot$)

(New Zealand, Sample period: 1986Q1 to 2005Q4)

Hypothesized No. of CVs	No Trend	Trend
	Trace Statistics	
Fewer than 1	**20.51** (0.01)	**30.40** (0.01)
Fewer than 2	**8.88** (0.00)	10.41 (0.11)
	λ-max Statistics	
Fewer than 1	11.63 (0.13)	**19.99** (0.04)
Fewer than 2	**8.88** (0.00)	10.41 (0.11)

Notes: See notes for Table 7.

Table 10: Weak Exogeneity Tests

(Canada, Australia and New Zealand)

Country (Sample period)	**LR test statistic (no trend)**	***P*-Value**	**LR test statistic (trend)**	***P*-Value**
Canada (1973Q1 to 2005Q4)	6.18	0.05	8.24	0.02
Australia (1985Q1 to 2005Q4)	4.44	0.11	5.95	0.05
New Zealand (1986Q1 to 2005Q4)	0.03	0.87	2.82	0.09

Notes:
(i) Weak exogeneity tests for $\ln(rfx)$, $\ln(comtot)$, and $\ln(enetot)$, for Canada and Australia. New Zealand weak exogeneity tests for $\ln(rfx)$ and $\ln(comtot)$.
(ii) Based on the benchmark model specification. The number of lags used in the test was selected based on a sequential modified likelihood-ratio test.
(iii) The likelihood-ratio (LR) test statistic follows a χ^2 distribution.

Appendix D: Estimation and Forecasting Results

Table 11: Prior and Posterior Distributions for the Benchmark Model

Canada, Sample period: 1973Q1-2005Q4

		Prior		Posterior	
Variables	**Density**	**Mean**	**S.D.**	**Mean**	**S.D.**
Constant	Normal	1.764	0.370	1.753	0.223
d_t	Normal	1.735	0.346	1.779	0.190
Speed of adj.	Normal	-0.165	0.035	-0.155	0.022
$\ln(comtot)_{t-1}$	Normal	-0.418	0.071	-0.421	0.042
$\ln(enetot)_{t-1}$	Normal	0.107	0.036	0.108	0.023
$d_t \ln(enetot)_{t-1}$	Normal	-0.378	0.075	-0.383	0.041
$Intdif_{t-1}$	Normal	-0.660	0.150	-0.647	0.101
σ	InvGam	—	—	0.019	0.001

Notes:
(i) We use the OLS point estimates and corresponding standard errors as the prior means and standard deviations, respectively.
(ii) The inverse Gamma prior has the shape $p(\sigma|s,v) \propto \sigma^{-v-1}\exp(-s^2v/2\sigma^2)$ with $s = 0.019$ and $v = 1$. The first and second moments of this prior do not exist.
(iii) d_t is a dummy variable that takes on the value 1 if $t > 1993Q3$ and 0 otherwise.

Table 12: Prior and Posterior Distributions for the Linear Multilateral Effects Model

Canada, Sample period: 1973Q1-2005Q4

		Prior		Posterior	
Variables	**Density**	**Mean**	**S.D.**	**Mean**	**S.D.**
Constant	Normal	2.109	0.549	2.160	0.293
d_t	Normal	1.455	0.325	1.463	0.185
Speed of adj.	Normal	-0.179	0.043	-0.168	0.024
$\ln(comtot)_{t-1}$	Normal	-0.487	0.109	-0.497	0.058
$\ln(enetot)_{t-1}$	Normal	0.111	0.033	0.112	0.022
$d_t \ln(enetot)_{t-1}$	Normal	-0.320	0.070	-0.323	0.040
$Intdif_{t-1}$	Normal	-0.593	0.151	-0.576	0.107
$UScabal_{t-1}$	Normal	0.135	0.219	0.146	0.129
$USfisbal_{t-1}$	Normal	0.259	0.113	0.246	0.079
σ	InvGam	—	—	0.019	0.001

Notes: See notes for Table 11.

Table 13: Prior and Posterior Distributions for the MSTV Model

Canada, Sample period: 1973Q1-2005Q4

		Prior				Posterior			
		$S_t = 0$		$S_t = 1$		$S_t = 0$		$S_t = 1$	
Variables	**Density**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**
Constant	Normal	2.109	0.549	—		2.086	0.286	—	
d_t	Normal	1.455	0.325	—		1.406	0.180	—	
Speed of adj.	Normal	-0.179	0.043	—		-0.161	0.024	—	
$\ln(comtot)_{t-1}$	Normal	-0.487	0.109	—		-0.486	0.055	—	
$\ln(enetot)_{t-1}$	Normal	0.111	0.151	—		0.114	0.023	—	
$d_t \ln(enetot)_{t-1}$	Normal	-0.320	0.070	—		-0.305	0.039	—	
$Intdif_{t-1}$	Normal	-0.593	0.151	-0.593	0.151	-0.581	0.128	-0.600	0.123
$UScabal_{t-1}$	Normal	0.135	0.219	0.000	0.010	0.456	0.189	-0.000	0.009
$USfisbal_{t-1}$	Normal	0.259	0.113	0.000	0.010	0.303	0.101	0.000	0.009
σ	InvGam	—	—	—	—	0.015	0.002	0.016	0.002
γ	Normal	-2.424	1.940	—		-2.390	1.558	—	
m_0	Beta	0.750	0.150	—		0.797	0.112	—	
m_1	Beta	0.500	0.150	—		0.503	0.131	—	
n_0	Beta	0.500	0.150	—		0.535	0.136	—	
n_1	Beta	0.750	0.150	—		0.820	0.103	—	

Notes:
(i) To identify $S_t = 0$ and $S_t = 1$, we (a) impose two restrictions on the transitory probability matrices (i.e., $m_0 > m_1$ and $n_1 > n_0$ and (b) set the prior means of the coefficients on $UScabal_{t-1}$, and $USfisbal_{t-1}$ in state 1 equal to zero.
(ii) The prior densities of the coefficients on $Const$, d_t, Speed of adj, $\ln(comtot)_{t-1}$, $\ln(enetot)_{t-1}$, $d_t \ln(enetot)_{t-1}$, $Intdif_{t-1}$, $UScabal_{t-1}$, and $USfisbal_{t-1}$ are identical to the corresponding prior densities of the linear multilateral effects model. We use the sample median and standard deviation of the US fiscal balance to GDP ratio in constructing our prior distribution for the threshold parameter γ.
(iii) The inverse Gamma prior has the shape $p(\sigma|s, v) \propto \sigma^{-v-1} \exp(-s^2 v/2\sigma^2)$ with $s = 0.019$ and $v = 1$. The first and second moments of this prior do not exist.
(iv) d_t is a dummy variable that takes on the value 1 if $t > 1993Q3$ and 0 otherwise.

Table 14: Prior and Posterior Distributions for the THR Model

Canada, Sample period: 1973Q1-2005Q4

		Prior				**Posterior**			
		$S_t = 0$		$S_t = 1$		$S_t = 0$		$S_t = 1$	
Variables	**Density**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**
Constant	Normal	2.109	0.549	—		2.122	0.286	—	
d_t	Normal	1.455	0.325	—		1.439	0.187	—	
Speed of adj.	Normal	-0.179	0.043	—		-0.156	0.023	—	
$\ln(comtot)_{t-1}$	Normal	-0.487	0.109	—		-0.492	0.056	—	
$\ln(enetot)_{t-1}$	Normal	0.111	0.151	—		0.113	0.022	—	
$d_t \ln(enetot)_{t-1}$	Normal	-0.320	0.070	—		-0.315	0.041	—	
$Intdif_{t-1}$	Normal	-0.593	0.151	-0.593	0.151	-0.549	0.124	-0.635	0.123
$UScabal_{t-1}$	Normal	0.135	0.219	0.000	0.010	0.405	0.184	-0.000	0.010
$USfisbal_{t-1}$	Normal	0.259	0.113	0.000	0.010	0.284	0.096	0.000	0.010
σ	InvGam	—	—	—	—	0.016	0.003	0.017	0.002
γ	Normal	-2.424	1.940	—		-2.388	1.400	—	

Notes:
(i) See the notes for Table 13. (ii) The THR model is a special case of the MSTV model where the following restrictions are jointly imposed: $m_0 = 1$, $m_1 = 0$, $n_0 = 0$, and $n_1 = 1$ jointly.

Table 15: Prior and Posterior Distributions for the Benchmark Model

Australia, Sample period: 1985Q1-2005Q4

		Prior		**Posterior**	
Variables	**Density**	**Mean**	**S.D.**	**Mean**	**S.D.**
Constant	Normal	2.715	1.045	2.716	0.683
Speed of adj.	Normal	-0.121	0.040	-0.108	0.029
$\ln(comtot)_{t-1}$	Normal	-0.549	0.222	-0.555	0.148
$\ln(enetot)_{t-1}$	Normal	0.034	0.241	0.040	0.154
$Rintdif_{t-1}$	Normal	-0.002	0.001	-0.002	0.001
σ	InvGam	—	—	0.039	0.003

Notes:
(i) See notes (i) and (ii) for Table 11.

Table 16: Prior and Posterior Distributions for the Linear Multilateral Effects Model

Australia, Sample period: 1985Q1-2005Q4

		Prior		Posterior	
Variables	**Density**	**Mean**	**S.D.**	**Mean**	**S.D.**
Constant	Normal	1.793	1.282	1.852	0.896
Speed of adj.	Normal	-0.101	0.039	-0.087	0.027
$\ln(comtot)_{t-1}$	Normal	-0.698	0.323	-0.719	0.208
$\ln(enetot)_{t-1}$	Normal	0.439	0.356	0.468	0.219
$Rintdif_{t-1}$	Normal	-0.001	0.002	-0.001	0.001
$UScabal_{t-1}$	Normal	0.676	0.311	0.679	0.207
$USfisbal_{t-1}$	Normal	0.690	0.309	0.679	0.201
σ	InvGam	—	—	0.037	0.003

Notes:
(i) See notes (i) and (ii) for Table 11.

