# OCACT Stochastic Model

Version 2004.1

Office of the Chief Actuary
U.S. Social Security Administration

March 24, 2006

# Overview

# Social Security Financial Analysis

- Annual Trustees Report: OCACT projects principal factors which affect the financial status of the OASDI program (75-year projection)

- Board of Trustees of the Federal Old-Age and Survivors Insurance (OASI) and Disability Insurance (DI) Trust Funds

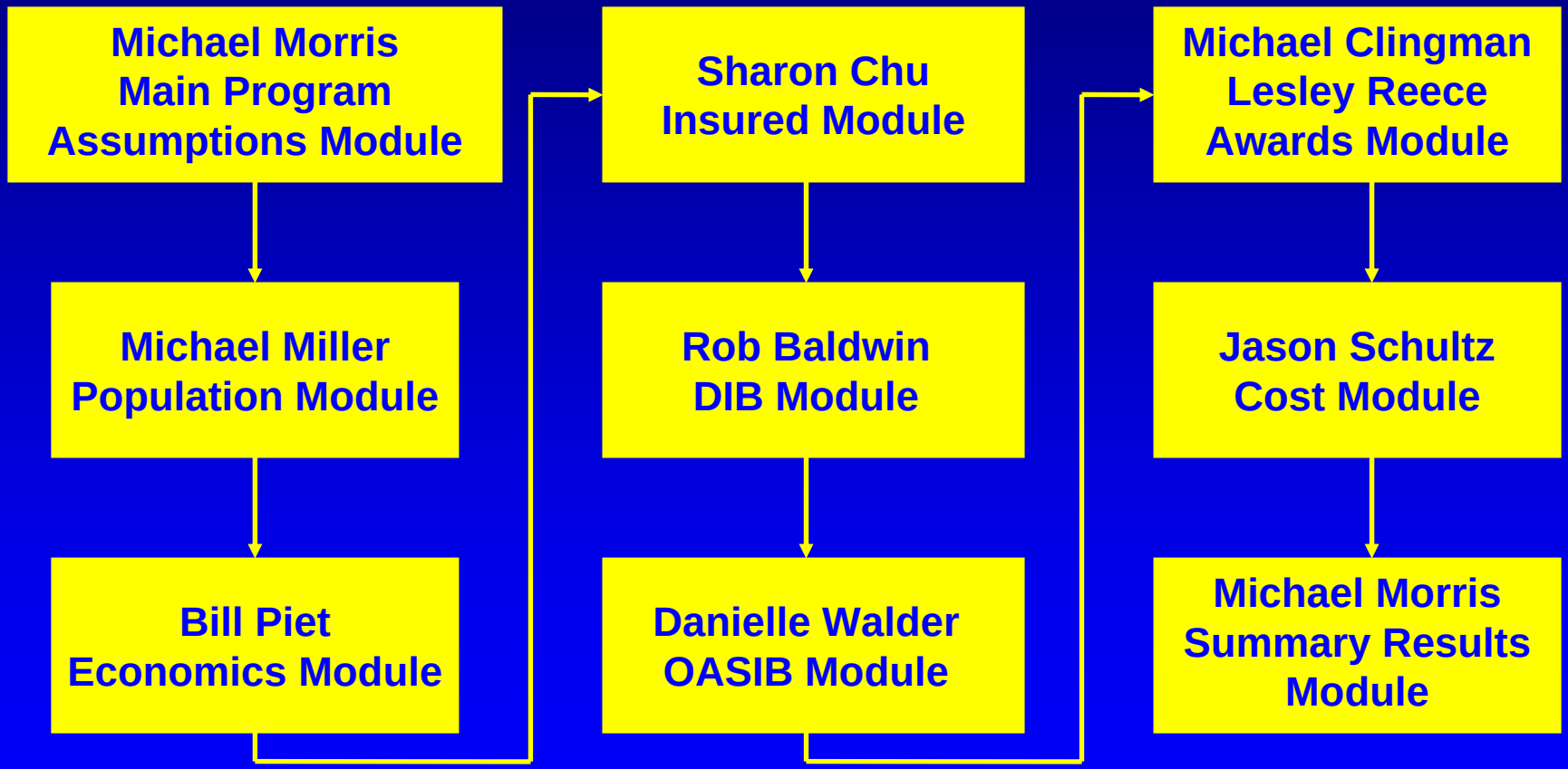
# Deterministic Model

- 3 sets of assumptions
  - Alt 1: low cost (more optimistic)
  - Alt 2: intermediate
  - Alt 3: high cost (more pessimistic)
- Sensitivities
- No indication of probabilities associated with each alternative

# Stochastic Model

- Assigns random variation for key assumptions
- Provides a probability distribution of possible outcomes around the intermediate case
- In the absence of random variation, the value of each input variable equals its value under the intermediate assumptions

# Programmers / Program Flow



# Demographic Equations

- Total Fertility Rate
- Rate of Mortality Improvement (42)
- Legal Immigration
- Legal Emigration
- Net Other Immigration

# Economic Equations

- Unemployment Rate
- Inflation Rate
- Real Interest Rate
- Growth Rate in the Real Average Wage

# Disability Equations

- Disability Incidence Rate (2)
- Disability Recovery Rate (2)

# Source Code Outline

```
do runNum = 1, totalRuns
   do year = TR_YEAR, lastProjYear
      call AssumptionsSubroutine(year)
      call PopulationSubroutine(year)
      call EconomicSubroutine(year)
      call InsuredSubroutine(year)
      call DIBSubroutine(year)
      call OASIBSubroutine(year)
      call AwardsSubroutine(year)
      call CostSubroutine(year)
      call SummaryResultsSubroutine(year, runNum)
   end do
end do
```

# Runtime Log

| Checkpoint Transition | Name of Transition | Time (in seconds) |
|---|---|---|
| 1==> 2 | Initialization | 4.67 |
| 2==> 3 | start simulation | 0.00 |
| 3==> 4 | assumptions subroutine | 0.57 |
| 4==> 5 | population subroutine | 0.50 |
| 5==> 6 | economic subroutine | 0.19 |
| 6==> 7 | insured subroutine | 0.03 |
| 7==> 8 | dib subroutine | 0.69 |
| 8==> 9 | oasib subroutine | 0.99 |
| 9==>10 | awards subroutine | 255.68 |
| 10==>11 | cost subroutine | 0.54 |
| 11==>12 | summary results subroutine | 0.08 |
| 12==>13 | status report | 0.05 |
| 13==> 3 | restart year loop | 0.00 |
| 13==>14 | end simulation | 0.00 |
| 14==>15 | wrapup | 0.22 |

| | |
|---|---|
| Total time of simulation: | 264.20 |
| Total number of runs: | 10 |
| Number of years per run: | 76 |
| Years simulated: | 2004 to 2079 |
| Approximate average time per run: | 26.42 |

# Interesting Facts

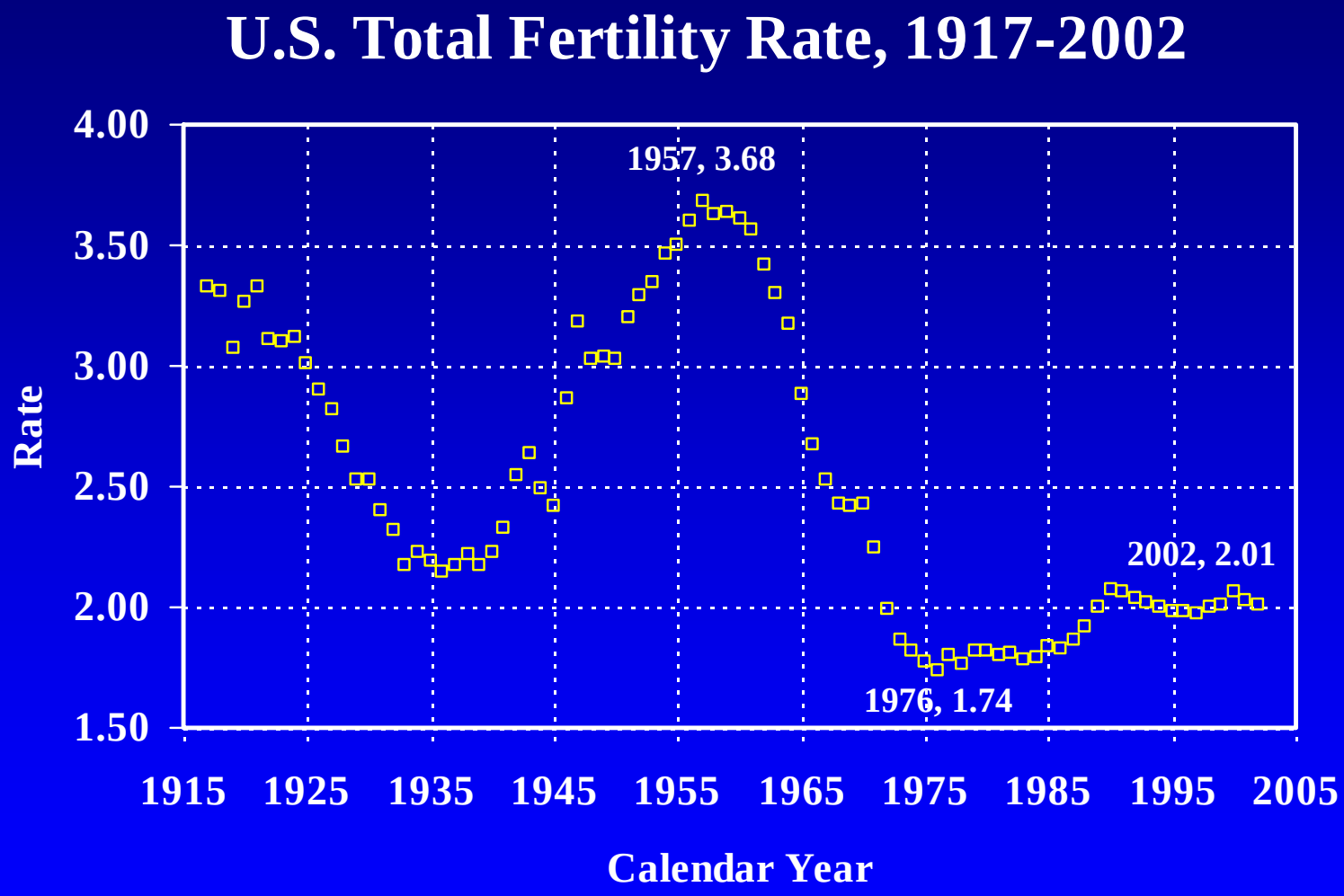
- First appearance:  2003 Trustees Report
- Model updated each year
- Compiled using Compaq Visual Fortran 6.1.A
  - 20 source code files
  - Approx. 26,000 lines of code
  - Over 160 data files
- Requires 425 MB of RAM
- Approx. 34 hours to run 5,000 simulations

# Examples of Equations

# Fertility Equation Development

- Data Sources
  - National Center for Health Statistics
  - U.S. Census Bureau
- Historical Period 1917 - 2002
- Age-specific birth rates for women, 14 - 49
- Total Fertility Rate = sum of age-specific birth rates in a given year.

