THE ARITHMETIC OF THE PUBLIC DEBT

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The federal government's budget situation has improved considerably over the past few years. In 1993-94, the budget deficit, calculated on the basis of the public accounts, was more than $42 billion (5.9% of Gross Domestic Product). The deficit dropped to $28.6 billion (3.7% of GDP) in 1995-96 and should be $19 billion (3% of GDP) in 1996-97 (see Table 1). The government has promised to keep the budget deficit under $17 billion (2% of GDP) in 1997-98 and $9 billion (1% of GDP) in 1998-99. There is every reason to believe, however, that the government will exceed its targets and the deficit will be less than projected in those two fiscal years. In fact, the deficit for 1996-97 will likely be between 13 and 16 billion dollars.

In 1997-98, Canada's financial requirements as a percentage of GDP (0.7%) will be lower than those of any other G7 country. The last time financial requirements as a percentage of GDP were lower was way back in 1969-70. It is reasonable to wonder why the government's financial requirements (as a percentage of GDP) are lower than its budget deficit. The difference is primarily attributable to the fact that financial requirements take into account available non-budgetary revenue, such as employee pension accounts. In 1998-99, Canada's financial requirements will be in a slight surplus position; the government will therefore not have to borrow any new money on credit markets.

Despite all the significant progress that has been made in recent years, Canada's public debt in relation to GDP (the debt/GDP ratio) has grown steadily. In 1996-97, the debt/GDP ratio is expected to be 74.4%, compared with 74% in 1995-96, 71.3% in 1993-94 and 58.4% in 1990-91. The ratio is not expected to start decreasing until the 1997-98 fiscal year; the debt/GDP ratio should stand at 73.1% by the end of that year and drop to 71.2% the following year.

Table 1
Summary Statement of Transaction:
Fiscal Outlook with Budget Measures[1]

	1994-1995	1995-1996	1996-1997	1997-1998	1998-1999
	(billions of dollars)				
Budgetary Revenues	123.3	130.3	135.5	137.8	144.0
Program Spending	118.7	112.0	109.0	105.8	103.5
Operating Balance	4.6	18.3	26.5	32.0	40.5
Public Debt Charges	42.0	46.9	45.5	46.0	46.5
Underlying Deficit	-37.5	-28.6	-19.0	-14.0	-6.0
Contingency Reserve				3.0	3.0
Deficit	-37.5	-28.6	-19.0	-17.0	-9.0
Net Public Debt	545.7	574.3	593.3	610.3	619.3
Non-Budgetary Transactions	11.6	11.4	13.0	11.0	10.0
Financial Requirements/Source	-25.8	-17.2	-6.0	-6.0	1.0
Per Cent of GDP					
Budgetary Revenues	16.5	16.8	17.0	16.5	16.6
Program Spending	15.9	14.4	13.7	12.7	11.9
Operating Balance	0.6	2.4	3.3	3.8	4.7
Public Debt Charges	5.6	6.0	5.7	5.5	5.3
Deficit	-5.0	-3.7	-2.4	-2.0	-1.0
Financial Requirements	-3.5	-2.2	-0.8	-0.7	0.1
Net Public Debt	73.0	74.0	74.4	73.1	71.2

[1] A positive number indicates a source of funds, a negative number a financial requirement.

Source: Government of Canada, *Budget 1997 - Budget Plan*, Ottawa, Department of Finance, 18 February 1997, p. 44.

Why will the debt/GDP ratio suddenly start to fall? The reason is fairly simple: in 1996-97, the federal government will have recorded a large operating balance relative to the size of the economy. "Operating balance" is the difference between budgetary revenues and program spending. In 1996-97, the government's budgetary revenues should total $135.5 billion, while program spending should be $109 billion; this means that the operating balance should be $26.5 billion or, more specifically, 3.3% of GDP (see Table 1).

This "operating balance/GDP" ratio holds the mathematical key to reducing the debt/GDP ratio. If the government wants to bring down the debt, it will have to keep recording positive operating balances in relation to the GDP. There is, however, a threshold under which even a positive operating balance relative to the GDP will not help reduce or even stabilize the debt/GDP ratio. Let us turn our attention to that threshold.

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Three variables have a bearing on the debt/GDP ratio: nominal growth, the cost of servicing the debt, and operating balance.

