A tri-modal model of Canadian inflation and its effect on indexing pension plan benefits

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Plan of Presentation

- The problem
- Deterministic approach: limits
- Stochastic approach
- Proposed model: Canadian data
- Results
- Conclusion
- Further areas of research

The Problem

- Providing an indexed annuity to a retiree: cost of the indexation protection
- Focus on inflation : assume a constant real return of 4% / year
- Reflect Canadian inflation historical data
- How much for partial indexation ?

Key Points

- Goal: price the cost of providing indexation to a retiree
- Constraints: recognize deductible, coinsurance and caps features in indexation formulas
- Format: express cost as a % of present value of non-indexed annuity

The Question

- Assuming a constant real return of 4%, calculate the cost of various indexation formula features (deductible, coinsurance)
- Express the cost in % of the present value of a non-indexed annuity for our typical participant
- Use or design an appropriate model

Potential Situations

- Negotiation of plan benefits: bargaining for cost of « CPI- 3% » formula
  - Inflation below 3%, 1992-1999;
  - cost = 0 for future protection ?
  - Other formulas, how much ?
- DC plan member asking for partial inflation protection in annuity price quote
- Adequacy of actuarial liability for current indexation formula benefits, changing economic environment

Annuity Data

- Participant: Male (60), Spouse: Female (57)
- Annuity Form: JL&S 60%
  - $ 1000/month for life
  - 60 % to spouse after death of participant
- Mortality: GAM83 table
- Valuation Date: Dec.31, 1999

Economic Assumptions

- Nominal Return= Inflation + Real Return
  - Inflation: **4** % /year
  - Real return: **4** % /year
- Deterministic: **8** % /year interest rate
- Inflation protection
  - According to indexation formula
  - Function of Consumer's Price Index (CPI)

Indexation Formula: Extremes

- 0% CPI
  - No additional cost, no protection
  - Frequent in Cdn private plans (ad-hoc)
- 100 % CPI
  - Costly, full protection to participant
  - Public Plans (C/QPP), Gov’t Employees
- Between these limits ?

Partial Indexation Formula

- Deductible
  - CPI-**3** %, CPI-**1**%
- Coinsurance
  - **50**% CPI, **75**% CPI
- Caps, limits
  - Min(**8**%, 75% CPI - 1%) , minimum **0** %
  - 100 % of Min(**4**%, CPI), minimum **0** %

Indexation Formula: Threshold

- Limited inflation protection, lower cost
- Split inflation in 2 (or more) parts:
  - 50 % of min (CPI, **3** %) **+**
  - 100 % of max(0%, CPI-**3** %)
- Partial protection in normal situations
- Full coverage for “catastrophic” situations

Deterministic Results: Example

- Non-indexed PV = $ **129,423** (Reference)
- Indexed PV, 1.0 % indexation
  - PV = $ 141,479
  - PV = 109.3 % of Reference
  - Cost of Indexation = **9.3** %
  - Inflation= 4 %, 25 % CPI or CPI-3 %

Deterministic Results

Inflation = 4 %, Nominal Return = 8 %, $1000/month
JL&S 60 % Annuity to Male Participant, Age 60

Indexation Rate (%)	Indexation Cost (% of reference value)
1.0	**9.3**
2.0	**20.0**
3.0	**32.6**
4.0	**47.3**
4.5	**55.7**

Deterministic Approach: Limits

- Indexation formula: are these similar ?
  - 50 % CPI = 2 %
  - CPI - 2 % = 2 %
  - 75% CPI –1 % = 2 %
  - 50 % min (CPI, 4 %) = 2 %
- Unable to put a value on “Insurance features” : add a loading (guess ?)

Model Features 1

- Not a deterministic method
- but use it for comparison
- Stochastic model: use 50-year CPI simulation

Model Features 2

- Initial state of inflation affects cost
  - Start in 1979,1989,1999…
  - Same value to “CPI-3%” indexation ?
- Various indexation formulas priced simultaneously
- 1000 simulations performed for each initial state of inflation: statistics on result

Stochastic Approach: Model

1. Define a long-term horizon: 50 years
2. Determine the annual inflation rate for years 2000-2049: one inflation path
3. Calculate for each year
   - indexation rate
   - nominal rate of return

Stochastic App: Model, Cont'd

4. Discount present value of annuity payments for each path
5. Do 1000 inflation paths: calculate distribution statistics of annuity
6. Calculate cost of indexation on the basis of relevant statistics

Canadian Inflation Data

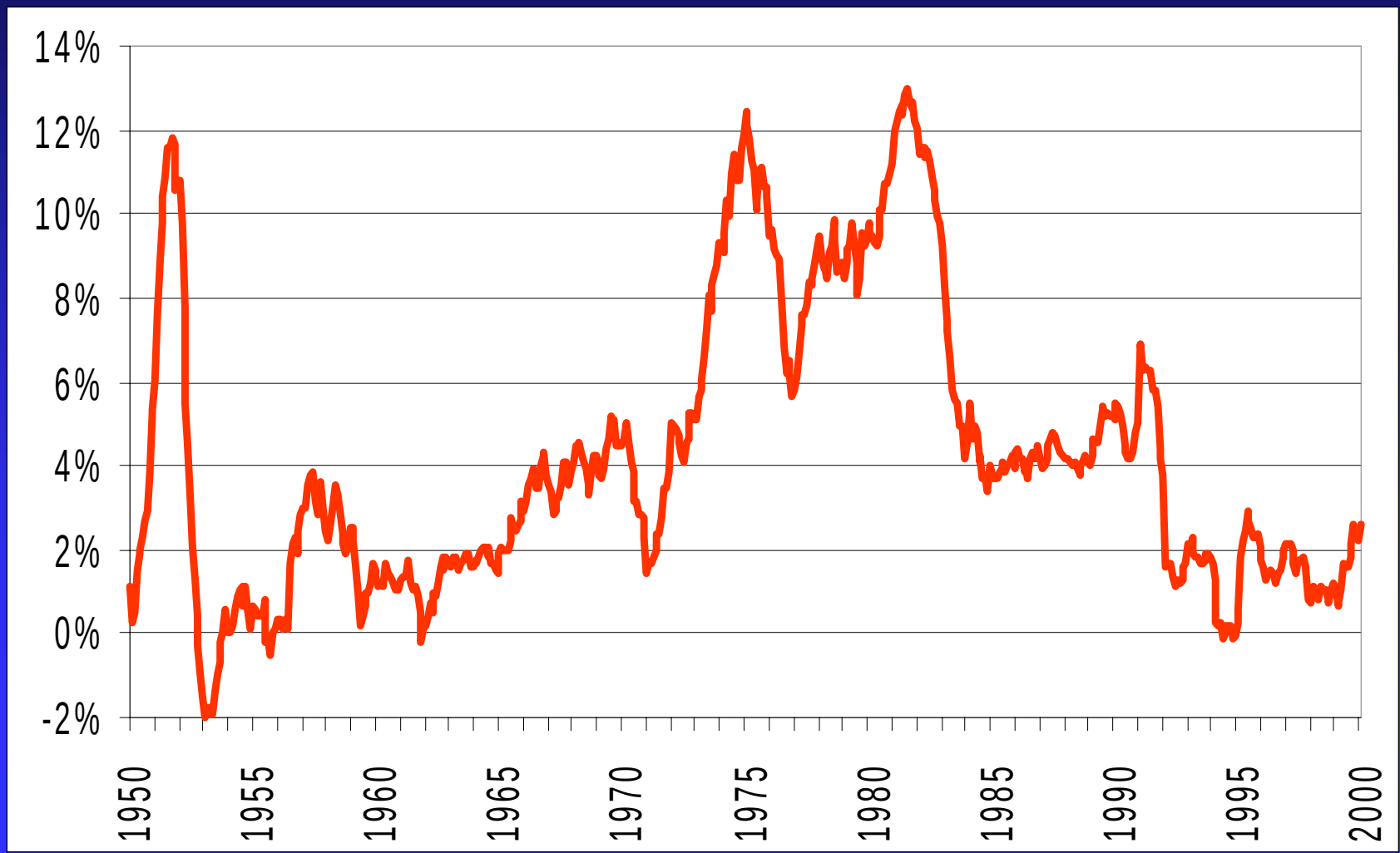
- Consumer's Price Index (CPI): 111.5
- Monthly data
- Various series: All Items, Region
- Change in basis (1992=100.0): 1-decimal precision problem
- Basket of goods revised from time to time

Inflation Rate: Some Details

- One rate per year: 3.5 %, -0.5%, 10.4 %
- CPI deemed a good measure (pensioners ?)
- Definition of CPI “rate of inflation”
  - Monthly data= Price Index
  - 12-month average or less (1,3 ?)
  - Avg ending in December, October ?

Canadian Inflation, 1950-1999

Each month= CPI(t) / CPI(t-12) –1; Dec. 49 to Dec. 99



1999 Canadian Inflation Rate

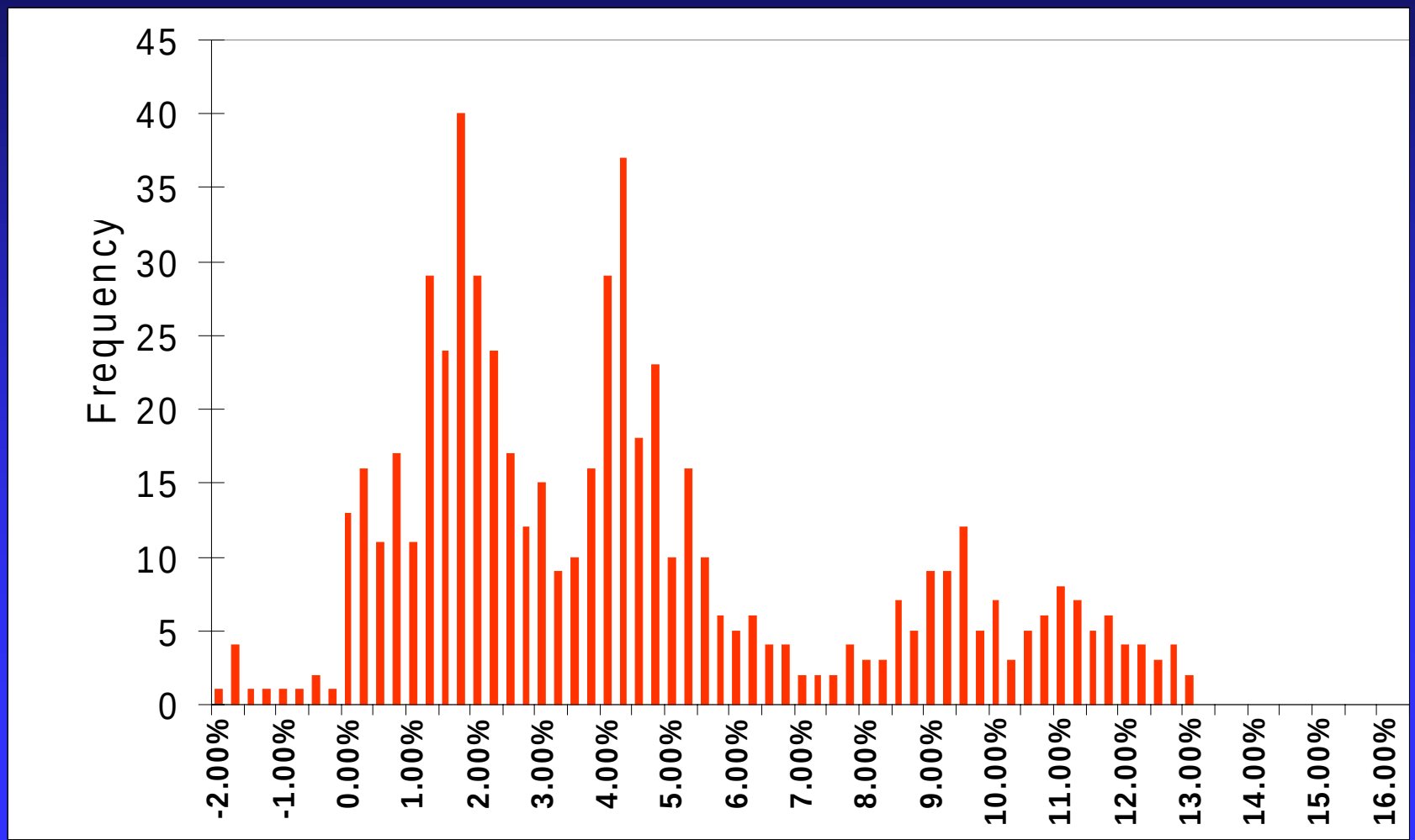
- **2.58** % = CPI(1999-12) / CPI(1998-12) -1
- **1.73** %, Dec. 31, 1999 12-month avg over Dec. 31, 1998 12-month avg
- **1.52** %, Oct. 31, 1999 12-month avg over Oct. 31, 1998 12-month avg (C/QPP)
- More than a 1 % variation according to inflation definition: material in short term