# Fertility Equation Development



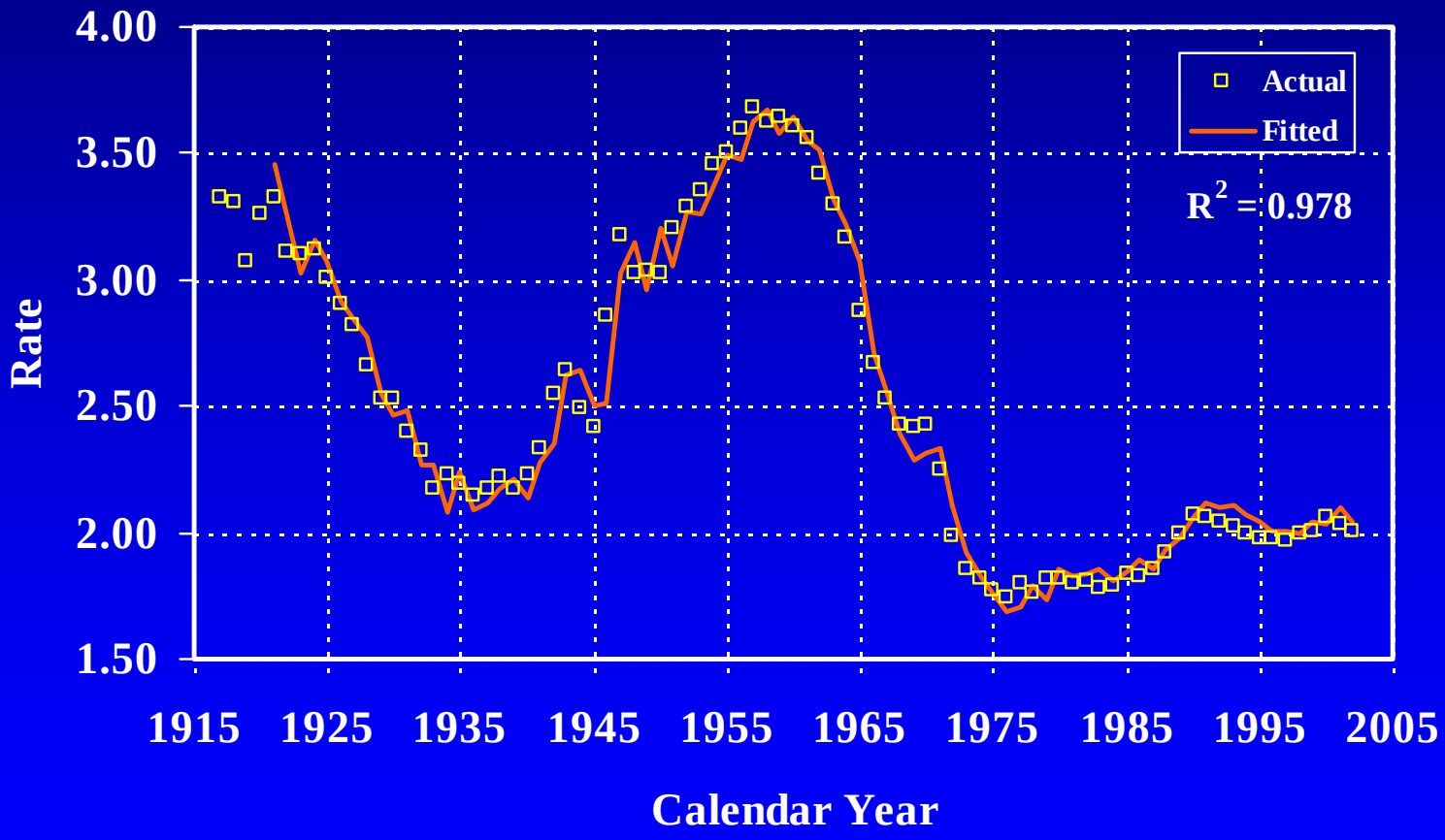
# Fertility Equation Specification

$$F_t = F_t^{TR} + 1.99 f_{t-1} - 1.51 f_{t-2} + 0.91 f_{t-3} - 0.42 f_{t-4} + \varepsilon_t - 0.67 \varepsilon_{t-1}$$

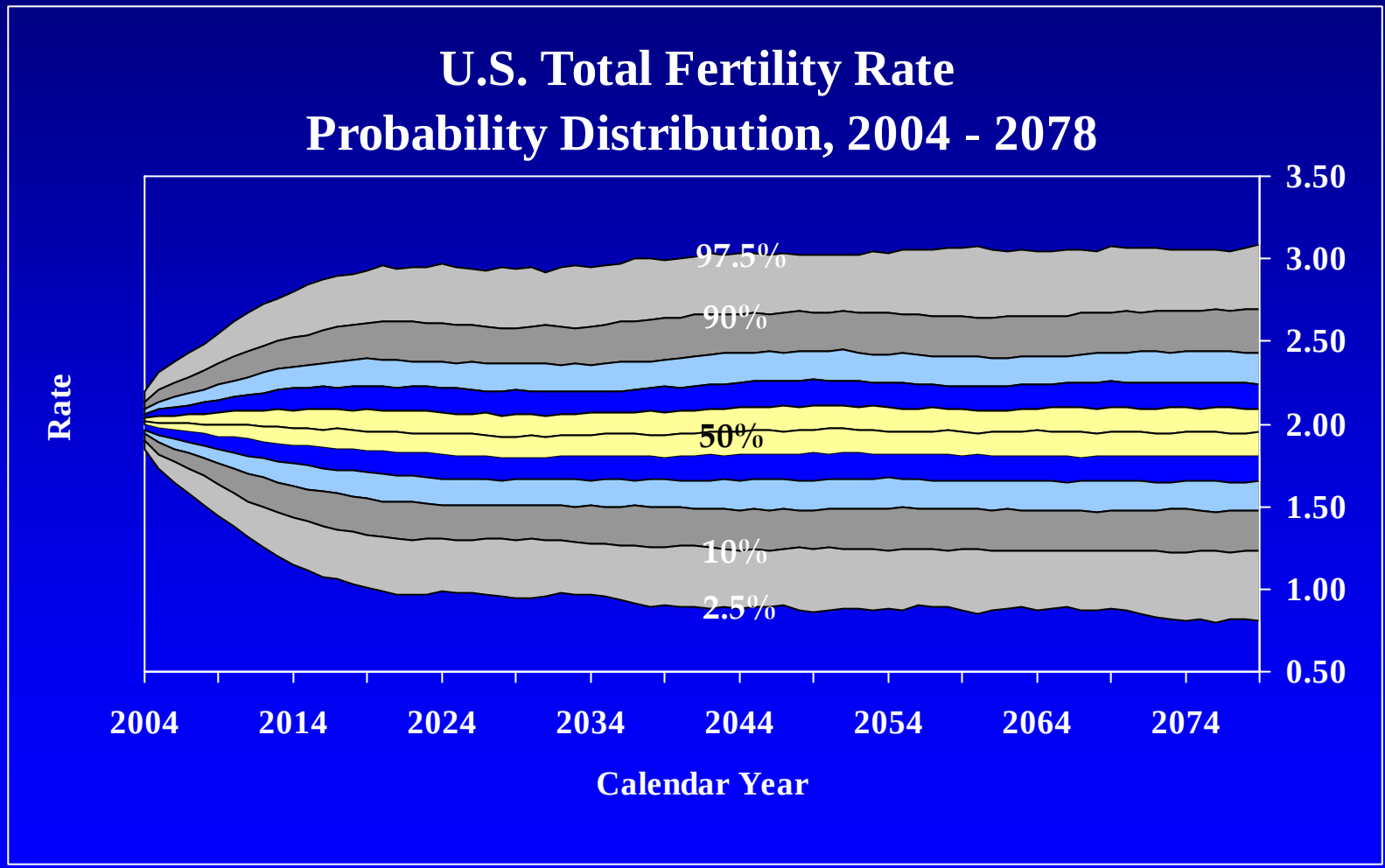
- $F_t$=Total Fertility Rate
- Regression performed with EViews software
- Modified ARMA(4,1)
  - Centered on 2004 Trustees Report Intermediate Assumption (TR04II)
  - TFR bounded below by 0.5 and above by 3.4

