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Depreciation Rates for the Productivity Accounts

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Table of contents

Abstract

This paper generates depreciation profiles for a diverse set of assets based on patterns of resale prices and retirements. In doing so, it explores the sensitivity of estimates of the growth in capital stock and capital services to alternate estimates of depreciation.

In the first instance, survival analysis techniques are used to estimate changes in valuation of assets over the course of their service life. In the second instance, a two-step procedure is utilized that first estimates the discard function for used assets (assets discarded at zero prices) and then uses the resulting estimates to correct for selection bias that arises when just positive used-asset prices are employed to estimate age-price profiles to produce depreciation rates. For the third method, a discard function and an asset efficiency function are jointly specified and estimated.

These three different methods produce depreciation profiles that follow convex patterns. Accelerated profiles are apparent for many individual assets in the machinery and equipment and structures classes.

We also compare the *ex post* estimates of length of life that are based on outcomes to *ex ante* expected lives and find they are much the same. We therefore choose *ex ante* lives along with information from the *ex post* rates on the rate of decline in an asset's value to generate a set of depreciation rates for use in the productivity accounts.

We then use our depreciation model to produce estimates of the growth in capital stock and capital services over the 1961 to 1996 period. We find that the resulting estimates of the growth in capital stock and capital services are quite similar to those previously produced.

1. Introduction

Studies of asset depreciation are illuminating, in part, because they enable national accountants to better characterize the evolution of an economy's productive capacity. The net capital stock available for production purposes is the gross capital stock minus the value of depreciation.

Depreciation estimates are also important for productivity measures. Multifactor productivity estimates depend on the growth of the economy's stock of capital assets. In the standard perpetual inventory framework, the stock of capital available to economic agents in any current period is simply the sum of current investment and cumulative net investment in past periods (i.e., gross accumulated capital stock less depreciation). Estimates of depreciation rates are used to turn the cumulative gross stock of capital into net capital stock. Disagreements about depreciation profiles may give rise to discordant statistical impressions of the amount of capital available to the production process. And to the extent that there is little evidence that can be used to discriminate among different depreciation profiles that are used to estimate net capital stock, estimates of depreciation are less useful to clients of a statistical agency—because the point estimates provided by these programs must be accompanied by large confidence intervals.

Accurate estimates of depreciation are also important when it comes to studies of investment behaviour.[1] Tax policies related to depreciation allowances are often aimed at influencing investment behaviour. Commenting on the consequences of the rules used for tax depreciation, Hulten and Wykoff (1981: 82) make the observation that "depreciation lives, without some factual basis, can lead to potentially serious distortions in the incentives to invest in various types of assets." Depreciation, and perceptions thereof, have substantial impacts on the economic system.

This paper is the second in a series that use new micro-level data on used-asset prices to estimate patterns of economic depreciation. As a first exercise, Gellatly, Tanguay, and Yan (2002) developed depreciation profiles and life estimates for 25 different machinery and equipment assets and 8 structures using data on used-asset prices for the period from 1988 to 1996. That paper compared the estimates that several alternate estimating frameworks produced. A particular framework, which used a duration model, was then chosen to provide estimates of depreciation that were then incorporated into estimates of the growth in capital stock and capital services that are employed in Statistics Canada's productivity program.

In this paper, we extend the used-asset price database from 1996 to 2001 and apply two additional estimation frameworks in order to produce perpetual inventory estimates of capital stock. This larger sample provides over 30,000 estimates of used prices on 49 individual assets which are aggregated into 29 different asset categories—categories that collectively comprise the non-residential portion of the capital stock.

As in the previous paper (Gellatly, Tanguay and Yan, 2002), we compare the *ex post* estimates of depreciation that are yielded by this approach to *ex ante* estimates that come from an alternate source of data—survey estimates of the 'expected' life of assets. The Statistics Canada

1. See Coen (1975).

investment survey that generates used-asset prices also provides estimates of the expected life of assets. These too can be used to produce estimates of depreciation and we do so here and compare them to the *ex post* results derived from the rate at which the prices of assets decline over time. Used-asset prices provide *ex post* information and tell us how assets worked out in practice. Use of “predicted” length of life estimates, when the investment is first made, makes use of *ex ante* estimates. As in our previous paper, we find a close similarity between the two and this gives us added confidence in the estimates that emerge from this analysis.

The principal objectives in this paper are: first, to develop a comprehensive profile of how asset values decline at different stages of service life; second, to ask whether the technique that uses *ex ante* information on length of asset life to estimate depreciation accords with *ex post* market outcomes; and third, to ask how alternate estimates of the depreciation rate affect our estimates of productivity growth.

But the primary contribution of this paper is to subject the extended data set to alternate estimation techniques in order to test the robustness of the results reported previously by Gellatly, Tanguay and Yan (2002). In the end, we decide to modify those results slightly.

The previous estimation framework modeled changes in asset value using estimation techniques that fall under the rubric of survival analysis. The previous results were based on a survival model that has been modified to produce estimates of depreciation. In this paper, we examine several alternative methods for estimating the depreciation rate. We first examine a two-step procedure (made popular by Hulten and Wykoff, 1981) that models the discard function of an asset and then uses the estimated function to correct the observed prices for assets that sell at positive prices for selection bias. The third procedure that is investigated here estimates the discard process and the selling price jointly in a simultaneous framework since joint estimation has several well-known desirable properties.

Each of these procedures differs in terms of the nature of the estimation framework and the demands placed on the data. The final choice of a summary statistic like a depreciation rate needs to take into account both properties of econometric estimation procedures when data are perfect and when they are imperfect. The final choice of estimates adopted here therefore considers both.

The structure of the paper is as follows. Section 2 reviews a range of theoretical and empirical issues that motivate this study. We discuss the properties of the data sample in Section 3. We develop the econometric models in Section 4. Monte Carlo simulations are used in Section 5 to help us evaluate the properties of the different estimation techniques. Estimates of depreciation rates are presented in Section 6. Estimates of capital stock based on the estimates of depreciation are evaluated in Section 7.

2. Foundations

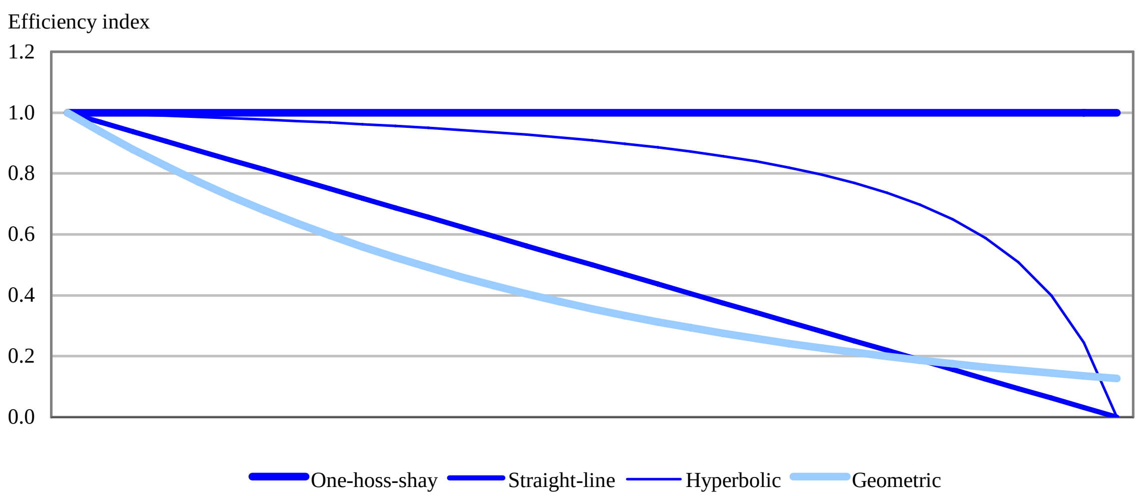
2.1 Efficiency and depreciation

As economic concepts go, depreciation is ubiquitous. A central characteristic of any system of production, depreciation, in its most common usage, refers to how the elements of an economic system erode with age. When it is desirable to do so, economic agents respond to this decline in productive capacity by reinvesting—businesses, in replacement technologies or plants and equipment; governments, in infrastructure and other public goods. These examples invoke images of depreciation as an observable, physical process, one that describes the rate at which productive assets are ingested, and dictates the pace of offsetting investments in maintenance and replacement.

Given that popular notions of depreciation are often beset by the above imagery, precise working definitions need to be set out at the onset. In particular, care must be taken to distinguish economic depreciation from physical, or capacity, depreciation. The crucial distinction between these two types rests with what is eroding or decaying—the production capabilities of the asset itself, or its subsequent economic value.

To make this distinction, we start by focusing on the evolution of an asset's productive efficiency, that is, its ability to produce goods and services over the course of its service life. The productive efficiency can be seen as the stream of earnings that the asset is able to produce over time. As the asset experiences wear and tear, its productive efficiency declines, and it undergoes a process of physical depreciation. We represent this process graphically using the set of efficiency profiles depicted in Figure 1 where we assume that the stream of earnings is known with certainty. In a subsequent section, we relax this assumption and treat the retirement date as being uncertain.

Figure 1 Comparative efficiency profiles



Source: Statistics Canada.

Following Hulten and Wykoff (1981), we consider four common efficiency profiles, beginning with the one-hoss-shay.[2] Assets with one-hoss-shay efficiency profiles undergo no physical depreciation over the course of their productive lifecycle. They retain their full ability to produce goods and services, and generate a constant stream of in-period revenue up until the end of their service life. A second class of assets may be characterized by a concave-to-the-origin efficiency profile. In this case, the decline in efficiency is more pronounced in later periods of service life than in earlier periods. A common representation of this process uses a hyperbolic curve. The third example is provided by assets that exhibit a straight-line efficiency profile, wherein their productive capacity, and in-period revenues, decline in progressive linear increments over their lifecycle. The fourth example involves assets that exhibit a profile whose earnings stream declines at a constant geometric rate.

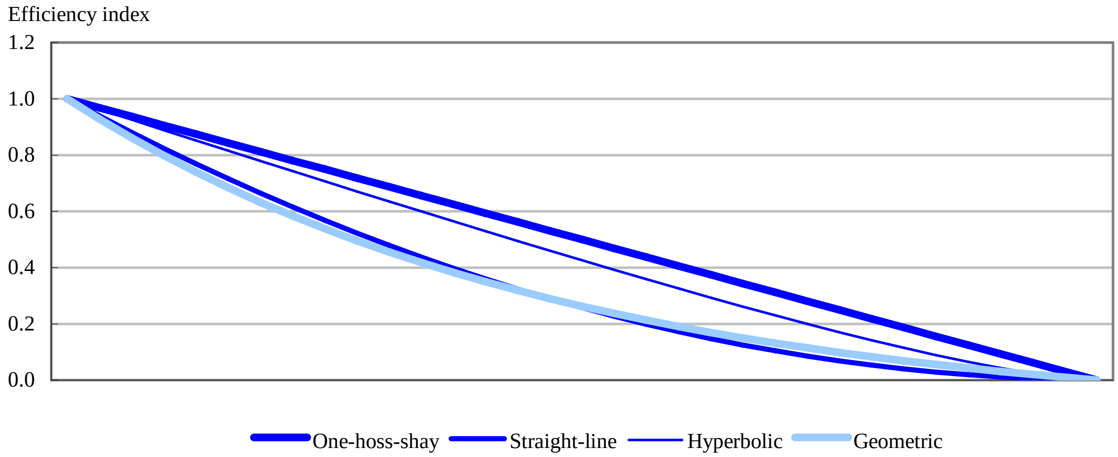
We now turn to consider the most commonly used economic depreciation profiles, defined as the decline in asset value (or asset price) associated with aging (Fraumeni, 1997). The asset value of an asset at any point in time should reflect the expected future earnings—the net present value of the future stream of earnings that is expected from owning the asset. The price decline that occurs each year in an asset's value reflects, in the first instance, the reduction in present value that occurs over a finite service life. Other things equal, an older asset has less opportunity to generate revenue than a younger asset—which reduces the economic value of the former. This decline in asset value will be accelerated if aging is accompanied by a loss of productive efficiency, as all capital assets that suffer wear and tear can be expected to return a lower stream of benefits in any single period. In Figure 2, we examine the patterns of economic depreciation that correspond to the efficiency profiles presented in Figure 1.[3]

The decline in present value is most clearly seen in the one-hoss-shay case. In the simplest of worlds where there is no discounting of future earnings streams, one-hoss-shay efficiency profiles will give rise to linear depreciation patterns, as older assets, while still generating the same in-period revenue as their younger counterparts, decline in value by a constant amount per period.[4] This "general non-equivalence" between asset efficiency and asset value over time is also apparent in the straight-line case. Linear efficiency profiles do not give rise to linear depreciation curves; rather, asset values in this case follow a more accelerated pattern with higher losses in value earlier in service life. Hyperbolic, straight line and geometric efficiency

2. Much of this comes directly from Hulten and Wykoff (1981).
3. These stylized relationships between asset efficiency and depreciation involve several simplifications - first, that service lives and efficiency patterns are known with certainty; second, that asset prices reflect the actualized value of its future stream of revenues where these revenues are a linear function of the capacity of the asset; and third, that there is no discounting of future returns.
4. We depict a linear depreciation profile here simply to illustrate the incremental decline in present value as the asset progresses through its service life. Note, however, that the depreciation curve corresponding to a one-hoss-shay efficiency profile will not be linear if: (i) the duration of service life is not known with certainty, or (ii) the value of the asset's productive capacity is discounted in future periods.

patterns give rise to a price-age profile that is convex to the origin.[5] We will return later to the way the depreciation curves of Figure 2 are mapped from Figure 1 efficiency profiles.

Figure 2 Corresponding depreciation profiles



Source: Statistics Canada.

These heuristic examples are worth stressing in view of Hulten and Wykoff's (1981: 90) observation that relationships between efficiency and depreciation represent "the most misunderstood relationship in all of depreciation theory." The requirement that national accountants adopt a consistent treatment of efficiency and economic depreciation has been voiced, more recently, by Jorgenson (1994). At issue is the extent to which rates of economic depreciation can be adequately used to proxy rates of physical replacement when making perpetual inventory estimates of capital stock.

The principal focus of this paper is on economic depreciation—the reduction in price or value associated with aging. Aging, however, is not equivalent to use, though we have tacitly treated it as such in the above examples. Asset values decline due to wear-and-tear and the reduction in present value as assets work their way through economic systems. As Fraumeni (1997) notes, changes in asset value are also driven by a continual process of "revaluation", that is, reductions in the value of older assets from period-to-period owing *inter alia* to increased obsolescence. There is an economic cost to holding older assets if new assets—assets that embody recent technological innovations—can lead to superior performance. Discussions of the methods used to estimate economic depreciation will often suggest that it is important to distinguish between the price effects of use and obsolescence, as it is not difficult to imagine circumstances in which the relative weight of the latter is a more significant determinant of overall price movements.

5. Once again, this efficiency-price relationship is conditional on several factors; see note 3. More importantly, the geometric efficiency frontier translates precisely into a geometric depreciation curve only in the case of an infinite lived asset. In the case of the geometric efficiency profile $(1-\delta)^y$, as depicted by Figure 1, the depreciation curve will be $\{(1-\delta)^y-(1-\delta)^T\}/\{1-(1-\delta)^T\}$ and indeed that, this expression collapses to the original geometric when T tends to infinity. When the asset has a fixed and finite life, the depreciation curve always reaches zero at the end of its service life while the efficiency profile will be truncated, and the depreciation curve is still convex but slightly more so than a geometric curve.