CPI Modeling

- Model CPI monthly values instead of inflation rate
- Apply specific transformation to past and projected data to get inflation rate
- Same model, even with varying definition of inflation rate
- Here: 12-month average ending Oct. 31

Inflation Distribution

- Frequency over last 50 years, monthly data

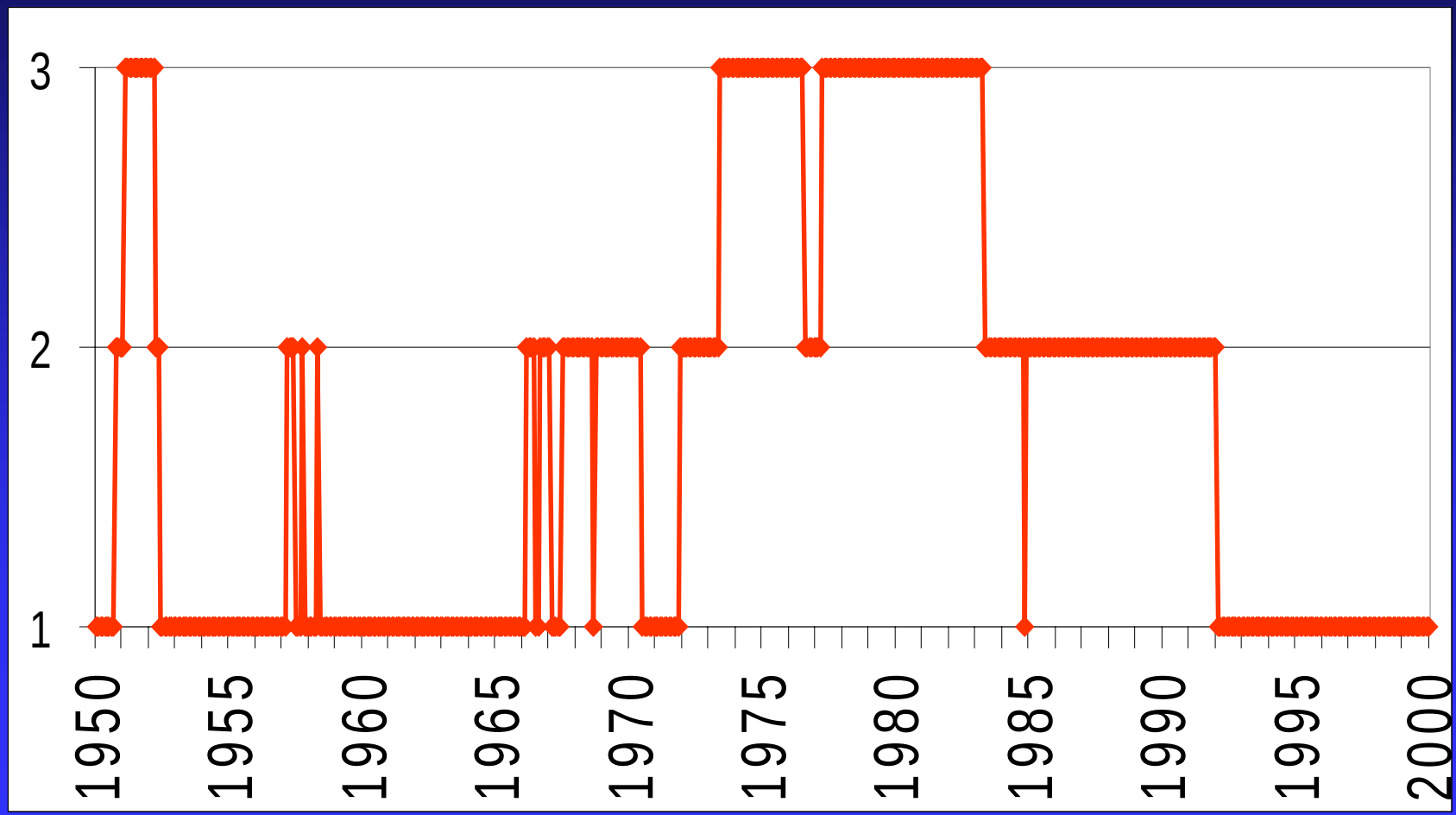


Inflation (CPI) Behavior

- 3 regimes: low, middle, high level
  - Barriers: 3.5 %, 7.0 % annual rate
  - Avoid fitting data to uni-modal distribution
  - Empirical data supports tri-modal
- Monthly switching between levels
  - 3×3 transition probability matrix based on current level and empirical data
- Conditional distribution of CPI monthly variation based on 50-year Canadian data

Cdn Inflation Level: 1, 2, 3

- Level 1: Infl.<3.5 %, Level 3: Infl.> 7.0 %



Relative Frequency of Level Variable

- Low inflation: 289/600 = **48.2** %
- Middle inflation: 186/600 = **31.0** %
- High inflation: 125/600 = **20.8** %
- “Low Inflation” ≈ 50 % chance
- 20 % Probability of High Inflation Level
- Not captured by economists’ estimated consensus on short term/long term median

Conditional Distribution of Inflation Monthly Change

- Large change: Level variable
  - low, middle, high inflation
- Small change: depends upon distribution of monthly variation, given the Level variable
- Empirical data regrouped by Level used to estimate parameters and shape of distribution

Inflation Level Transitions

- Dec. 1949 to Dec. 1999, monthly data
- Number of level-to-level transitions

Fr \ To	**1**	**2**	**3**	Total
1	279	10	0	289
2	10	173	3	186
3	0	3	122	125
Total	289	186	125	**600**

Inflation Level Transitions Probabilities

- Based on 50-year monthly data
- Percentage based on initial level

Fr \ To	1	2	3	Total
1	96.5	3.5	0	100 %
2	5.4	93.0	1.6	100 %
3	0	2.4	97.6	100 %

Shape and Parameters of Distr.

- Assumption: normal distribution for monthly change
- No time correlation inside each distribution
- Estimate mean, variance
- Check if normal with chi-square test, grouped data

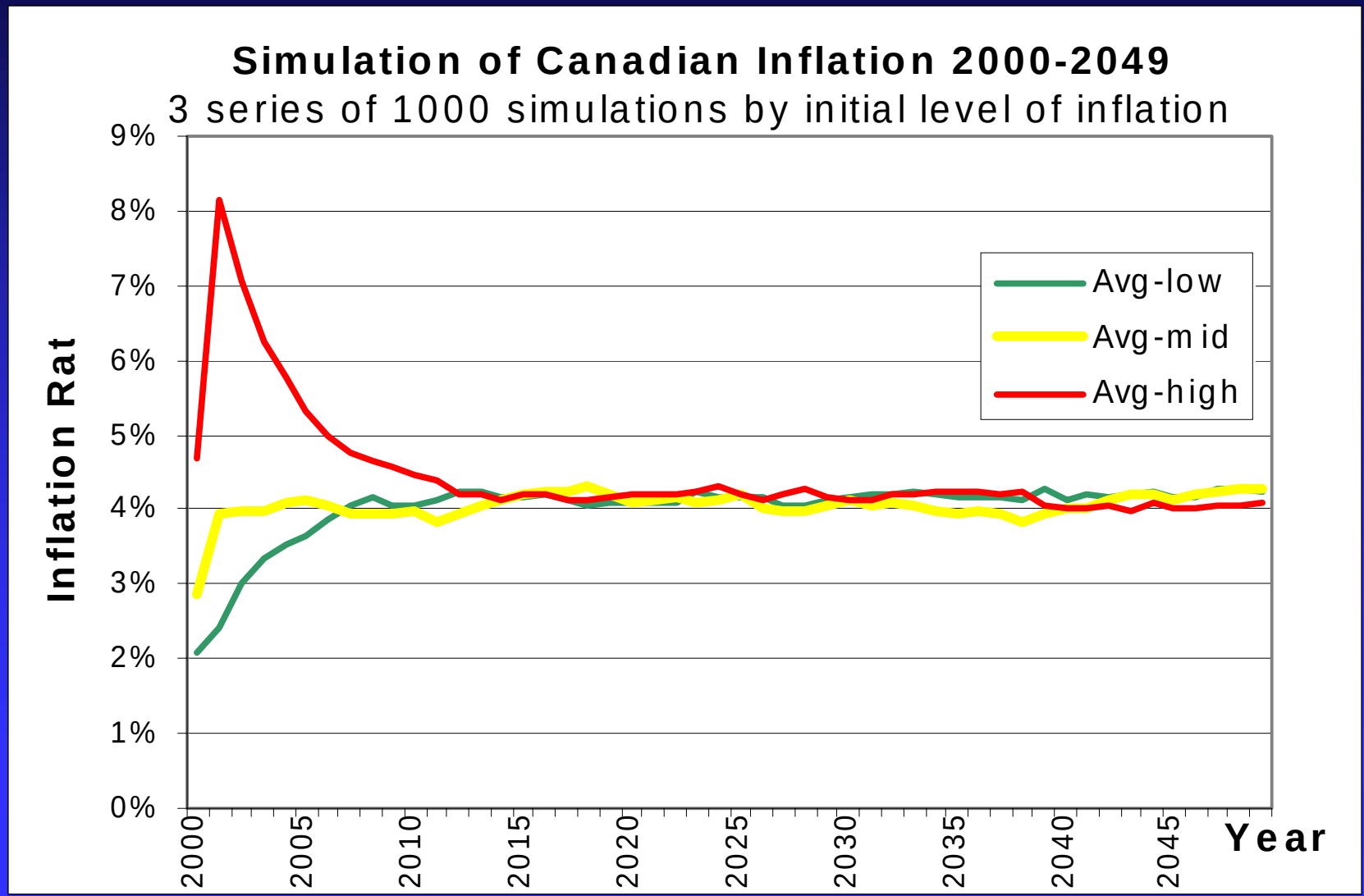
Parameters by Distribution

	Mean	Standard Deviation	Normal ? (χ^2 Test)
Low	**0.001412**	**0.002975**	**Yes,**
Middle	**0.003559**	**0.003140**	**Yes ,but…**
High	**0.007564**	**0.004345**	**Yes**

Running the model: part 1

- Provide mean-variance parameters
- Determine Initial Level: Low, Mid, High
- Provide past CPI data, # of months and delay for averaging purposes
- Generate the future monthly CPI data for 50 years: 1000 times per initial level
- Calculate the inflation rate for each year, each scenario of 50 years

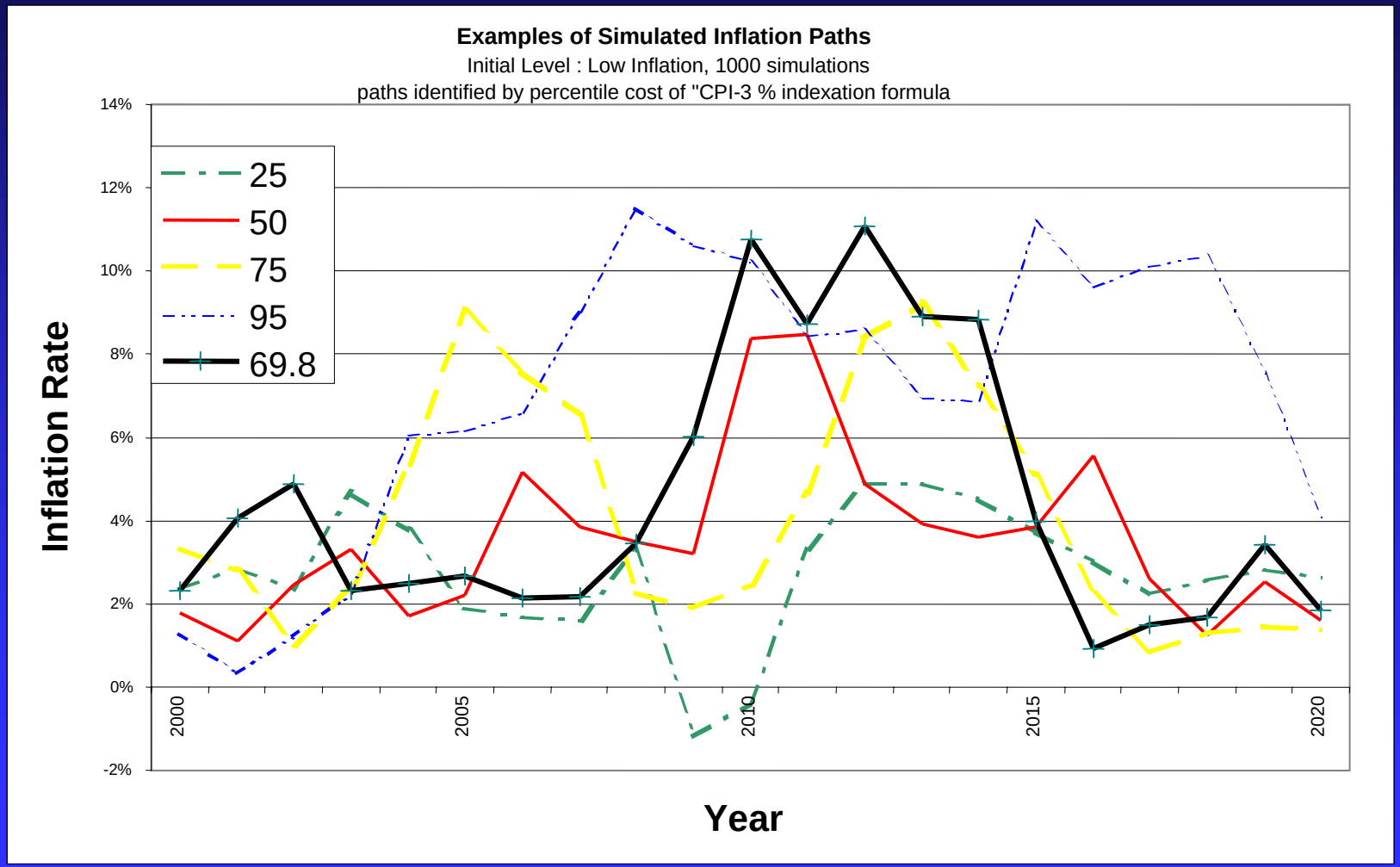
50-year Projected Inflation Path



Short/Long Term Inflation Avg.