# Fertility Equation Fit

## U.S. Total Fertility Rate, 1917-2002



# Fertility Equation Results



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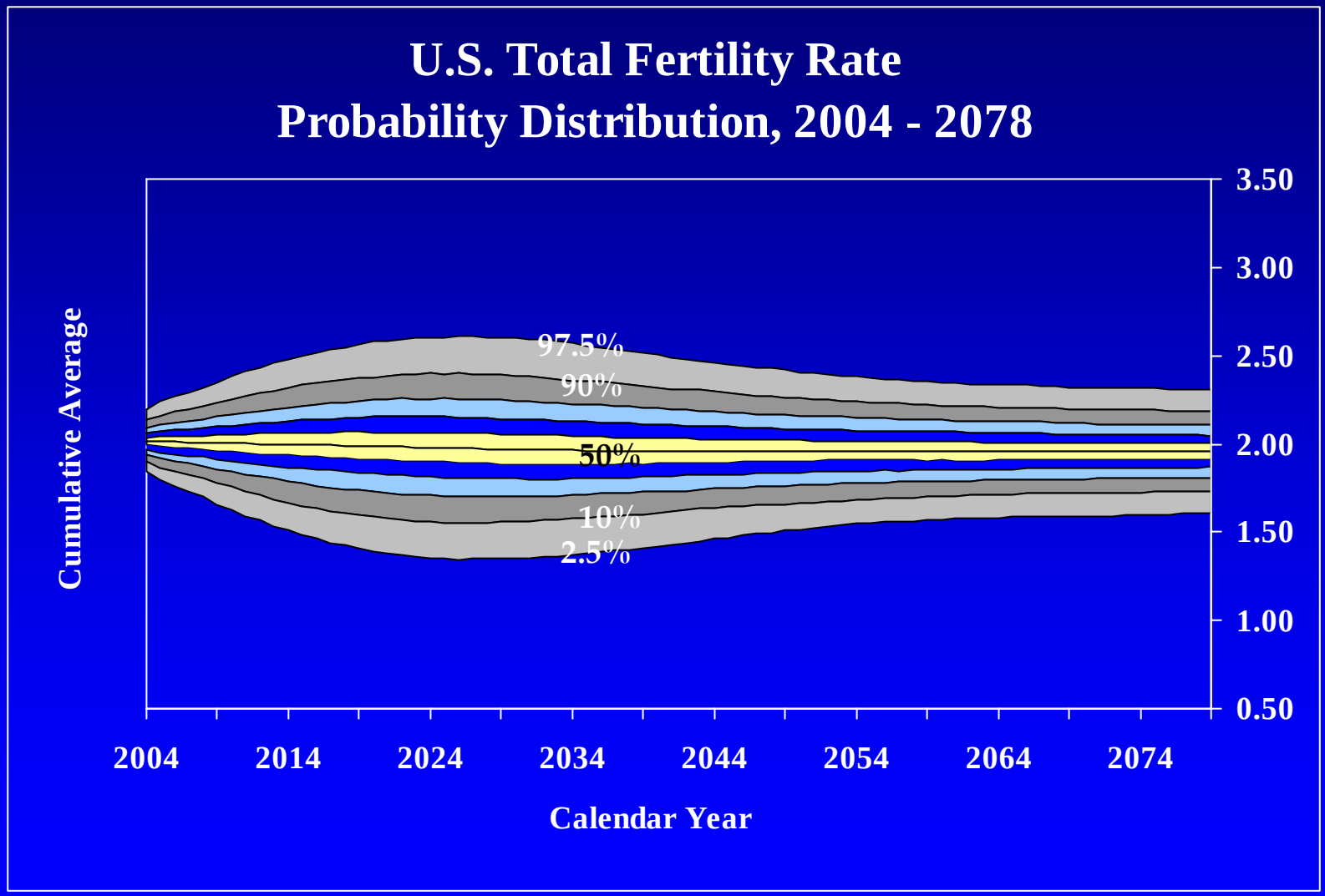

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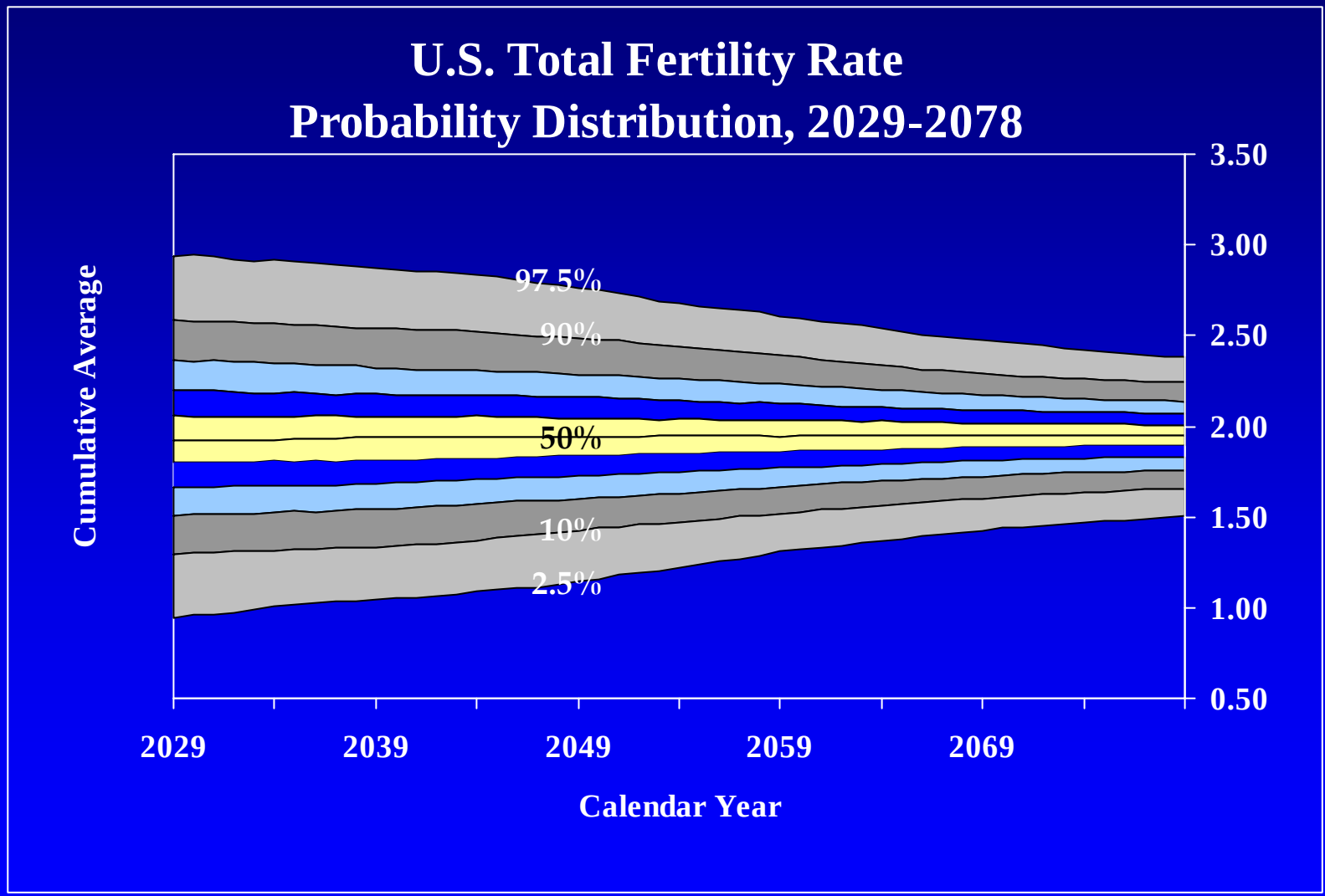
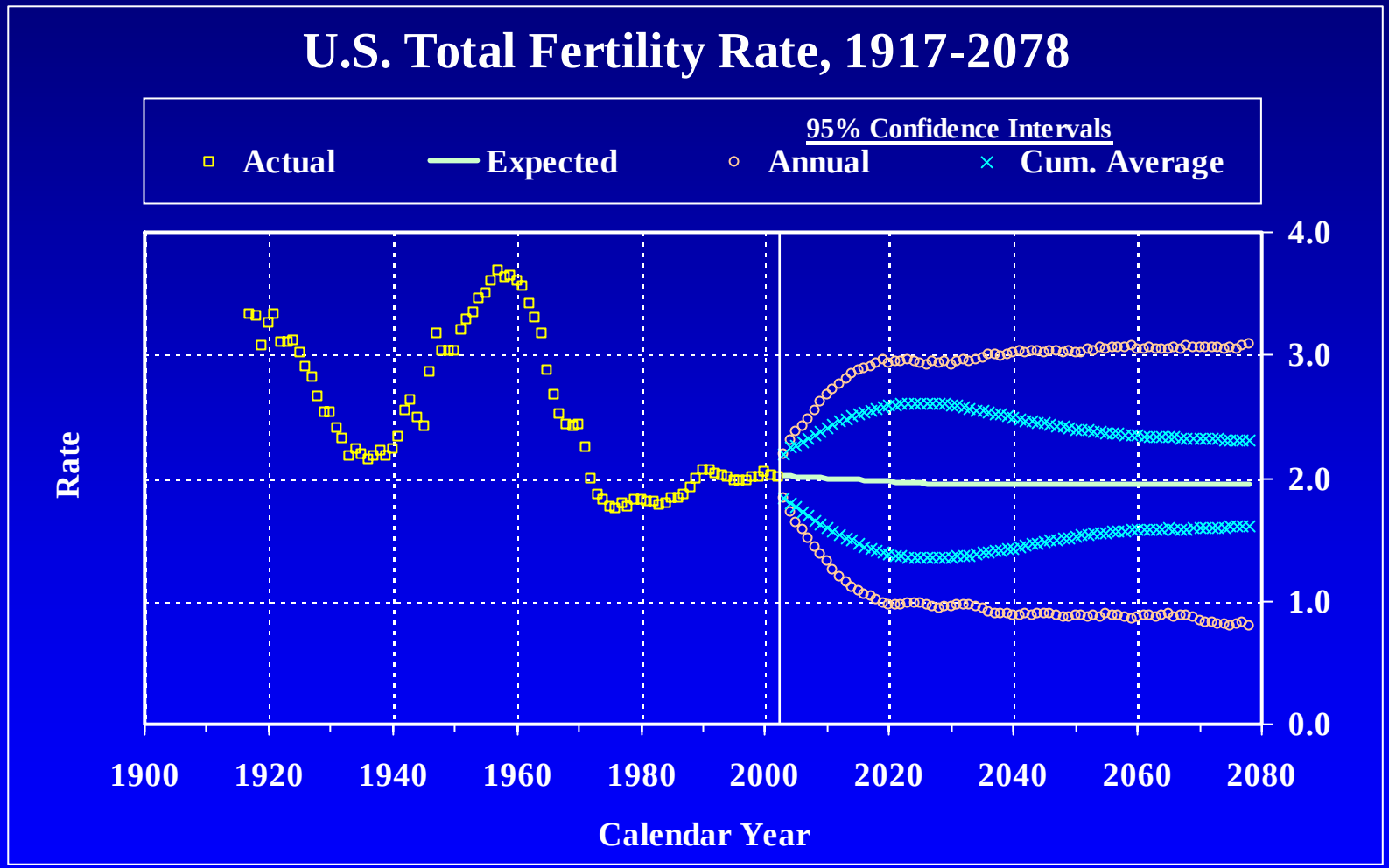