Personal computers may undergo relatively little physical depreciation over their service life, and yet they may experience large declines in resale value due to the rapid onset of obsolescence.

Herein, we treat aging and obsolescence as basic determinants of the same process—in that both effect changes in the price of an asset over its lifecycle.

2.2 Straight-line and geometric forms of depreciation

This section discusses how depreciation estimates are commonly derived when estimates are available of the length of life (T). It focuses on two specific forms of depreciation: straight-line and geometric. While much analytical interest rests with the latter, straight-line depreciation is a useful starting point, and is applied extensively in a national accounting framework. In this section, the length of life of an asset is treated as non-stochastic—as known with certainty.

The simple algebra of straight-line and geometric depreciation is outlined by Fraumeni (1997). We present much of her discussion below.

Straight-line patterns assume equal dollar value depreciation at all stages of an asset's lifecycle. Per-period depreciation for a dollar of investment takes the form

$$D = \frac{1}{T} \qquad (1)$$

where T is service life. Although the dollar loss is equal from period-to-period, the rate of depreciation—that is, the percent change in asset value from period-to-period—increases progressively over the course of an asset's service life. For a marginal dollar of investment, this rate is

$$\delta_i = \frac{1}{T-(i-1)}, \text{ for all periods } i = 1,...,T. \qquad (2)$$

Geometric depreciation represents the conceptual counterpoint to the straight line case. Geometric profiles hold the rate of depreciation, not the period-to-period dollar amount, fixed over the course of an asset's service life.[6] Geometric profiles are accelerated—with higher dollar depreciation in early periods—giving rise to the convex age-price profile depicted in Figure 1.

Per-period depreciation is defined as

$$D_i = \delta(1-\delta)^{(i-1)} \qquad (3)$$

where δ is the constant (age invariant) rate of depreciation.

The majority of empirical research on asset depreciation has concentrated on the geometric form. In early studies, geometric patterns were often assumed. Evidence that geometric rates are

6. For an overview of the geometric distribution, see Hastings and Peacock (1975).

generally appropriate for a wide range of asset types is found in Hulten and Wykoff (1981) and Koumanakos and Hwang (1988).[7]

In practice, geometric rates are analytically expedient for two reasons: (1) they can be estimated indirectly via accounting methods; and (2) their constant-rate property allows them to be used as a proxy for the replacement rate in standard perpetual inventory models of capital stock. We address the first of these points below.

Direct estimates of δ can be derived from information on resale prices or on the length of life of the asset (T). For many years, the latter method was the most common and T was determined from accounting information—often associated with tax laws. In the absence of sufficient price information, geometric rates can be calculated indirectly from estimates of the length of life (T) of an asset derived from the tax code as

$$\delta = \frac{DBR}{T} \tag{4}$$

where T is service life and DBR must be chosen and is referred to as the declining-balance rate. The value of the declining-balance rate determines, other things equal, the extent to which asset values erode more rapidly early in the lifecycle (Fraumeni, 1997). Higher values of the declining-balance rate bring about higher reductions in asset value earlier in service life, giving rise to more convex (i.e., accelerated) depreciation profiles.[8]

The rate of depreciation is calculated indirectly by Equation (4). When the estimate of T is based on *ex ante* expectations of service life, the depreciation rate can be described as *ex ante*. In Canada, service life estimates can be derived from the expectations of survey respondents regarding an asset's useful life. The Investment and Capital Stock Division captures in its annual investment survey, the expected length of life on all new investments that are reported to Statistics Canada.

There has been considerable debate over whether the assumptions embodied in the calculation of geometric rates are empirically appropriate. Some researchers have questioned whether the heavy losses in asset value that are often observed early in asset life are consistent with constant, geometric rates. It should be stressed that constant rates do not, in and of themselves, preclude highly accelerated depreciation profiles; rather, the issue is simply whether these rates are, on

7. For a survey of the empirical literature, see Fraumeni (1997); for a discussion of empirical methods, see Jorgenson (1994).

8. The concept of a finite service life requires adaptation in the geometric case. To see this, note that service life T is not finite, in the same sense that it can be considered finite in the straight-line case. Straight-line patterns depreciate at a constant dollar amount until the economic value of an asset is exhausted, that is, up until the point of retirement. In contrast, the geometric patterns given by Equations (3) and (4) are infinite in that some (progressively declining) portion of asset value continues to survive *after* service life T. This "surplus" or remaining asset value is not necessarily trivial. Consider a hypothetical asset which has a mean service life of 25 years. If we base our estimates of geometric depreciation on the double-declining-balance rate (DBR=2), 13.5% of asset value survives beyond the mean retirement age. This "infinite" characteristic of geometric forms has occasioned the use of truncation techniques that assume the remaining value of the asset is suddenly all lost at the discard point T. This leads to sudden and often large losses in value at T. This problem however exists only in a world of certainty and disappears in a world of uncertainty that takes into account the random nature of T.

net, sensible representations of the change in asset value in every period. A key aspect of this debate centers on choosing (by estimation or otherwise) an appropriate value for the declining-balance rate (DBR). Even if constant-rate, geometric age-price profiles are empirically justified, the choice of particular values for DBR and T is still at issue. If T is chosen from the tax code, the estimate thereof may differ from actual lives if the tax code does not use accurate length of lives—as it may deliberately do if it is trying to stimulate investment. If T is taken from a survey, it involves other problems. Firms are required, in advance, to predict how long an asset may last—and may error in a systematic way. Much concern also rests with the apparent ad hoc nature of the declining-balance rate DBR. While estimates of service life T often derive from expert sources, assumptions about the declining-balance rate are not always transparent. We elaborate at greater length on this issue below and argue that the choice of DBR need not be done arbitrarily—rather the choice can be informed by theory and information derived from *ex post* data on asset prices.

Double-declining-balance rates (DDBR)—which set the value of the DBR equal to 2—have been extensively used in practice. In their estimates of capital stock, Christensen and Jorgenson (1969) employ double-declining-balance rates to estimate rates of economic depreciation. Statistics Canada's productivity program has historically based its estimates of geometric depreciation on a double-declining rate. One advantage of the DDBR is that it provides a "conceptual bridge" back to the straight-line case, anchoring the midpoints of the depreciation schedules at an equivalent age point. Indeed, the average depreciation rate in the straight-line case will match the constant rate derived from a DBR of 2.

To see this, we can examine a simple measure of central tendency. Defining μ as the midpoint of the geometric curve (the expected life of a dollar invested in an asset), then

$$\mu = \frac{1}{\delta}, \tag{5}$$

or equivalently from Equation (4), when δ is chosen as DBR/T

$$\mu = \frac{T}{DBR}. \tag{6}$$

Now $\frac{T}{2}$ also represents the midpoint of the linear depreciation schedule (the point at which a dollar is half-way depreciated) of an asset whose length of life is T. Thus, if the DBR in the geometric formula is set equal to 2, the linear depreciation world, often used by accountants, can be brought into congruency with a geometric world—so that an average dollar in the geometric world lasts the same amount of time as it takes a dollar to lose half its' length of life, which, as the next section shows, is just the expected life of a dollar invested in an asset in the straight-line world.

Recent estimates of geometric depreciation used by the Bureau of Economic Analysis assume a lower value for the declining-balance rate for many individual assets (DBR=1.65 for machinery and equipment and 0.91 for structures). Based on the empirical research of Hulten and Wykoff (1981), these values will, other things equal, produce lower rates of geometric depreciation than the double-declining case.

The basis for the Hulten-Wykoff estimates of the DBR warrant some discussion here. In a study for the Office of Tax Analysis of the Department of the Treasury, the authors generate direct estimates of geometric depreciation for a large variety of assets, based on samples of used-asset prices, and then base subsequent estimates of δ (for assets for which no price information was available) on the geometric accounting method described by Equation (4), using arbitrary estimates of T developed from the tax code. This two-stage procedure enabled the authors to produce a set of depreciation estimates that was consistent with the asset classes used by the U.S. National Income and Product Accounts. To produce geometric rates of depreciation from Equation (4), Hulten and Wykoff calculated average values for the declining-balance rate (DBR) using their price-based estimates of δ and exogenous information on service life from the tax code. This yielded average DBR values of 1.65 for machinery and equipment and 0.91 for structures—average DBR values based on asset categories for which price information was directly available.[9] In cases where no price information on other assets was available, the authors then combined these estimates of DBR with asset-specific information on tax-code service life T to produce indirect estimates of δ.[10] The estimates of DBR so produced were only meant to be useful for filling in their data set, not to be used for alternate estimates of T, such as those which Statistics Canada's survey produces from direct questions to firms on their expected length of life.

One advantage of the data available to us is that we can ask whether *ex ante* depreciation rates that derive from a geometric accounting framework are consistent with the *ex post* rates produced by the econometric models developed here. We examine this in Section 6.2 by asking whether summary measures of asset life that are derived from econometric estimates of depreciation (which derive from price information collected over a 13-year period) are consistent with recent survey evidence on expected service life which can be used to estimate δ via Equation (4).

2.3 Efficiency and economic depreciation in a world of certainty

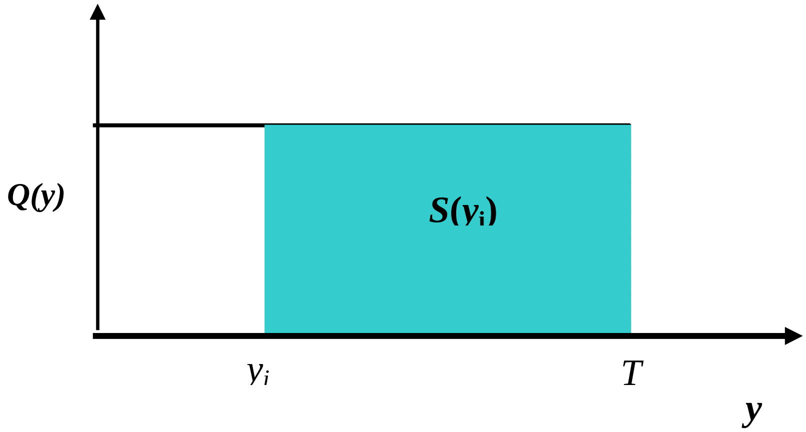
In the previous section, we presented a heuristic description of the manner in which depreciation rates can be derived from information on the service lives associated with assets. We depict convex depreciation rates from a geometric type and the linearly declining price curve that results from different approaches to the same phenomenon—but ones that can be reconciled. In reality, there may be less difference between the two than Figure 2 suggests when the stochastic nature of the asset discard process is taken into account. In this section, we model the relationships more formally because of the importance of understanding the role of uncertainty when developing the estimation framework that is used in this paper.

To start with, we pick only the one-hoss-shay case in which there is no reduction in the asset's capacity over the course of its productive life.

9. As Hulten and Wykoff (1981: 94) note, the asset categories for which they were able to calculate depreciation rates directly from price information represent a substantial share of total NIPA (National Income and Product Accounts) investment expenditures—42% of investment in non-residential structures and 55% of investment in producers' durable equipment.
10. For a useful discussion of the Hulten-Wykoff methodology, see Fraumeni (1997).

The relative value of the full stream of services to be yielded by the asset at time y_i is expressed by the ratio of the shaded area on the total rectangle defined by the length $0 - T$ where T is the length of life of the asset. This ratio declines linearly as y_i approaches T.[11]

Figure 3 One-hoss-shay efficiency profile



Source: Statistics Canada.

Let $Q(y)$ refer to the efficiency index for specific ages y. The variable y expresses the time at which an atom of value embodied in the asset is lost. $f(y)$ refers to the loss of value per unit of time. Use of the asset for one period exhausts the constant amount of value that the asset could potentially produce. We normalize over T so that $f(y)$ has the characteristic of a density function.

$$f(y) = \frac{Q(y)}{\int_0^T Q(y)\,dy} \text{ for } 0 < y < T,\ 0 \text{ elsewhere.} \tag{7}$$

From Figure 3, we can deduce that the $f(y)$ derived from a constant $Q(y)$ will be a uniform distribution between 0 and T. The loss of value will be spread equally over the asset's useful life.

We have:

$$f(y) = 1/T \text{ for } 0 < y < T,\ 0 \text{ elsewhere} \tag{8}$$

and the expectation will be provided by:

$$E(y) = \int_0^T y\, f(y)\,dy = \left. \frac{y^2}{2T} \right|_0^T = T/2\,. \tag{9}$$

11. We ignore the fact that this value should be discounted and treat the stream as the simple sum of future earnings. This issue is discussed in Appendix B.

The expected life of a dollar invested in the asset will be the half of the expected life of the asset itself.

Now the expected life of a dollar invested is just the average time over which a dollar of investment is lost. Its inverse is just the average rate of depreciation.[12] From Equation (9), it is therefore apparent that the average depreciation is just $2/T$ and the declining-balance rate (DBR) should be chosen as 2 in this instance, to provide an average rate of depreciation when T is known. More generally, the average rate of depreciation can always be calculated as the inverse of $E(y)$.

The average depreciation concept is required for the linear world because depreciation rates actually increase over time The average across time is required if we are to produce a summary statistic that will be employed in the standard perpetual inventory formula that assumes a constant rate of depreciation that is used to estimate capital stock from a stream of investment flows.

The cumulative density function (c.d.f.) of $f(y)$, denoted by $F(y)$, or the c.d.f. expresses the total proportion of initial value lost since the beginning of the asset's service life.

Consequently, economic depreciation can be expressed by 1 minus $F(y)$ which provides $S(y)$, the so-called survival function.

We have:

$$S(y) = 1 - \int f(y)dy = 1 - F(y) \; . \tag{10}$$

When the profile is constant, the economic depreciation is a linear decreasing function, as was shown in Figure 2.

The constant capacity profile is often modified to provide for a gradual reduction of capacity produced by an asset early in life with a rapid increase in that decline as the asset approaches its useful length of life *T*. This type of modification produces a concave capacity curve. One functional form that takes on a concave capacity profile and is used by the Bureau of Labor Statistics (BLS) is the hyperbolic function, which is written as

$$Q(y) = (T - y)/(T - \beta y) \tag{11}$$

where β is a shape parameter. β's upper limit is 1 which produces the case of constant capacity to the end of life *T*. For $0<\beta<1$, the capacity curve will be concave (see Figure 4). If $\beta = 0$, it becomes linear decreasing. For negative values of β, the capacity curve becomes convex.
The density of the hyperbolic capacity profile will be:

$$f(y) = \frac{(T - y)\beta^2}{(T - \beta y)T\left[(1-\beta)\ln(1-\beta) + \beta\right]} \text{ for } 0 < y < T, 0 \text{ elsewhere.} \tag{12}$$

12. This can be seen directly in the case of the geometric depreciation function where δ=1/E(y). It is shown to be more generally true in Tanguay (2005).

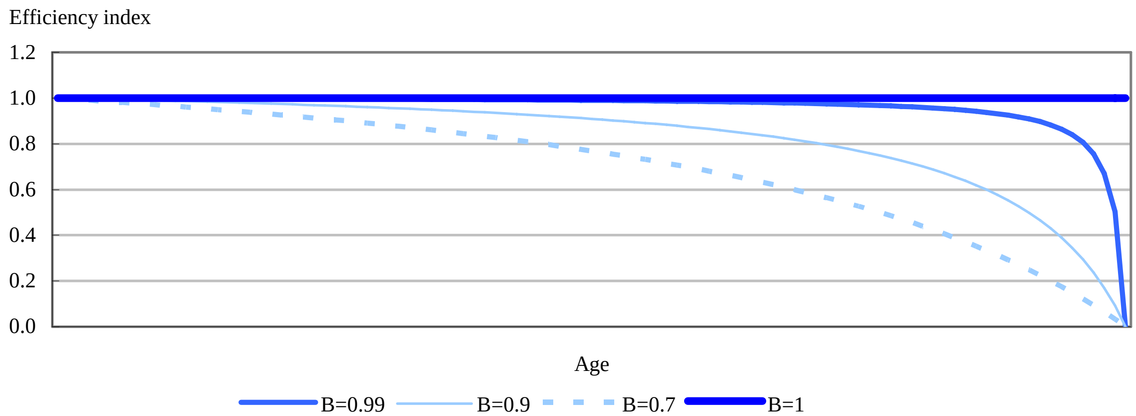
When β=1, $f(y)$ collapses to the density of a uniform distribution.