Table 17: Prior and Posterior Distributions for the MSTV Model

Australia, Sample period: 1985Q1-2005Q4

		Prior				Posterior			
		$S_t = 0$		$S_t = 1$		$S_t = 0$		$S_t = 1$	
Variables	**Density**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**
Constant	Normal	1.793	1.282	—		1.772	0.834	—	
Speed of adj.	Normal	-0.101	0.039	—		-0.093	0.027	—	
$\ln(comtot)_{t-1}$	Normal	-0.698	0.323	—		-0.721	0.188	—	
$\ln(enetot)_{t-1}$	Normal	0.439	0.356	—		0.463	0.228	—	
$Rintdif_{t-1}$	Normal	-0.001	0.002	-0.001	0.002	-0.000	0.001	-0.001	0.002
$UScabal_{t-1}$	Normal	0.676	0.311	0.000	0.010	0.731	0.212	0.000	0.009
$USfisbal_{t-1}$	Normal	0.690	0.309	0.000	0.010	0.883	0.243	-0.000	0.009
σ	InvGam	—	—	—	—	0.027	0.004	0.043	0.010
γ	Normal	-2.424	1.940	—		-1.285	1.736	—	
m_0	Beta	0.750	0.150	—		0.847	0.097	—	
m_1	Beta	0.500	0.150	—		0.541	0.141	—	
n_0	Beta	0.500	0.150	—		0.425	0.129	—	
n_1	Beta	0.750	0.150	—		0.736	0.140	—	

Notes:
(i) See notes for Table 13.

Table 18: Prior and Posterior Distributions for the THR Model

Australia, Sample period: 1985Q1-2005Q4

		Prior				Posterior			
		$S_t = 0$		$S_t = 1$		$S_t = 0$		$S_t = 1$	
Variables	**Density**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**
Constant	Normal	1.793	1.282	—		1.844	0.797	—	
Speed of adj.	Normal	-0.101	0.039	—		-0.095	0.026	—	
$\ln(comtot)_{t-1}$	Normal	-0.698	0.323	—		-0.747	0.188	—	
$\ln(enetot)_{t-1}$	Normal	0.439	0.356	—		0.475	0.210	—	
$Rintdif_{t-1}$	Normal	-0.001	0.002	-0.001	0.002	-0.001	0.001	-0.001	0.002
$UScabal_{t-1}$	Normal	0.676	0.311	0.000	0.010	0.744	0.198	0.000	0.010
$USfisbal_{t-1}$	Normal	0.690	0.309	0.000	0.010	0.805	0.217	0.000	0.009
σ	InvGam	—	—	—	—	0.028	0.003	0.049	0.017
γ	Normal	-2.424	1.940	—		0.387	1.580	—	

Notes:
(i) See notes for Table 13.

Table 19: Prior and Posterior Distributions for the Benchmark Model

New Zealand, Sample period: 1986Q1-2005Q4

		Prior		Posterior	
Variables	**Density**	**Mean**	**S.D.**	**Mean**	**S.D.**
Constant	Normal	4.449	1.623	3.992	0.690
Speed of adj.	Normal	-0.102	0.045	-0.087	0.034
$\ln(comtot)_{t-1}$	Normal	-0.823	0.347	-0.728	0.148
$Rintdif_{t-1}$	Normal	-0.003	0.002	-0.003	0.002
σ	InvGam	—	—	0.042	0.003

Notes:
(i) See notes (i) and (ii) for Table 11.

Table 20: Prior and Posterior Distributions for the Linear Multilateral Effects Model

New Zealand, Sample period: 1986Q1-2005Q4

		Prior		Posterior	
Variables	**Density**	**Mean**	**S.D.**	**Mean**	**S.D.**
Constant	Normal	2.601	2.065	2.551	1.141
Speed of adj.	Normal	-0.079	0.037	-0.068	0.024
$\ln(comtot)_{t-1}$	Normal	-0.341	0.449	-0.300	0.250
$Rintdif_{t-1}$	Normal	0.000	0.002	0.000	0.001
$UScabal_{t-1}$	Normal	0.762	0.258	0.758	0.186
$USfisbal_{t-1}$	Normal	1.009	0.250	1.011	0.169
σ	InvGam	—	—	0.036	0.003

Notes:
(i) See notes (i) and (ii) for Table 11.

Table 21: Prior and Posterior Distributions for the MSTV Model

New Zealand, Sample period: 1986Q1-2005Q4

		Prior				Posterior			
		$S_t = 0$		$S_t = 1$		$S_t = 0$		$S_t = 1$	
Variables	**Density**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**
Constant	Normal	2.601	2.065	—		2.900	1.152	—	
Speed of adj.	Normal	-0.079	0.037	—		-0.056	0.023	—	
$\ln(comtot)_{t-1}$	Normal	-0.341	0.449	—		-0.369	0.261	—	
$Rintdif_{t-1}$	Normal	0.000	0.002	0.000	0.002	0.000	0.002	-0.001	0.002
$UScabal_{t-1}$	Normal	0.762	0.258	0.000	0.010	0.825	0.226	-0.000	0.009
$USfisbal_{t-1}$	Normal	1.009	0.250	0.000	0.010	1.184	0.208	0.000	0.009
σ	InvGam	—	—	—	—	0.029	0.004	0.031	0.007
γ	Normal	-2.424	1.940	—		-1.527	1.736	—	
m_0	Beta	0.750	0.150	—		0.867	0.077	—	
m_1	Beta	0.500	0.150	—		0.596	0.134	—	
n_0	Beta	0.500	0.150	—		0.489	0.134	—	
n_1	Beta	0.750	0.150	—		0.788	0.121	—	

Notes:
(i) See notes for Table 13.

Table 22: Prior and Posterior Distributions for the THR Model

New Zealand, Sample period: 1986Q1-2005Q4

		Prior				Posterior			
		$S_t = 0$		$S_t = 1$		$S_t = 0$		$S_t = 1$	
Variables	**Density**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**	**Mean**	**S.D.**
Constant	Normal	2.601	2.065	—		2.638	1.138	—	
Speed of adj.	Normal	-0.079	0.037	—		-0.062	0.024	—	
$\ln(comtot)_{t-1}$	Normal	-0.341	0.449	—		-0.333	0.257	—	
$Rintdif_{t-1}$	Normal	0.000	0.002	0.001	0.002	0.001	0.002	-0.001	0.002
$UScabal_{t-1}$	Normal	0.762	0.258	0.000	0.010	0.822	0.208	0.000	0.010
$USfisbal_{t-1}$	Normal	1.009	0.250	0.000	0.010	1.130	0.188	0.000	0.009
σ	InvGam	—	—	—	—	0.031	0.003	0.035	0.010
γ	Normal	-2.424	1.940	—		0.584	1.418	—	

Notes:
(i) See notes for Table 13.

Table 23: Goodness of Fit

Canada, Sample period: 1973Q1-2005Q4

Statistics	Benchmark	Multilateral	MSTV	THR
Prior Probability, $\pi_{i,0}$	1/4	1/4	1/4	1/4
Marginal Likelihood, $\ln P(y^T \| \mathcal{M}_i)$	321.286	322.183	324.489	323.152
Posterior Probability, $\pi_{i,T}$	0.029	0.071	0.712	0.187
Posterior Odds Ratio, $\pi_{i,T}/\pi_{Benchmark,T}$	1.000	2.452	24.604	6.465
DIC	-654.356	-657.346	-666.010	-661.676

Notes:

(i) The marginal data density is calculated based on Geweke's harmonic mean estimator.

Table 24: Goodness of fit: The MSTV Model versus the Random Walk

Canada, Sample period: 1973Q1-2005Q4

Statistics	Random Walk	MSTV
Prior Probability, $\pi_{i,0}$	1/2	1/2
Marginal Likelihood, $\ln P(y^T \| \mathcal{M}_i)$	310.807	324.489
Posterior Probability, $\pi_{i,T}$	0.000	1.000
Posterior Odds Ratio, $\pi_{i,T}/\pi_{randomwalk,T}$	1.000	874629.655
DIC	-624.816	-666.010

Notes:
(i) See note for Table 23.

Table 25: Goodness of Fit

Australia, Sample period: 1985Q1-2005Q4

Statistics	Benchmark	Multilateral	MSTV	THR
Prior Probability, $\pi_{i,0}$	1/4	1/4	1/4	1/4
Marginal Likelihood, $\ln P(y^T \| \mathcal{M}_i)$	143.023	146.873	148.315	148.786
Posterior Probability, $\pi_{i,T}$	0.001	0.083	0.351	0.563
Posterior Odds Ratio, $\pi_{i,T}/\pi_{Benchmark,T}$	1.000	46.998	198.711	318.318
DIC	-293.403	-301.939	-307.233	-308.228

Notes:
(i) See note for Table 23.

Table 26: Goodness of fit: The MSTV Model versus the Random Walk

Australia, Sample period: 1985Q1-2005Q4

Statistics	Random Walk	MSTV
Prior Probability, $\pi_{i,0}$	1/2	1/2
Marginal Likelihood, $\ln P(y^T \| \mathcal{M}_i)$	142.225	148.315
Posterior Probability, $\pi_{i,T}$	0.002	0.998
Posterior Odds Ratio, $\pi_{i,T}/\pi_{randomwalk,T}$	1.000	428.209
DIC	-287.269	-307.233

Notes:
(i) See note for Table 23.

Table 27: Goodness of Fit

New Zealand, Sample period: 1986Q1-2005Q4

Statistics	Benchmark	Multilateral	MSTV	THR
Prior Probability, $\pi_{i,0}$	1/4	1/4	1/4	1/4
Marginal Likelihood, $\ln P(y^T \| \mathcal{M}_i)$	131.215	141.315	140.906	140.376
Posterior Probability, $\pi_{i,T}$	0.000	0.486	0.323	0.190
Posterior Odds Ratio, $\pi_{i,T}/\pi_{Benchmark,T}$	1.000	24332.778	16172.51	9519.861
DIC	-270.478	-291.535	-298.152	-292.564

Notes:
(i) See note for Table 23.