# Fertility Equation Results



# Mortality Equation Development

- Data Sources
  - National Center for Health Statistics
  - U.S. Census Bureau
  - Centers for Medicare & Medicaid Services
- Historical Period 1900 - 2000
- Central death rates for 42 age-sex groups

# Mortality Equation Specification

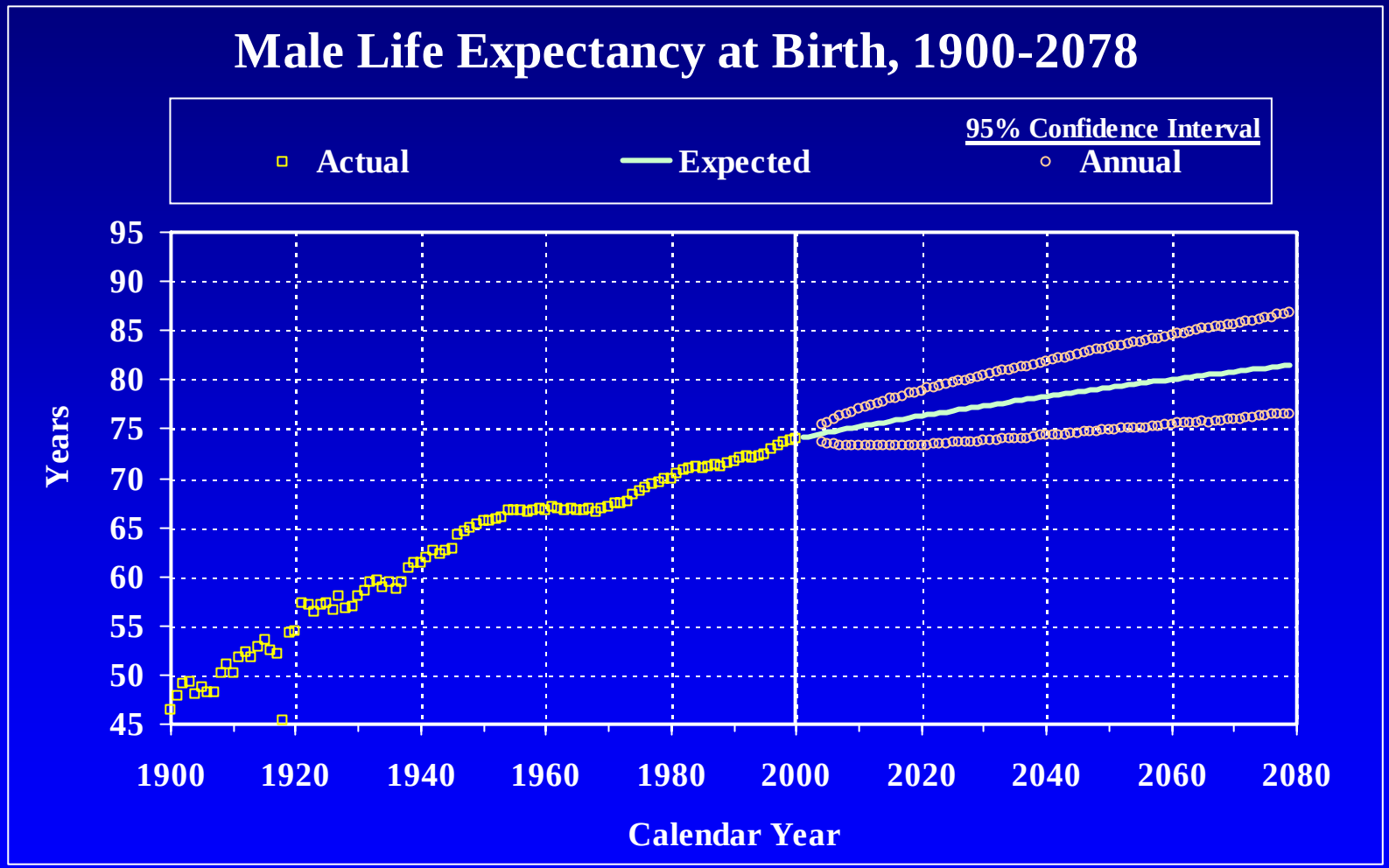
$$MR_{k,t} = MR_{k,t}^{TR} + \phi_k mr_{k,t-1} + \varepsilon_{k,t}$$

- $MR_{k,t}$=Rate of decrease in central death rate
- Modified AR(1)
- Random error among 42 age-sex groups correlated using Cholesky decomposition

# Cholesky Decomposition

- Use fitted equations to compute residuals
- Compute variance-covariance matrix of residuals, **V**
- Perform Cholesky decomposition, **V = LL'**
- Generate a vector of independent standard normal random variables, $\vec{x}$
- Multiply lower triangular matrix by vector, $\mathbf{L}\vec{x} = \vec{\varepsilon}$

# Mortality Equation Results



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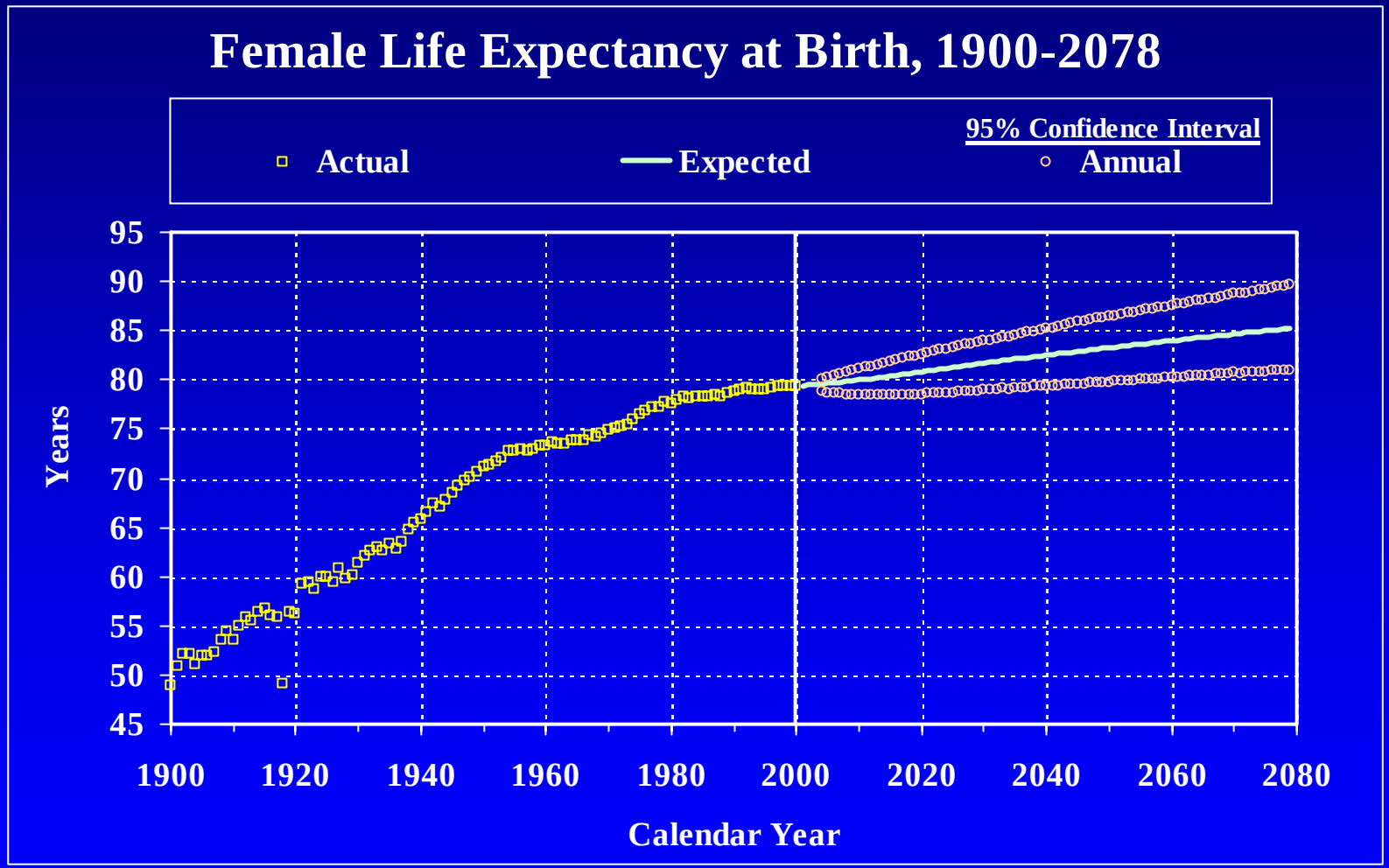


# Economic Equations

- Unemployment Rate, CPI and Real Interest Rate estimated using Vector Autoregression (VAR) with 2 lags.
  - Unemployment rates expressed as log-odds ratios.
  - Percent change in CPI transformed into logs with lower bound of -3%.