The c.d.f. of y, $F(y)$ will be:

$$F(y) = \frac{T(1-\beta)\ln(T-\beta y)+y\beta}{T\left[(1-\beta)\ln(1-\beta)+\beta\right]}. \tag{13}$$

As expected, when β=1, the expression collapses to the linear form $F(y)=y/T$. When β=0 the above expression is indeterminate, but it converges to a quadratic.

Figure 4 Hyperbolic capacity profiles



Source: Statistics Canada.

The expected life of one dollar of investment is:

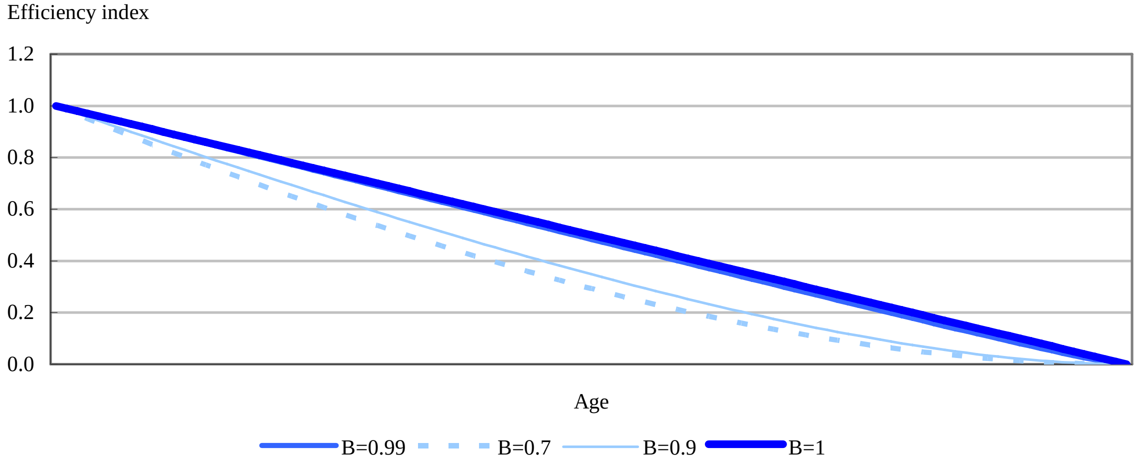
$$E(y) = T\left[\frac{1}{\beta} - \frac{\beta}{2\left\{(1-\beta)\ln(1-\beta)+\beta\right\}}\right] \tag{14}$$

from which the DBR can be computed by $T/E(y)$

$$DBR = \frac{\beta\left[(1-\beta)\ln(1-\beta)+\beta\right]}{\left[\left[(1-\beta)\ln(1-\beta)+\beta\right]-\frac{1}{2}\beta\right]}. \tag{15}$$

Depreciation patterns yielded by this survival function depend on the value of β. Figure 5 provides some examples of economic depreciation curves derived from various values of β. When $\beta < 1$, the depreciation curve is always convex.

Figure 5 Economic depreciation curves mapped by hyperbolic capacity profiles



Source: Statistics Canada.

In this paper, we make use of an alternate, more tractable functional form to represent a concave capacity profile, that is:

$$f(y) = \frac{k+1}{kT}\left[1 - \left(\frac{y}{T}\right)^k\right]. \tag{16}$$

The efficiency profile mapped by this function will be concave for any value of k varying from 1 (linear declining) to infinite (one-hoss-shay). The expectation of y will be:

$$E(y) = T\left[\frac{k+1}{2(k+2)}\right]. \tag{17}$$

This means that the DBR associated with Equation (17) is:

$$DBR = \left[\frac{2(k+2)}{k+1}\right]. \tag{18}$$

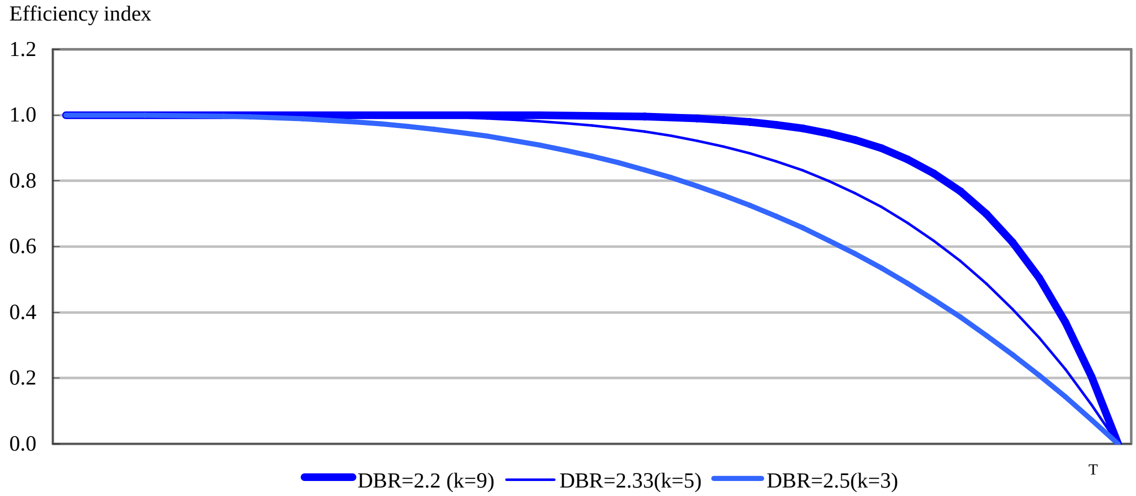
Equation (18) provides an easy way to build a mapping between the parameters of the capacity profile and the DBR. Its value will be between 2 and 3. This range also holds for any functional form as long as the underlying efficiency profile is concave.

The c.d.f. related to Equation (16) is:

$$F(y) = \frac{k+1}{kT}\left[y - \frac{y^{k+1}}{(k+1)T^k}\right]. \tag{19}$$

Different capacity profiles using this functional form and the DBR linked to them are presented in Figure 6.

Figure 6 Concave efficiency profiles and the declining-balance rates (DBR) related



Source: Statistics Canada.

2.4 Efficiency and economic depreciation in a world of uncertainty

In reality, the value of the time of discard *(T)* is not known with certainty because some assets will be retired before T and others will be retired after T. T should therefore be treated as random. In this section, we show that when we do so, the price profiles should follow a curve that is convex—even when the efficiency profile of an asset is constant.

We also note that as soon as we recognize that T is random, we become interested in the average depreciation concept because in a world of uncertainty, the rate of depreciation will be different for assets that have lasted different lengths of time. The asset that dies in its first year suffers a 100% depreciation rate. The asset that lasts many years before suddenly being discarded has a much lower depreciation rate. An average of all the experiences is required to provide a representative rate for use in the standard perpetual inventory formula that assumes a constant geometric rate and that is used by Statistics Canada to estimate capital stock from a stream of investment flows.

To start, we treat a population of assets as each having an efficiency profile coming from a one-hoss-shay, but being discarded at different times t. Time of discard is modeled as a random variable having a mean of T but having a variance around T. The distribution is usually taken to be asymmetric—skewed to the left—with more units being discarded before T than after. As we shall see, the assumption of randomness, along with a particular skewness, produces a price curve that is convex, as is the geometric function discussed above.

Let us refer to t as a specific realization of this process. T is not observed but t is. Given this feature, the density function described by $f(y)$ should be considered, instead, as the conditional density $f(y \mid T = t)$.

In order to simplify notation, we will now only refer to t and write the conditional density $f(y\,|\,t)$. It describes the loss of value for a specific duration.

We use Bayes' theorem to build the joint density of y and t denoted $f(t,y)$.

$$f(y) = \int_{-\infty}^{+\infty} f(t,y)\,dt = \int_{-\infty}^{+\infty} f\left(y\middle|t\right) f\left(t\right) dt\,. \tag{20}$$

A variant of this relation is provided by:[13]

$$E(y) = E[E(y\,|\,t)]\,. \tag{21}$$

Because the conditional expectation $E(y\,|\,t)$ depends only on t and not on y, the expectation of y can be derived without having to compute its density. We simply 'weigh' the conditional expectation of y using the density of t.

The following example is illustrative.

Suppose that $f(y\,|\,t)$ is provided, as in Equation (8), by $(1/t)$ and that $f(t)$ is Weibull with parameters λ and ρ.

$$f(t) = \lambda^{\rho} \rho t^{(\rho-1)} e^{-\left((\lambda t)^{\rho}\right)}. \tag{22}$$

A Weibull function is a commonly used functional form that captures distributions that are skewed and has the advantage that only two parameters are required for its specification. Its first two moments are simple functions of these parameters and are relatively easy to estimate. For now, we will suppose that ρ is greater than 1, which imposes an increasing hazard rate. This assumption corresponds to what we should normally expect for a survival model describing physical durations.

Given the fact that y, the duration of atoms of value embodied in a given asset, will always be lower than or equal to t, we can write the joint p.d.f. of t and y:

$$f(t,y) = \begin{cases} \lambda^{\rho} \rho t^{(\rho-2)} e^{-\left((\lambda t)^{\rho}\right)} & \text{for} \quad 0 \le y \le t \le +\infty, \\ 0, & \text{otherwise.} \end{cases} \tag{23}$$

To compute the marginal density of y, $f(y)$, we integrate this function over t between y and $+\infty$. This provides:[14]

$$f(y) = \int_{y}^{+\infty} \lambda^{\rho} \rho t^{(\rho-2)} e^{-\left((\lambda t)^{\rho}\right)} \partial t = \lambda \Gamma\left[1 - 1/\rho, (\lambda y)^{\rho}\right]. \tag{24}$$

Where $\Gamma\left[a,z\right]$ is the complementary incomplete gamma function.[15]

13. See Hogg and Craig (1995).
14. See Tanguay (2005).
15. Opus cited.

At $t = 0$, $f(y)$ collapses to $\lambda\,\Gamma(1\text{-}1/\rho)$.

The survival function, estimated at point s, is:

$$S(s) = \int_{s}^{+\infty} \lambda\Gamma\left[1-1/\rho,(\lambda y)^{\rho}\right] = e^{-\left((\lambda s)^{\rho}\right)} - \lambda s\,\Gamma\left[(\tfrac{1}{\rho}-1),(\lambda s)^{\rho}\right]. \quad (25)$$

It can be shown (Tanguay, 2005) that the expectation of y is:

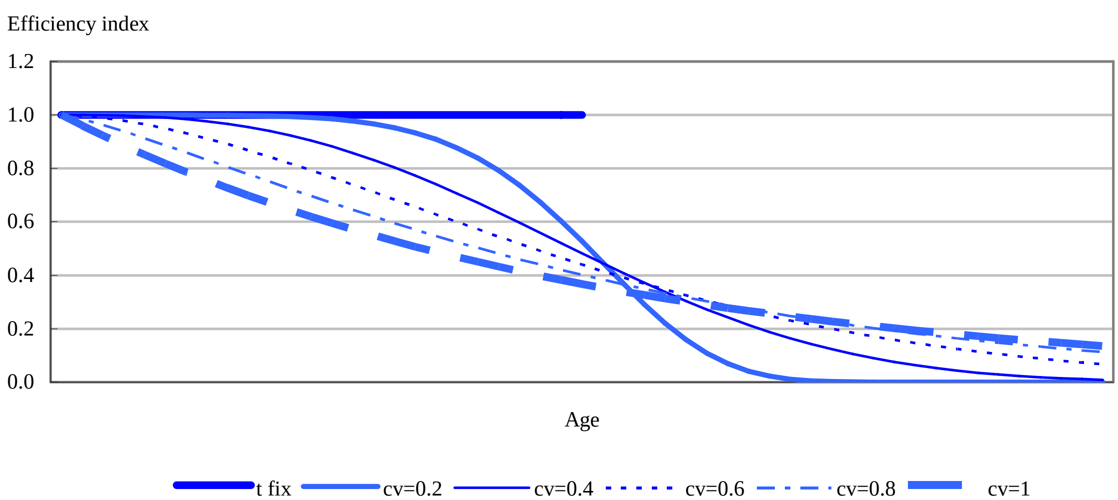
$$E(y) = \frac{1}{2}\frac{1}{\lambda}\Gamma\left[1+1/\rho\right],$$

which can be calculated with estimates of the Weibull parameters of ρ and λ.

Expected capacity is no longer constant in this model despite the fact that each asset is still assumed to follow a constant capacity. Figure 7 plots expected capacity over time for different Weibull distributions. Alternate distributions are defined in terms of the size of the coefficient of variation yielded by different values of ρ and λ. The larger is the coefficient of variation of the expected duration (a function of ρ and λ), the more convex the expected capacity.

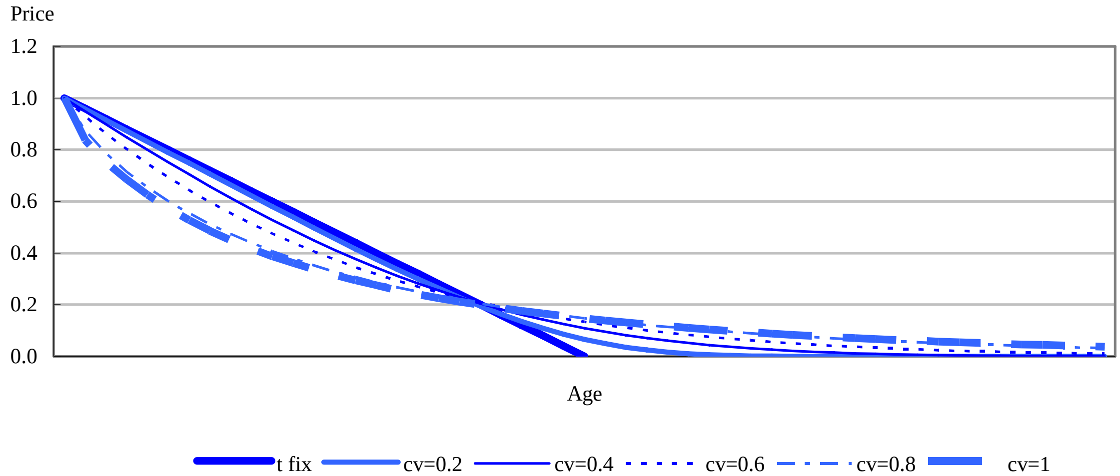
With expected capacity now a convex function of time, the expected value of the asset also follows a convex trajectory. Figure 8 depicts the economic depreciation that is generated by alternate Weibull functions.

Figure 7 Expected capacity under Weibull durations



Notes: cv= coefficient of variation.
Conditional capacity is constant.
Source: Statistics Canada.

Figure 8 Economic depreciation under Weibull durations



Notes: cv= coefficient of variation.
Conditional capacity is constant.
Source: Statistics Canada.