Nominal growth (g) means growth in the economy, which directly affects the relative size of the debt. The more the economy grows, the smaller is the debt in proportion. In other words, if the economy grows faster than the debt changes, the debt/GDP ratio may decrease.

Obviously, the debt/GDP ratio will decrease from one year to another if the debt increases more slowly than the GDP or, in other words, if the proportional change in the debt is less than nominal growth.[1]

The change in the debt (ΔD) is simply the interest to be paid (i) on the debt (D) minus the operating balance (OB).[2] The cost of servicing the debt therefore also has a direct effect on the size of the debt. The higher the interest rate, the faster the debt grows.

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(1)   This can be expressed as follows: $\Delta D/D < \Delta GDP/GDP$, where ΔD = change in debt, D = debt and ΔGDP = change in GDP. The equation can be further simplified as: $\Delta D/D < g$, where g = nominal growth.

(2)   In other words, $\Delta D = (i \times D) - OB$, where i = cost of interest on the debt and OB = operating balance.

The operating balance also has an impact on change in the debt. The higher the balance, the more it can offset the high cost of servicing the debt and the more slowly the debt will grow.

What this means is that the rate of nominal growth (g), the interest rate (i) applicable to debt servicing and the operating balance (OB) all have a bearing on the relative size of the debt.[3]

The combined effect of these three variables can be expressed algebraically. The result is an equation setting out the conditions in which the debt/GDP ratio will start declining.

$$\textbf{(i - g)} \times \textbf{D/GDP} < \textbf{OB/GDP}.$$

There are only two ways to solve this equation. The first is to tackle the left side. This assumes that nominal growth (g) is greater than the average rate of interest (i) on the public debt. The chances of this happening in Canada in the near future are very slim. Moreover, the government cannot easily influence either of these two exogenous variables, at least in the short term. The second solution is to tackle the right side; here, everything depends on the government's budget choices. If the government decides to incur a large enough positive operating balance as a percentage of GDP, the debt/GDP ratio will decrease.

The federal government's operating balance has increased steadily in the past few years. In 1995-96, for example, the balance was equal to 2.4% of GDP, up sharply from what it had been previously. The last time the balance was higher than 2.4% of GDP was 1956-57. According to government projections, the operating balance as a percentage of GDP will be 3.3% in 1996-97, 3.8% in 1997-98 and 4.7% in 1998-99. The last time the operating balance was higher than 4.7% of GDP was in 1948-49 (7.2% of GDP).

Table 2 and Figure 1 illustrate the impact of different operating balances on the relative size of the debt. The simulations in Table 2 assume a nominal growth rate of 4.9%

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(3)   The relative size of the debt will therefore decrease if $\Delta D/D < g$ or $((i \times D) - OB)/D < g$. Moving the denominator D to the other side of the equation gives $(i \times D) - OB < c \times D$. Rearranging the variables ultimately gives $(i - g) \times D < OB$, the condition required for the debt/GDP ratio to decrease. Dividing by the GDP gives $(i - g) \times D/GDP < OB/GDP$, or the condition as a percentage of GDP needed for the debt/GDP ratio to decrease.

Table 2
Impact of Different Operating Balances on the Debt/GDP Ratio

Simulation No. 1	1997-1998	1998-1999	1999-2000	2000-2001	2001-2002	2002-2003
GDP (billions of dollars)	835	874.2	915.3	958.4	1003.4	1050.6
Public Debt	610.3	638.8	670.8	703.2	737.3	773.1
Operating Balance	32	17.5	18.3	19.2	20.1	21.0
Debt/GDP Ratio	0.731	0.73	0.73	0.73	0.73	0.74
Nominal Growth	0.049	0.047				
Interest on Debt	0.077					
Operating Balance/GDP	**0.02**					

Simulation No. 2	1997-1998	1998-1999	1999-2000	2000-2001	2001-2002	2002-2003
GDP (billions of dollars)	835	874.2	915.3	958.4	1003.4	1050.6
Public Debt	610.3	616.2	620.6	623.4	624.2	622.9
Operating Balance	32.0	41.1	43.0	45.0	47.2	49.4
Debt/GDP Ratio	0.731	0.705	0.678	0.650	0.622	0.593
Nominal Growth	0.049	0.047				
Interest on Debt	0.077					
Operating Balance/GDP	**0.038**	**0.047**				