# Economic Equations Specification

- The 3-Variable VAR

$$U_t = U_t^{TR} + 0.96u_{t-1} - 0.30u_{t-2} + 0.40i_{t-1} - 0.08i_{t-2} + 0.75r_{t-1} + 0.61r_{t-2} + \varepsilon_{1t}$$

$$I_t = I_t^{TR} - 0.77u_{t-1} + 0.72u_{t-2} + 0.60i_{t-1} + 0.30i_{t-2} - 4.85r_{t-1} + 1.80r_{t-2} + \varepsilon_{2t}$$

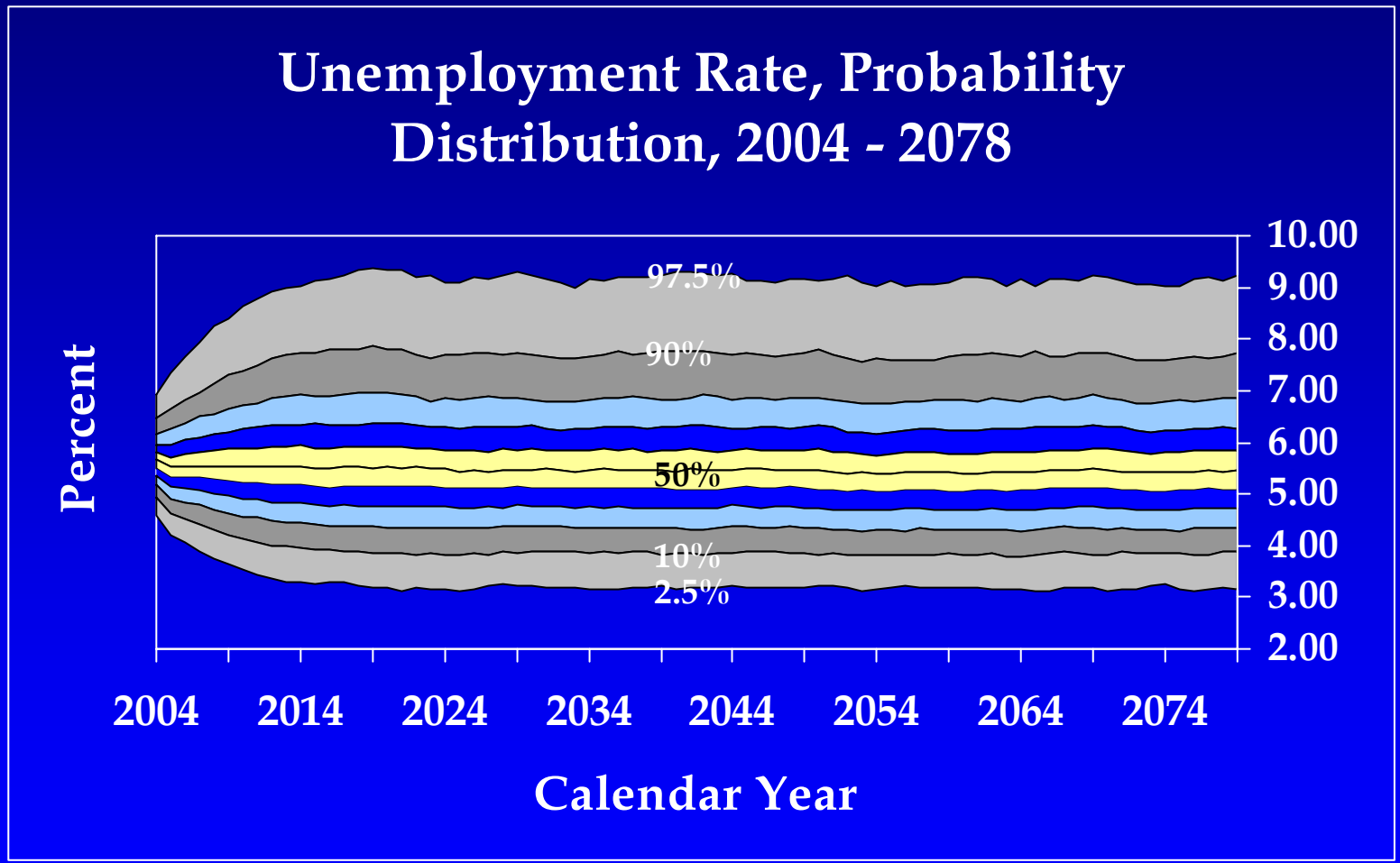
$$R_t = R_t^{TR} + 0.06u_{t-1} - 0.05u_{t-2} + 0.03i_{t-1} - 0.03i_{t-2} + 1.23r_{t-1} - 0.32r_{t-2} + \varepsilon_{3t}$$

# Cholesky Decomposition

$$\mathbf{L}\vec{x} = \vec{\varepsilon}$$

$$\begin{bmatrix} 0.113 & 0 & 0 \\ -0.050 & 0.139 & 0 \\ 0.001 & -0.007 & 0.010 \end{bmatrix} \begin{bmatrix} 0.854 \\ -0.041 \\ 2.285 \end{bmatrix} = \begin{bmatrix} 0.097 \\ -0.049 \\ 0.024 \end{bmatrix}$$

# Unemployment Rate Equation Results



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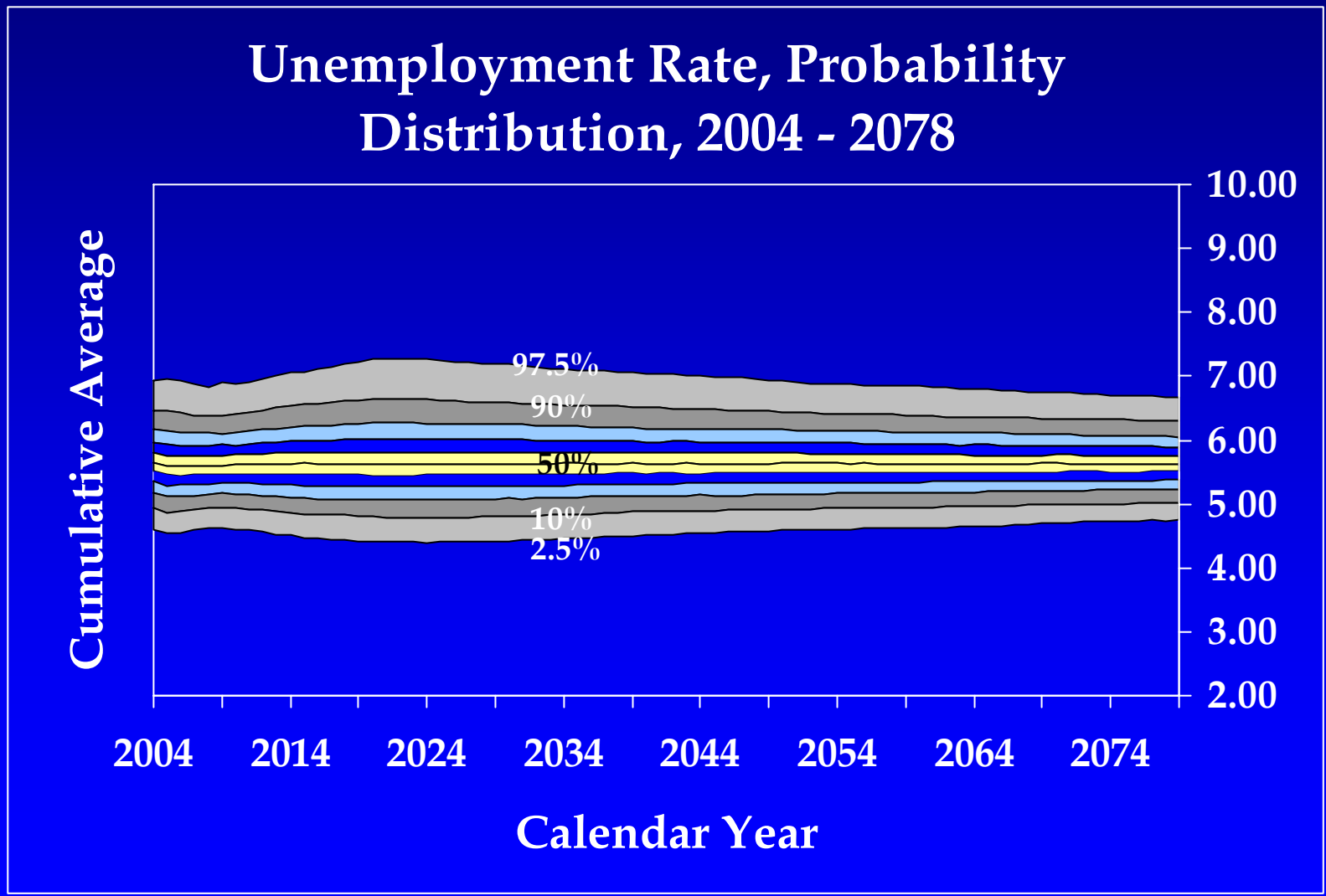
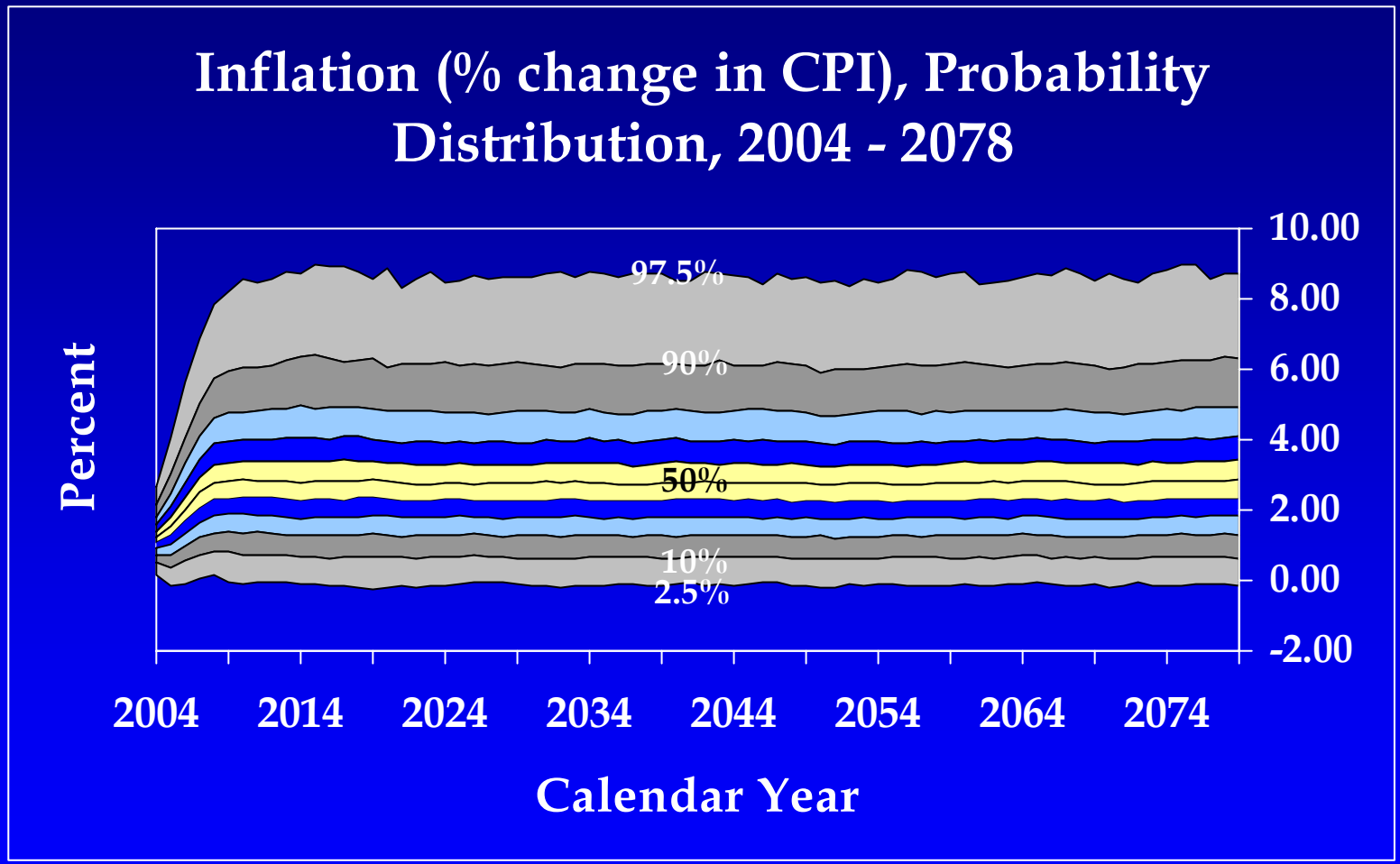