In the previous example, we have chosen a Weibull function to represent the discard process and a constant capacity function. Other choices are available. For example, if *f(t)* is gamma of parameter 2, $1/\lambda$, and if $f(y\,|\,t)$ is uniform, then *f(y)* will collapse to a simple exponential.

$$f(t) = \lambda^2 t e^{-\lambda t}$$
$$\Rightarrow f(t,y) = \lambda^2 e^{-\lambda t} \tag{26}$$

and using $U = \lambda t$ and $dt = dU/\lambda$, we have:

$$\int_{y}^{+\infty} \lambda^2 e^{-\lambda t} dt = \int_{\lambda y}^{+\infty} \lambda e^{-U} dU = \lambda \left. -e^{-U} \right|_{\lambda y}^{+\infty} = \lambda e^{-\lambda y}. \tag{27}$$

We can extend those results to situations where capacity profiles are concave and consequently, where the DBR exceeds the value of 2. If *f(t)* is Weibull and $f(y\,|\,t)$ is defined by Equation (16) we have:

$$f(y) = \frac{\lambda(k+1)}{k}\left\{\Gamma\left(1-1/\rho,(\lambda y)^{\rho} - (\lambda y)^{(\rho-1)} E_{\left(\frac{k+1}{\rho}\right)}\left[(\lambda y)^{\rho}\right]\right\} \tag{28}$$

where E_v is an exponential integral of order v. This function can be solved for integer values of v (Tanguay, 2005). The derived survival function will be:

$$S(s) = \int_{s}^{+\infty} f(y)dy = \frac{(k+1)}{k}\left\{ e^{-\left((\lambda s)^{\rho}\right)} - \lambda s\,\Gamma\left[(\tfrac{1}{\rho}-1),(\lambda s)^{\rho}\right] - \tfrac{1}{\rho} E_{\left(\frac{k+1+\rho}{\rho}\right)}\left[(\lambda s)^{\rho}\right]\right\}. \tag{29}$$

The parameters of Equation (29) can be estimated from real data using a linear interpolation between integer values of E_v.

3. Data source

The data used for this study come from Statistics Canada's annual Capital and Repair Expenditures Survey, an establishment-based survey undertaken by the Investment and Capital Stock Division, in which respondents are asked to report on their sales and disposals of fixed assets. This microdatabase provides the basic data used in this analysis.

Respondents to the survey operate in a broad mix of goods and services industries. The survey provides detailed information on asset type, gross book value, sale price, and age of each asset disposed of. The gross book value includes the original investment value plus the capitalized improvements incurred over the life of the asset. Deflators for investment assets were used in order to express all price information in real dollars. The machinery and equipment (M&E) assets cover close to 80% of the M&E stock; building assets cover 57% of total assets in this category but a much higher percentage when institutional buildings are removed. On the other hand, engineering assets account for only 13% capital stocks in this category.

The basic unit used in this paper is a survival ratio of the value of the original asset, observed at some age s. For all observations i in the sample, the survival ratio is calculated as:

$$R_i = \frac{SP_i}{GBV_i} \qquad (30)$$

where SP is the selling or discard price of the asset at age t, and GBV is its gross book value. Both numerator and denominator are expressed in constant dollars. R_i is thus the share of asset value that remains when the asset is sold at some reported age t. If the asset has been retired without a sale, R_i equals 0, corresponding to a zero selling price.

Studies that use market prices to estimate depreciation profiles must address issues of data reliability.[16] Traditionally, used-asset samples have not contained information on retirements, which, in turn, will severely bias the estimation of depreciation profiles. In their 1981 study, Hulten and Wykoff controlled for retirements by weighting their price data by assumed survival probabilities. Ad hoc adjustments of this sort are not required herein, as information on discards is included directly in our database.

In the previous paper that examined depreciation rates (Gellatly, Tanguay and Yan, 2002), the database employed used-asset prices that covered an 11-year reporting period (1985 to 1996). The sample used for the analysis was generated in several stages. The initial base sample had 53,802 observations on 240 separate assets. The new database that was used for this study added observations—from 1996 to 2001. Many of these observations, however, could not be used due to data limitations. We first removed observations that were missing information on age and/or initial book value. We then excluded institutional assets from the sample (e.g., schools, hospitals, universities).

16. Once again, for a general discussion of these issues, see Fraumeni (1997).

A filtering process was developed to exclude those observations where discards at a zero price did not make sense, which involved both a general routine and specific edits where anomalies stood out.[17] This reduced the dataset to 30,235 observations on 119 assets. We then restricted the analysis to individual assets with more than 75 observations. This further reduced the sample of usable observations to 32,048 observations on 49 assets.

The new database that was used for this study added observations—from 1996 to 2001. In addition, new editing procedures were employed. After the additional filtering process was applied to the new database, over 30,000 observations on used assets were available for estimation. This extended database consists of observations where the price of disposition is positive (13,718) and those where it was zero (18,330). Machinery and equipment assets accounted for 92% of the assets in the first case and 87% in the second case.

We also discovered that there was, in some cases, a concentration of non-zero prices near zero. We felt that many of those observations were, in reality, describing a scrap value, not the value of assets surviving. Therefore, we classified these as discards. To do so, we used a conservative lower bound of 0.06 below which a price ratio was considered to indicate a retirement.

We also encountered a problem with digits preference in the respondents, since there was a concentration of durations on rounding values like 5, 10, 15 and 20 years. This is a typical problem in many surveys and arises because some respondents tend to round the duration values they report. These patterns of age-rounding can affect the accuracy of estimates. Accordingly, we adopted the correction for digit preference that is described in Gellatly, Tanguay and Yan (2002).

While the database provides a unique opportunity to estimate depreciation curves with used-asset price data, it should be recognized that the validity of using used-asset prices depends on these prices reflecting the value of representative assets—that they do not represent "lemons".[18] If assets sold in resale markets are inferior to those that owners retain for production, the observed prices are biased downwards. The extent to which the lemon issue limits the utility of used-asset studies is dependent inter alia upon one's preconceptions about the ability of markets to solve information problems. For instance, the emergence of market intermediaries that provide used-asset information to prospective buyers will reduce the severity of these information asymmetries. In addition, the existence of different market segments, corresponding to different quality types, also reduces the impact of the lemon problem. Herein, we take the view that used-asset prices can tell us much about economic depreciation. We should stress that the edit strategy eliminates some of the more apparent lemons—observations with extremely low resale values, relative to like assets, early in their service life.

Another important problem is that for many assets, there is no effective second-hand price market. The specificity of some investment projects, their spatial immobility or the transaction cost that would be involved in their sale may make them useful only in their current use. In those situations, firms will probably value the price of keeping the assets in production differently from what an outsider would agree to pay for it. When there are transactions in the resale market

17. See Appendix A for details.
18. See Akerlof (1970).

for this kind of asset, the prices may reflect distress values and not ongoing business value.[19] Consequently, economic depreciation cannot be estimated exclusively by resale prices.

In order to take into account potential problems with the use of used prices, we limit our estimations to the assets (mainly machinery and equipment) where the resale market is reasonably active. For example, in engineering construction, less than 40% of the observations had positive prices and of those, about the half had a price ratio lower than 6%. Consequently, we removed engineering construction from our estimation procedure. We kept only a few classes for buildings, where there were a reasonable number of transactions—but we note that we might expect our econometric framework to do less well here. The observations on which we focus consist primarily of assets classified as machinery and equipment (about 31,000). The data allow us to estimate depreciation rates directly for 22 major asset categories out of the 155 assets tracked by Statistics Canada for its investment program.

Finally, concerns over representativeness often come to the fore when results are based on small samples. Hulten and Wykoff (1981) and Koumanakos and Hwang (1988) notwithstanding, much of empirical work on asset depreciation has been based on small samples for limited numbers of assets. Herein, our database confers certain advantages, as we are able to amass a large and diverse set of price information based on a comprehensive annual investment survey taken by the national statistical agency. The mean number of observations is 665, the minimum is 77 and the maximum is 1,400 per asset.

We provide the characteristics of the final sample in Table 1.

19. Cases of bankruptcies or mergers are examples of those particular situations. In the first case, assets are likely to be underestimated while in the second, they may (but not always) be overestimated.

Table 1 Data sample

Asset code	Description	Asset prices	Asset prices	Expected life
		Number of observations original	Number of observations extended	Number of observations
1001	Plants for manufacturing	791	1,016	4,579
1006	Warehouses, refrigerated storage, freight terminals	268	348	1,634
1008	Maintenance garages, workshops, equipment storage facilities	151	186	1,076
1013	Office buildings	626	774	4,138
1016	Shopping centres, plazas, malls, stores	202	362	1,375
1099	Other industrial and commercial	168	195	753
3002	Telephone and cablevision lines, underground and marine cables	162	206	551
3003	Communication towers, antennae, earth stations	128	154	524
6001	Office furniture, furnishing (e.g., desks, chairs)	2,651	3,345	12,611
6002	Computers, associated hardware and word processors	2,484	2,930	13,438
6003	Non-office furniture, furnishings and fixtures (e.g., recreational equipment, etc.)	875	883	13,438
6004	Scientific, professional and medical devices	871	570	3,673
6005	Heating, electrical, plumbing, air conditioning and refrigeration equipment	365	406	2,564
6006	Pollution abatement and control equipment	124	116	697
6007	Safety and security equipment (including firearms)	118	88	978
6009	Motors, generators, transformers, turbines, compressors and pumps of all types	571	693	2,587
6010	Heavy construction equipment (e.g., loading, hauling, mixing, paving, grating)	596	627	1,175
6011	Tractors and other field equipment (truck tractors - see 6203)	459	477	753
6012	Capitalized tooling and other tools (hand, power, industrial)	707	831	3,169
6013	Drilling and blasting equipment	127	153	295
6201	Automobiles and major replacement parts	2,554	2,214	3,881
6202	Buses (all types) and major replacement parts	204	234	420
6203	Trucks, vans, truck tractors, truck trailers and major replacement parts	3,086	4,100	5,868
6205	Locomotives, rolling stock, street and subway cars, other rapid transit and major parts	207	247	397
6206	Ships and boats and major replacement parts	104	123	299
6207	Aircraft, helicopters, aircraft engines and other major replacement parts	223	288	792
6299	Other transportation equipment	209	229	517
6401	Computerized material handling equipment		138	721
6402	Computer-assisted process for production process	539	1,024	4,381
6403	Computer-assisted process for communication and related equipment	267	586	3,747
6601	Non-computer assisted process for material handling	1,001	1,406	3,396
6602	Non-computer assisted process for production process	2,918	3,688	7,125
6603	Non-computer assisted process for communication and related equipment	595	669	2,384
8999	Other machinery and equipment (not specified elsewhere)	745	1,090	4,596

Source: Statistics Canada.

4. Estimation framework

As Jorgenson (1994: 1) observes, “the challenge facing economic statisticians” engaged in research on economic depreciation “is to employ asset-price information effectively.” Econometric studies that use vintage prices to estimate depreciation curves build on the pioneering work of Hall (1971) and Hulten and Wykoff (1981). Hall introduced an analysis of variance model in which prices were estimated as a function of age- and time-specific dummies. A major contribution of Hall’s model is the estimation of non-linear time and age effects.[20] Hulten and Wykoff extended econometric (and non-linear) research on depreciation schedules by utilizing a Box-Cox model which tests for the appropriateness of various functional forms (rectangular, straight-line, and geometric). Koumanakos and Hwang (1988) applied this Box-Cox approach to their analysis of Canadian assets using 1987 data on used-asset prices.

In what follows, we develop three different specifications—referred to as METHOD1, METHOD2, and METHOD3—to estimate both a discard function to obtain an estimate of the length of life of an asset and an age-price profile to obtain a depreciation rate and an estimate of the declining-balance rate (DBR). The previous sections have shown that the price-age profile that we are interested in estimating will take on some type of convex form. There are a number of candidates that can be chosen for this exercise. In our estimates, we chose three different forms—an Exponential, a Weibull and the general form as outlined in Equation (29). All are arbitrary. The first two are related to one another. Extensively used in duration analysis, the Weibull distribution is a flexible parametric form, characterized by two parameters, that allows for variable, age-variant rates of depreciation, but can be restricted to produce constant (exponential) rates that are directly comparable to the geometric rates commonly used in depreciation accounting. The third form was chosen, not by simply following the tradition that uses a Weibull, but by asking what the form would look like if there was a Weibull discard function and a general concave efficiency profile. The derived equation that characterizes the resulting age-price profile requires us to estimate three parameters.

Despite these differences, it should be acknowledged that the different functional forms chosen may not yield significant differences in the variable being estimated here. It is important to remember that each approach is being used to derive the *average* depreciation rate of an asset as outlined in the previous sections. For our purposes, we want to know what happens, on average, to depreciation over the length of life of an asset, not how the depreciation rate changes across the entire lifetime of an asset. And as we show below, the estimate of the average depreciation rate produced below depends less on the functional form chosen and more on making sure that the data used are representative of the entire population of asset transactions.

20. A more thorough review of the empirical work noted here—particularly Hall (1971) and Hulten and Wykoff (1981)—is found in Jorgenson (1994). Our discussion here is by no means exhaustive. For a more comprehensive review, see Jorgenson (1994) and Fraumeni (1997).

4.1 Survival model (METHOD1)

We begin by considering asset valuation within the standard maximum-likelihood framework.

Let y define a dummy variable describing the two possible life states for a given asset, and let

$y = 1$ when the asset is dead or retired (its sale value equals zero)
$y = 0$ if otherwise.

The likelihood of an observation $\ell(t)$ is

$$\ell(t) = f(t)^{y} S(t)^{(1-y)} \tag{31}$$

where $f(t)$ is the density function, and $S(t)$ the survival function.[21]

Equation (31) is best applied to situations in which the event being modelled can be described using binary life states. (e.g., "alive" or "dead"). If the asset is "dead", the likelihood function reduces to the density function, and gives the probability of death at age t. If the asset is still "alive", the likelihood reduces to the survival function, and gives the probability that it survives until t. The log-likelihood of observing a sample of n observations then takes the form

$$\ln L = \sum_{i=1}^{n} \left[y_i \ln f(t_i) + (1 - y_i) \ln S(t_i) \right]. \tag{32}$$

We now modify Equation (31) based on the set of observed survival ratios R_i (defined previously by Equation [30]). Each individual atom of value has its own duration, and R_i expresses the proportion of them that survives at some age t while $1 - R_i$ is the proportion lost. Each individual asset is therefore considered as a specific cohort of values. The log-likelihood of a sample of n observations (cohorts) becomes

$$\ln L = \sum_{i=1}^{n} \left[(1 - R_i) \ln f(t_i) + R_i \ln S(t_i) \right]. \tag{33}$$

The log-likelihood formulation given by Equation (33) has an intuitive interpretation. R_i, the price ratio, represents the amount of asset value that survives to some age t, multiplied by a corresponding survival probability $S(t_i)$, while $1 - R_i$ represents the amount of value lost, multiplied by its failure probability $f(t_i)$.

While well suited to many survival applications, Equation (33) needs to be modified in order to produce estimates of economic depreciation. The use of the standard density formulation $f(t_i)$ assumes that asset values remain unchanged in all periods prior to being sold or retired. Embedded, then, in Equation (33) are profiles that are conceptually similar to a "one-hoss-

21. This is consistent with the standard model of survival. See for example, Cox and Oakes (1984), and Nelson (1982).

shay”—with asset values remaining at their maximum survival ratio prior to some age period (the point of transaction t) at which some partial or total loss in value is observed. Since this is too restrictive an assumption, we modify Equation (32) to adjust for continuous depreciation by replacing the density term $f(t_i)$ with the cumulative density $F(t_i)$. While the density term $f(t_i)$ assumes that the loss in asset value occurs at t, the cumulative density $F(t_i)$ assumes that reductions in value occur *before* time t.