Table 28: Goodness of fit: The MSTV Model versus the Random Walk

New Zealand, Sample period: 1986Q1-2005Q4

Statistics	Random Walk	MSTV
Prior Probability, $\pi_{i,0}$	1/2	1/2
Marginal Likelihood, $\ln P(y^T \| \mathcal{M}_i)$	133.098	140.906
Posterior Probability, $\pi_{i,T}$	0.000	0.999
Posterior Odds Ratio, $\pi_{i,T}/\pi_{randomwalk,T}$	1.000	2460.619
DIC	-268.947	-298.152

Notes:
(i) See note for Table 23.

Figure 10: Probability of State 0 Occurring in the MSTV Model for Canada

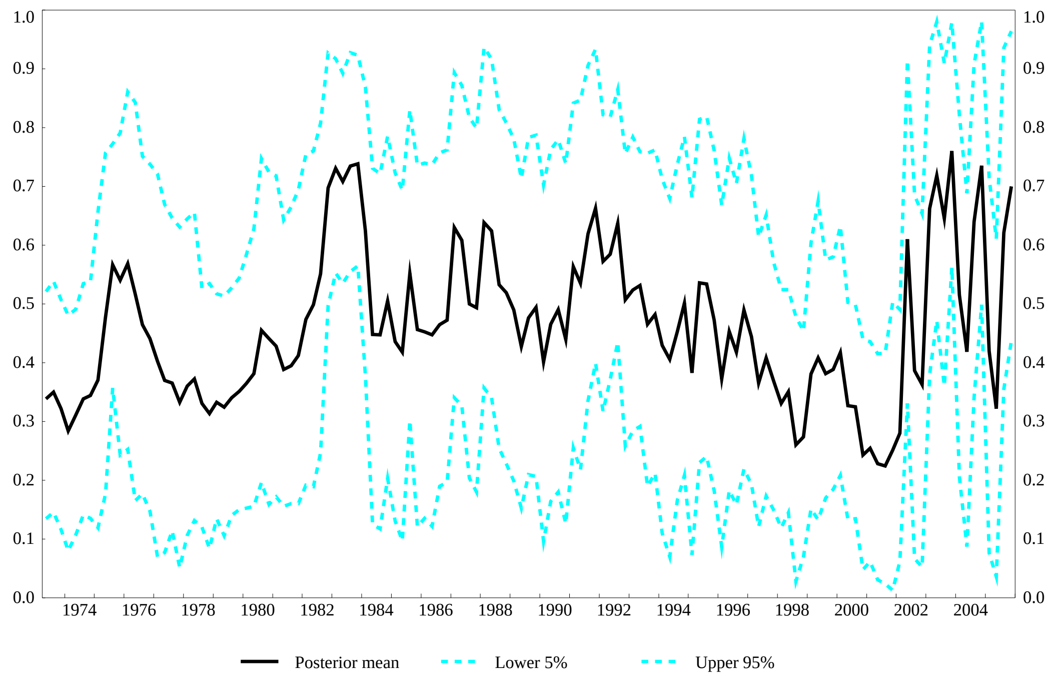


Figure 11: The Evolution of the Threshold Variable and the Probability of State 0 Occurring (Posterior Mean) in the MSTV Model for Canada

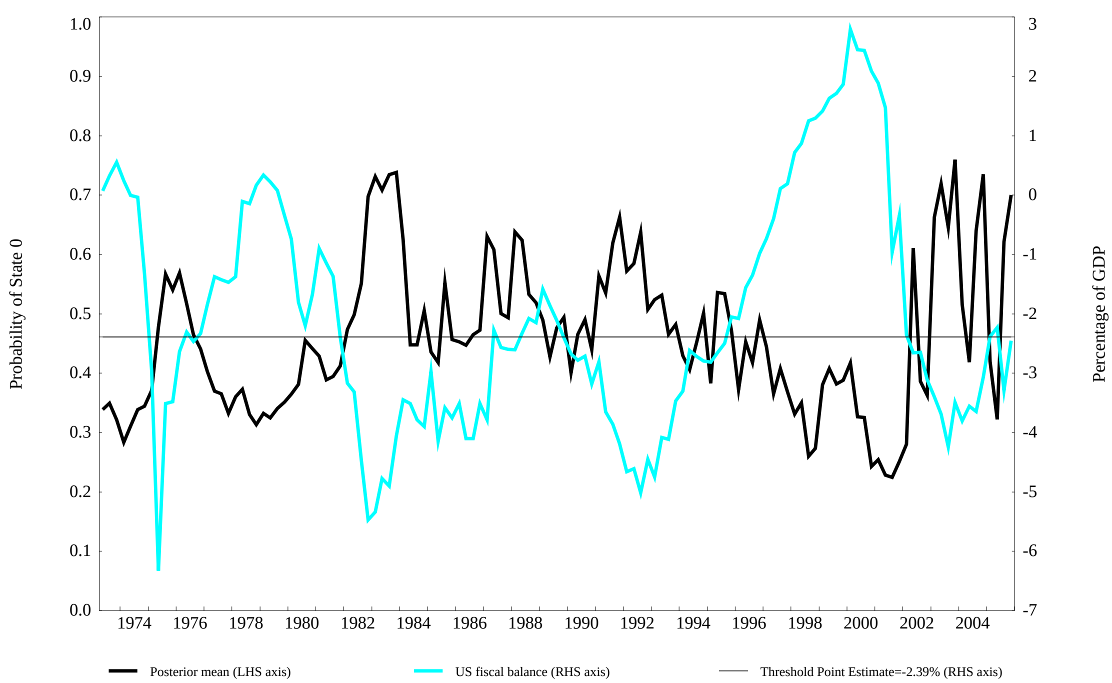


Figure 12: The Evolution of the Multilateral Variables in the Two Regimes Identified by the Threshold Model for Canada

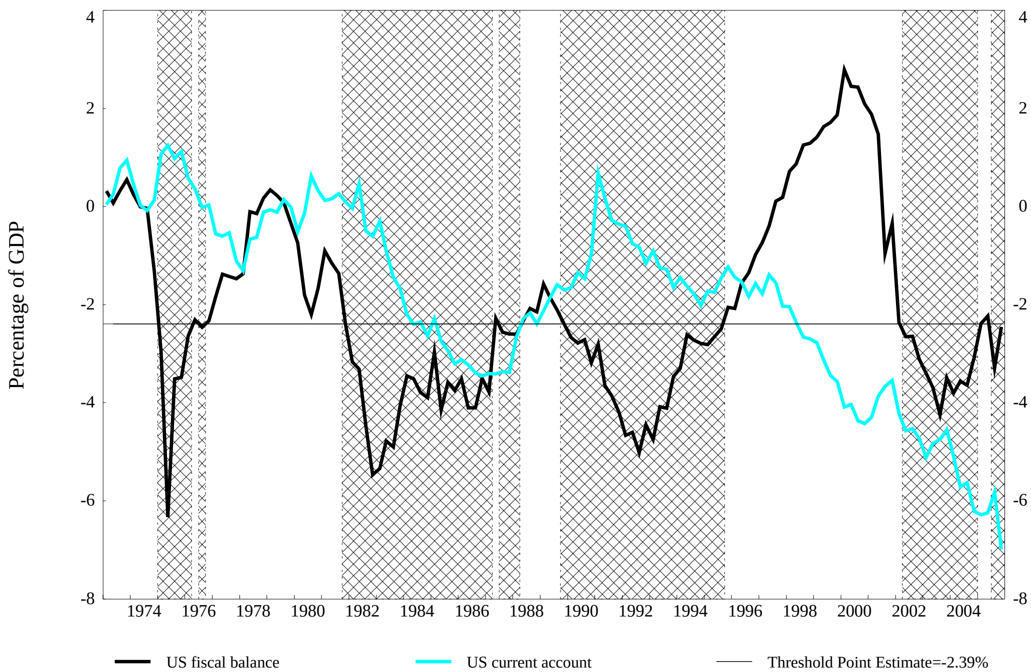


Figure 13: Probability of State 0 Occurring in the MSTV Model for Australia

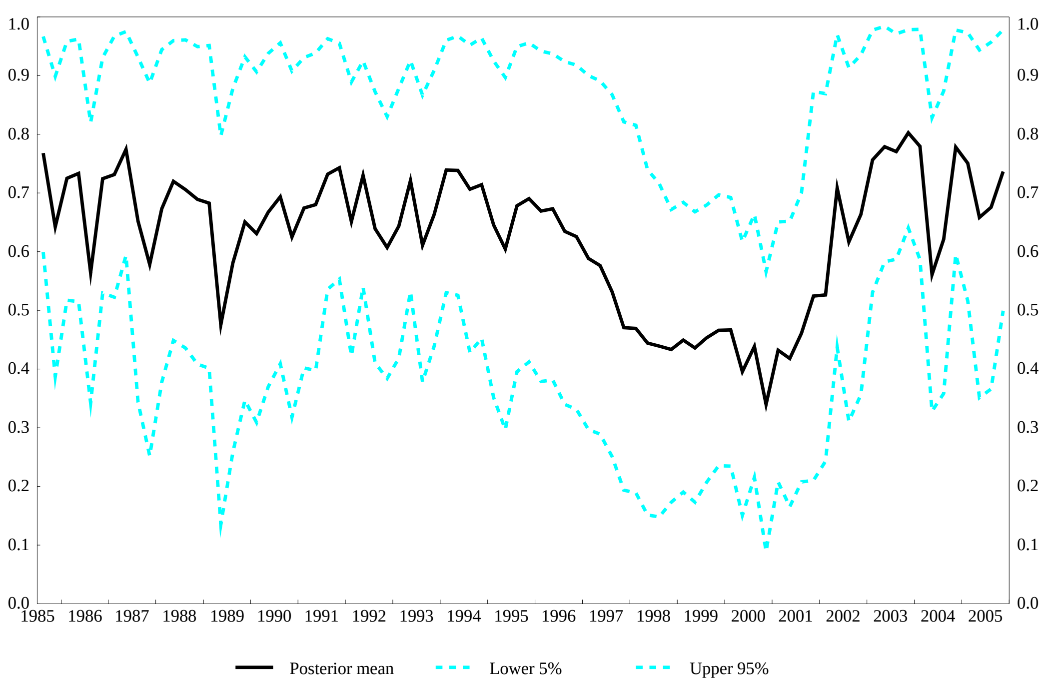


Figure 14: The Evolution of the Threshold Variable and the Probability of State 0 Occurring (Posterior Mean) in the MSTV Model for Australia