# CPI Equation Results



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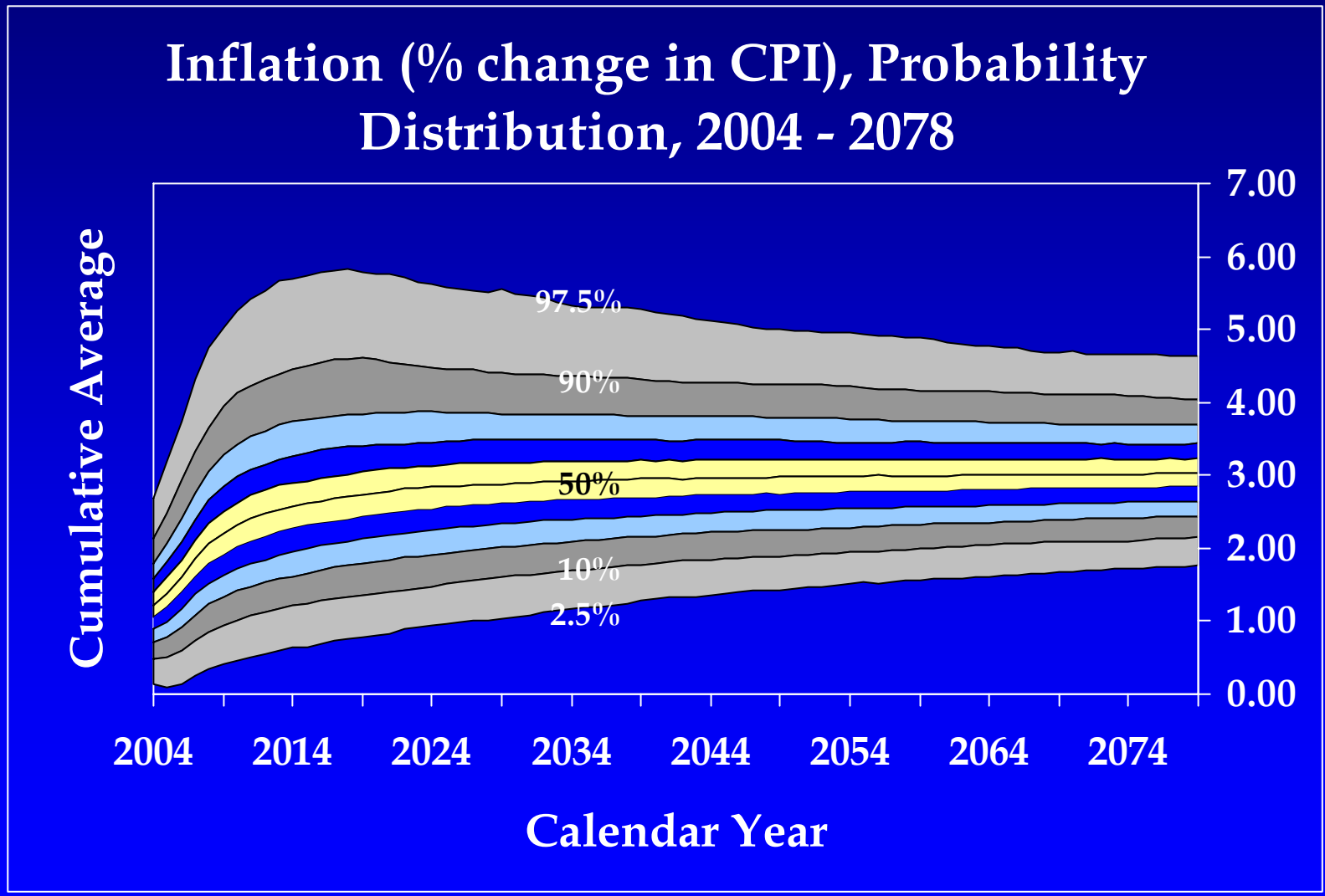
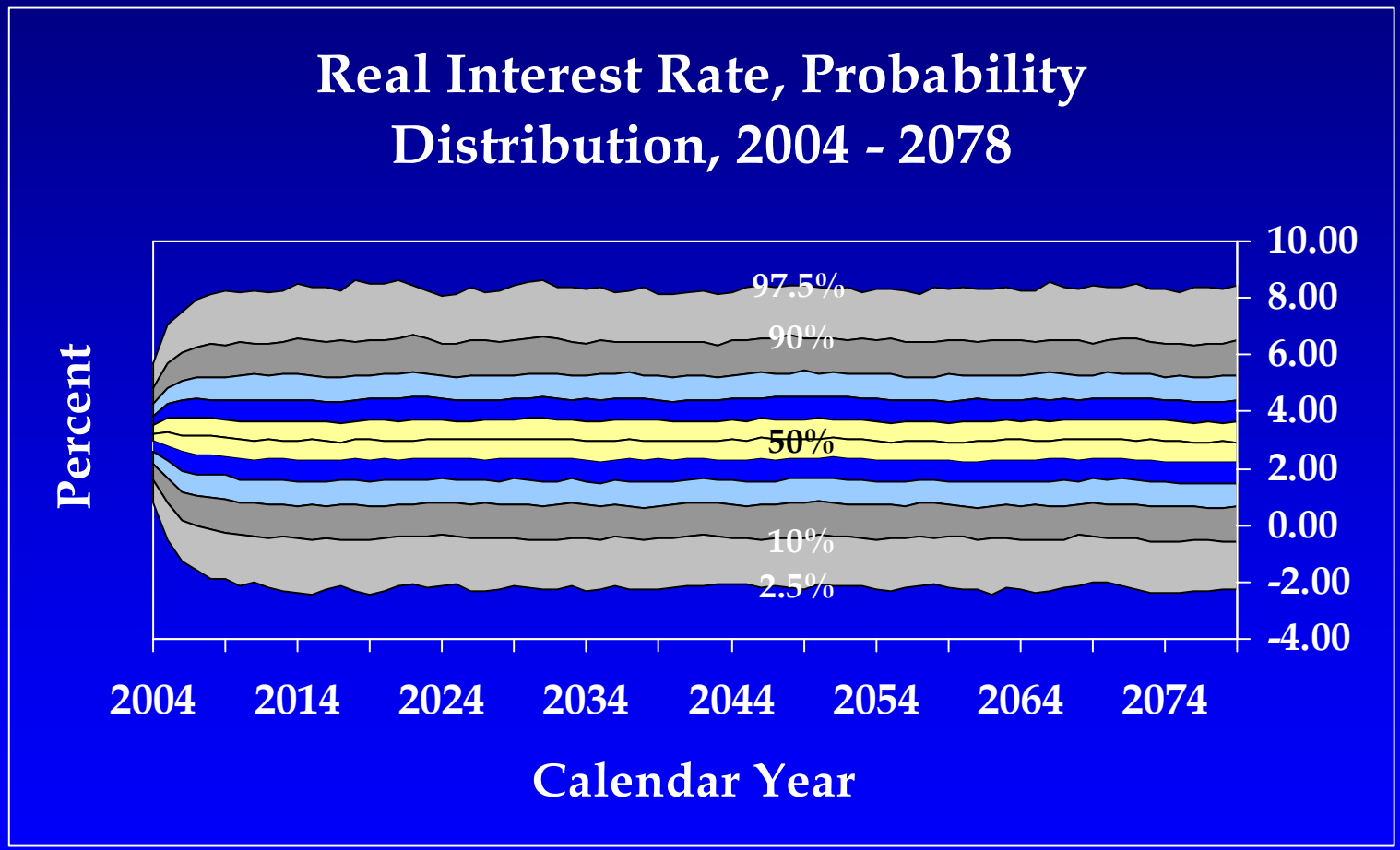