The estimating equation becomes

$$\ln L = \sum_{i=1}^{n} \left[(1 - R_i)\ln F(t_i) + R_i \ln S(t_i)\right] \tag{34}$$

where $F(t_i)$ is the probability that asset values will decline at some point prior to t.[22] To estimate the above model using the Weibull distribution, we express the cumulative density $F(t_i)$ and survivor functions $S(t_i)$ as

$$S(t) = \exp[-(\lambda t)^{\rho}] \tag{35}$$

$$F(t) = 1 - \exp[-(\lambda t)^{\rho}]. \tag{36}$$

Restricting the parameter ρ to a value of one will produce the exponential or continuous geometric version of the model with the survivor and cumulative density functions

$$S(t) = \exp(-\lambda t) \tag{37}$$

$$F(t) = 1 - \exp(-\lambda t). \tag{38}$$

We use the exponential version of the Weibull since our previous results (Gellatly et al., 2002) found that the exponential could not be rejected for a large number of assets.

Estimation of Equation (34) based on individual survival ratios R_i assumes that depreciation schedules are not correlated with the size—or dollar value—of the asset. To account for dollar value differences across observations, we weight each observation by its share of total asset value, multiplied by the number of observations in the asset sample.[23] Denoting this weight as w_i, we can rewrite Equation (34) as

$$\ln L = \sum_{i=1}^{n} w_i \left[(1 - R_i)\ln F(t_i) + R_i \ln S(t_i)\right]. \tag{39}$$

---

22. This is similar to binary response models where the level of response (time) is observation specific. Our formulation resembles one of the prototypes listed by Lagakos (1979)—in which observations share a common survival distribution, but different censoring experiences. In our framework, the likelihood function is both left- and right-censored, and the usual indicator variable y is replaced by a survival ratio R_i.

23. Accordingly, the sum of the weights is equal to the total number of observations. No artificial degrees of freedom are created.

4.2 Two-step technique (METHOD2)

The second estimator that will be examined employs the used-asset price data and corrects it for selection bias using a two-step procedure. This two-step approach has been used, among others, by Hulten and Wykoff.[24] In their path-breaking estimates, they only had price data on used assets and little in the way of information on the discard pattern. That is, they lacked information on the actual discards that were not being observed in used-asset markets that only collected price data for transactions that yielded positive values. In the absence of these data, they made assumptions about the mean length of life and the distribution of discards around this point in order to adjust downward the positive prices that were observed in order to average in the missing observations on assets that were discarded at a zero price.

Our database allows us to estimate directly the discard process. The modelling framework adopted for this consists of directly estimating the age-price profiles using data on used-asset prices, adjusting the estimates for the censored sample bias using a retirement distribution.[25] Contrary to most studies, which calibrate a retirement distribution around a mean service life, retirement probabilities in this study are directly estimated using information on retirement (that is, transactions characterized by zero prices) and sales of used assets. All the observations (both positive and zeros) are used to estimate the actual discard function and then this is used to correct the estimators for a proportion of discards at each point of time.[26]

This means that an assumption is required about the discard or retirement pattern, the survival function of t. For the present estimates, it is assumed that the retirement distributions follow a Weibull specification. The cumulative *(D)* and density *(f)* probability functions for retirement are respectively:

$$D(t;\lambda,\rho) = 1 - Sv(t;\lambda,\rho) = 1 - \exp\left[-(ts)^{\rho}\right] \tag{40}$$

$$f(t;\lambda,\rho) = \lambda\rho(\lambda t)^{\rho-1}\exp\left[-(ts)^{\rho}\right]. \tag{41}$$

The parameters that need to be estimated are the scale parameter, λ, and the shape parameter, ρ of the Weibull distribution.

To start, we let c be a binary variable that takes the value 1 for complete durations, 0 otherwise. The log-likelihood function becomes:

$$l_t = c\log\left[f(t;\boldsymbol{\theta})\right] + (1-c)\log\left[S(t;\boldsymbol{\theta})\right] \tag{42}$$

where $\boldsymbol{\theta}$ represents the parameters to be estimated.

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24. Hulten and Wykoff (1981), opus cited.
25. It is worth noting that this approach differs substantially from the two-step Model developed among others, by Heckman. The correction is introduced here directly on the dependant variable instead of being added to the explanatory variables as in the traditional Heckman approach. That facilitates linear transformation that would otherwise be difficult to accomplish.
26. We also experimented with a version that only used the discard points (the zeros) but discarded this because it involves clear censoring biases that have long been established in the econometrics literature.

In the case of a Weibull specification, this becomes:

$$l_t = c\left[\rho \log(\lambda) + \log(\rho) + (\rho - 1)\log(t)\right] - (\lambda t)^{\rho} . \tag{43}$$

The parameters of the above equation are estimated using a maximum likelihood estimator (ML). The ML estimates of λ and ρ are used to adjust the original price ratios R via Equation (44):

$$R_a = R \exp\left(-\left(\hat{\lambda} t\right)\hat{\rho}\right). \tag{44}$$

In the second step, we estimate the price-age model itself. Here, R_a becomes the dependant variable. We use a second Weibull specification to represent the age-time profile. In order to avoid confusion with the formulation used for the discard process, we replace the parameters λ and ρ of this second Weibull by δ and α. Taking the natural logarithm of the Weibull survivor function and multiplying by (-1) yields:

$$-\log(R_a) = (\delta t)^{\alpha} . \tag{45}$$

Transforming this into linear form, we have

$$\log[-\log(R_a)] = \alpha[\log\delta + \log t] + u . \tag{46}$$

Accordingly, a regression of the form

$$y = a + bt + u \tag{47}$$

will yield estimates of the Weibull parameters where

$$\delta = \exp(a/\alpha) \tag{48}$$

and

$$\alpha = b \tag{49}$$[27]

and u is the error term.

The parameters of this model can be estimated with the least squares technique. *GLS* or *FGLS*[28] may be required because the error term is typically heteroscedastic, the variance of $\log(R_a)$ decreasing in the space of $\log(t)$.

Once again, we can generate an exponential (constant-rate) variant of this model simply by restricting the value of the Weibull parameter α.[29] When α equals unity, the rate of depreciation occurs at a constant rate—defined by the exponential hazard rate δ. This rate represents the linkage between the survival framework that is used here and other geometric accounting methods described by Equation (34), as the exponential distribution is simply the continuous-time version of the geometric.

27. For a useful discussion, see Lawless (1982).
28. Generalised least squares or feasible generalised least squares. See Davidson and MacKinnon (1993) for a description.
29. Gellatly, Tanguay and Yan (2002) test whether this restriction is justified.

The observations for the discard function are weighted by the gross book value of the asset. The weights serve as proxy for quantities, which are measured by the gross book value (*GBV*) in constant dollars. These weights are necessary to account for the consolidated reporting of the Capital and Repair Expenditures Survey (several transactions may be reported as a single response) and for the fact that some assets have more capital embedded in them (example: a two-floor building versus a twenty-floor building).

We estimate the discard function using a maximum likelihood technique that takes into account the digit preference problem found in the database. The existence of digit preferences means that the independent variable (time) is measured with error. We deal with this problem by substituting a new variable for age where a digit preference problem was identified.[30] Second, since the dependent variable is bounded by 0 and 1, we expect the error term to be heteroscedastic and we use a feasible generalised least squares technique.

The main advantage of the two-step approach over that previously used is that it makes use of an estimate of the discard process—rather than the actual observations provided by the sampling procedure of the survey. This is an advantage to the extent that actual discards observed are not fully representative of the population. It must be recognized that the survey in its present form was not designed to obtain the type of representative sample of used asset prices that is required to estimate an average depreciation rate. Although it clearly provides a large number of discards, it may not provide the coverage in each cell that is required for estimation (in our case, by age of asset). Specification and estimation of the discard model provides an alternate method to using actual observations. However, it must be recognized that this technique may introduce a source of error if the discard process is not really Weibull as is assumed by the estimation process. More importantly, this approach ignores the link that exists between the discard model, the capacity profiles and the depreciation curves. As shown previously, a Weibull discard function is not necessarily consistent with a Weibull price profile. In fact, the Weibull discard function along with several common efficiency profiles does not yield a Weibull survival curve—the curve used in our two-step procedure.

4.3 Simultaneous technique (METHOD3)

The third approach that is adopted here involves the simultaneous estimation of the discard and the survival function. There are several potential advantages of doing so. It has been demonstrated that the shape of the survival density function will depend on the shape of both the discard function and the efficiency function and that those shapes are likely to be different. A simultaneous framework will force the estimators to respect the consistencies between the two processes generating t and y given that those processes are related. This consistency can be imposed even in presence of specification error, when we do not know the exact form of the discard model. This simultaneous framework is the logical methodological counterpart of the joint density approach. It does not require more working assumptions than the two-step method,

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30. This is done using a drop into a uniform distribution centered on the age and bounded according to the estimated magnitude of the rounding provided by the first step (i.e., for 10, the age is replaced by a drop into a uniform distribution centered on 10 and bounded between $10-ma, 10+ma$ where ma is the estimated rounding parameter).

except that the links between the two processes (discard and economic depreciation) are made explicit.

For example, if we observe realizations of the random variable t and empirical survival functions of y, the system could take the form:

$$(i) \quad l_t = f(t;\boldsymbol{\theta}) \tag{50}$$

$$(ii) \quad l_y = S\left(y;\boldsymbol{\theta},\eta\right) \tag{51}$$

where l_t and l_y stand respectively for the likelihood functions of t and y, θ for a vector of parameters common to both functions and η, the parameter defining the shape of the capacity profile, which is specific to l_y.

The fact that some parameters are shared by the two equations force the consistencies mentioned above. The first equation expresses the physical duration t while the second corresponds to survival of y which determines the resale price of used assets. When the price is zero, the information is complete in terms of duration but left censored in terms of value. When the price is non zero, the data are right censored in terms of duration but provide more information on $S(y)$. The simultaneous estimation framework exploits the complementarities between the information on y and t.

The specification used to estimate the discard function is Weibull and is provided by Equation (41).

We used two alternate assumptions about the nature of the efficiency frontier. In the first case, we assume that it has a uniform distribution (a one-hoss-shay efficiency profile).

In this case, the survival curve of the value is defined in Equation (25) and the log likelihood function takes the form:

$$ly = W(1-c)\left[\left(1-R_a\right)\log\left[1-e^{-\left((\lambda s)^{\rho}\right)}+\lambda s\Gamma\left[1-\tfrac{1}{\rho},(\lambda s)^{\rho}\right]\right]+R_a\log\left[e^{-\left((\lambda s)^{\rho}\right)}-\lambda s\Gamma\left[1-\tfrac{1}{\rho},(\lambda s)^{\rho}\right]\right]\right] \tag{52}$$

W is the weight. The second variant assumes a concave efficiency curve which was derived in Equation (28). The log of the likelihood function is:

$$ly = W(1-c)\begin{bmatrix}\left(1-R_a\right)\log\left[1-e^{-\left((\lambda s)^{\rho}\right)}+\lambda s\Gamma\left[1-\tfrac{1}{\rho},(\lambda s)^{\rho}\right]+\tfrac{1}{\rho}\mathrm{E}_{\left(\frac{k+1+\rho}{\rho}\right)}\left[(\lambda s)^{\rho}\right]\right]+\\ R_a\log\left[e^{-\left((\lambda s)^{\rho}\right)}-\lambda s\Gamma\left[1-\tfrac{1}{\rho},(\lambda s)^{\rho}\right]-\tfrac{1}{\rho}\mathrm{E}_{\left(\frac{k+1+\rho}{\rho}\right)}\left[(\lambda s)^{\rho}\right]\right]\end{bmatrix}. \tag{53}$$

In estimating the simultaneous framework, we employ the same solution to the digit preference that we refer to in Section 4.1.

Finally, it should be noted that all of the relationships that have been outlined between the capacity profile, discard process, and the price-survival functions neglect the issue of discounting. The value of an asset depends not on the sum of all future earnings but on the sum of future discounted earnings.

All three techniques estimate depreciation rates by drawing a line through a locus of points that describe how the price of an asset declines over time. In all three cases, the rate at which this price declines is a function of the physical depreciation of the asset and the discount rate that is used to discount future earnings. All three techniques therefore embed the discount rate in the rate of decline that is estimated. Whether this is a significant problem depends on whether this seriously distorts the estimate of depreciation.

The attached note contained in Appendix B describes how the discount rate enters into determining the price-time profile that is being estimated. It shows the impact of excluding the discount rate (as all three methods do) in a world of both certainty and uncertainty. The note argues that while discounting 'contaminates' any estimate of depreciation that comes from estimates using time-age profiles, the impact of neglecting the discount rate is likely to be relatively small.

5. Evaluation of methods by Monte Carlo simulations

The three methods outlined in the previous section differ in several respects. The first approach does not model the discard process directly. The second approach models the discard process but does not establish the link between the discard process and the survival curve. It is a variant of an approach used by Hulten and Wykoff (1981) who did not have data on discards and had to incorporate assumptions about the nature of the discard process into their data to reduce selection bias using a two-step procedure. In our case, data exist on the discard process and we proceed in the two-step procedure only to provide a parallel with the earlier approach—though as we argue, there is no need to do so and a simultaneous estimation procedure offers certain advantages. The third method is more efficient in that it simultaneously uses information on both discards and price-time profiles to estimate the average depreciation rate. The third method is the most elegant in that it shows how specification of both the discard process and the efficiency curve of each asset yield the survival curve that is being estimated with prices of used assets.

It should be recalled that we are trying here to estimate the average depreciation rate. That is, ultimately we are not interested in asking how that depreciation rate changes over time. Rather, we intend to find the average and apply it to all assets regardless of age. Increasing complexity of functional forms is really only important if we were trying to ascertain how the depreciation rate changed over time. For some purposes, we might want to know just how depreciation rates vary over time, or how they have changed, and we might have more data to disentangle these various influences. But the estimates that are being generated here need only provide summary statistics adequate for use in the productivity program—where we want to find an average depreciation rate to use in a geometric function.

We are nevertheless interested in the precision of the underlying average depreciation estimates because, in the end, we are forced to choose one or a combination of several methods to produce the estimates of depreciation that will be used in the productivity program. In order to investigate the precision of each method, we make use of Monte Carlo simulations. In doing so, we generate a set of observations from specific functional forms for both the discard and the efficiency frontier and then employ each of the three methods outlined here to test how well each method estimates the underlying depreciation rate.

For this exercise, artificial data are produced using the Generalized Residuals approach[31]and a Weibull specification for *f(t).* For each run or replication, parameters ρ and λ are randomly chosen within their usual range of respectively [1-3] and [0.02-0.50]. The correlation between the parameters as estimated from real data is very weak (0.027). Therefore, the two parameters are chosen independently from one another. In each experiment, we use about 2,000 replications[32] and two types of procedures for drawing observations that exhibit a positive price when disposal occurs—random and non-random procedures. These are situations where the very fact that an asset was discarded for a positive price indicates that the asset has more potential for use and thus the observation is censored as far as information on the actual length of life is concerned. We choose two alternate techniques to generate the non-zero price observations because the data collection technique was not designed to produce a random sample of assets that are disposed of but that have a positive price. Assets observed with a non-zero price are captured only when an asset is sold—and this event may not be a random event. Therefore, the observations in our sample may not result from a random sampling of assets that are in production.[33]

Figures 9 to 13 contain the results for Monte Carlo simulations. Scatter plots of the results were built with the horizontal axis corresponding to the true parameter's values while the vertical axis corresponds to the estimates. Each point on the scatter plots corresponds to the estimates of one specific run. The main diagonal corresponds to situations where the true values of the parameters are reflected in the estimates while the dots around it correspond to estimates using different methods. For a given estimator, clusters of dots close to the diagonal depict a good performance. When the estimators are less efficient, their dispersion around the true value is wider. A concentration of clusters over or under the diagonal reveals upward or downward bias.