Simulation No. 3	1997-1998	1998-1999	1999-2000	2000-2001	2001-2002	2002-2003
GDP (billions of dollars)	835	874.2	915.3	958.4	1003.4	1050.6
Public Debt	610.3	622.3	633.6	644.1	653.5	661.8
Operating Balance	32	35.0	36.6	38.3	40.1	42.0
Debt/GDP Ratio	0.731	0.712	0.692	0.672	0.651	0.630
Nominal Growth	0.049	0.047				
Interest on Debt	0.077					
Operating Balance/GDP	**0.04**					

Simulation No. 4	1997-1998	1998-1999	1999-2000	2000-2001	2001-2002	2002-2003
GDP (billions of dollars)	835	874.2	915.3	958.4	1003.4	1050.6
Public Debt	610.3	613.6	615.1	614.5	611.6	606.2
Operating Balance	32	43.7	45.8	47.9	50.2	52.5
Debt/GDP Ratio	0.731	0.702	0.672	0.641	0.610	0.577
Nominal Growth	0.049	0.047				
Interest on Debt	0.077					
Operating Balance/GDP	**0.05**					

Figure 1

Debt/GDP Ratio

Operating Balance and Debt/GDP Ratio

OB/GDP = 2%	OB/GDP = 3.8/4.7%	OB/GDP = 4%	OB/GDP = 5%		
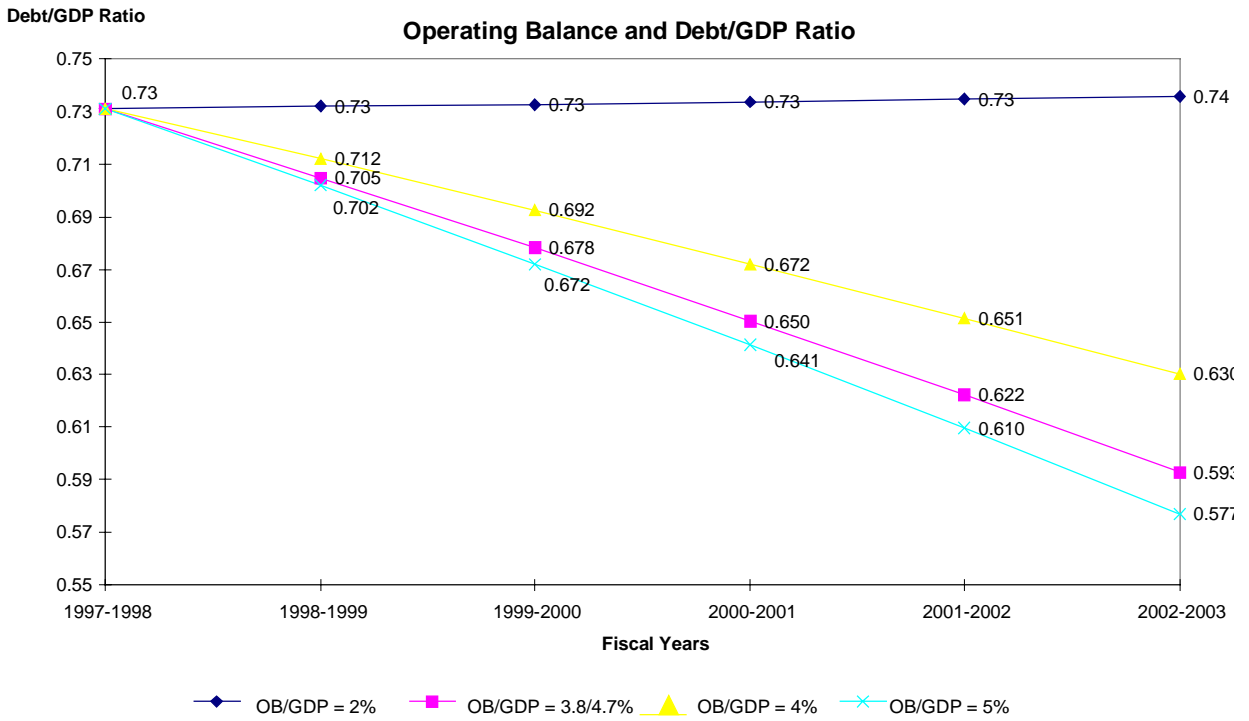
for 1997-98 and 4.7% for 1998-99 (private-sector projections) and an average rate of interest on the public debt of 7.7% for the entire period in question. The first simulation shows that a balance of 2% would simply stabilize the public debt relative to the GDP at around 73%. The second simulation uses the operating balances as a percentage of GDP given in the last budget, assuming that the 4.7% operating balance would not change after 1998-99. The third and fourth simulations use an operating balance as a percentage of GDP of 4% and 5% respectively. The four simulations clearly show that the higher the operating balance, the faster the decrease in the debt as a percentage of GDP.