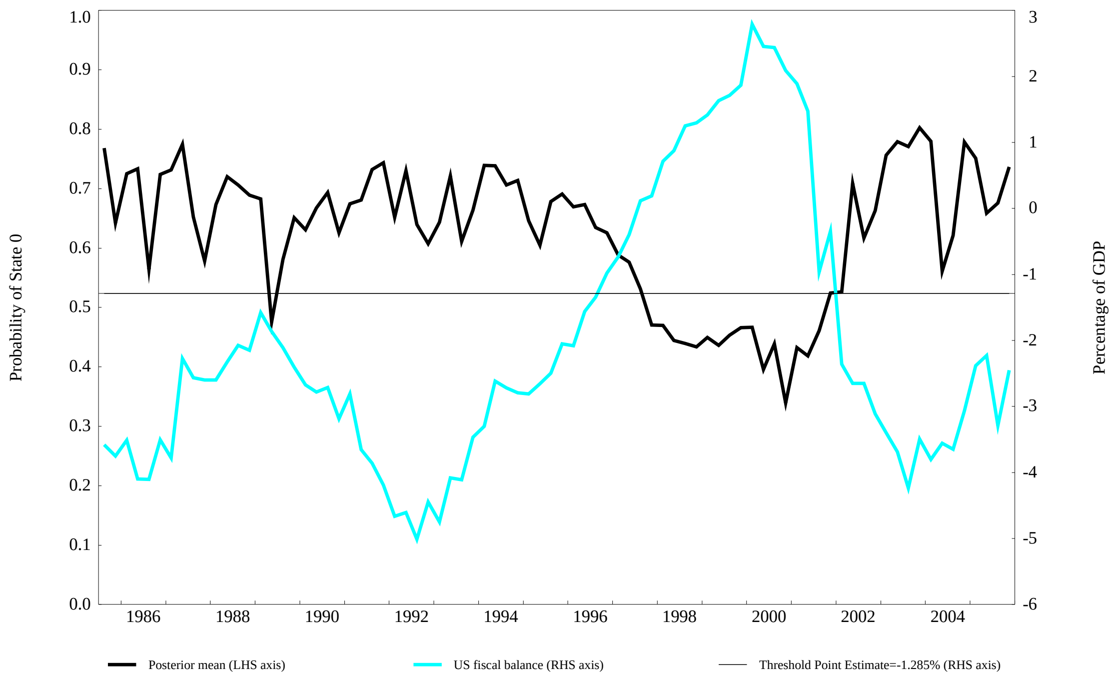


Figure 15: The Evolution of the Multilateral Variables in the Two Regimes Identified by the Threshold Model for Australia

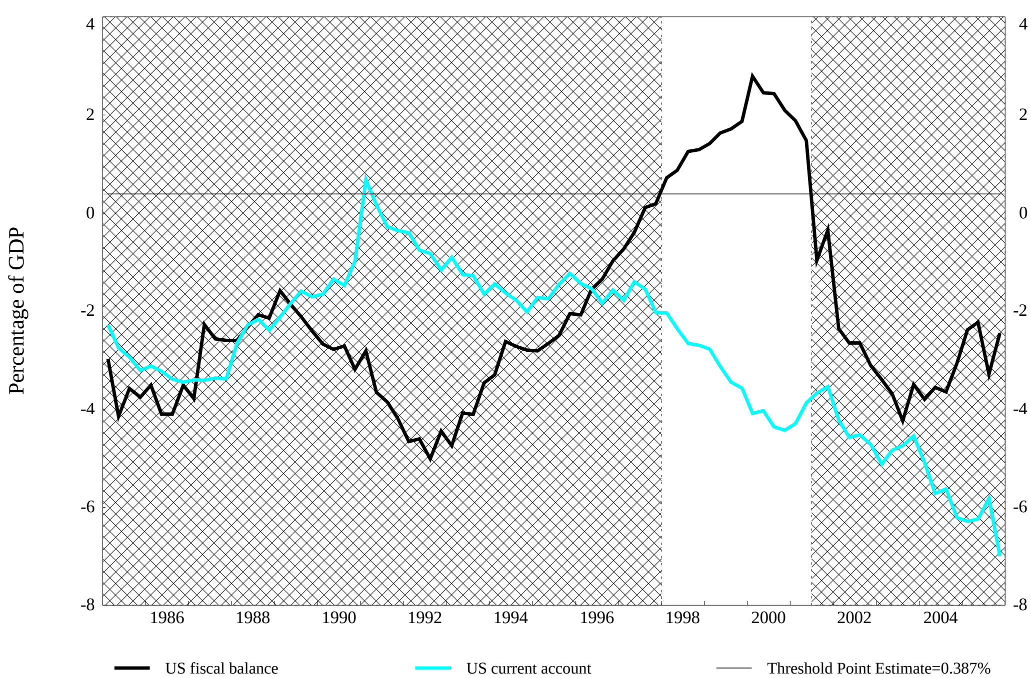


Figure 16: Probability of State 0 Occurring in the MSTV Model for New Zealand

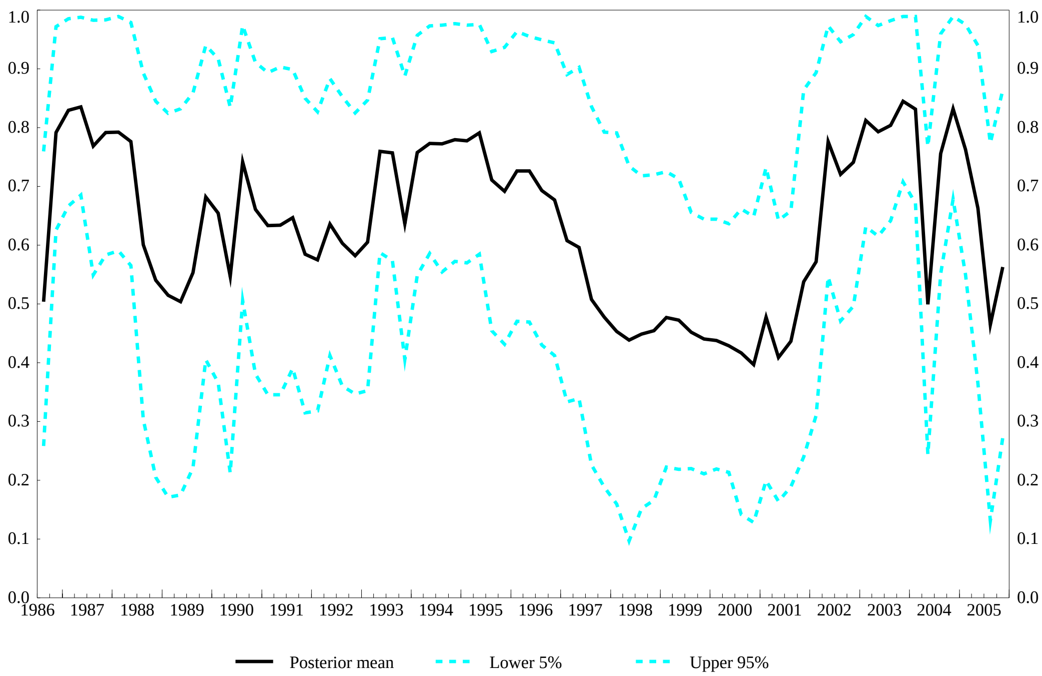


Figure 17: The Evolution of the Threshold Variable and the Probability of State 0 Occurring (Posterior Mean) in the MSTV Model for New Zealand

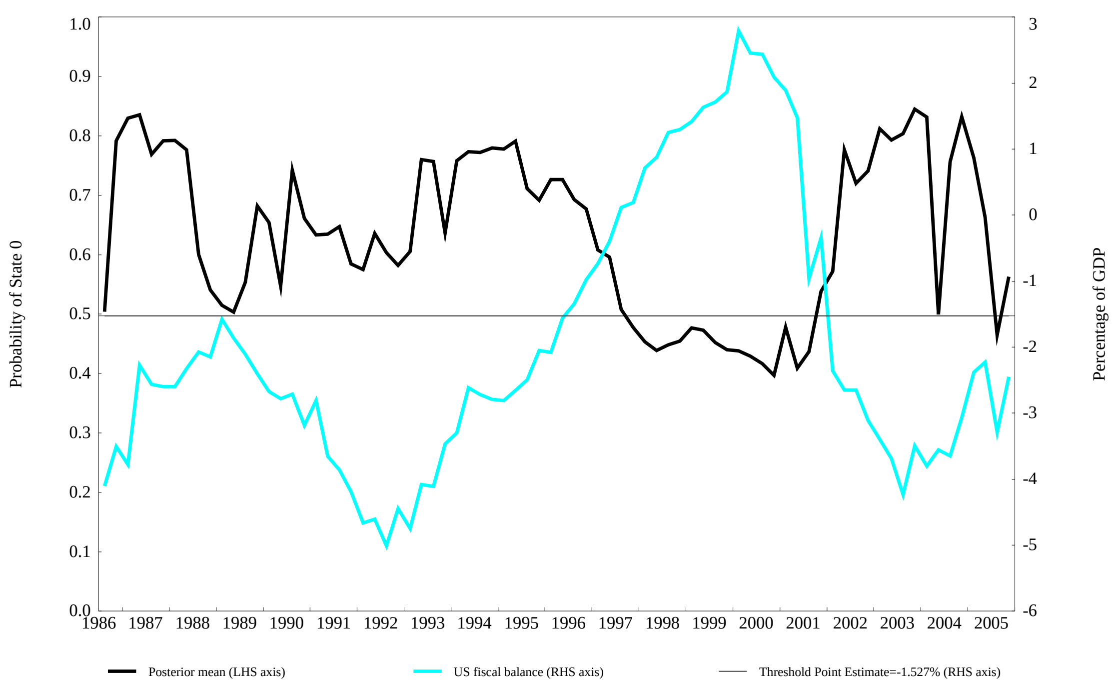


Figure 18: The Evolution of the Multilateral Variables in the Two Regimes Identified by the Threshold Model for New Zealand