The models estimated are:

METHOD1—the maximum likelihood survival model;
METHOD2—the two-step approach with the discard component estimated from all spells (both the zero and positive-price observations);
METHOD3—Simultaneous approach.

31. For details on Generalised Residuals approach, see Lancaster (1985a and 1985b) and Appendix of Gellatly, Tanguay and Yan (2002).
32. This is in the range proposed by Davidson and Mackinnon (1993) for a simple estimator comparison.
33. For details about generating the random and non-random censoring, see Tanguay (2005).

We employed two versions of the two-step approach. The first one was estimated only from complete durations—that is, only the zero prices are used to estimate the length of life. In the second version of the two-step approach, the discard model takes into account the non-zero prices, which become survivors in terms of the duration model. The latter is always superior to the former and is the version reported here.

The first experiment consists of generating the joint density using a Weibull discard function and a constant efficiency profile. This formulation implicitly imposes a DBR of 2. We focus both on the results when the non-zero prices are derived from a random and a non-random process. The results are reported in Figure 9.

METHOD1 is the estimator that exhibits the poorest performance in terms of both the dispersion of the estimators and their departure from the real values. METHOD2—the two-step method—performs better.[34] METHOD3—the simultaneous approach—is superior to the other techniques. Its estimates are less biased and more efficient (the variance is smaller around the diagonal).

The second Monte Carlo run uses the same models but adds a measurement error to the price ratio. We introduce an error component ε_{S_y} which is heteroscedastic and generated by:

$$\varepsilon_{\mathrm{Sy}} \approx 0.2\,[N\,(S(y))(1 - S(y))]$$

where N is a draw from a standard normal distribution. The results are presented in Figure 10. Once again, METHOD3—the simultaneous estimate—produces the best results. It is robust to the type of measurement errors introduced here.

The third experiment tests whether specification error matters. It is presented in Figure 11. Again observations were generated by a Weibull duration combined with constant capacity profiles. However, the model estimated was defined as a Gamma of parameter 2 on the duration side which, with a constant capacity profile, produces an exponential depreciation curve (see Equation [27]). The same specification error—the use of an exponential for the survival function—was employed in all three cases.

In this experiment, simultaneous estimates no longer always outperform the two-step procedure. Under the assumptions of constant capacity, a random positive-price sampling procedure, and absence of measurement error, the two-step procedure becomes the preferable option. It is better to use the specification of Equations (26) and (27) even if they do not correspond to the actual functional forms. The gain in efficiency offsets misspecification error.

Unfortunately, the conditions that favour the two-step approach are limited. They are not valid when the positive-price sampling procedure is not random, as can be seen from the second line of Figure 12. In this case, the simultaneous estimate is superior to the two-step procedure. The second line of Figure 12 demonstrates that the two-step procedure loses its efficiency when we move away from a random positive-price sampling procedure. Referring back to Figures 10

34. For METHOD2, we use a version of the price curve that is derived from Equation (25). The estimated model is

$$\log\left(R_a\right) = \log\left[e^{-\left((\lambda s)^{\rho}\right)} - \lambda\, s\, \Gamma\left[\tfrac{1}{\rho}, \left(\lambda\, s\right)^{\rho}\right]\right] + u \ .$$

and 11, we also see the superiority of the simultaneous procedure when the positive-price sampling procedure is non-random—though the bias of the simultaneous procedure is opposite to that of the two-step procedure.

It should also be noted that when the capacity profile is constant, small sample properties of the two-step procedure may also be better than those for the simultaneous procedure, as we can see in Table 2, as long as the positive-price sampling procedure is random. For samples of 100, the two-step procedure performs better in versions 1, 3 and 5. However at 200 observations the two-step improves on the simultaneous only in version 3.

Table 2 Coefficient of variation of depreciation rate from Monte Carlo simulation

	Version 1	Version 2	Version 3	Version 4	Version 5	Version 6	Version 7	Version 8	Version 9
n=100*									
2. GTY**	0.494	1.216	0.916	2.374	0.539	1.317	…	…	…
3. Two steps	0.130	0.479	0.031	0.928	0.152	0.477	…	…	…
4. Simultaneous	0.201	0.102	0.129	0.164	0.202	0.106	…	…	…
n=200*									
2. GTY	0.488	0.964	1.152	1.631	0.521	1.010	3.023	1.959	1.906
3. Two steps	0.249	0.203	0.018	0.608	0.255	0.213	0.039	0.632	0.576
4. Simultaneous	0.112	0.088	0.110	0.153	0.111	0.089	0.007	0.075	0.077
5. Reweighted simultaneous	…	…	…	…	…	…	…	0.053	0.061
n=400*									
2. GTY	0.432	1.165	1.029	2.066	0.475	1.251	…	…	…
3. Two steps	0.185	0.254	0.024	0.874	0.204	0.255	…	…	…
4. Simultaneous	0.099	0.008	0.105	0.127	0.101	0.079	…	…	…

… not applicable

Notes: * n is the sample size

** GTY stands for the results obtained using Gellatly, Tanguay and Yan technique (METHOD1).

*** N is the number of replications

Version 1 Constant capacity profiles – No Measurement error – No Specification error – Censoring Random N=2000***
Version 2 Constant capacity profiles – No Measurement error – No Specification error – Censoring Not Random N=2000***
Version 3 Constant capacity profiles – No Measurement error – Specification error – Censoring Random N=2000***
Version 4 Constant capacity profiles – No Measurement error – Specification error – Censoring Not Random N=2000***
Version 5 Constant capacity profiles – Measurement error – No Specification error – Censoring Random N=2000***
Version 6 Constant capacity profiles – Measurement error – No Specification error – Censoring Not Random N=2000***
Version 7 Concave capacity profiles – No Measurement error – Specification error – Censoring Random N=2300***
Version 8 Concave capacity profiles – No Measurement error – Specification error – Censoring Not Random N=2300***
Version 9 Concave capacity profiles – Measurement error and Specification error – Censoring Not Random N=2300***

Source: Statistics Canada.

In Figure 13, we relax the assumption of constant capacity profiles and once more investigate the impact of misspecification. Durations were generated using as a joint density, the product of a Weibull duration and a concave capacity using Equation (12) and again, two types of sampling procedures for assets discarded at positive prices. Misspecification was introduced by using different models in estimation than in generation of the data.[35]

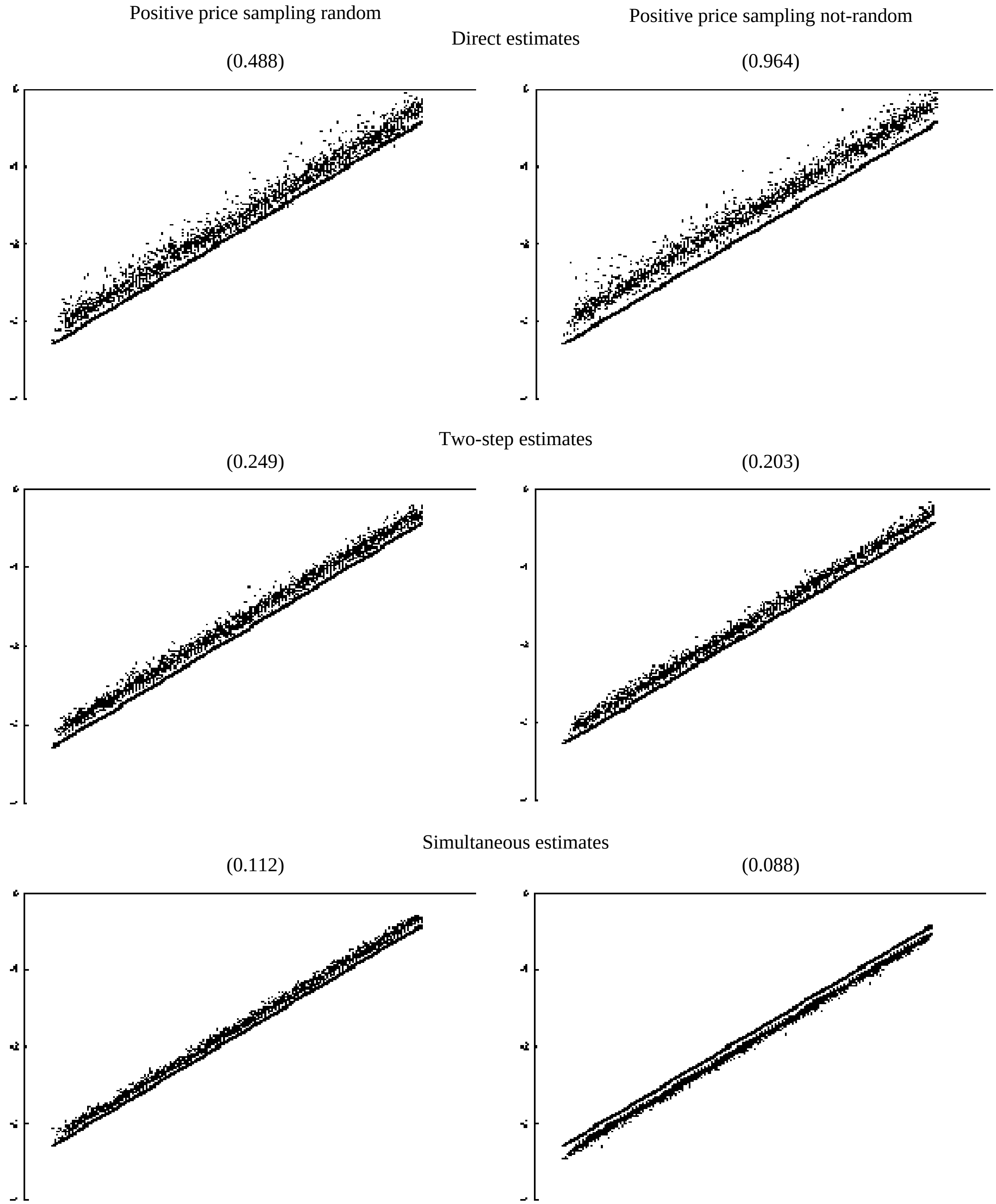
Once again, when the positive-price sampling procedure is random, the simultaneous estimator is the most efficient estimator. When positive-price sampling is non-random, the simultaneous technique is less biased and more efficient—but its efficiency is less than when positive-price sampling occurs in a random fashion.

The impact of a non-random pattern of positive-price sampling was disquieting and led us to develop a way to compensate for the loss of efficiency that results from this feature.[36] When we impose this reweighting and reestimate the depreciation rate, we improve the efficiency of our estimates substantially (Figure 13 and Table 2, line 5).

35. The Weibull specification was used for METHOD1 and METHOD2 and Equation (53) for METHOD3.

36. The method is described in Appendix B.

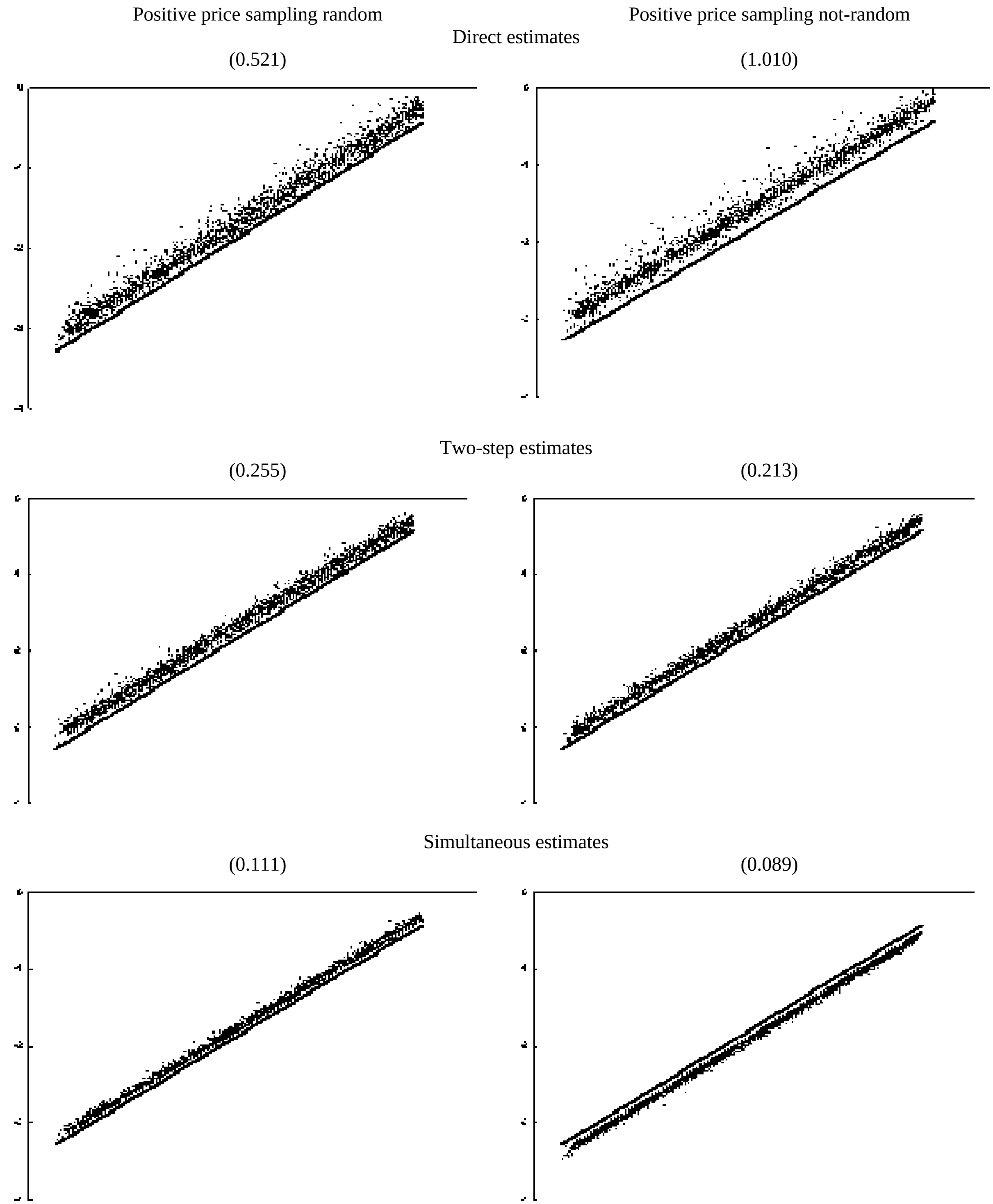
Figure 9 Log of depreciation rate when *f*(*t*) is Weibull and capacity profiles are constant



Notes: No specification error – No measurement error.
N=2000, n=200, (c.v.).
N is the number of replications.
n is the sample size.
c.v. = coefficient of variation.