The recent drops in interest rates have already begun to have an impact on the cost of servicing the public debt. As medium- and long-term securities mature, they are refinanced at lower interest rates. Table 3 and Figure 2 assume that the average interest rate will gradually decrease from 7.7% to 6.8% beginning in 2002-03. In this new context, the debt/GDP ratio would decrease more quickly if operating balances were to stay the same.

IMPACT ON POLITICAL CHOICES

Now that the government has met its deficit reduction objectives, it has raised questions about the appropriate relative size of the debt. Some have argued that decreasing the debt/GDP ratio should be the primary objective of the next budgetary policies, while others maintain that keeping the debt/GDP ratio stable is the target to pursue.

The traditional economic argument is that the government should endeavour to generate large operating balances in times of economic growth. Such a budgetary policy would make it possible to control the size of the debt when the next economic downturn comes, at which time the operating balance will most likely decrease. Moreover, decreasing the debt/GDP ratio when conditions permit would make it easier for the government to borrow money in the next slowdown and thereby stabilize the economy.

Deteriorating economic conditions (for example, slower economic growth and higher interest rates) would force the government to come up with increasingly large operating balances in order to control the size of the debt. For example, assuming an average interest rate of 8.5%, nominal growth of 3%, and a debt to GDP ratio of 73%, the government would

Table 3
Impact of Different Operating Balances on the Debt/GDP Ratio

Simulation No. 5	**1997-1998**	**1998-1999**	**1999-2000**	**2000-2001**	**2001-2002**	**2002-2003**
GDP (billions of dollars)	835	874.2	915.3	958.4	1003.4	1050.6
Public Debt	610.3	638.0	665.6	693.0	720.8	748.8
Operating Balance	32	17.5	18.3	19.2	20.1	21.0
Debt/GDP Ratio	0.73	0.73	0.73	0.72	0.72	0.71
Nominal Growth	0.049	0.047				
Interest on Debt	0.077	0.074	0.072	0.07	0.069	0.068
Operating Balance/GDP	**0.02**					

Simulation No. 6	**1997-1998**	**1998-1999**	**1999-2000**	**2000-2001**	**2001-2002**	**2002-2003**
GDP (billions of dollars)	835	874.2	915.3	958.4	1003.4	1050.6
Public Debt	610.3	614.4	615.6	613.6	608.8	600.8
Operating Balance	32.0	41.1	43.0	45.0	47.2	49.4
Debt/GDP Ratio	0.731	0.703	0.673	0.640	0.607	0.572
Nominal Growth	0.049	0.047				
Interest on Debt	0.077	0.074	0.072	0.07	0.069	0.068
Operating Balance/GDP	**0.038**	**0.047**				

Simulation No. 7	**1997-1998**	**1998-1999**	**1999-2000**	**2000-2001**	**2001-2002**	**2002-2003**
GDP (billions of dollars)	835	874.2	915.3	958.4	1003.4	1050.6
Public Debt	610.3	620.5	628.6	634.2	637.8	639.2
Operating Balance	32	35.0	36.6	38.3	40.1	42.0
Debt/GDP Ratio	0.731	0.710	0.687	0.662	0.636	0.608
Nominal Growth	0.049	0.047				
Interest on Debt	0.077	0.074	0.072	0.07	0.069	0.068
Operating Balance/GDP	**0.04**					