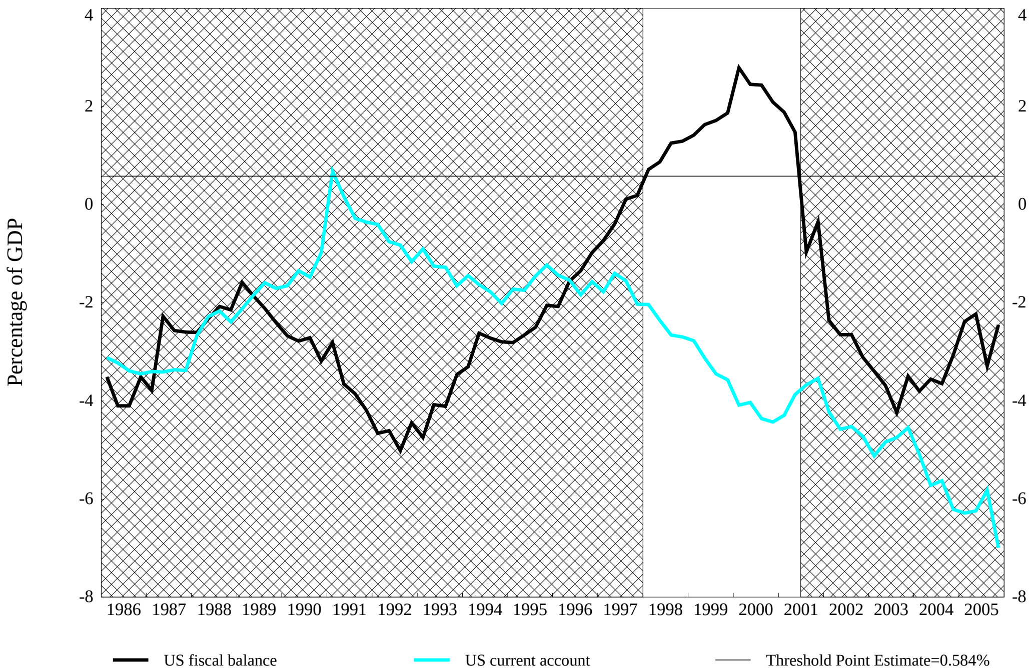


Figure 19: Posterior Distributions for In-Sample, Dynamic Simulations
Canada, Sample period: 1973Q3-2005Q4

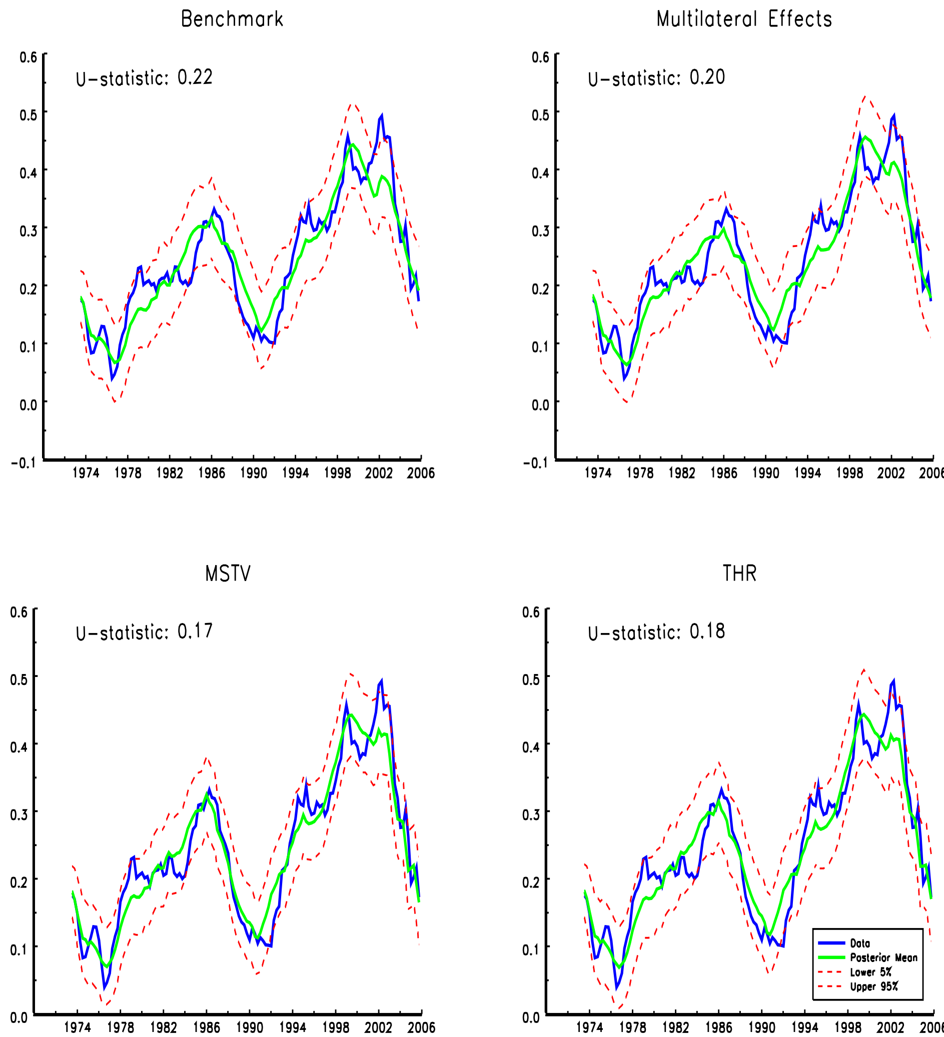


Figure 20: Posterior Distributions for In-Sample, Dynamic Simulations
Australia, Sample period: 1985Q3-2005Q4

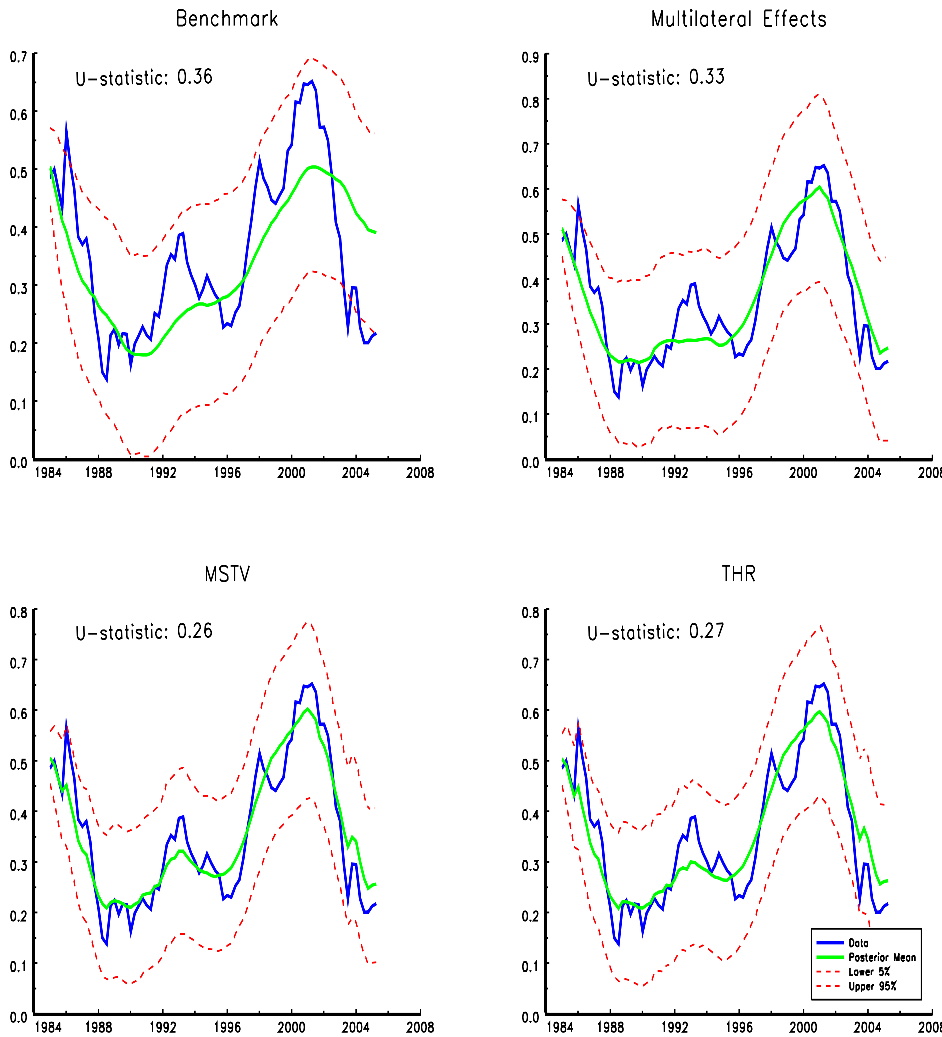


Figure 21: Posterior Distributions for In-Sample, Dynamic Simulations
New Zealand, Sample period: 1986Q3-2005Q4