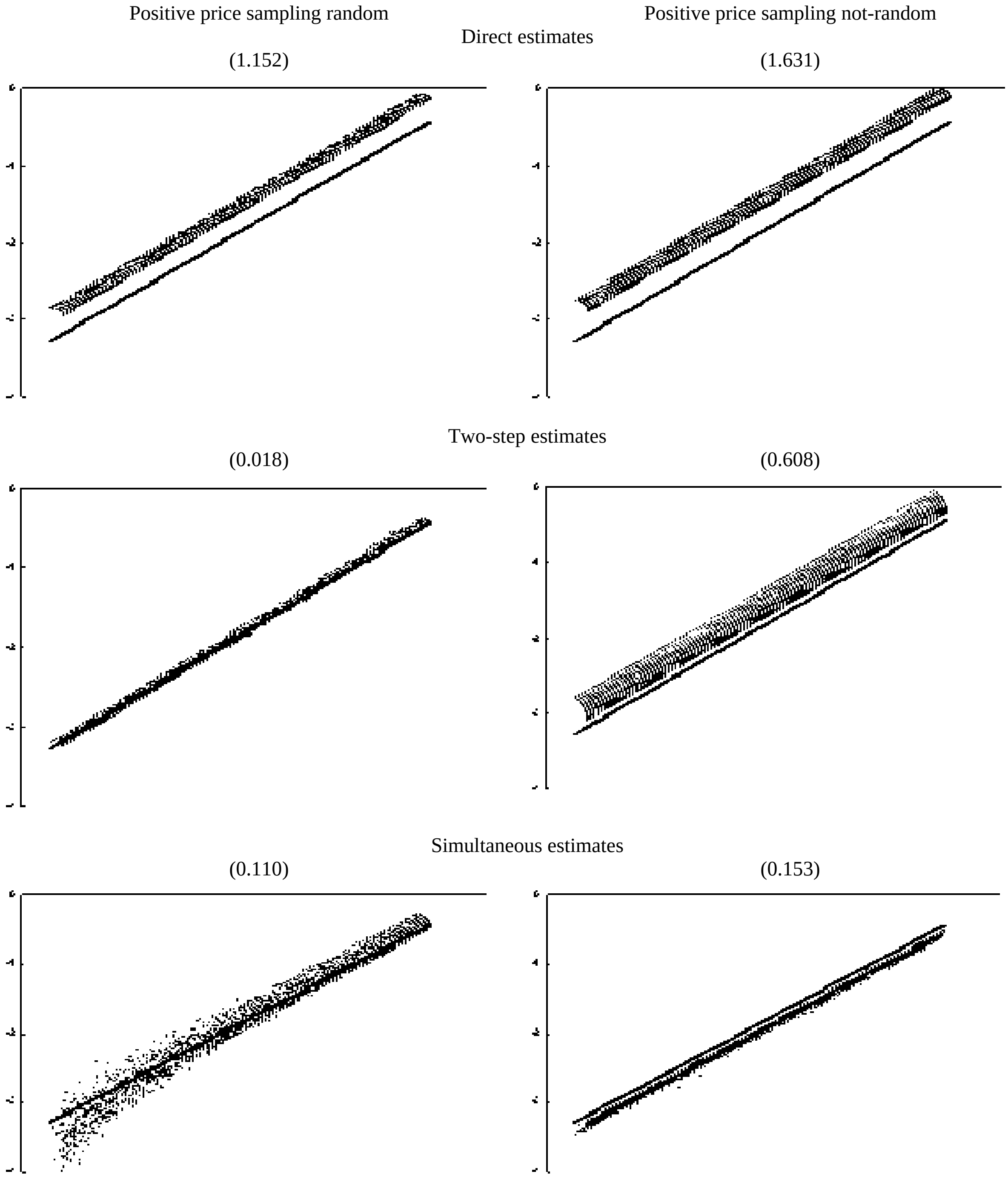
Source: Statistics Canada.

Figure 10 Log of depreciation rate when *f*(*t*) is Weibull and capacity profiles are constant



Notes: No specification error – Measurement error on *S(y)*.
N=2000, n=200, (c.v.).
N is the number of replications.
n is the sample size.
c.v. = coefficient of variation.
Source: Statistics Canada.

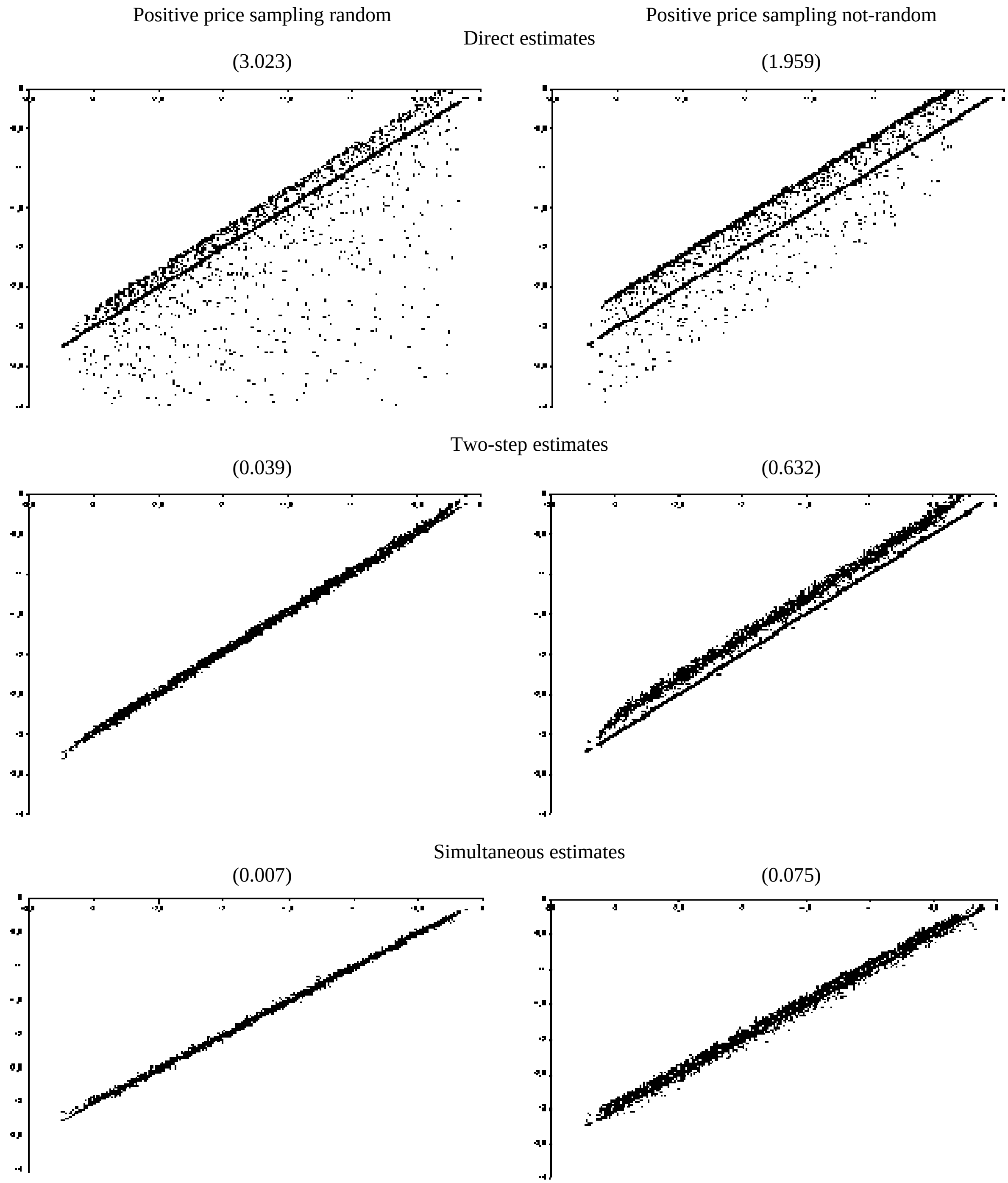
Figure 11 Log of depreciation rate when *f*(*t*) is Weibull and capacity profiles are constant



Notes: Specification error – No measurement error.
N=2000, n=200, (c.v.).
N is the number of replications.
n is the sample size.
c.v. = coefficient of variation.

Source: Statistics Canada.

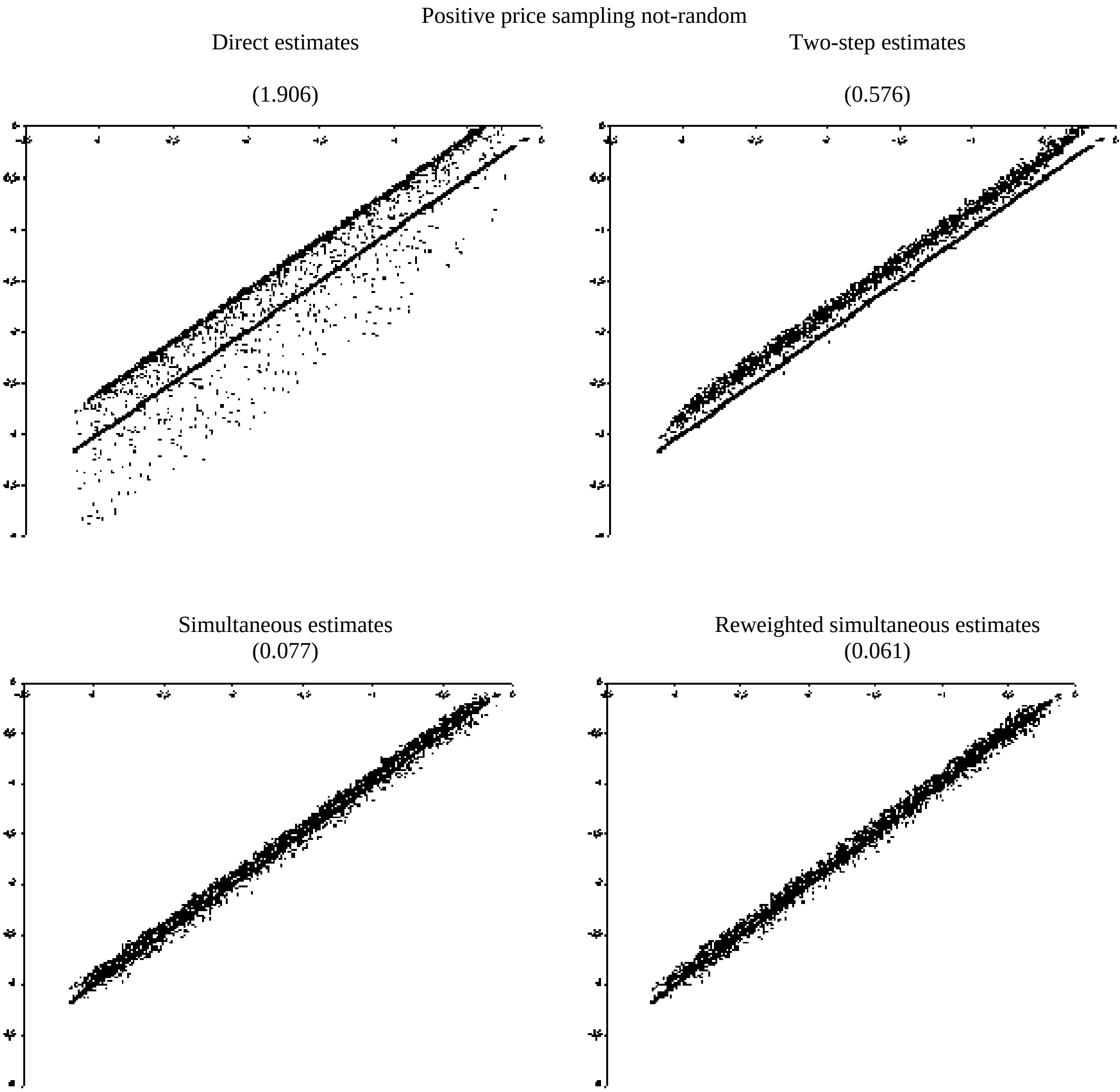
Figure 12 Log of depreciation rate when $f(t)$ is Weibull and capacity profiles are concave



Notes: Specification error – No measurement error.
N=2000, n=200, (c.v.).
N is the number of replications.
n is the sample size.
c.v. = coefficient of variation.

Source: Statistics Canada.

Figure 13 Log of depreciation rate when *f*(*t*) is Weibull and capacity profiles are concave



Notes: Specification error – No measurement error.
N=2000, n=200, (c.v.).
N is the number of replications.
n is the sample size.
c.v. = coefficient of variation.

Source: Statistics Canada.

6. Empirical results

In the previous sections, we have outlined the various approaches that were used in this investigation and indicated, in broad form, the nature of the estimation techniques used. Each of these techniques makes use of the prices of assets that are being discarded or otherwise disposed of—a large database of over 30,000 observations.

While rich in detail, there are potential problems that need to be resolved. The editing processes that were described above deal with those that can be manually resolved. A second problem that was carefully considered revolved around the randomness of the sample.

Analysts need always to keep in mind that the data they are using may not have been generated in a random way and that the sampling technique may have produced a sample produces serious bias in its estimates. A classical survey design process is aimed at reducing these problems. But even here, problems may arise during the survey process. And survey methodologists have designed methods to use post-survey reweighting to address the problem.

The data with which we are working potentially suffers from non-randomness as a result of the 'purposive' sampling process used to generate the data. As a result, the data may not be ideal for estimation. One manifestation of this problem is the 'lumpiness' at certain ages of the asset that sometimes is seen in the data. In these cases, we clearly do not obtain a wide range of observations to estimate depreciation rates ranging from very young ages to very old ages. Or, if we think of the other dimension of our data, the price ratio ranging from 0 to 1, we often see more observations in some groupings than in others.

To address this problem, we reweight our observation set (see Appendix C). When we impose this reweighting and reestimate the depreciation rate, we improve the efficiency of our estimates substantially (Figure 13 and Table 2, line 5).

6.1 Estimates of ex post rates of depreciation

While the simulations provide us with an insight into how the estimates are likely to differ across different estimation techniques, it is only by comparing the estimates that are generated by each technique that we can evaluate the size of actual differences. For this purpose, we make use of exactly the same database of used-asset prices and discards for each of three estimation methods. We compare the results across the three estimators—what we call METHOD1, METHOD2 and METHOD3—the survival, the two-step and the simultaneous procedures. We also present the results from our earlier paper (Gellatly, Tanguay, and Yan, 2002) that also uses METHOD1.

While we experiment with two versions of METHOD2 to estimate the length of life of assets, one that makes use of both actual discards and the transactions with positive prices, and one that only makes use of discards, we believe that the Monte Carlo evidence supporting the former is sufficiently persuasive to favor adopting the technique that makes full use of all information for our purposes—both positive and zero values of used-asset prices.

Although we refer here to METHOD3 in generic terms, this method does not generate a unique estimate. As the earlier section demonstrated, it requires the specification of both the discard function and the asset efficiency profile, and different specifications yield different depreciation rates. We experimented with two assumptions regarding the functional form that we believe are reasonable. In both cases, we assumed a Weibull distribution for discards. But in the first case, we make use of constant efficiency profiles—the type of profiles that are generated by light-bulbs (Equation [5]). In the second case, we make use of a concave profile (see Equation [53]). In the first case, the assumption of a constant capacity profile implicitly assumes a declining-balance estimate (DBR) of 2 (Equation [9]). In the second case, the estimation of the parameter k that determines the concavity of the efficiency frontier yields a DBR estimate between 2 and 3 (Equation [18]). The differences between the depreciation estimates yielded by the two techniques were relatively small, and therefore we report only those estimates that are derived from allowing the DBR to be estimated, unless a nested test does not reject the hypothesis that the DBR does not differ from 2.

As noted above, we have reweighted the data that were used for all three methods. This was done in two stages. First, we visit our sample in the time or age dimension. Then, we do the same in the price-survival dimension.[37] The reweighting in the age or time dimension has little impact on the relative values of the depreciation estimates yielded by each method. The reweighting in the price-survival dimension has more impact on the estimates. This suggests that the sample of discards is reasonably representative of the population—but the sample of positive prices is not.

