Optimal Asset Allocation

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- ## Risk premium estimate

  - ### Historical excess returns

  - ### Prospective approach 1: Breakdown of returns

  - ### Prospective approach 2: Implicit risk premium

- ## Asset allocation

  - ### Return and risk of generic portfolios

  - ### Constrained optimizations

  - ### Sensitivity analysis

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- Historical excess returns
- Approach 1: Breakdown of returns
- Approach 2: Implicit risk premium

➢ Objective: Optimal asset allocation

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Historical Returns
1900-2000 (Dimson, March and Staunton) – Percentages

Country	Geometric Return		Excess Return	Standard Deviation	Inflation
	Equities	Bonds			
South Africa	12.0	6.3	5.7	19.7	4.8
Germany	9.7	2.8	6.7	28.4	5,1*
Australia	11.9	5.2	6.3	18.9	4.1
Belgium	8.2	5.1	3.1	20.7	5.5
Canada	**9.7**	**5.0**	**4.5**	**17.8**	**3.1**
Denmark	8.9	6.8	2.0	16.9	4.1
Spain	10.0	7.5	2.3	20.3	6.1
United States	10.1	4.8	5.0	20.0	3.2
France	12.1	6.8	4.9	21.6	7.9
Ireland	9.5	6.0	3.2	17.4	4.5
Italy	12.0	6.7	5.0	30.0	9.1
Japan	12.5	5.9	6.2	33.2	7.6
Netherlands	9.0	4.1	4.7	21.4	3.0
United Kingdom	10.1	5.4	4.4	16.7	4.1
Sweden	11.6	6.2	5.2	22.1	3.7
Switzerland	7.6	5.1	2.7	17.9	2.2
World average			**4.5**		
World weighted average			**4.6**	**14.5**	

* For Germany, years 1922-23 are excluded.

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Actual Historical Returns
1951-2000 (Dimson, Marsh and Staunton, 2002) – Percentages

Country	Geometric Return		
	Equities	**Bonds**	**Excess Return**
South Africa	6.7	0.3	6.4
Germany	9.1	3.7	5.1
Australia	6.2	0.1	6.1
Belgium	6.8	2.9	3.8
Canada	**7.0**	**2.5**	**4.5**
Denmark	6.5	3.8	2.7
Spain	5.5	0.9	4.5
United States	8.5	1.8	6.6
France	8.3	4.4	3.7
Ireland	7.9	1.9	5.9
Italy	5.2	1.9	3.3
Japan	9.1	3.0	5.9
Netherlands	9.2	1.0	8.2
United Kingdom	8.6	1.6	6.8
Sweden	9.5	1.6	7.8
Switzerland	7.2	1.9	5.3
World average	**7.6**	**2.1**	**5.4**
World weighted average	**8.2**	**2.8**	**5.3**

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Actual Historical Returns

Importance of the horizon: 1802-2001 (Siegel, 2002, United States) – Percentages

Holding Period	Equities		Bonds		% Equities > Bonds	Risk	
	Min.	Max.	Min.	Max.		Equities	Bonds
1 year	-38.6	66.6	-21.9	35.1	**61**	18.1	8.6
2 years	-31.6	41.0	-15.9	24.7	**65**	13.0	6.4
5 years	-11.0	26.7	-10.1	17.7	**71**	7.5	5.2
10 years	-4.1	16.9	-5.4	12.4	**80**	4.3	4.0
20 years	1.0	12.6	-3.1	8.8	**92**	2.9	3.1
30 years	2.6	10.6	-2.0	7.4	**99**	1.5	2.6

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Actual Historical Returns

1956-2002: Impact of years 2001 and 2002 – Percentages

	Canada			United States		
	S&P/TSX	**Bonds***	**Treasury Bills**	**S&P 500**	**Bonds****	**Treasury Bills**
1956-2002						
Geometric return	4.5	3.6	2.5	5.8	2.6	1.6
Standard deviation	15.8	8.8	1.5	14.9	8.7	0.9
Excess return		0.8	1.9		3.1	4.2
1956-2000						
Excess return		1.8	2.7		4.6	5.4

* Canada: Long-term SC bondholder indices (1956-1979) and long-term Canada SC (1980-2002)

** United States: Rate of interest on federal long-term liabilities adjusted for the duration (1956-1972) and Lehman Brothers Long Term US Treasury (1973-2002)

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- ## Tremendous volatility of historical excess annual returns

  - ### Imprecise historical data
  - ### Significant confidence intervals

- ## Historical results versus prospective risk premium

  - ### Significant difference depending on whether looking at 2000 or 2002

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- Actual equities return: $RRA_t = RDiv_t + RGC_t$