Initial Level	15 years 2000-2014	35 years 2015-2049	50 years 2000-2049
Low	**3.67%**	**4.17%**	**4.02%**
Mid	**3.92%**	**4.11%**	**4.05%**
High	**5.17%**	**4.15%**	**4.46%**

Examples Of Simulated Paths: CPI-3%



Running The Model: Part 2

- Use each inflation scenario as input
- Other inputs
  - Expected cash-flows (annuity)
  - Real rate of return scenario (4% constant)
  - Indexation formulas (15 in parallel)
- Compute present value
- Also used for deterministic valuation

Indexation Formulas: 1

- Cost of indexation found for each path as
  - P.V. (formula) – P.V. (formula 1),
  - i=2, 3,…14, 15
- Formula 1 = Non indexed pension
- 2, 3, 4, 5 = 25%, 50%, 75%, 100% CPI
- 6, 7, 8 = CPI – 1%, CPI - 2%, CPI - 3%

Indexation Formulas: 2

- Formula 9 = 75% CPI – 1% (Ontario)
- 10, 11= 75% CPI – 2% , 75% CPI – 3%
- 12 = two-stage formula
  - 50% « Min(CPI, 3%) » +100% « CPI-3% »
- 13 = 100% Min (CPI, 6%)
- 14 = 100% Min (CPI, 4%)
- 15 = 100% Min (CPI, 3%)

Running The Model: Part 3

- Express each present value (PV) result as % of deterministic PV at 8 % interest
- Calculate the cost of indexation, for each scenario, as cost of Indexed P.V. – non-indexed PV.
- Rank results, calculate percentiles

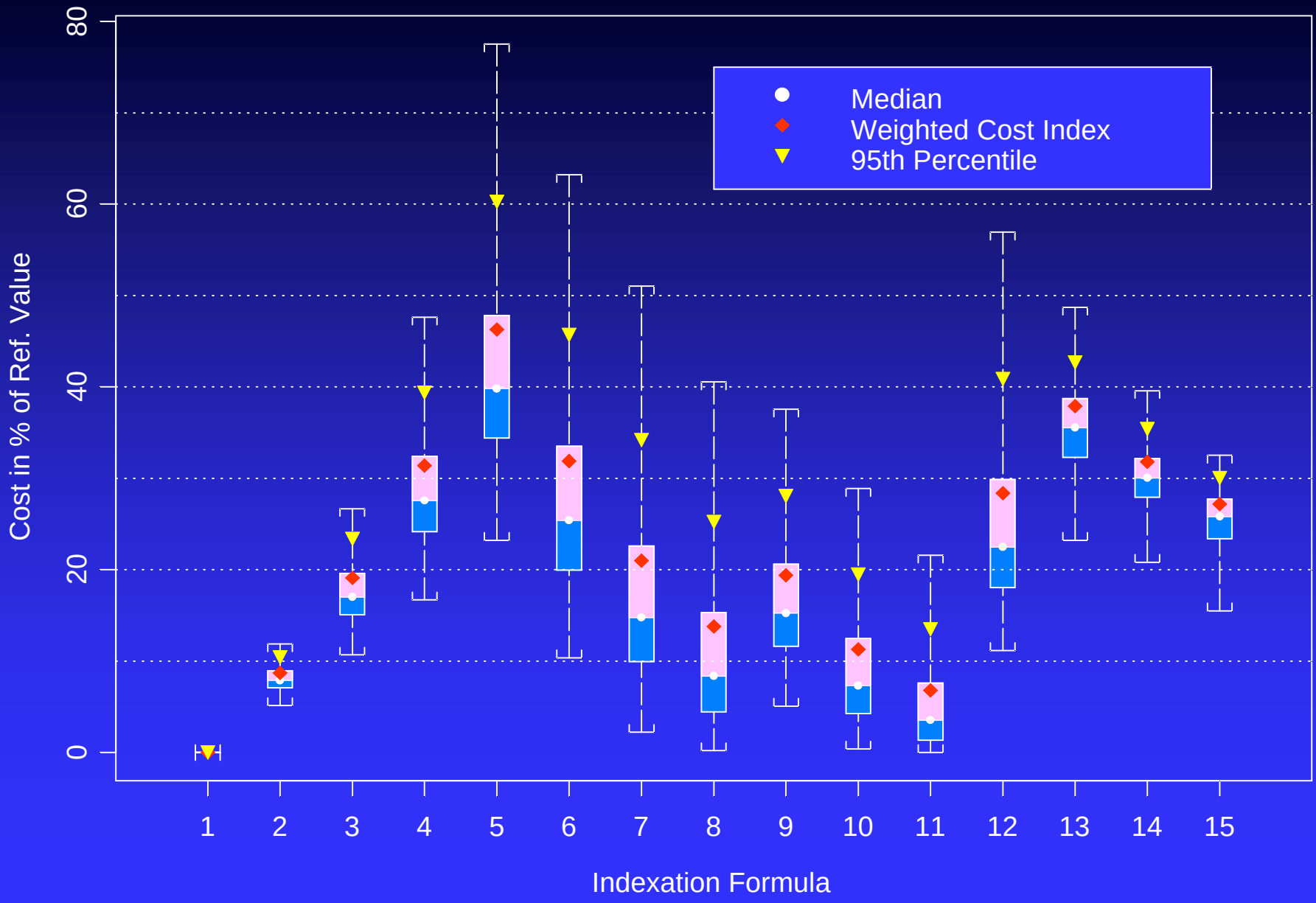
Weighted Cost Index

- Cost distribution reduced to one value
- Wish to capture cost of high inflation: Not average nor median only
- Recognize central tendency: larger weight on median
- Recognize upper percentiles: judgment as to number and weights
- Weighted Cost Index uses 3 values as follows:

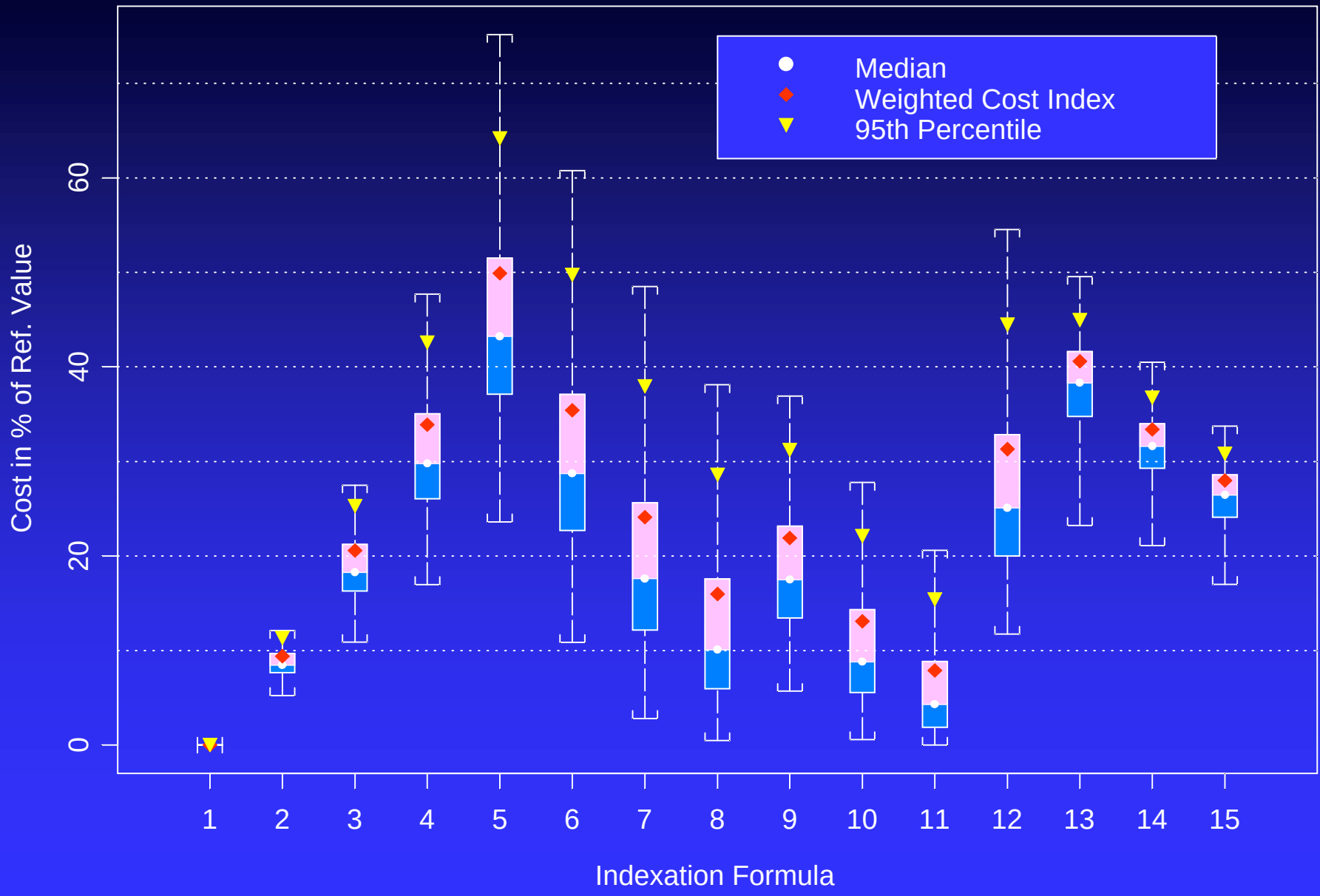
$$50\% \times 50^{th} + 30\% \times 75^{th} + 20\% \times 95^{th}$$