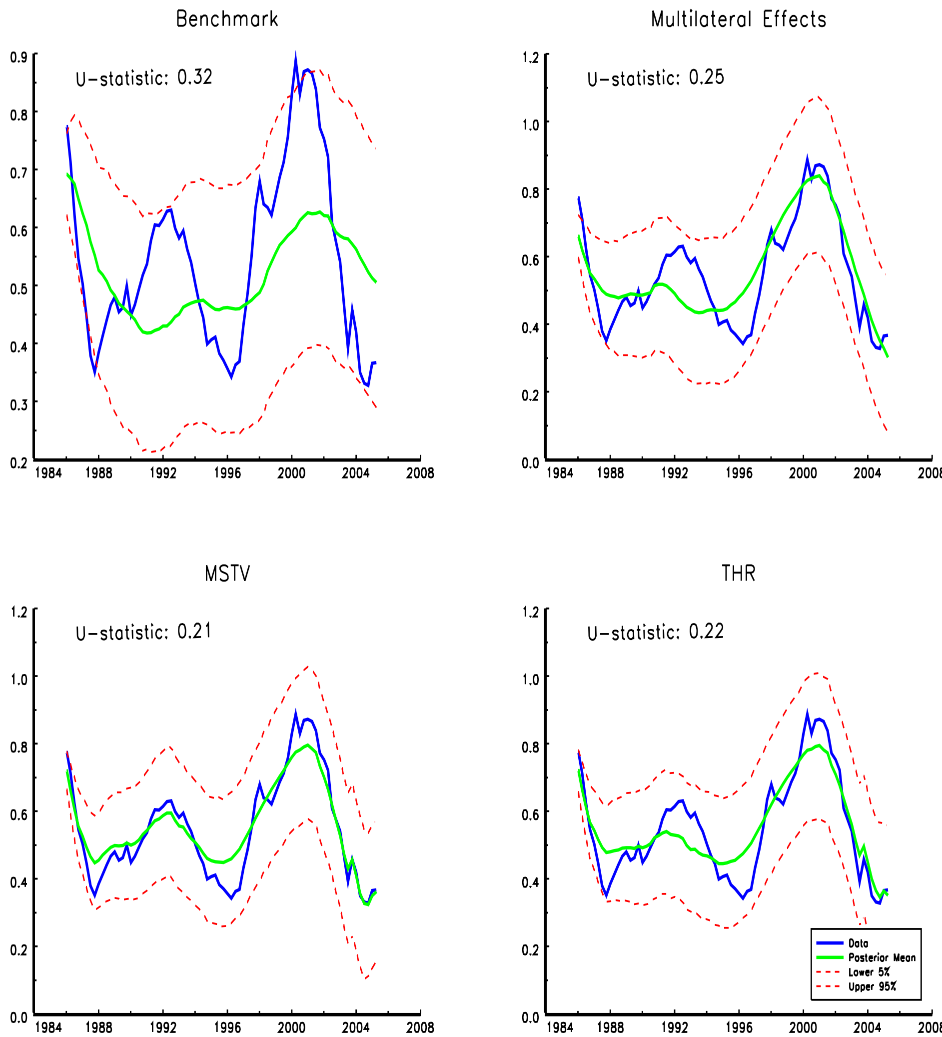


Figure 22: Out-of-Sample Forecasting Performance
Canada, Forecasting period: 2000Q4-2005Q4

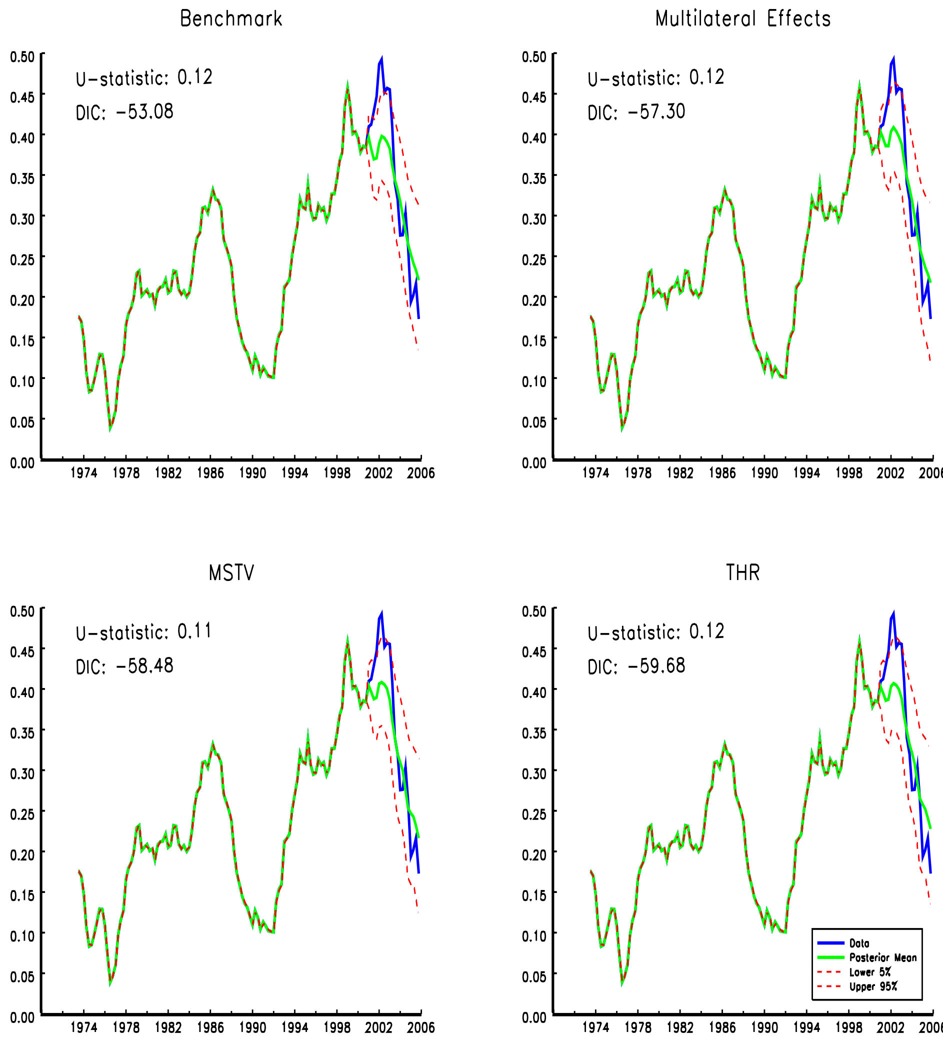


Figure 23: Out-of-Sample Forecasting Performance
Australia, Forecasting period: 2000Q4-2005Q4

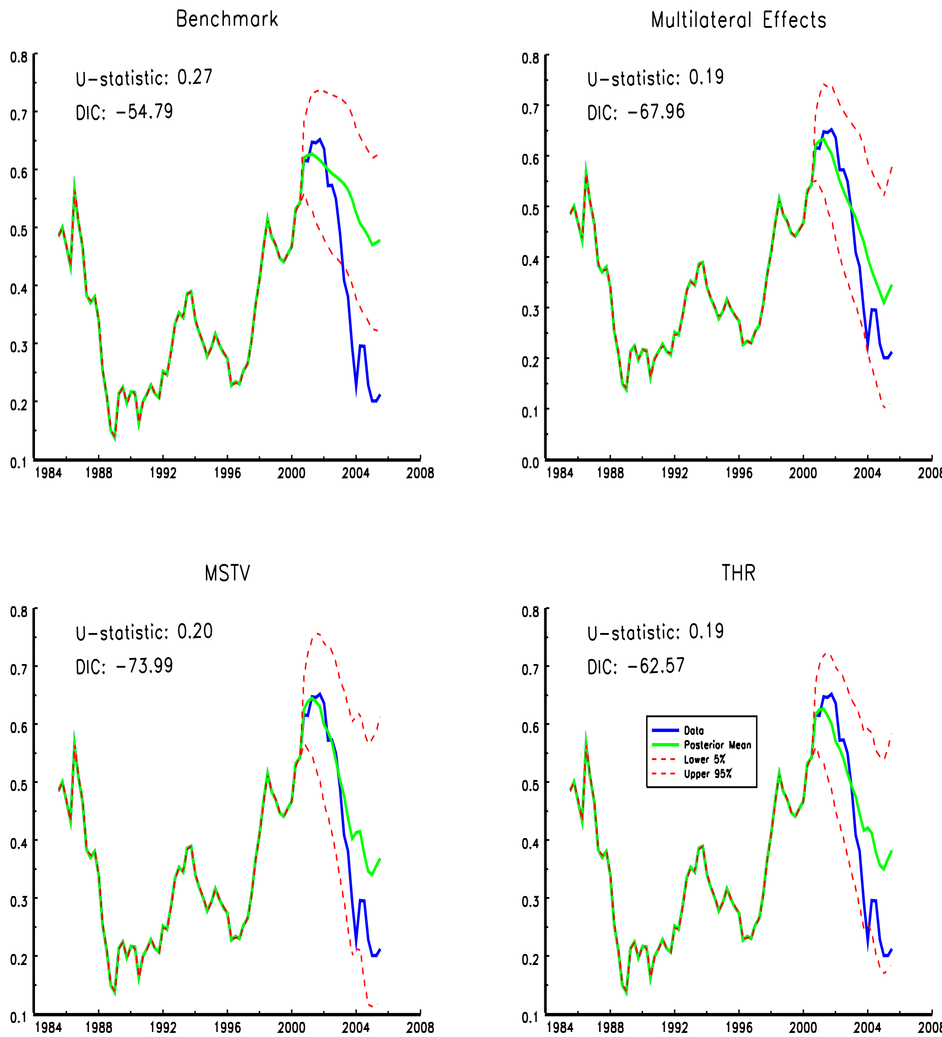
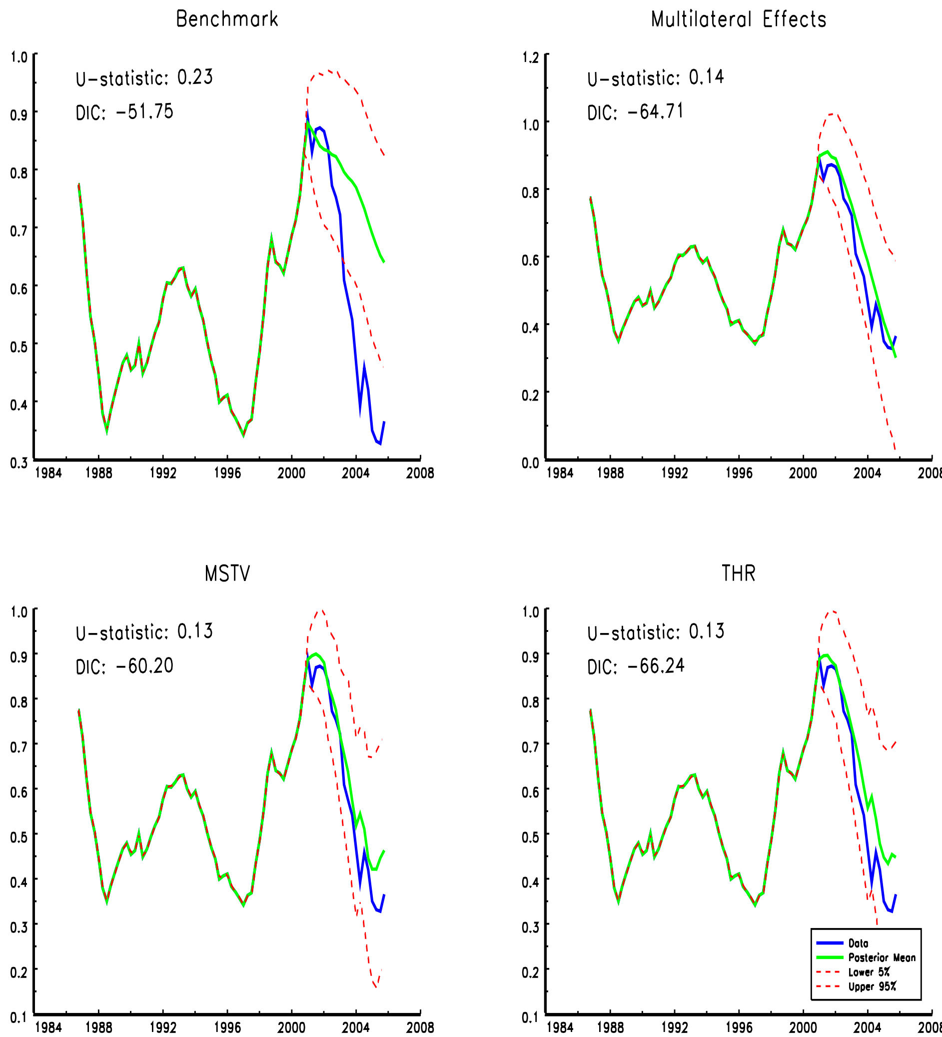