    Where: $Rdiv_t$: dividend yield

    RGC_t: actual capital gains real return

- $RGC_t \approx g_{C/B,t} + g_{RB,t}$

    Where: $g_{C/B,t}$: growth in multiple price/earnings

    $g_{RB,t}$: actual growth in earnings

	Example 1			Example 2		
	Year 0	Year 1		Year 0	Year 1	
C/B	10	10		10	10.20	
RB	1	1.02		1	1	
$g_{C/B,t}$			0%			2%
$g_{RB,t}$			2%			0%
$RGC_t = g_{C/B,t} + g_{RB,t}$			2%			2%

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Breakdown of Dividend Yield

Canadians and Americans in 1956-2000 – Percentages

	S&P/TSX		S&P 500	
	Annual Average	**Contribution**	**Annual Average**	**Contribution**
Total actual yield	**5.4**		**7.1**	
Dividend yield	3.3	61	3.5	50
Capital gain	2.1	39	3.5	50
Capital gain breakdown				
Actual growth in earnings	1.5		1.9	
Changes in the price/earnings ratio	0.5		1.6	

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Breakdown of Stock Market Yield
Canadians and Americans in 1956-2002 – Percentages

	S&P/TSX		S&P 500	
	Annual Average	**Contribution**	**Annual Average**	**Contribution**
Total actual yield	**4.5**		**5.8**	
Dividend yield	3.2	72	3.4	59
Capital gain	1.2	28	2.3	41
Capital gain breakdown				
Actual growth in earnings	-0.6		0.5	
Changes in price/earnings ratio	1.8		1.9	

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- ## Lag in real growth in the United States
  - ### GDP: 3.3% (1.9% per capita)
  - ### Earnings: 1.5%
  - ### Dividends: 1.1%
- ## Why?
  - ### Transfer of wealth to employees and managers?
  - ### Problem with the calculation of indices?
    - Discontinuity in earnings and dividends: replacing of value securities by growth stock
- ## Choice: real economic growth

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- Has there been a structural change in business distribution policies?

- Replacing dividends by cash through the buy back of equity

  - United States

    - 3.59% of earnings in 1972 versus 41.8% in 2000
    - In 2000, $172 B for dividends versus $194 B for buybacks

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- Approach taken: implicit risk premium
    - Constraint: intrinsic value (v) = price (p)
    - Intrinsic value = discounting cash flows (CF)
    - Unknown: discount rate (k)
    - *Risk premium = k_t - YTM_t*
        Where YTM_t : *yield to maturity*

- Example:
$$v_t = p_t = \sum_{s=1}^{\infty} \frac{FM_{t+s}}{(1+k)^{t+s}}$$

P_{2003} = 1000
FM = 80 (perpetuity)
K = 8%
YTM_{2003} = 5%
Premium = 8% - 5% = 3%

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- ## Different valuation models

  - Discounting dividends (Gordon)

  - Discounting cash flows (MA)

  - Sum of the book value of equity + discounting of future economic value added (EBO)

- ## EBO model: evaluation model based on available inputs

  - Book value of equity

  - Earnings projections

  - Potential problem: biased projections

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$$v_t = bv_t + \sum_{s \equiv 1}^{\infty} \frac{e_{t+s} - k_t \times bv_{t+s-1}}{(1+k_t)^{t+s}}$$

Where v_t = intrinsic value of the index at time t

bv_t = book value of equity at time t

e_{t+s} = earnings projections at time $t+s$

k_t = YTM_t+Pr

Note:

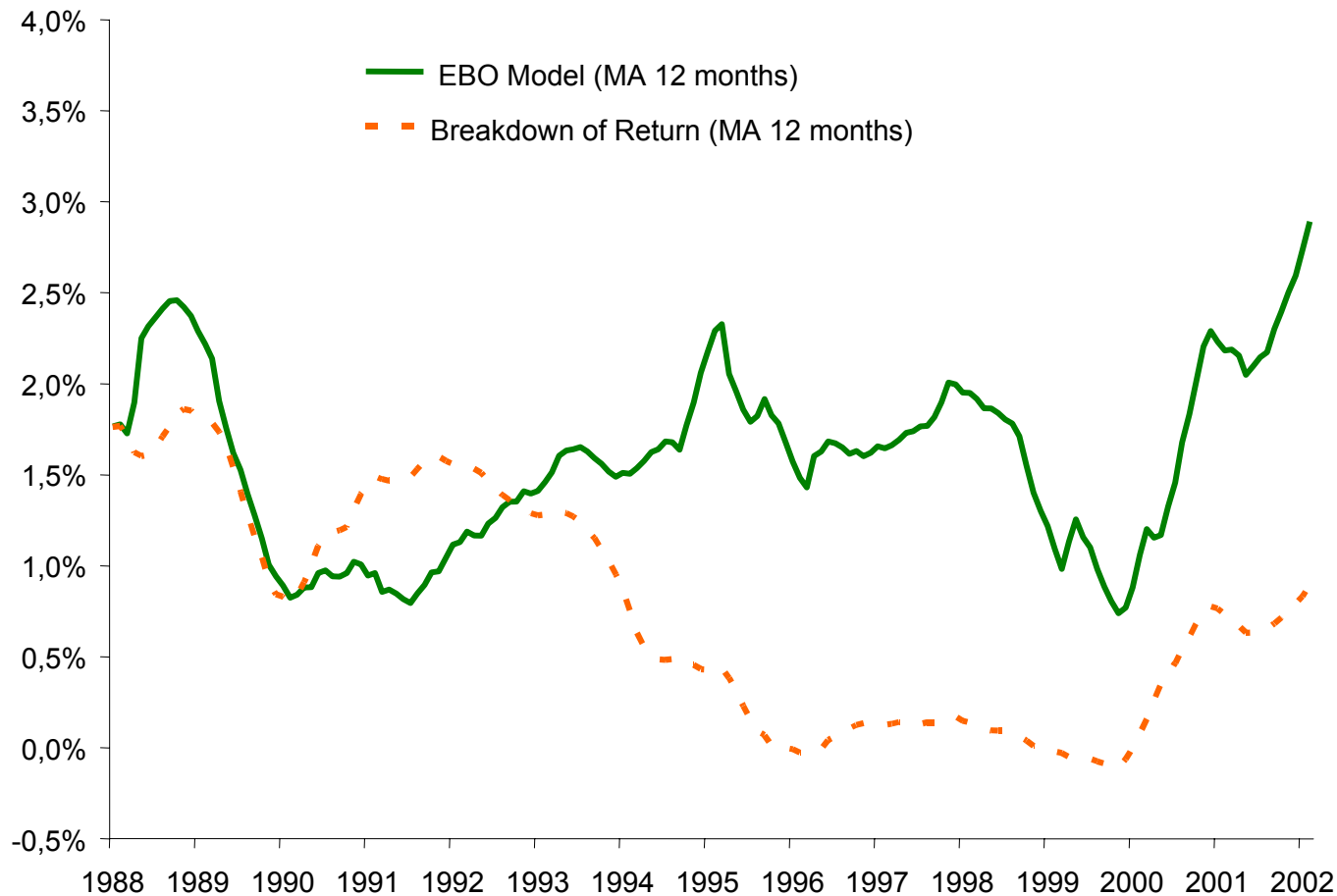"Abnormal" earnings or economic value added = $e_{t+s}-k_t \times bv_{t+s-1}$

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EBO Model
Example: Implicit premium – January 2003

	Canada	as % of Price	United States	as % of Price
Price of index	848.2		858.3	
EPS_{t+1}	44.3	5.2%	44.0	5.1%
EPS_{t+2}	57.3	6.8%	51.3	6.0%
EPS_{t+3}	67.1	7.9%	59.1	6.9%
g (long term)	2.0%		2.0%	
bv_t	527.5	62.2%	320.9	37.4%
k_t	8.2%		7.6%	
YTM_t	5.0%		4.1%	
$Premium_t$	3.2%		3.6%	

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Comparing Approaches 1 and 2
Risk premium in Canada

- Minimum threshold
  - Approach 1: Breakdown of return
    - Prospective risk premium = 1.3%
    - Buyback correction ≈ +0.5%
- Maximum threshold
  - Approach 2: Implicit risk premium
    - Projected risk premium = 3%
    - Optimism bias correction ≈ -1%
- Conclusion: 2% prospective premium
- Impact on the allocation of assets in a reference portfolio

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Projecting Stock Exchange Returns
EBO approach

Country	Implicit Premium (12 month MA)	Projected Local Return	Interest Rate Differential	Projected Hedged Return
	(1)	(2)	(3)	(4) = (2) + (3)
Canada	3.0%	8.0%	0.0%	8.0%
United States	2.9%	6.6%	1.3%	7.9%
United Kindgom	2.0%	6.1%	0.8%	6.9%
Germany	3.8%	7.7%	1.0%	8.8%
Italy	2.8%	6.9%	1.0%	7.9%
France	3.4%	7.3%	1.0%	8.4%
Japan	4.5%	5.3%	4.2%	9.5%
Australia	2.4%	7.4%	-0.1%	7.3%
Netherlands	4.2%	8.2%	1.0%	9.2%
Sweden	3.5%	7.9%	0.5%	8.4%
EAFE				8.3%

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	Projected Return	Projected Risk
Short-term securities	3.6%	1.0%
Bonds	4.8%	6.6%
Canadian equities	8.0%	16.4%
American equities	7.9%	16.9%
Foreign equities	8.3%	19.1%
Real estate	8.5%	13.4%

The correlation matrix is presented in Annex A.