Stochastic Results

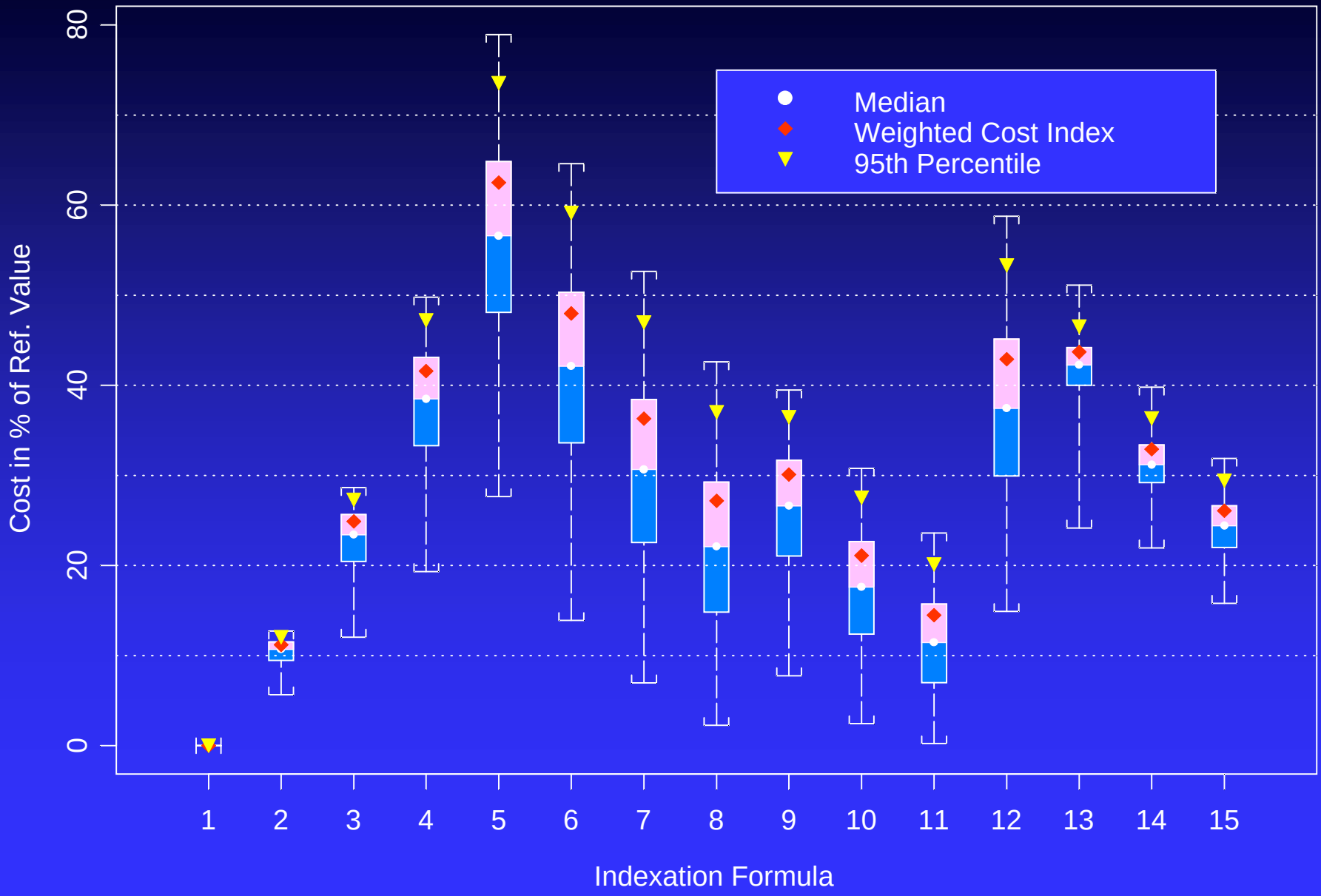
Cost of Indexation Formula, Initial Inflation Level: Low



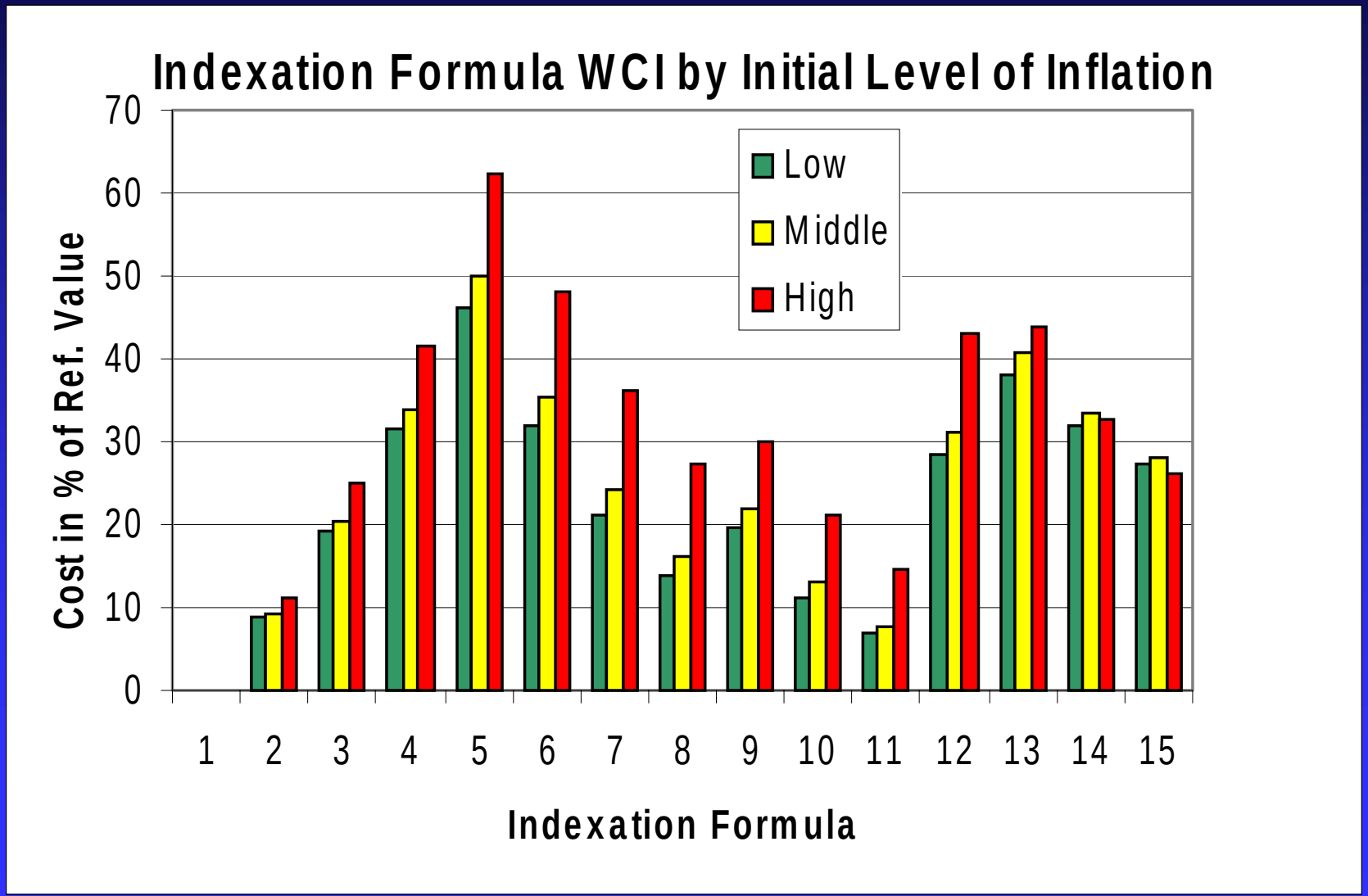
Cost of Indexation Formula, Initial Inflation Level: Middle



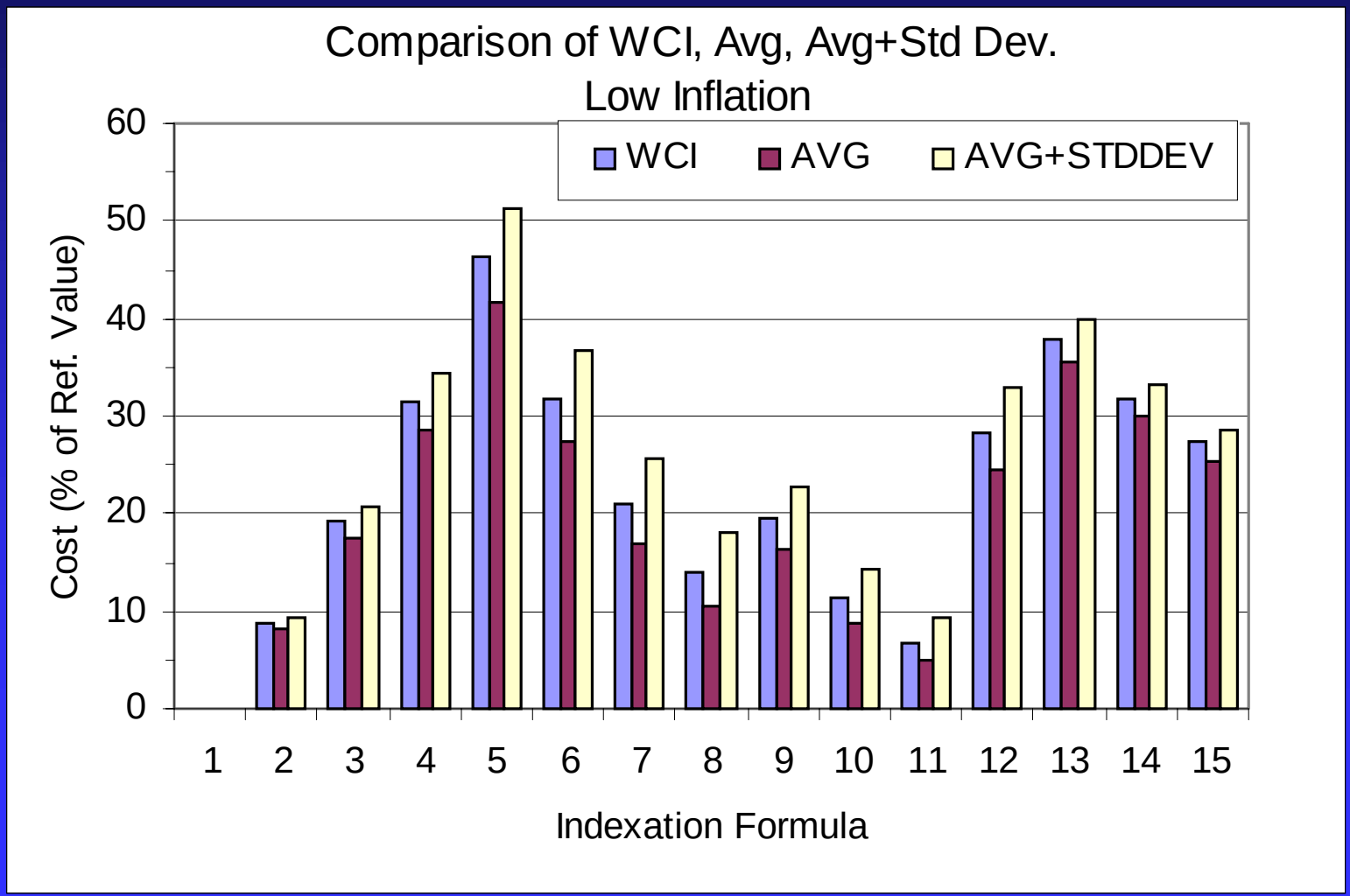
Cost of Indexation Formula, Initial Inflation Level: High



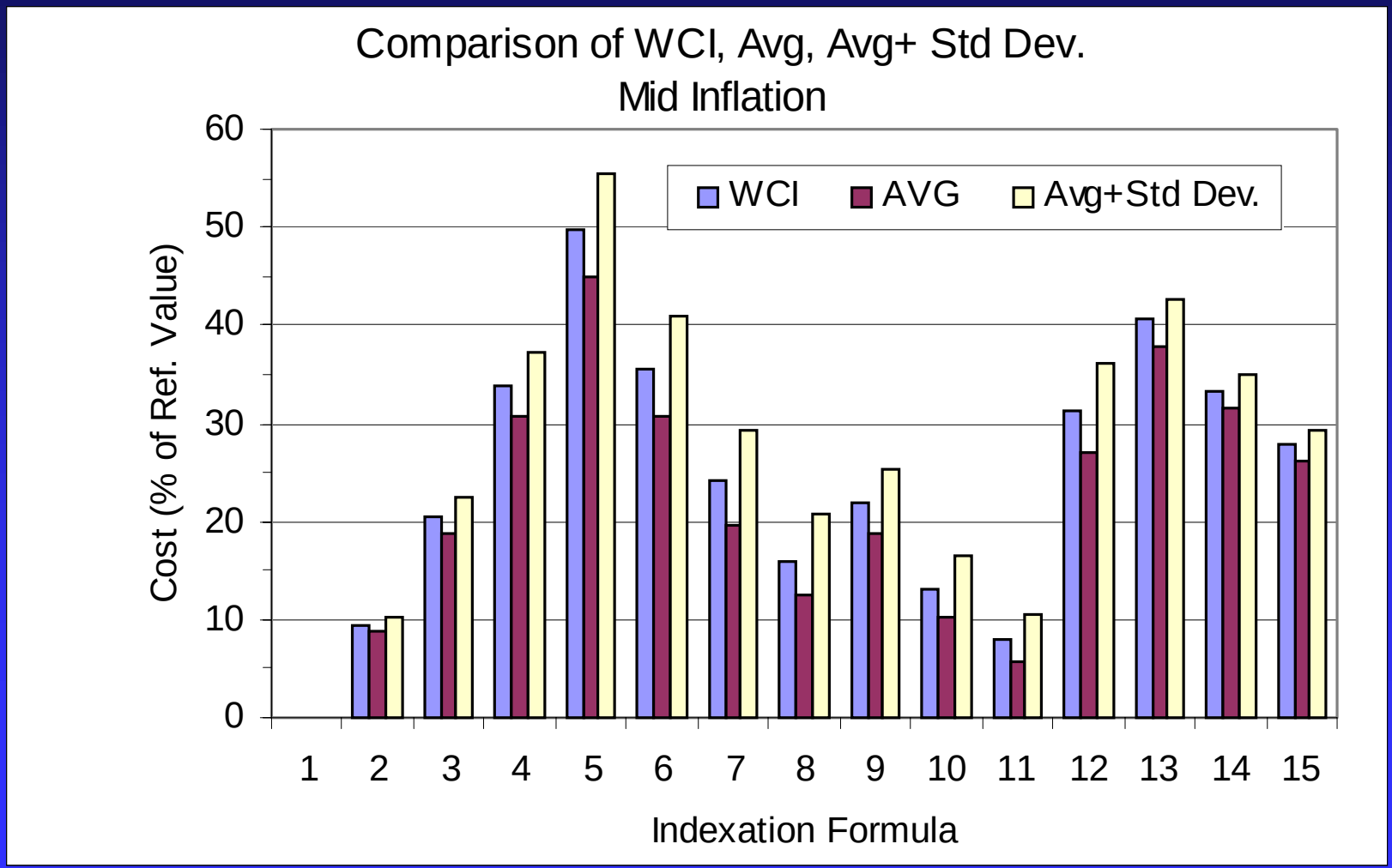
Indexation Cost As a Function of Initial Inflation Level