# Real Interest Rate Equation Results



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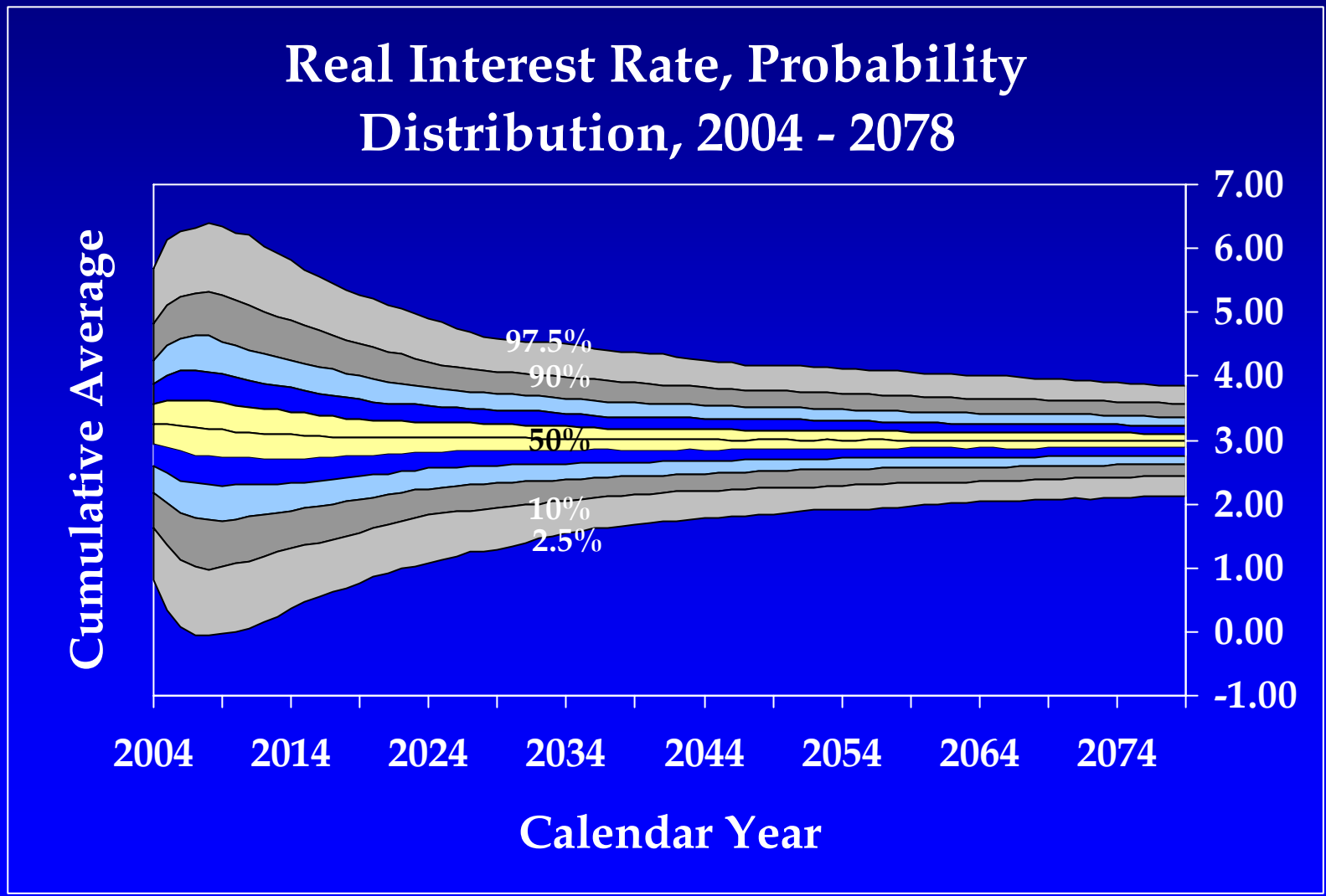


# Principal Measures of Financial Status

# Cost Estimates

Projection methods are based on techniques used in OCACT's deterministic model

- Insured population (including disability insured)
- Number of DI and OASI beneficiaries
- Average award benefit
- Average in-current-pay benefit

This allows us to determine the year-by-year scheduled benefits, the largest component of the cost of the OASDI program

# Year-by-Year Trust Fund Operations

- Income
  - Tax contributions
  - Taxation of benefits
  - Interest income
- Outgo
  - Scheduled benefits
  - Administrative expenses
  - Railroad interchange

EOY Assets = BOY Assets + Income - Outgo

# Annual Values

For a given trust fund path, in each year we may compute:

- Income rate
- Cost rate
- Annual balance
- Trust fund ratio

# Summarized Values

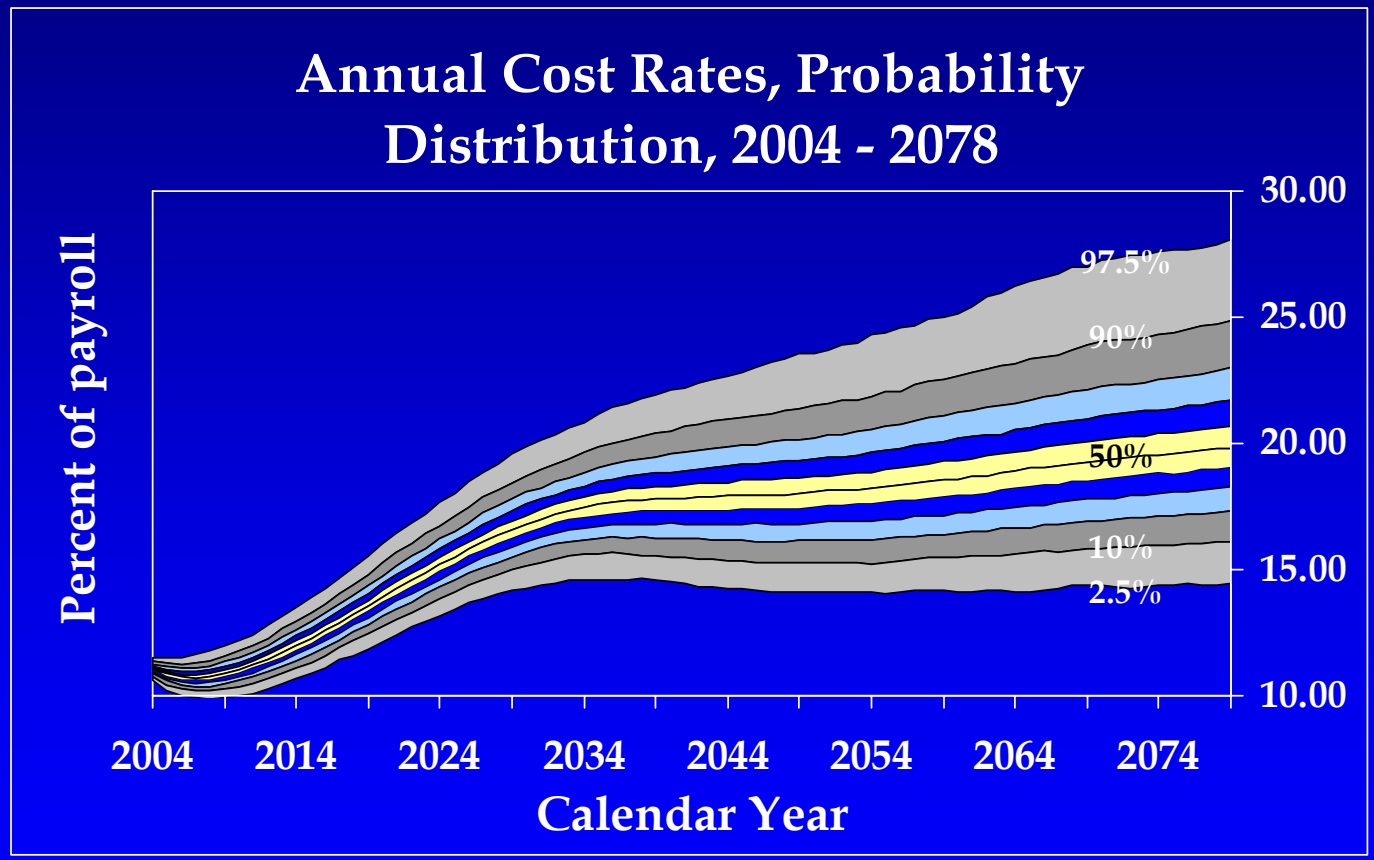
For each trust fund path, we may compute the following values:

- Summarized income rate
- Summarized cost rate
- Actuarial balance
- Open-group unfunded obligation
- Exhaustion year

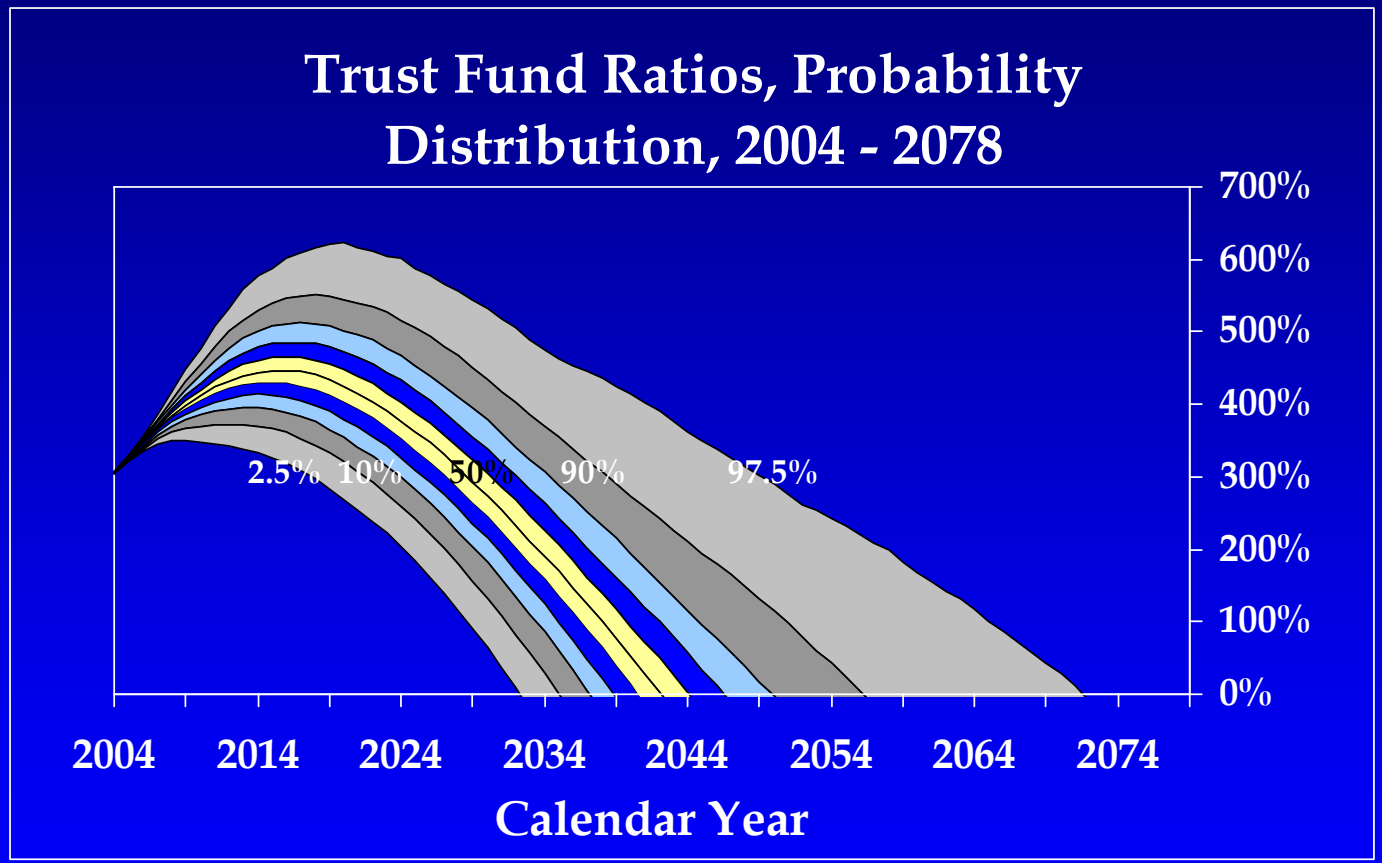
# Results for OASDI Based on 2004 Trustees Report