Figure 24: Out-of-Sample Forecasting Performance
New Zealand, Forecasting period: 2000Q4-2005Q4



Appendix E: Technical Appendix

Inference on the Probabilities of the States of the Economy

We draw an inference on the probability of the state of the economy, S_t, conditional on the threshold variable, q_{t-1}, and the information set at date $t-1$, Ψ_{t-1}, $P[S_t = i|q_{t-1}, \Psi_{t-1}]$ for $i = \{0, 1\}$. Suppose that $q_{t-1} \leq \gamma$. In this case, the probability of state zero occurring is as follows:

$$
\begin{aligned}
P[S_t = 0|q_{t-1} \leq \gamma, \Psi_{t-1}] &= \sum_{j=0}^{1} P[S_t = 0, S_{t-1} = j|q_{t-1} \leq \gamma, \Psi_{t-1}] \\
&= \sum_{j=0}^{1} P[S_t = 0|S_{t-1} = j, q_{t-1} \leq \gamma, \Psi_{t-1}] P[S_{t-1} = j|q_{t-1} \leq \gamma, \Psi_{t-1}] \\
&= m_0 P[S_{t-1} = 0|q_{t-1} \leq \gamma, \Psi_{t-1}] + (1 - n_0) P[S_{t-1} = 1|q_{t-1} \leq \gamma, \Psi_{t-1}] \\
&= m_0 \frac{P[S_{t-1} = 0, q_{t-1} \leq \gamma|\Psi_{t-1}]}{P[q_{t-1} \leq \gamma|\Psi_{t-1}]} + (1 - n_0) \frac{P[S_{t-1} = 1, q_{t-1} \leq \gamma|\Psi_{t-1}]}{P[q_{t-1} \leq \gamma|\Psi_{t-1}]} \\
&= \frac{m_0}{F^q(\gamma)} P[S_{t-1} = 0, q_{t-1} \leq \gamma|\Psi_{t-1}] + \frac{1 - n_0}{F^q(\gamma)} P[S_{t-1} = 1, q_{t-1} \leq \gamma|\Psi_{t-1}] \\
&= \frac{m_0}{F^q(\gamma)} P[S^*_{t-1} = 0|\Psi_{t-1}] + \frac{1 - n_0}{F^q(\gamma)} P[S^*_{t-1} = 2|\Psi_{t-1}] \qquad (17)
\end{aligned}
$$

The probability of state one occurring when $q_{t-1} \leq \gamma$, $P[S_t = 1|q_{t-1} \leq \gamma, \Psi_{t-1}]$, is $1 - P[S_t = 0|q_{t-1} \leq \gamma, \Psi_{t-1}] = (1 - m_0)P[S^*_{t-1} = 0|\Psi_{t-1}]/F^q(\gamma) + n_0 P[S^*_{t-1} = 2|\Psi_{t-1}]/F^q(\gamma)$. Suppose next that $q_{t-1} > \gamma$. In this case, the probability of state zero occurring is as follows:

$$
\begin{aligned}
P[S_t = 0|q_{t-1} > \gamma, \Psi_{t-1}] &= \sum_{j=0}^{1} P[S_t = 0, S_{t-1} = j|q_{t-1} > \gamma, \Psi_{t-1}] \\
&= \sum_{j=0}^{1} P[S_t = 0|S_{t-1} = j, q_{t-1} > \gamma, \Psi_{t-1}] P[S_{t-1} = j|q_{t-1} > \gamma, \Psi_{t-1}] \\
&= m_1 P[S_{t-1} = 0|q_{t-1} > \gamma, \Psi_{t-1}] + (1 - n_1) P[S_{t-1} = 1|q_{t-1} > \gamma, \Psi_{t-1}] \\
&= m_1 \frac{P[S_{t-1} = 0, q_{t-1} > \gamma|\Psi_{t-1}]}{P[q_{t-1} > \gamma|\Psi_{t-1}]} + (1 - n_1) \frac{P[S_{t-1} = 1, q_{t-1} > \gamma|\Psi_{t-1}]}{P[q_{t-1} > \gamma|\Psi_{t-1}]} \\
&= \frac{m_1}{1 - F^q(\gamma)} P[S_{t-1} = 0, q_{t-1} > \gamma|\Psi_{t-1}] + \frac{1 - n_1}{1 - F^q(\gamma)} P[S_{t-1} = 1, q_{t-1} > \gamma|\Psi_{t-1}] \\
&= \frac{m_1}{1 - F^q(\gamma)} P[S^*_{t-1} = 1|\Psi_{t-1}] + \frac{1 - n_1}{1 - F^q(\gamma)} P[S^*_{t-1} = 3|\Psi_{t-1}], \qquad (18)
\end{aligned}
$$

and the probability of state one occurring when $q_{t-1} > \gamma$, $P[S_t = 1|q_{t-1} > \gamma, \Psi_t]$, is $1 - P[S_t =$

$0|q_{t-1} > \gamma, \Psi_{t-1}] = (1-m_1)P[S^*_{t-1} = 1|\Psi_{t-1}]/(1-F^q(\gamma)) + n_1 P[S^*_{t-1} = 3|\Psi_{t-1}]/(1-F^q(\gamma))$.

In the case of the threshold model, the state probabilities (17) and (18) degenerate to zero or one. This becomes clear when we set $m_0 = n_1 = 1$ and $m_1 = n_0 = 0$. On the one hand, the probability of state zero occurring when $q_{t-1} \leq \gamma$, equation (17), is as follows:

$$\begin{aligned}
P[S_t = 0|q_{t-1} \leq \gamma, \Psi_{t-1}] &= \frac{1}{F^q(\gamma)} P[S^*_{t-1} = 0|\Psi_{t-1}] + \frac{1}{F^q(\gamma)} P[S^*_{t-1} = 2|\Psi_{t-1}] \\
&= \frac{1}{F^q(\gamma)} \{P[S_{t-1} = 0, q_{t-1} \leq \gamma|\Psi_{t-1}] + P[S_{t-1} = 1, q_{t-1} \leq \gamma|\Psi_{t-1}]\} \\
&= \frac{1}{F^q(\gamma)} \{P[q_{t-1} \leq \gamma|\Psi_{t-1}]\} \\
&= 1.
\end{aligned}$$

The above equation implies that in the threshold model, if $q_{t-1} \leq \gamma$, the state of the economy is zero with probability one. On the other hand, it becomes clear from equation (18) that $P[S_t = 0|q_{t-1} > \gamma, \Psi_{t-1}] = 0$ under the restricted transition probabilities. Therefore, when $q_{t-1} > \gamma$ in the threshold model, the state of the economy is one with probability one.

Measures of Overall Statistical Fit

Let $\mathcal{M}_i$ denote specification $i \in \{1, 2, ..., M\}$ where M is the number of models of interest. We evaluate the overall statistical fit of model i, $\mathcal{M}_i$, by computing its posterior model probability, $\pi_{i,T}$. We assign the same prior model probability $\pi_{i,0} = 1/M$ across all models. Then, posterior model probability $\pi_{i,T}$ is given by:

$$\pi_{i,T} = \frac{\pi_{i,0} P(y^T|\mathcal{M}_i)}{\sum_{i=1}^{M} \pi_{i,0} P(y^T|\mathcal{M}_i)} \tag{19}$$

where $P(y^T|\mathcal{M}_i) = \int L(y^T|\Theta_i, \mathcal{M}_i)P(\Theta_i)d\Theta_i$ is the marginal likelihood of model $\mathcal{M}_i$.[33] We then calculate the posterior odds ratio of model $\mathcal{M}_i$ versus model $\mathcal{M}_j$, $\pi_{i,T}/\pi_{j,T}$ for $i \neq j$. A value larger than 1 indicates that model $\mathcal{M}_i$ matches the data y^T better than model M_j, and vice versa.