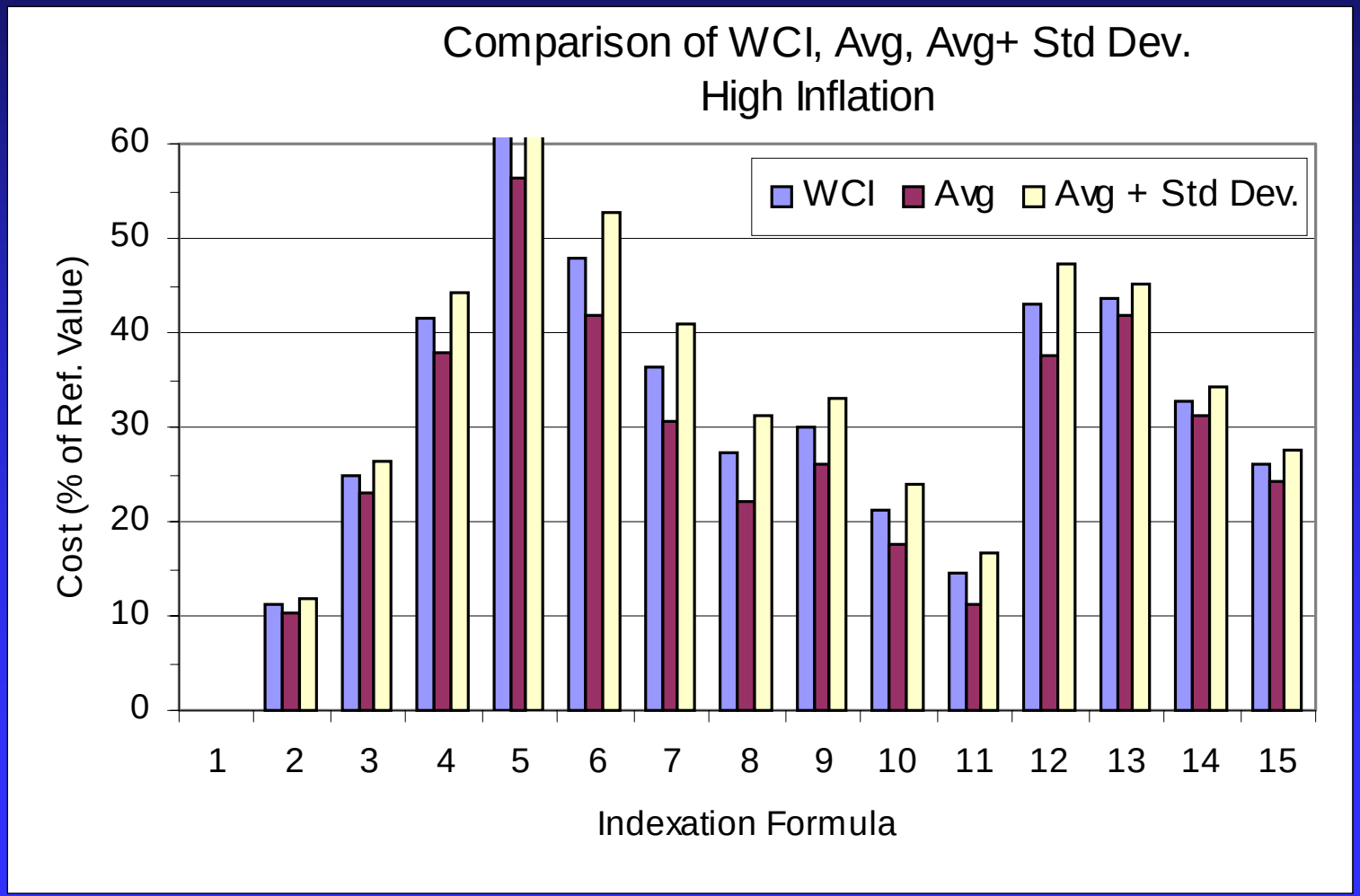
Other Cost Measures: Low Inflation

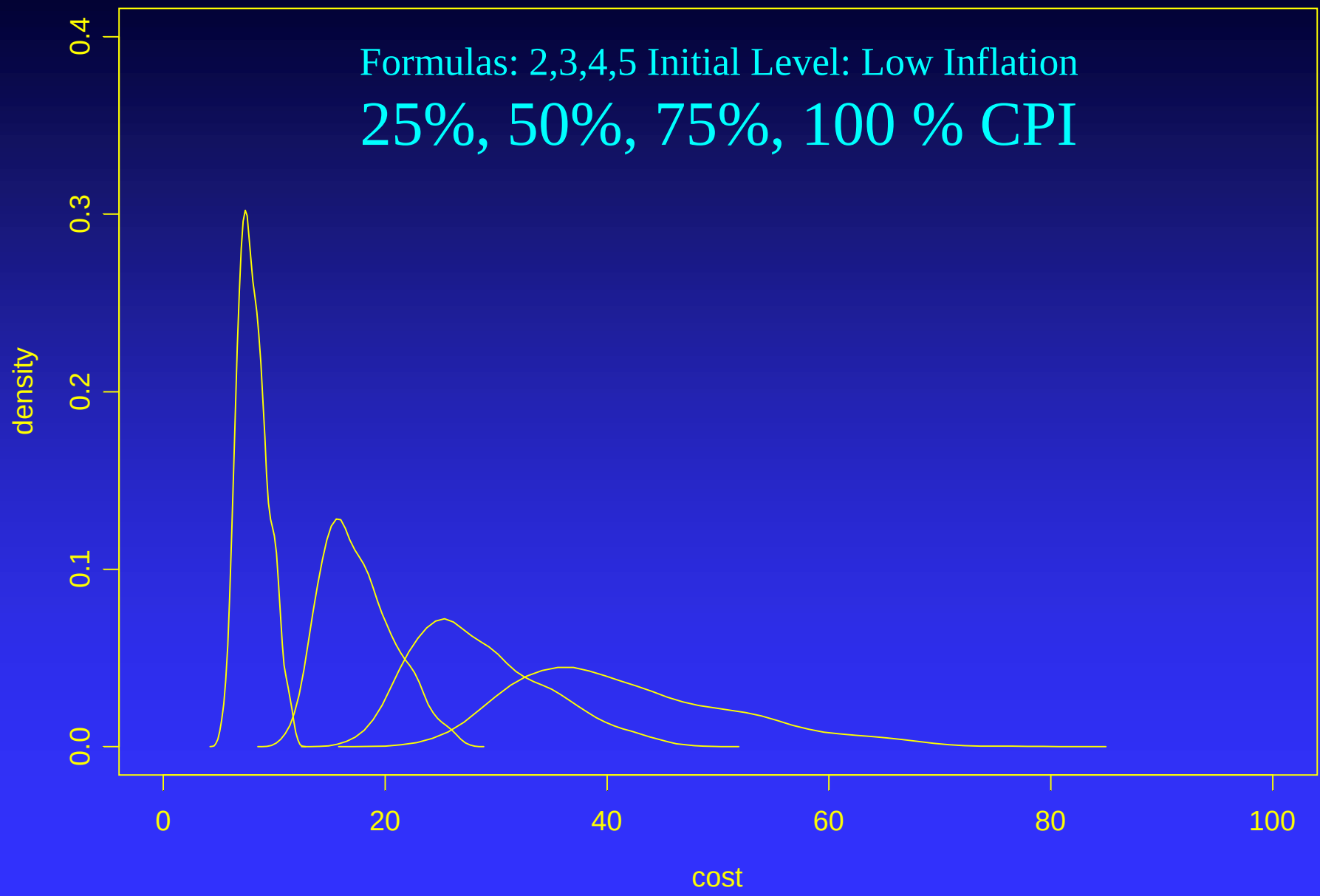


Other Cost Measures: Medium Inflation



Other Cost Measures: High Inflation




Formulas: 2,3,4,5 Initial Level: Low Inflation
25%, 50%, 75%, 100 % CPI
density
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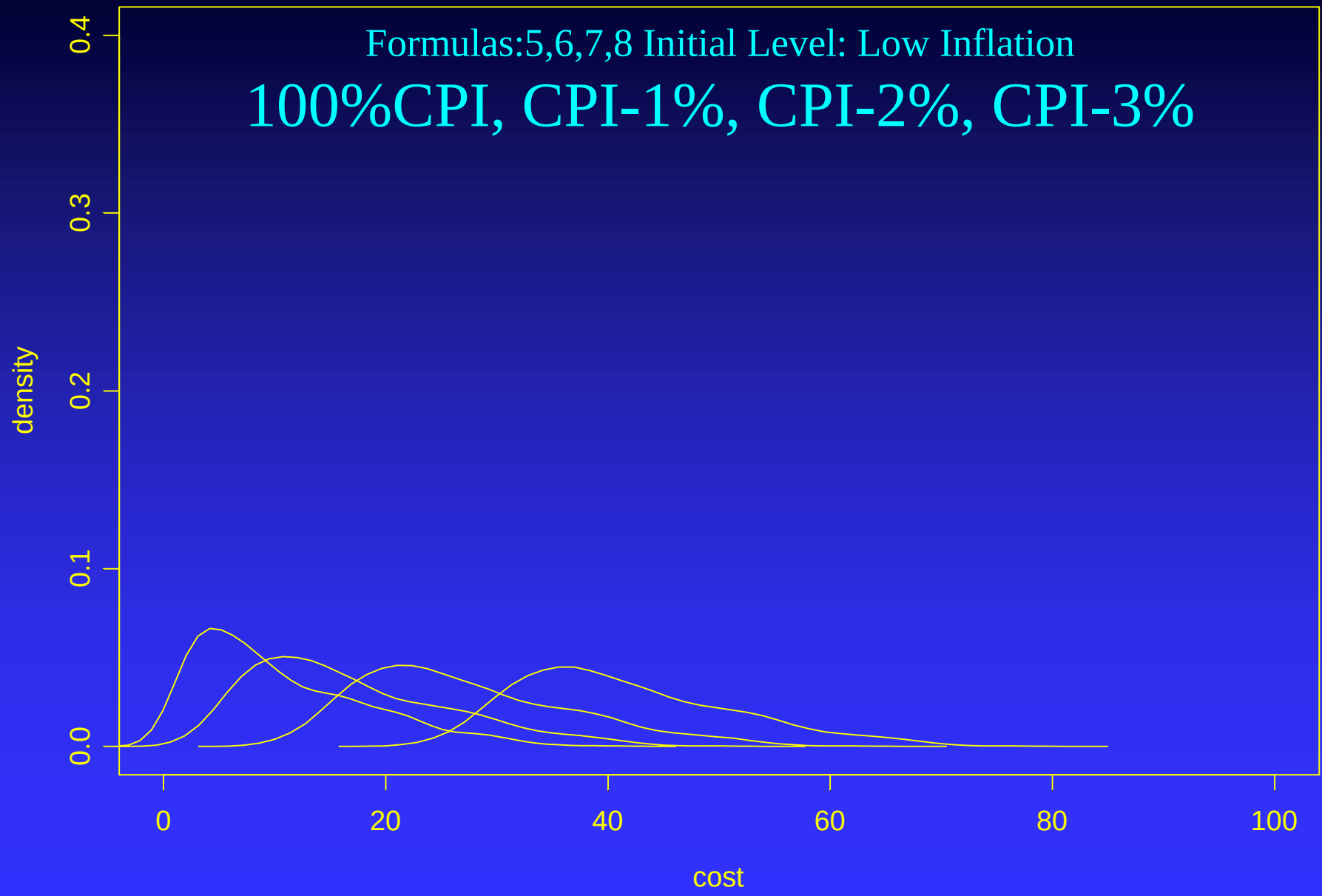
Formulas:5,6,7,8 Initial Level: Low Inflation
100%CPI, CPI-1%, CPI-2%, CPI-3%
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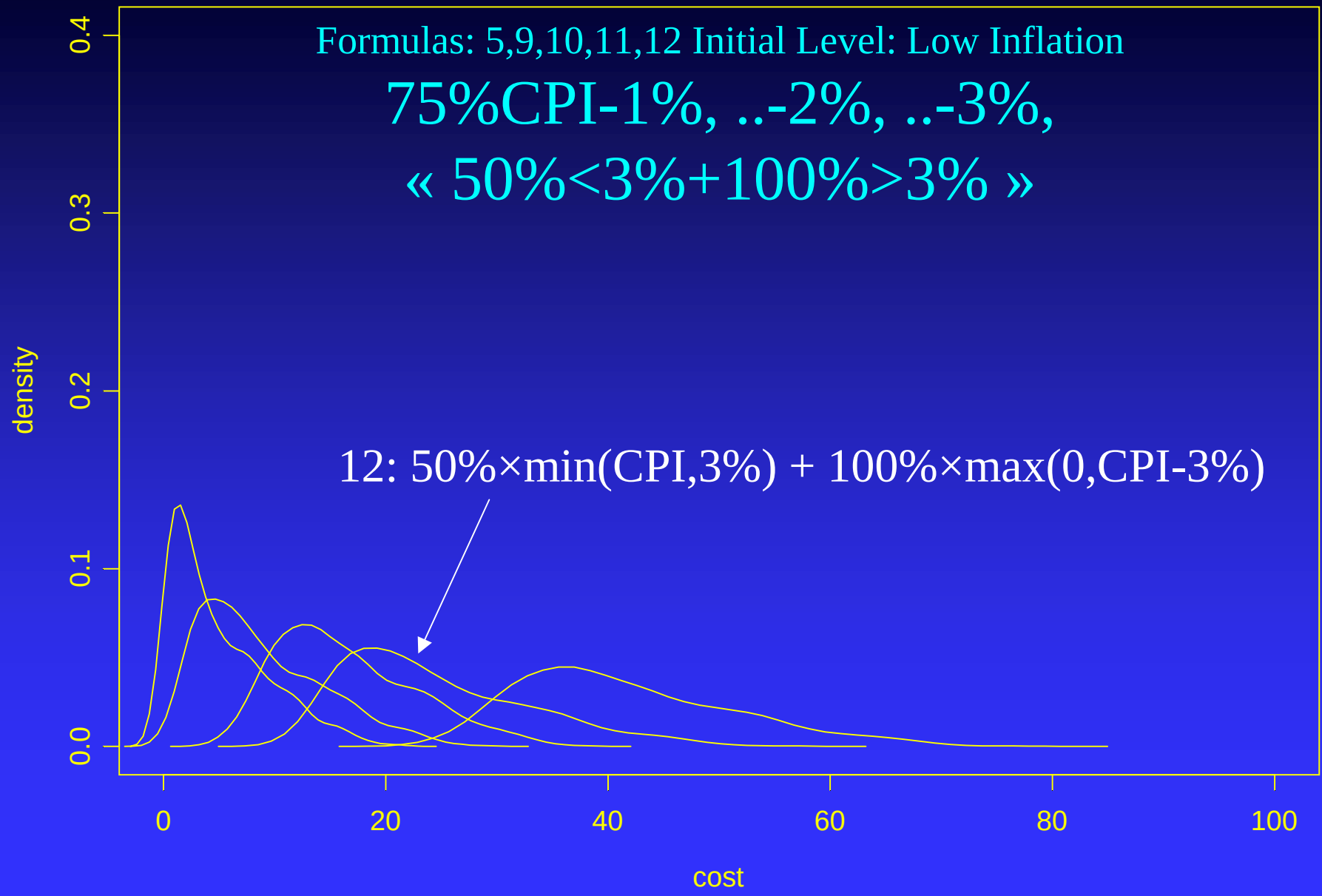
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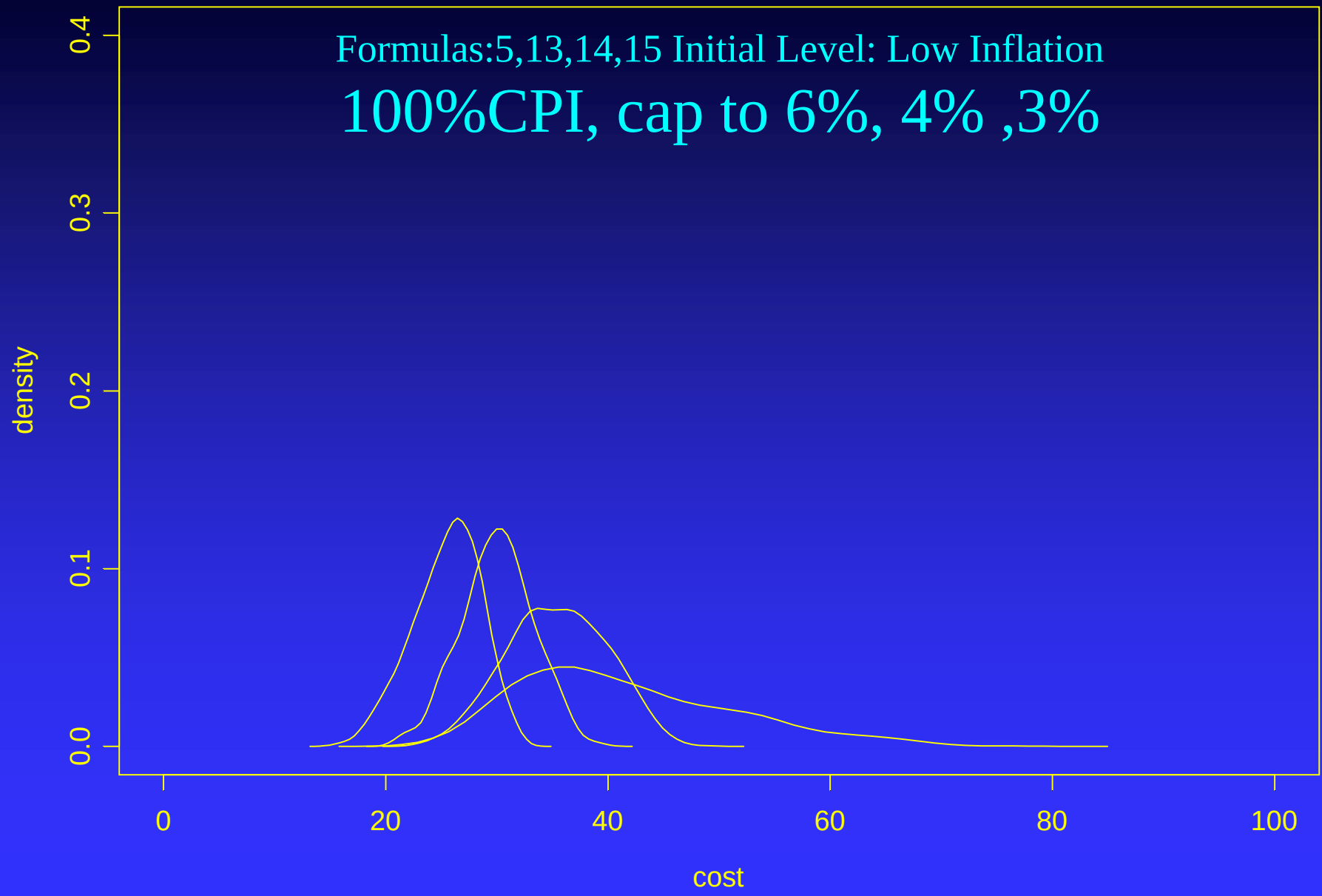