| | Deterministic Model | | | Percentiles from Stochastic Model | | | | |
|---|---|---|---|---|---|---|---|---|
| | TR04 II (Alt 2) | TR04 I (Alt 1) | TR04 III (Alt 3) | 50th | 10th | 90th | 2.5th | 97.5th |
| Actuarial Balance | -1.89 | 0.41 | -4.96 | -1.98 | -3.25 | -0.85 | -4.02 | -0.33 |
| Summarized Income Rate (percent of payroll) | 13.84 | 13.72 | 13.98 | 13.84 | 13.69 | 13.99 | 13.62 | 14.08 |
| Summarized Cost Rate (percent of payroll) | 15.73 | 13.30 | 18.94 | 15.83 | 14.65 | 17.15 | 14.08 | 17.96 |
| Open Group Unfunded Obligation (1/1/04 trillions PV $) | 3.7 | -1.1 | 10.3 | 4.0 | 1.5 | 7.1 | 0.4 | 9.2 |
| Exhaustion Year | 2042 | - | 2031 | 2042 | 2035 | 2056 | 2032 | 2071 |

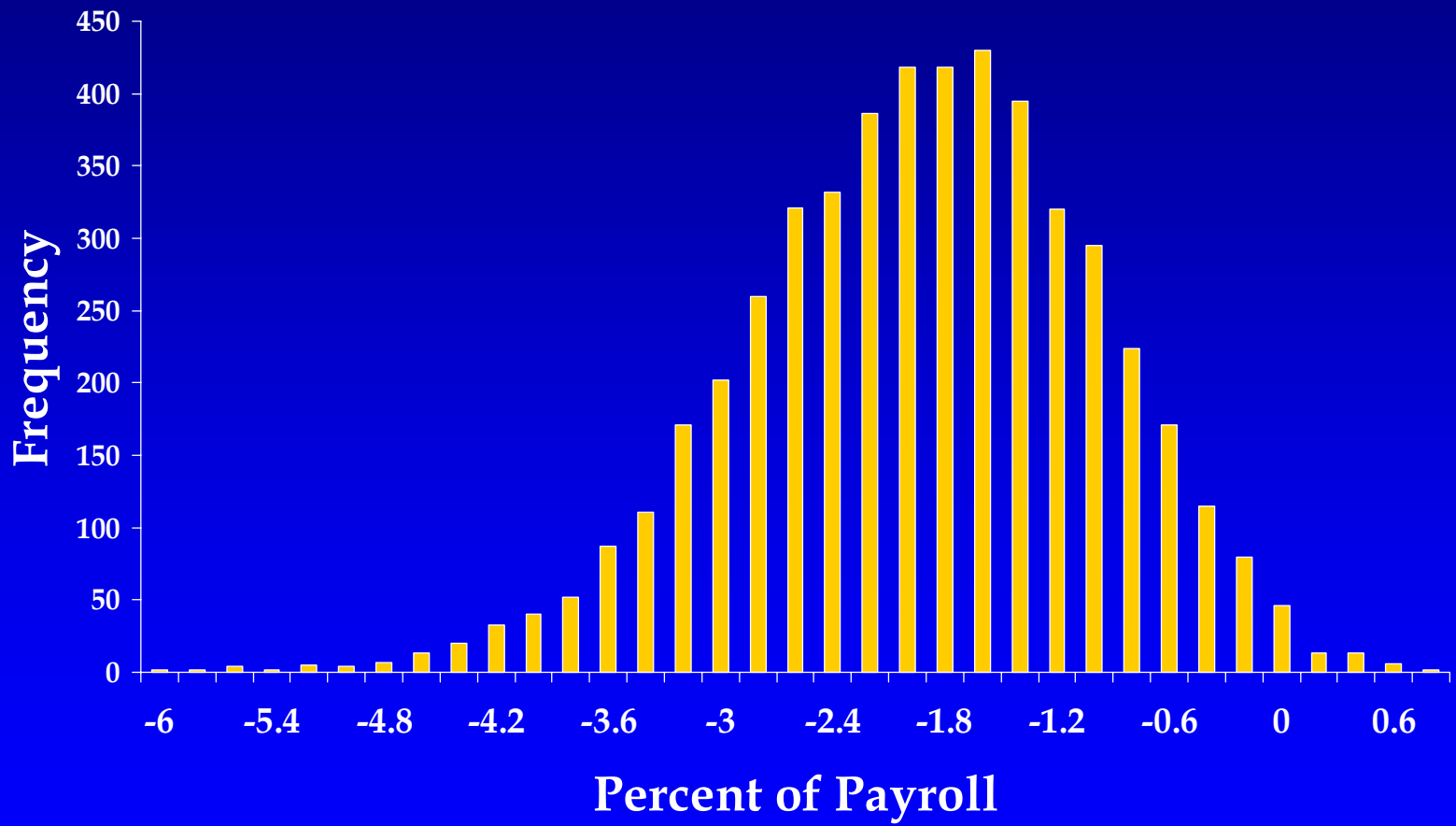
# Annual Cost Rates



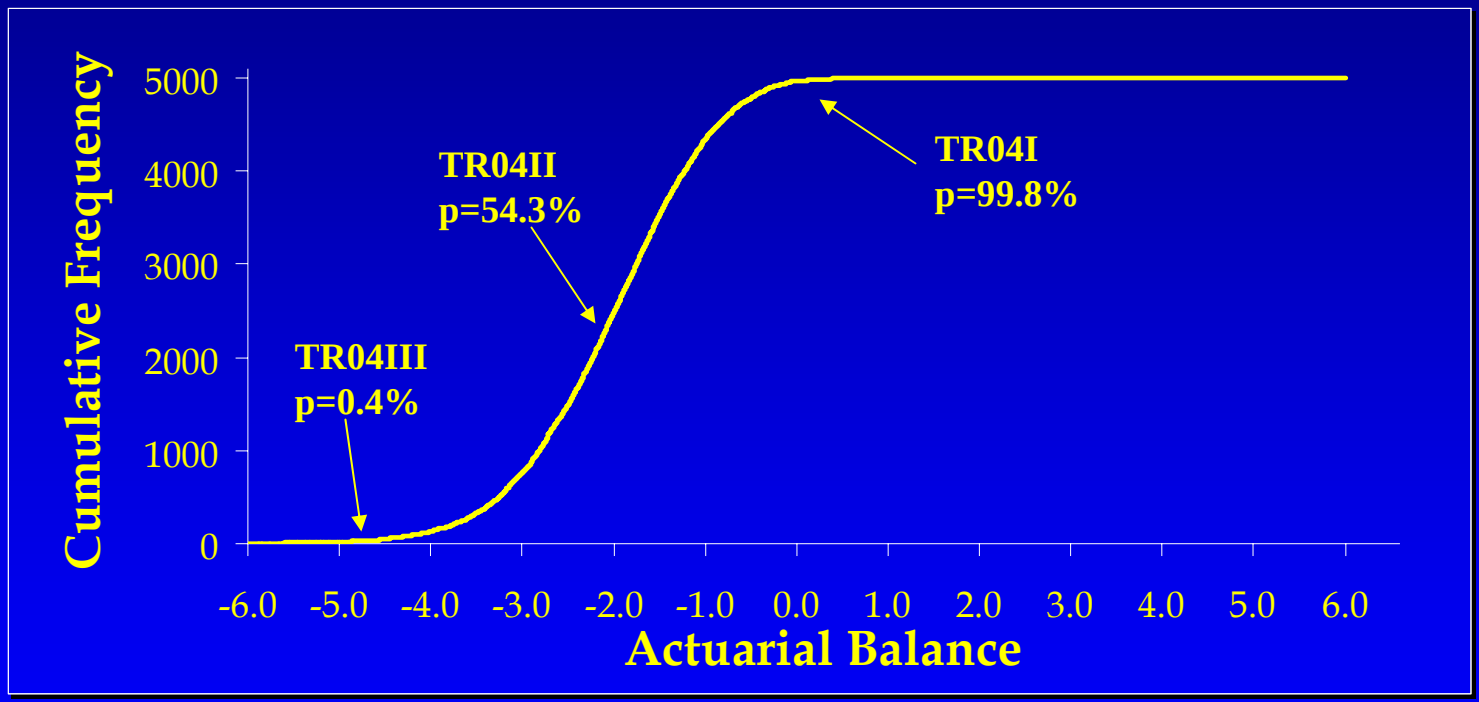
# Trust Fund Ratios



# Histogram of Actuarial Balances



# Cumulative Frequency Distribution



# Improvements to the Stochastic Model

# Possible Extensions to Model

- Additional stochastic variables
- Using alternative equations
- Introducing structural variation in the long-range central tendencies of variables

# Possible Extensions to Model

Could model additional variables, for example:

- labor force participation rate
- productivity
- marriage rates

# Possible Extensions to Model

Interplay between variables: three economic variables are correlated and are estimated using a vector autoregressive model. The interaction of other variables could also be considered.

# Possible Extensions to Model

- For example, each run would vary the value of the ultimate assumption (such as the average total fertility rate over the last 50 year of the projection period) around an amount equal to the Trustees assumption.
- Add variation to the parameter values.

# Further Information

**Check our website:**

http://www.ssa.gov/OACT/stochastic/index.html

**Actuarial Study on our Stochastic Model:**

http://www.ssa.gov/OACT/NOTES/as117/as117.pdf

**Technical questions:**

Tony Cheng 410-966-5732

Michael Miller 410-965-7776