To compare competing models, we also exploit the deviance information criterion (DIC), following the suggestion of (Gelman et al., 2004, p183).[34] To introduce the DIC, let $D(y^T; \Theta_i, \mathcal{M}_i)$

[33]We approximate the marginal likelihood with the modified harmonic-mean estimator proposed by Geweke (1999).

[34]Berg et al. (2004) apply the DIC to their comparison of stochastic volatility models.

denote *the deviance* of model i given the data y^T, which is defined as follows:

$$D(y^T;\Theta_i,\mathcal{M}_i) \equiv -2\ln L(y^T|X^T,\Theta_i,\mathcal{M}_i).$$

Let $D_{\hat{\Theta}_i(y^T)}(y^T)$ denote the deviance of model i which is evaluated at the posterior mean of the parameters Θ_j,$\hat{\Theta}_i(y^T)$,

$$D_{\hat{\Theta}_i(y^T)}(y^T) \equiv D(y^T;\hat{\Theta}_i(y^T),\mathcal{M}_i),$$

and $D_{avg}(y^T)$ denote the average deviance of model i for data y^T over the posterior distribution of the parameters Θ_i,

$$D_{avg}(y^T) \equiv N^{-1}\sum_{l=1}^{N} D(y^T;\Theta_i^l,\mathcal{M}_i),$$

where Θ_i^l is the lth draw from the posterior distribution $P(\Theta_i|\Psi_u,\mathcal{M}_i)$. The DIC then is defined as

$$DIC \equiv 2D_{avg}(y^T) - D_{\hat{\Theta}_i(y^T)}(y^T).$$

: The smaller the DIC of model i, the better the overall fit of the model with respect to the data y^T.

In-sample, dynamic simulation

We also evaluate and compare our models with respect to the their performance in in-sample, dynamic simulations. Noting that $P[S_t^*|\Psi_t]$ in equation (16) is the probability of state S_t^* conditional on information up to period t, we can also derive the probability of state S_t^* conditional on the model and the entire sample, $P[S_t^*|\Psi^T,\mathcal{M}_i]$, for each posterior draw Θ_i of model $\mathcal{M}_i$ using Kim's (1994) smoothed inferences. Given Θ_i and $P[S_t^*|\Psi^T,\mathcal{M}_i]$, we can construct the in-sample, dynamic simulation of the dependent variable y_t, $\hat{y}_t(\Theta_i,\mathcal{M}_i)$, as follows:

$$\hat{y}_t(\Theta_i,\mathcal{M}_i) = \begin{cases} X_t\theta_0 + e_{0,t} & \text{with probability } P[S_t^*=0|\Psi^T,\mathcal{M}_i] + P[S_t^*=1|\Psi^T,\mathcal{M}_i] \\ X_t\theta_1 + e_{1,t} & \text{with probability } P[S_t^*=2|\Psi^T,\mathcal{M}_i] + P[S_t^*=3|\Psi^T,\mathcal{M}_i] \end{cases} \tag{20}$$

where $\theta_{0,i}$ and $\theta_{1,i}$ are included in the posterior draw Θ_i.

The posterior mean of the in-sample, dynamic simulation, y_t^*, is then given by $y_t^* = \int \hat{y}_t(\Theta_i,\mathcal{M}_i)P(\Theta_i|y^T,\mathcal{M}_i)d\Theta_i$. We also calculate the posterior mean of the Theil U-statistic U_i^* using:

$$U_i^* = \int U(\Theta_i, \mathcal{M}_i) P(\Theta_i | y^T, \mathcal{M}_i) d\Theta_i,$$

where

$$U(\Theta_i, \mathcal{M}_i) = \{T^{-1} \sum_{t=1}^{T} [y_t - \hat{y}_t(\Theta_i, \mathcal{M}_i)]^2 / var(y)\}^{1/2}.^{35}$$

Out-of-sample forecasting performance

Finally, we also evaluate and compare the out-of-sample forecasting performance of our four different models.[36] As explained by Geweke (1999), we can show that the higher the marginal likelihood of model i, $P(y^T|\mathcal{M}_i)$, the better the model's accuracy in out-of-sample forecasting. To see this, consider model i's predictive density of $y_{u+1}, y_{u+2}, \cdots, y_t$ conditional on the information up to period u, $P(y_{u+1}, \cdots, y_t | \Psi_u, \mathcal{M}_i)$:

$$\begin{aligned} P(y_{u+1}, \cdots, y_t | \Psi_u, \mathcal{M}_i) &= \int P(\Theta_i | \Psi_u, \mathcal{M}_i) \prod_{s=u+1}^{t} P(y_s | \Psi_{s-1}, \Theta_i, \mathcal{M}_i) d\Theta_i \\ &= \frac{\int P(\Theta_i | \mathcal{M}_i) \prod_{s=1}^{t} P(y_s | \Psi_{s-1}, \Theta_i, \mathcal{M}_i) d\Theta_i}{P(y^u | \mathcal{M}_i)} \\ &= \frac{P(y^t | \mathcal{M}_i)}{P(y^u | \mathcal{M}_i)}. \end{aligned} \tag{21}$$

In other words, the predictive density $P(y_{u+1}, \cdots, y_t | \Psi_u, \mathcal{M}_i)$ is the ratio of the marginal data density of y^t to that of y^u, $P(y^t|\mathcal{M}_i)/P(y^u|\mathcal{M}_i)$. Note that the marginal data density of the entire data set y^T, $P(y^T|\mathcal{M}_i)$, is identical to the predictive density $P(y_1, \cdots, y_T | \Psi_0, \mathcal{M}_i)$ because Ψ_0 is the empty set. Then from equation (21), we can write

$$\begin{aligned} P(y^T | \mathcal{M}_i) &= P(y_1, \cdots, y_T | \Psi_0, \mathcal{M}_i) \\ &= \frac{P(y^T | \mathcal{M}_i)}{P(y^0 | \mathcal{M}_i)} \\ &= \prod_{s=1}^{T} \frac{P(y^s | \mathcal{M}_i)}{P(y^{s-1} | \mathcal{M}_i)} \\ &= \prod_{s=1}^{T} P(y_s | \Psi_{s-1}, \mathcal{M}_i). \end{aligned} \tag{22}$$

[35] The Theil U-statistic does not suffer from a scaling problem, as do the root mean squared error and the mean average error. See, for example, Greene (2000, p310).

[36] One of the advantages of MS models over THR models is that it is easier to generate the out-of-sample forecasts in the former compared to the latter when the forecast horizon is longer than one period. See Granger and Teräsvirta (1993, chap. 8).

Notice that $P(y_s|\Psi_{s-1}, \mathcal{M}_i)$ is the predictive density of the data y_s: the probability of the data y_s conditional on the one-period past information Ψ_{s-1} and model i. Hence, equation (22) implies that given the data y^T, a model generating a higher marginal likelihood has a higher accuracy for all the one-period-ahead out-of-sample forecasts.

To compare models with respect to their out-of-sample predictive power for a subsample, we also exploit the DIC. To construct the DIC for this case, suppose that we want to evaluate the out-of-sample forecasting ability of model i for the future data points, $y_{u+1}, y_{u+2}, \cdots, y_t$, conditional on the information set up to period u.[37] The deviance for the future data points, $D(y_{u+1}, y_{u+2}, \cdots, y_t; \Theta_i, \mathcal{M}_i)$ is defined as -2 times the log-likelihood:

$$D(y_{u+1}, y_{u+2}, \cdots, y_t; \Theta_i, \mathcal{M}_i) \equiv -2 \ln P(y_{u+1}, y_{u+2}, \cdots, y_t|\Psi_u, \Theta_i, \mathcal{M}_i).$$

Let $D_{\hat{\Theta}_i(\Psi_u)}(y_{u+1}, y_{u+2}, \cdots, y_t)$ and $D_{avg}(y_{u+1}, y_{u+2}, \cdots, y_t)$, respectively, denote the deviance of the future data points $y_{u+1}, y_{u+2}, \cdots, y_t$ which is evaluated at the posterior mean of the parameters Θ_j conditional on the information set Ψ_u, $\hat{\Theta}_i(\Psi_u)$:

$$D_{\hat{\Theta}_i(\Psi_u)}(y_{u+1}, y_{u+2}, \cdots, y_t) \equiv D(y_{u+1}, y_{u+2}, \cdots, y_t; \hat{\Theta}_i(\Psi_u), \mathcal{M}_i),$$

and the average deviance of $y_{u+1}, y_{u+2}, \cdots, y_t$ over the posterior distribution of the parameters Θ_i conditional on the information set Ψ_u:

$$D_{avg}(y_{u+1}, y_{u+2}, \cdots, y_t) \equiv N^{-1} \sum_{l=1}^{N} D(y_{u+1}, y_{u+2}, \cdots, y_t; \Theta_i^l, \mathcal{M}_i),$$

where Θ_i^l is the lth draw from the posterior distribution $P(\Theta_i|\Psi_u, \mathcal{M}_i)$. The DIC then is defined as:

$$DIC \equiv 2D_{avg}(y_{u+1}, y_{u+2}, \cdots, y_t) - D_{\hat{\Theta}_i(\Psi_u)}(y_{u+1}, y_{u+2}, \cdots, y_t),$$

where a smaller DIC for model i implies a better out-of-sample forecasting performance.

[37]Following the exercise by Meese and Rogoff (1983), which is conventional in the empirical exchange rate literature, we deal with all exogenous variables as being deterministic and known when we construct the out-of-sample forecasts on the log of the real exchange rate.