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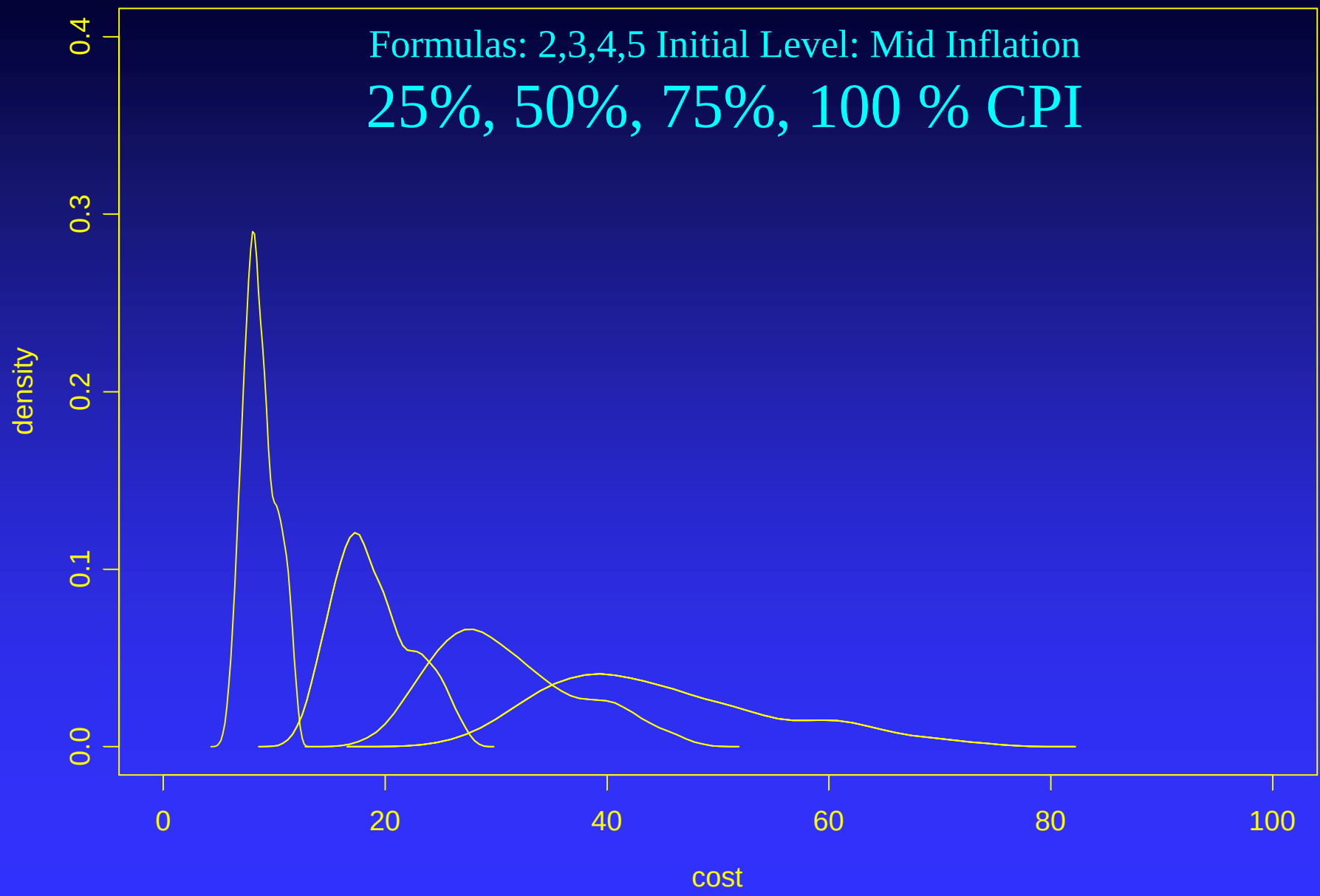
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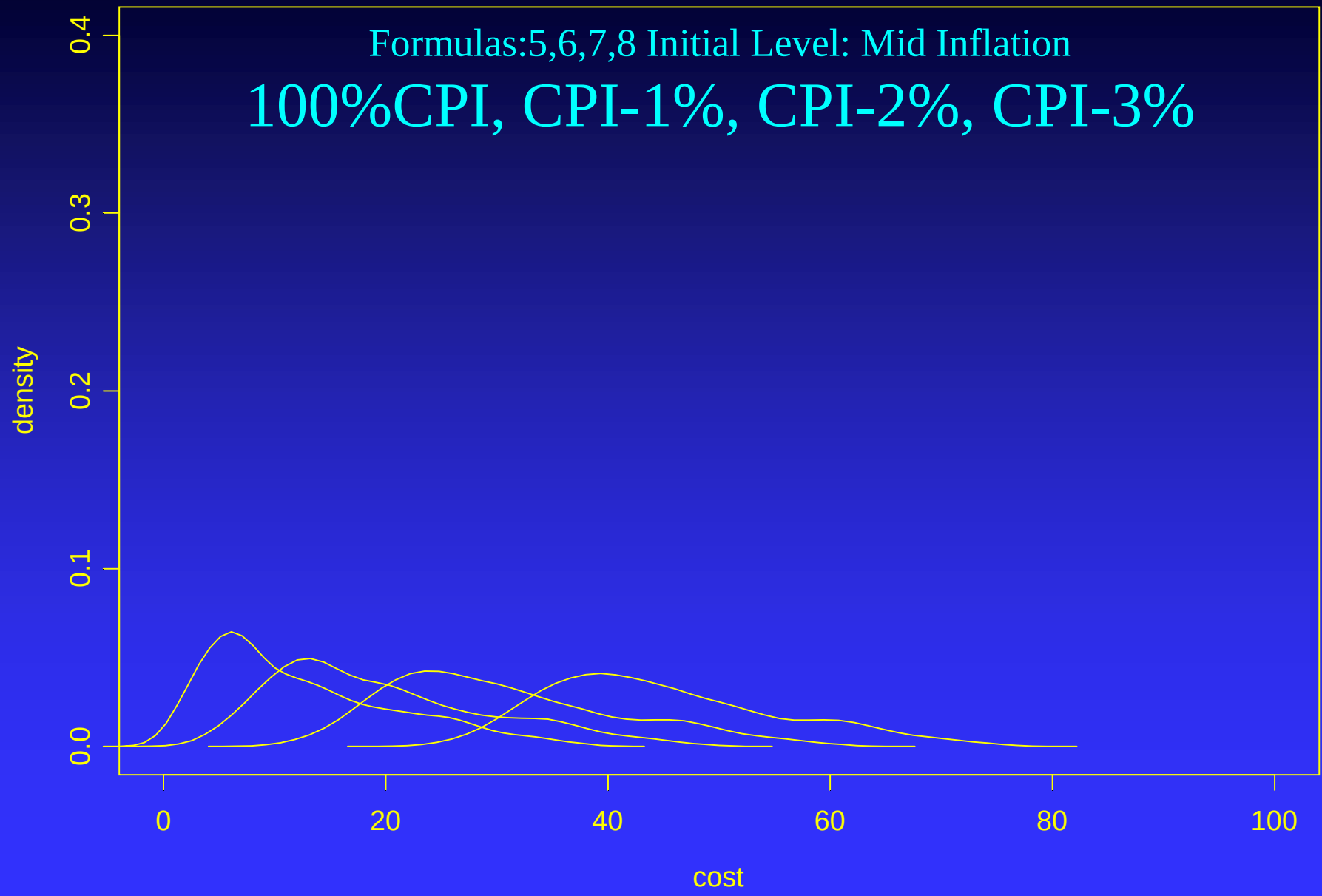
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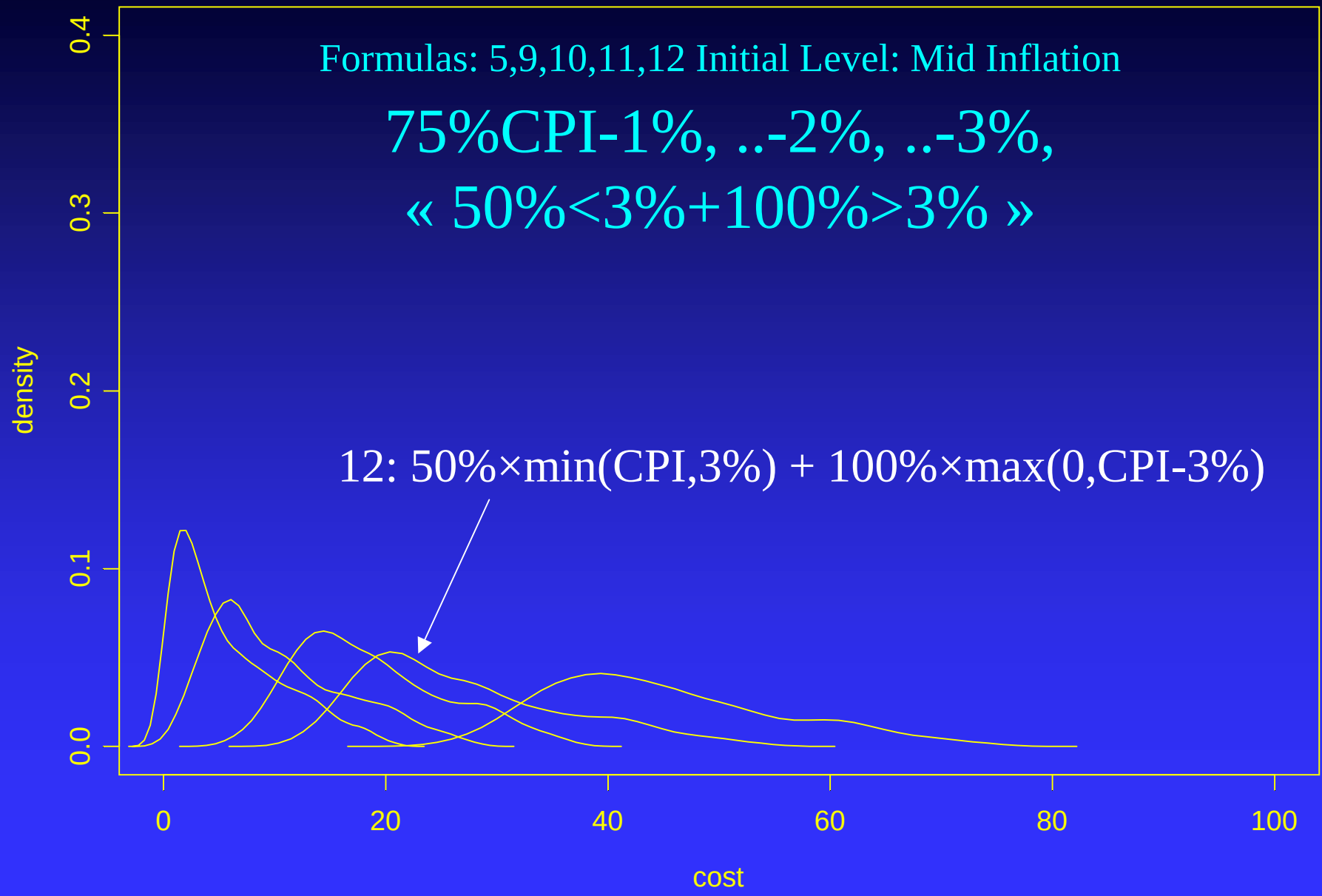
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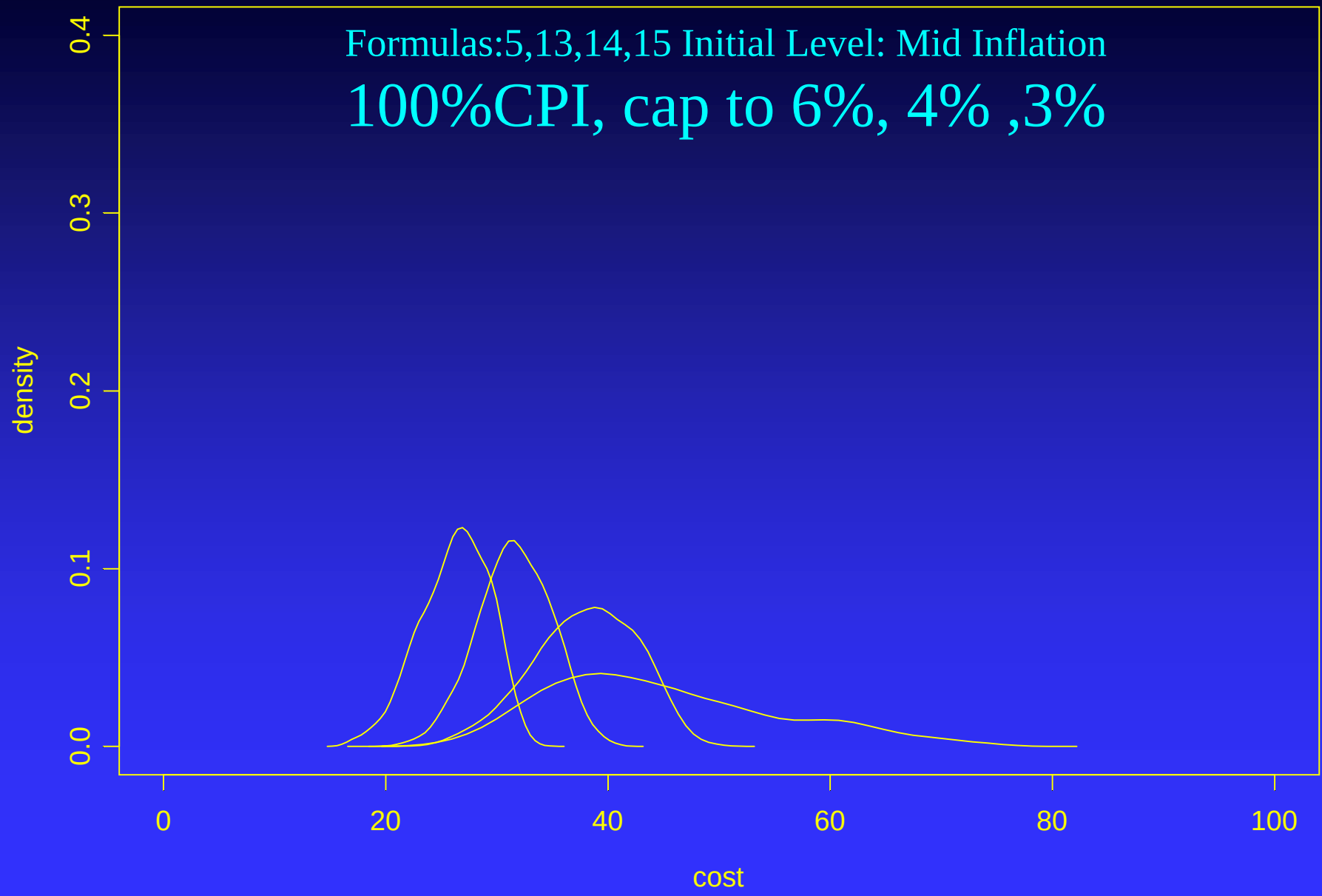
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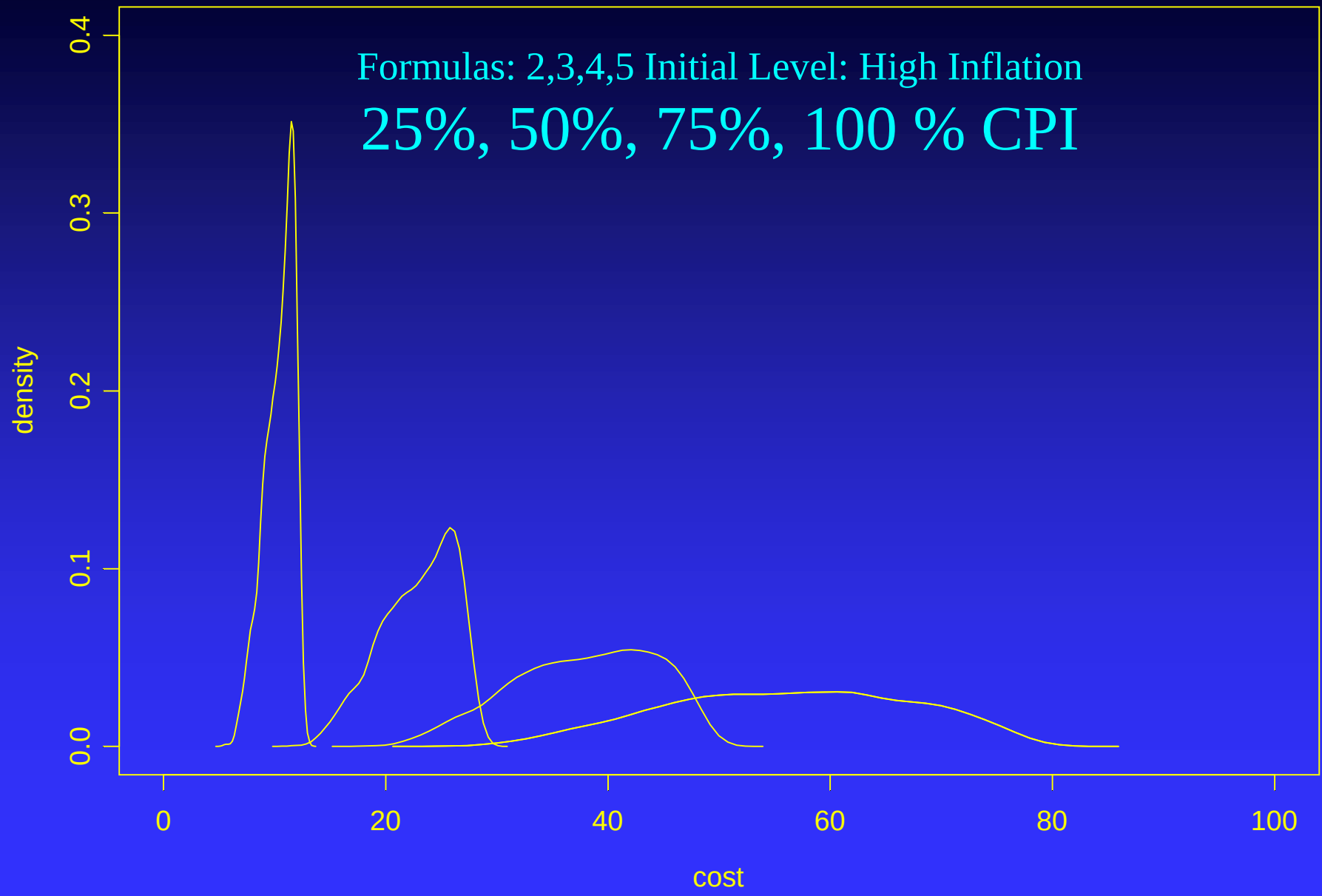
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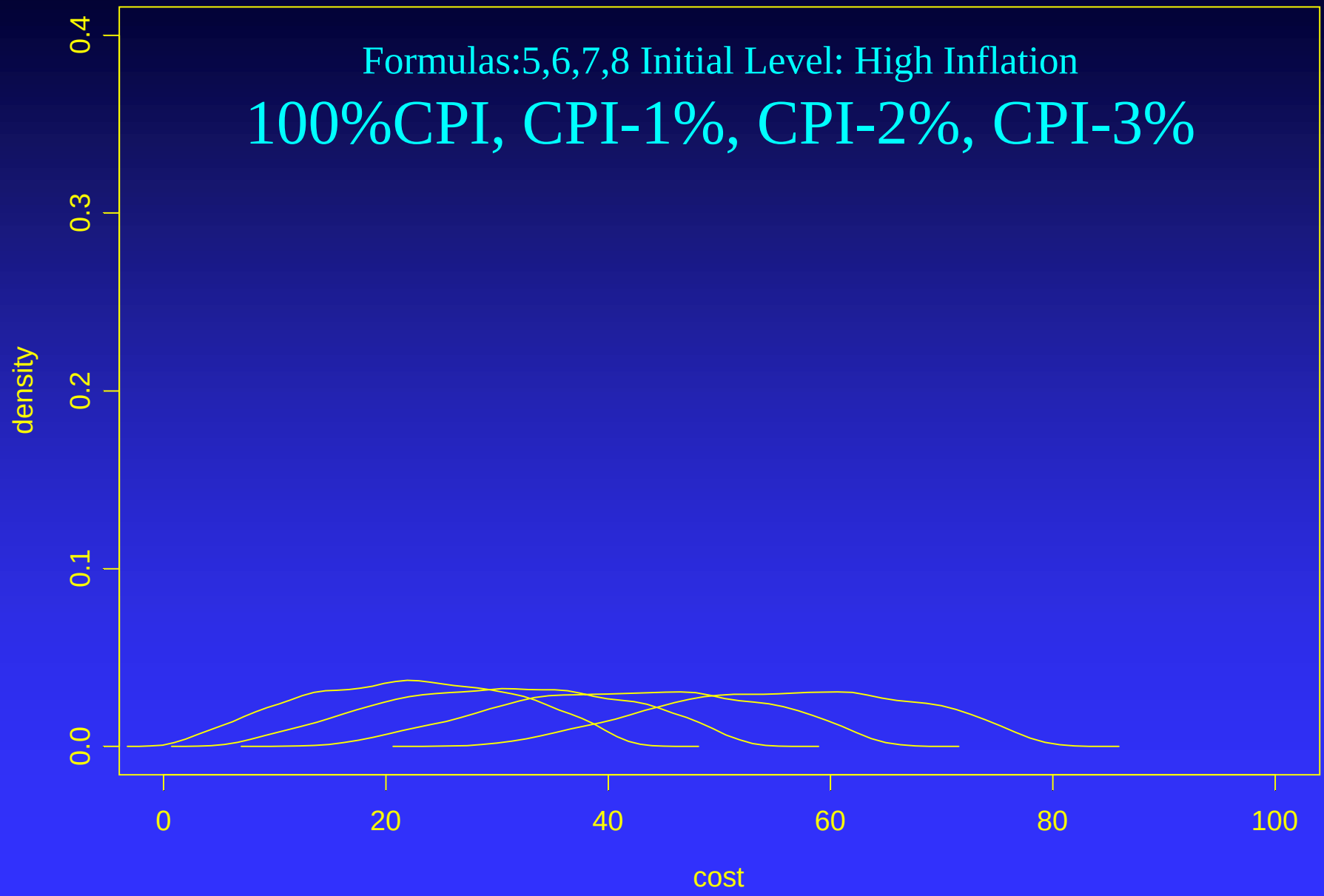