Simulation No. 8	**1997-1998**	**1998-1999**	**1999-2000**	**2000-2001**	**2001-2002**	**2002-2003**
GDP (billions of dollars)	835	874.2	915.3	958.4	1003.4	1050.6
Public Debt	610.3	611.7	610.0	604.8	596.4	584.4
Operating Balance	32	43.7	45.8	47.9	50.2	52.5
Debt/GDP Ratio	0.731	0.700	0.666	0.631	0.594	0.556
Nominal Growth	0.049	0.047				
Interest on Debt	0.077	0.074	0.072	0.07	0.069	0.068
Operating Balance/GDP	**0.05**					

Figure 2

Operating Balance and Debt/GDP Ratio
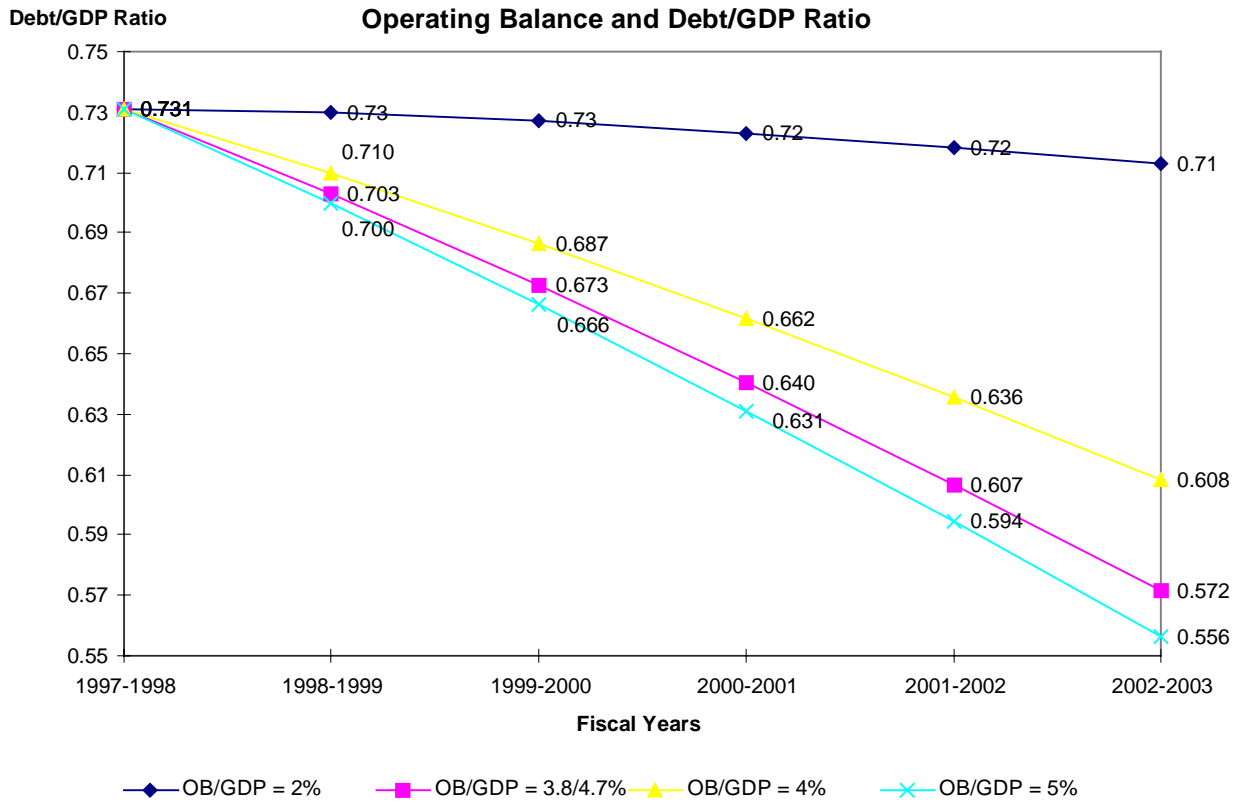
need an operating balance equal to 4% of GDP (i - g) $\times$ D/GDP), that is, (8.5% - 3%) $\times$ 0.73, simply to stabilize the debt. In an economic downturn, it would be very hard for the government to achieve that kind of balance. It is easier to work on reducing the debt in times of economic growth and lower interest rates when the government does not have to worry about stabilizing the economy.