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Composition of Generic Reference Portfolios

Asset Category	Portfolio 1	Portfolio 2	Portfolio 3
Short-term securities	2.0%	2.0%	2.0%
Bonds	48.0%	38.0%	28.0%
Fixed income securities	**50.0%**	**40.0%**	**30.0%**
Canadian equities	15.0%	17.0%	19.0%
American equities	14.0%	17.0%	20.0%
Foreign equities	14.0%	17.0%	20.0%
Real estate	7.0%	9.0%	11.0%
Variable income securities	**50.0%**	**60.0%**	**70.0%**

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	Portfolio 1	Portfolio 2	Portfolio 3
Total return	6.4%	6.8%	7.1%
Risk	8.2%	9.1%	10.1%
Sharpe Ratio	**0.35**	**0.35**	**0.35**

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	Lower Thresholds	Upper Thresholds
Short-term securities	0.0%	20.0%
Bonds	25.0%	50.0%
Canadian equities	15.0%	40.0%
American and foreign equities	0.0%	30.0%
Real estate	0.0%	15.0%

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Composition of Optimal Portfolios for Different Risk Levels

	Minimum Risk	6% Risk	8% Risk	10% Risk	Maximum Return
Short-Term Securities	20.0%	8.3%	0.0%	0.0%	0.0%
Bonds	50.0%	50.0%	41.7%	25.0%	25.0%
Fixed Income Securities	**70.0%**	**58.3%**	**41.7%**	**25.0%**	**25.0%**
Canadian equities	15.0%	21.7%	28.2%	32.6%	30.0%
American and foreign equities	0.0%	5.0%	15.1%	27.4%	30.0%
Real Property	15.0%	15.0%	15.0%	15.0%	15.0%
Variable Income Securities	**30.0%**	**41.7%**	**58.3%**	**75.0%**	**75.0%**
	100.0%	100.0%	100.0%	100.0%	100.0%

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Projected Return and Risk for Optimal Portfolios

Optimal Portfolios

	Minimum Risk	6% Risk	8% Risk	10% Risk	Maximum Return
Total Return	5.6%	6.1%	6.8%	7.4%	7.7%
Risk	4.6%	6.0%	8.0%	10.0%	11.3%
Sharpe Ratio	0.44	0.42	0.40	0.38	0.36

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Probabilities Related to Certain Return Thresholds

	Optimal Portfolios				
	Minimum Risk	**6% Risk**	**8% Risk**	**10% Risk**	**Maximum Return**
One Year Horizon					
More than 10%	17%	26%	34%	39%	40%
Less than 4%	36%	36%	37%	37%	38%
Less than 0%	11%	15%	20%	23%	25%
Five Year Horizon					
More than 10%	2%	7%	18%	27%	29%
Less than 4%	22%	22%	22%	23%	24%
Less than 0%	1%	1%	3%	5%	6%
Ten Year Horizon					
More than 10%	1%	2%	10%	19%	22%
Less than 4%	13%	13%	14%	15%	16%
Less than 0%	1%	1%	1%	1%	2%

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Sensitivity Analysis

Composition of optimal portfolios for an 8% risk level

	Optimal Portfolios		
	EBO Scenario -1%	EBO Scenario	EBO Scenario +1%
Short-term securities	0.0%	0.0%	0.0%
Bonds	42.2%	41.7%	41.4%
Canadian equities	27.1%	28.2%	28.8%
American and foreign equities	15.8%	15.1%	14.8%
Real estate	15.0%	15.0%	15.0%
Projected return	6.2%	6.7%	7.3%
Risk	8.0%	8.0%	8.0%

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Annex A - Correlation Matrix

Ten-year horizon - Annual

	Short-Term Securities	Bonds	Canadian Equities	American Equities	Foreign Equities	Quebec World	Real Estate
Short-Term Securities	1.00						
Bonds	0.25	1.00					
Canadian Equities	-0.06	0.10	1.00				
American Equities	0.12	0.48	0.59	1.00			
Foreign Equities	-0.03	0.22	0.68	0.70	1.00		
Quebec World	0.07	0.60	0.62	0.93	0.85	1.00	
Real Estate	0.23	-0.32	0.20	0.02	0.30	0.04	1.00

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