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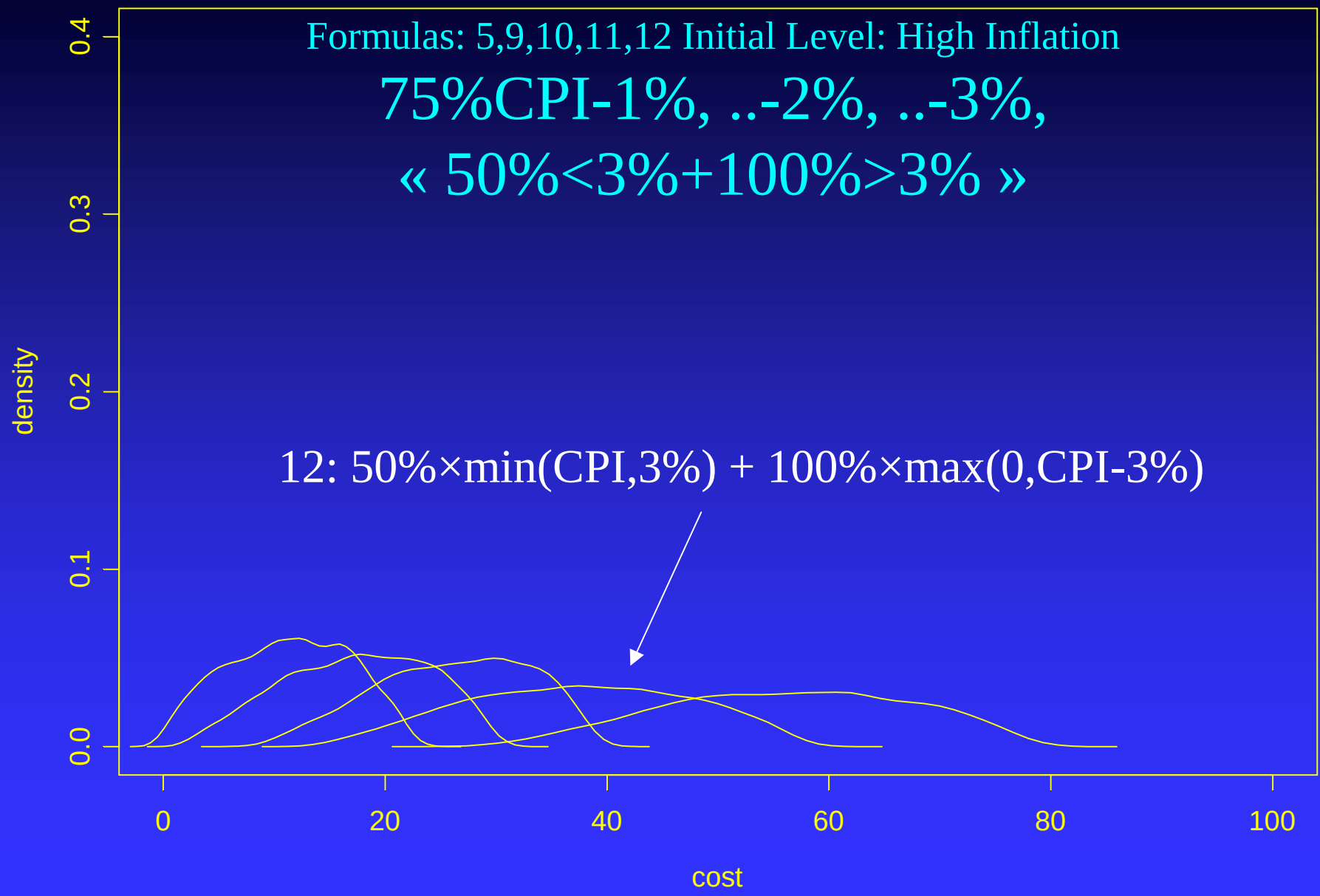
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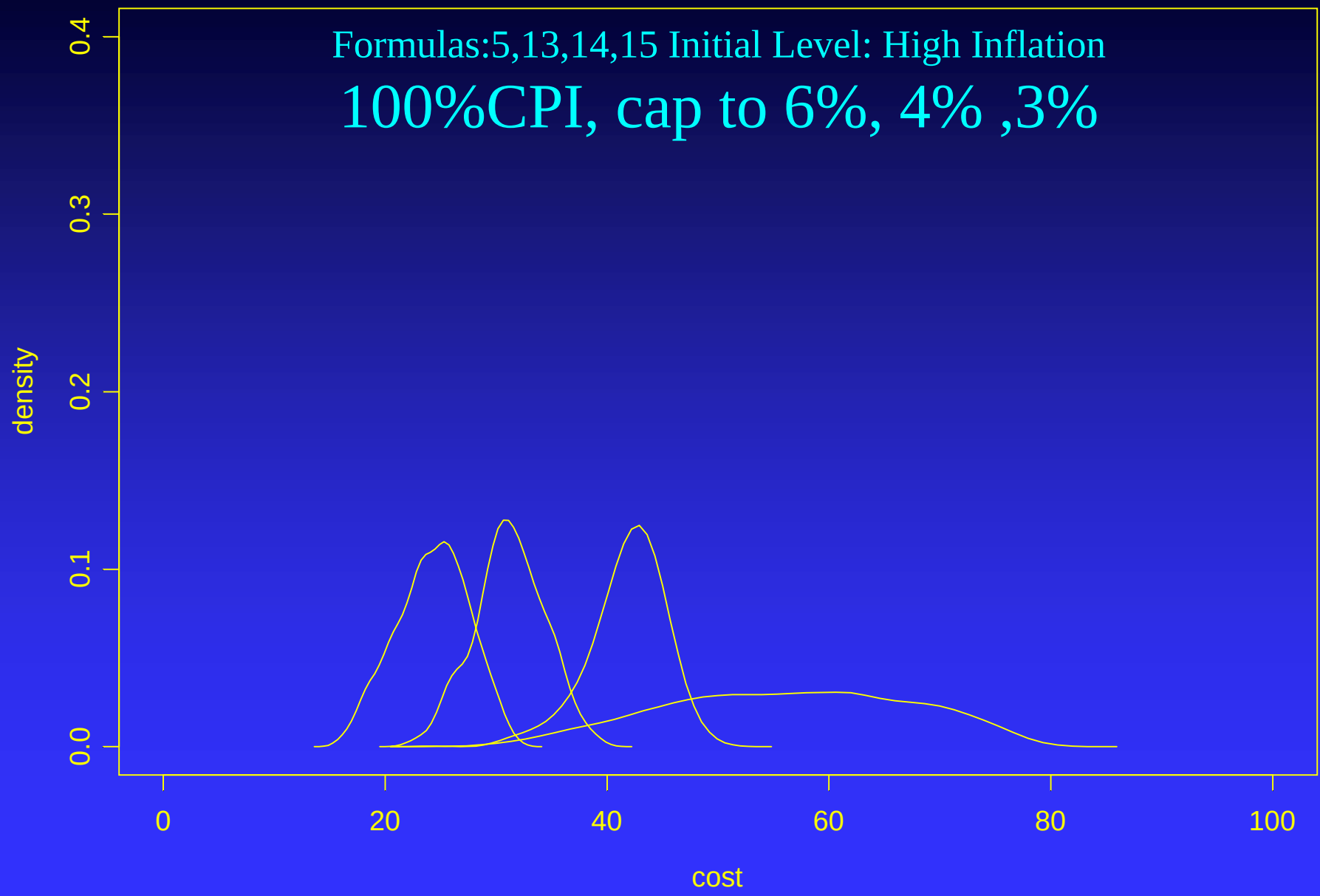
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Results

- Initial level affects results for 10 years
- Cost distribution is wide:WCI accounts for it
- Traditional approach: calculate margin added to indexation rate
- Uni-modal stochastic approach: simpler but
  - initial condition ?
  - Mean-reverting to “normal” state ?

Conclusion

- Features of tri-modal model for inflation
  - Another view at past data and expectations
  - Allows pricing of various indexation formulas and other valuation purposes
  - Recognizes current level of inflation in distribution of indexation cost
  - Weighted Cost Index is one way to capture distribution

Further Areas of Research

- Link with average salary increase
- Replace constant 4% real return with constant asset mix
- Link asset categories real return to inflation
- Expand cash-flows to include typical population (all pensioners, actives)
- Relationship between investment, benefit and funding policies
- Other Countries: support for tri-modal model ?