Without this reweighting, the depreciation estimates for METHOD1 and METHOD2 are about the same and average about 5 percentage points higher than METHOD3. After the reweighting is applied, the two estimates are close to that of METHOD3, which is virtually unaffected by the reweighting. Reweighting makes the estimated average depreciation estimate less sensitive to the method chosen.

In Tables 3 and 4, we report the estimates of the average depreciation rate by type of asset in our sample, based on each of the three econometric methods outlined above. Table 3 contains estimates for machinery and equipment. Table 4 contains the estimates for various structures or buildings. We use only those assets for whom we estimated depreciation rates in our previous paper (Gellatly, Tanguay and Yan, 2002). Our use of different estimation techniques allows us to gauge the extent to which depreciation profiles are sensitive to different operational versions of our survival model.

37. For more details, see Tanguay (2005).

Table 3 Depreciation rates for machinery and equipment using alternate methods

Asset category	Original data	New data		
	METHOD1	METHOD1	METHOD2	METHOD3
Office furniture, furnishing (e.g., desks, chairs)	0.303	0.230	0.212	0.259
Computers, associated hardware and word processors	0.588	0.472	0.403	0.531
Non-office furniture, furnishings and fixtures (e.g., recreational equipment, etc.)	0.213	0.199	0.194	0.233
Scientific, professional and medical devices (including measuring, controlling, laboratory equipment)	0.256	0.269	0.236	0.222
Heating, electrical, plumbing, air conditioning and refrigeration equipment	0.270	0.227	0.162	0.172
Pollution abatement and control equipment	0.222	0.223	0.178	0.123
Safety and security equipment (including firearms)	0.455	0.373	0.189	0.211
Motors, generators, transformers, turbines, compressors and pumps of all types	0.250	0.135	0.131	0.129
Heavy construction equipment (e.g., loading, hauling mixing, paving, grating)	0.192	0.144	0.166	0.177
Tractors of all types and other field equipment (truck tractors - see 6203)	0.192	0.130	0.158	0.183
Capitalized tooling and other tools (hand, power, industrial)	0.500	0.261	0.224	0.242
Drilling and blasting equipment	0.217	0.189	0.193	0.192
Automobiles and major replacement parts	0.238	0.239	0.257	0.303
Buses (all types) and major replacement parts	0.200	0.092	0.149	0.149
Trucks, vans, truck tractors, truck trailers and major replacement parts	0.238	0.191	0.215	0.238
Locomotives, rolling stock, street and subway cars, other rapid transit and parts	0.161	0.133	0.169	0.103
Ships and boats and major replacement parts	0.110	0.074	0.111	0.098
Aircraft, helicopters, aircraft engines and other major replacement parts	0.067	0.081	0.079	0.084
Other transportation equipment	0.400	0.171	0.268	0.201
Computer-assisted process for production process	0.303	0.164	0.176	0.172
Computer-assisted process for communication and related equipment	0.323	0.253	0.229	0.222
Non-computer assisted process for material handling	0.303	0.143	0.168	0.196
Non-computer assisted process for production process	0.303	0.144	0.152	0.155
Non-computer assisted process for communication and related equipment	0.769	0.256	0.196	0.232
Other machinery and equipment	0.185	0.175	0.160	0.172
Mean	0.290	0.199	0.191	0.200
Standard error of mean	0.031	0.018	0.012	0.017

Source: Statistics Canada.

Table 4 Depreciation rates for buildings using alternate methods

Asset category	Original data	New data		
	METHOD1	METHOD1	METHOD2	METHOD3
Plants for manufacturing, processing and assembling goods	0.130	0.097	0.086	0.091
Warehouses, refrigerated storage, freight terminals	0.062	0.088	0.065	0.071
Maintenance garages, workshops, equipment storage facilities	0.135	0.085	0.083	0.085
Office buildings	0.076	0.074	0.062	0.059
Shopping centers, plazas, malls, stores	0.052	0.151	0.107	0.145
Other industrial and commercial	0.078	0.094	0.078	0.093
Telephone and cablevision lines, underground and marine cables	0.385	0.212	0.118	0.127
Communication towers, antennae, earth stations	0.128	0.119	0.099	0.114
Mean	0.131	0.115	0.087	0.098
Standard error of mean	0.038	0.016	0.007	0.010

Source: Statistics Canada.

Comparisons of the set of results using METHOD1 on the original database and the new database allow us to evaluate both the impact of extending the number of observations in the database and applying a reweighting technique to take into account the effect of non-random sampling. Both have had an impact. The mean depreciation rate for machinery and equipment goes from 29.0% using the old database to 24.5% with the new database but the old weighting technique and to 19.9% using the new database and the new weighting technique. The mean depreciation rate for buildings has gone from 13.1% to 11.5% as a result of both the new data points and the new weighting techniques.

Changes were a little larger in the case of several assets. In case of computers and office furniture for example, this was caused by a large revision in the price index series and this affected the computations of the price ratios.

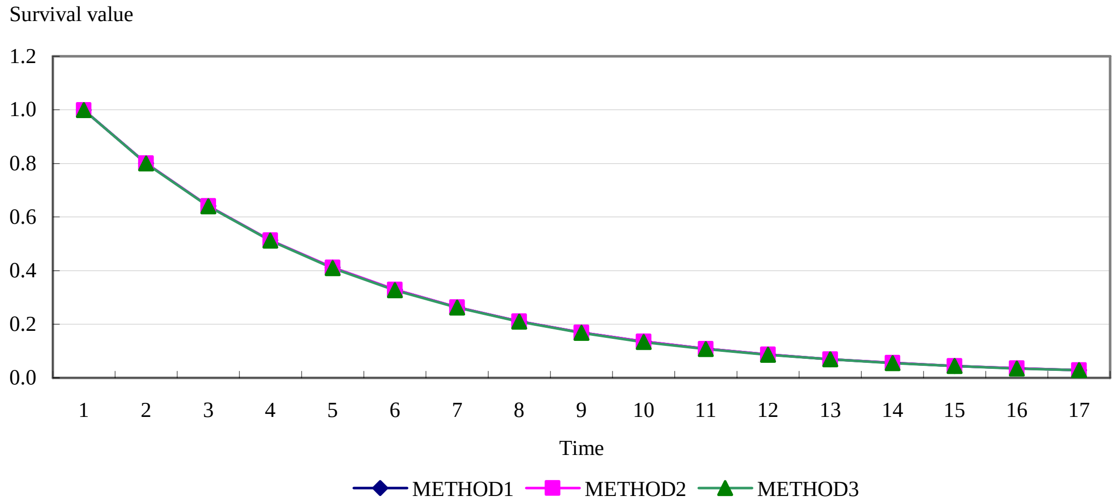
The estimates of depreciation rates provided by METHOD2 are, on average, in the same range as those provided by METHOD1. The mean depreciation rate for machinery and equipment was 19.1% for METHOD2 and 19.9% for METHOD1. These differences are not significant.

The mean depreciation rate for buildings using METHOD1 was 11.5%, while for METHOD2, it was 8.7% and for METHOD3, 9.8%. Again, these differences, especially between METHOD2 and METHOD3, are small.

Many assets in the machinery and equipment class exhibit substantial reductions in asset value early in their service life regardless of the technique that is used. All the techniques yield average depreciation rates around 20%. In keeping with our earlier results, this implies that $E(y)$, the expected life of a dollar invested is around five years. We graph the profile of an average machinery and equipment asset in Figure 14 using the average depreciation rates derived from METHOD1, METHOD2, and METHOD3.

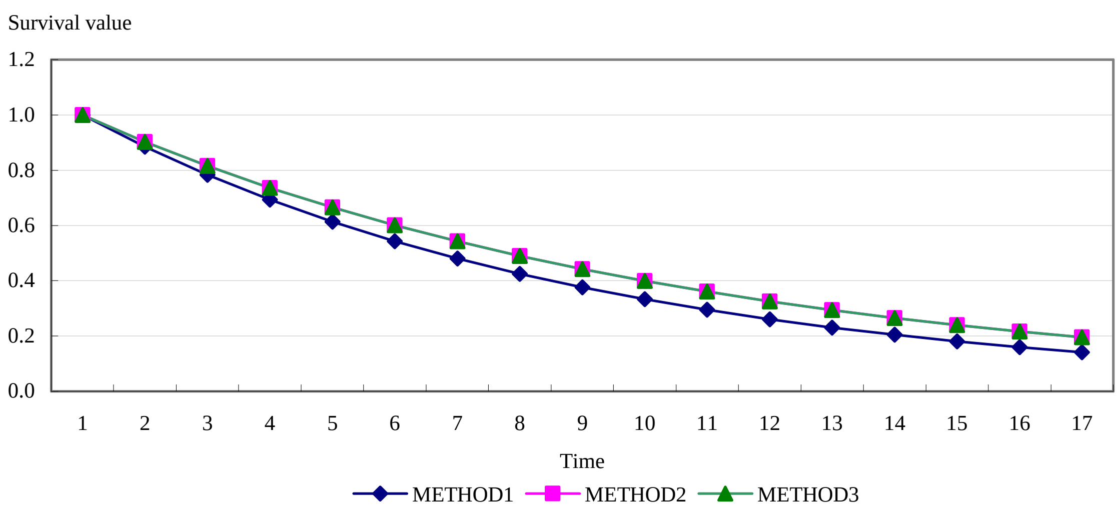
High levels of depreciation are not unique to assets in the machinery and equipment class. Manufacturing plants—the asset category with the largest number of observations in this class—have an average depreciation rate of around 10%, which implies that the expected life of a dollar invested is 10 years. Office buildings have an average depreciation rate that is a little above 6%. In this case, the expected length of life of a dollar is on average about 20 years. We graph the profile of an average asset from those buildings for which we have estimates in Figure 15 using the average depreciation rates derived from METHOD1, METHOD2, and METHOD3.

Figure 14 Depreciation profiles for machinery and equipment



Source: Statistics Canada.

Figure 15 Depreciation profiles buildings



Source: Statistics Canada.

High levels of depreciation among structures are not, in our view, counterintuitive. On this, the case of plants is illustrative. First, there is evidence (Baldwin, 2005), that the average length of life of a firm operating a plant is around 10 years. Second, many of the characteristics of plants are idiosyncratic. In many situations, the investments that have been made in a plant for one user are not readily transferable from existing owners to new owners, that is, from current-use to future-use. The substantial death rate of most manufacturing plants means that the embedded special investment is likely to be lost when the death of the enterprise occurs.[38]

As noted above, the average depreciation rates that are generated by METHOD2 and METHOD3 are quite similar. But the estimates for individual assets differ—though not greatly. The absolute differences for the machinery and equipment assets range from 0.1 to 13.7 percentage points with a median of 2. The absolute differences for the buildings range from 0.2 to 3.4 percentage points with a median of 0.9. These differences are rarely statistically significant. The standard errors for each of METHOD1, METHOD2 and METHOD3 are included in Table 5 for the machinery and equipment asset categories and in Table 6 for the building asset categories.[39] They average 1.8% for machinery and equipment and around 1% for buildings when METHOD3 is used. This implies that a 95% confidence interval would on average have a width of around 4 percentage points for the machinery and equipment assets (around a mean of 20) and 2 percentage points for the building assets (around a mean of 10). A more precise test of the differences between the depreciation estimates for METHOD2 and METHOD3 by asset is provided in Tables 5 and 6. There is no significant difference for 26 of 32 assets.

38. This problem of value loss when an asset shifts from one usage to another affects the asset value of other assets besides buildings. Ramey and Shapiro (2001) discuss what happened to used assets in the aerospace industry when they were sold to firms outside the immediate original user group.

39. Variance estimates were based on a delta method for METHOD1 and METHOD3. However, this technique could not be applied to the two-step procedure (METHOD2) since no direct estimation of the Fisher information matrix is provided in a two-step procedure. Therefore, we had to adapt the approach developed by Murphy and Topel (1985), for a two-step model with non-linear specification. An adaptation was required to take into account that the second step involves only a sub-sample of the initial data.

Table 5 Standard errors for machinery and equipment and test for differences between METHOD2 and METHOD3

Asset category	Standard error			Test for differences	
	METHOD1	METHOD2	METHOD3	T	Probability
Office furniture, furnishing (e.g., desks, chairs)	0.006	0.007	0.056	0.828	0.408
Computers, associated hardware and word processors	0.013	0.016	0.122	1.039	0.299
Non-office furniture, furnishings and fixtures (e.g., recreational equipment etc.)*	0.009	0.011	0.016	1.998	0.046
Scientific, professional and medical devices (including measuring, controlling, laboratory equipment)	0.016	0.011	0.012	0.867	0.386
Heating, electrical, plumbing, air conditioning and refrigeration equipment	0.018	0.015	0.010	0.514	0.607
Pollution abatement and control equipment	0.036	0.045	0.011	1.181	0.240
Safety and security equipment (including firearms)	0.072	0.038	0.039	0.398	0.692
Motors, generators, transformers, turbines, compressors and pumps of all types	0.008	0.008	0.007	0.206	0.837
Heavy construction equipment (e.g., loading, hauling mixing, paving, grating)	0.008	0.010	0.004	0.995	0.320
Tractors of all types and other field equipment (truck tractors - see 6203)	0.008	0.015	0.013	1.272	0.204
Capitalized tooling and other tools (hand, power, industrial)	0.013	0.006	0.013	1.236	0.217
Drilling and blasting equipment	0.022	0.018	0.009	0.073	0.942
Automobiles and major replacement parts*	0.007	0.007	0.019	2.281	0.023
Buses (all types) and major replacement parts	0.008	0.046	0.016	0.003	0.997
Trucks, vans, truck tractors, truck trailers and major replacement parts*	0.004	0.004	0.007	2.785	0.005
Locomotives, rolling stock, street and subway cars, other rapid transit and major parts*	0.013	0.015	0.004	4.299	0.000
Ships and boats and major replacement parts	0.009	0.010	0.008	1.059	0.292
Aircraft, helicopters, aircraft engines and other major replacement parts	0.007	0.005	0.008	0.501	0.617
Other transportation equipment	0.016	0.040	0.011	1.632	0.104
Computer-assisted process for production process	0.007	0.005	0.006	0.443	0.658
Computer-assisted process for communication and related equipment	0.016	0.021	0.015	0.297	0.767
Non-computer assisted process for material handling*	0.005	0.003	0.008	3.363	0.001
Non-computer assisted process for production process	0.003	0.002	0.002	1.121	0.262
Non-computer assisted process for communication and related equipment	0.015	0.017	0.012	1.762	0.079
Other machinery and equipment (not specified elsewhere)	0.007	0.005	0.018	0.622	0.534
Mean	0.014	0.015	0.018	1.231	0.381

* denotes significant difference between METHOD2 and METHOD3
Source: Statistics Canada.

Table 6 Standard errors for buildings and test for differences between METHOD2 and METHOD3

Asset category	Standard error			Test for differences	
	METHOD1	METHOD2	METHOD3	T	Probability
Plants for manufacturing, processing and assembling goods	0.004	0.002	0.008	0.646	0.519
Warehouses, refrigerated storage, freight terminals	0.007	0.004	0.008	0.801	0.424
Maintenance garages, workshops, equipment storage facilities	0.009	0.005	0.021	0.116	0.908
Office buildings	0.004	0.002	0.003	0.880	0.379
Shopping centers, plazas, malls, stores*	0.010	0.005	0.013	2.809	0.005
Other industrial and commercial	0.009	0.005	0.007	1.865	0.064
Telephone and cablevision lines, underground and marine cables	0.026	0.012	0.007	0.643	0.521
Communication towers, antennae, earth stations including dishes for state, etc.	0.014	0.011	0.008	1.130	0.260
Mean	0.010	0.006	0.009	1.111	0.385

* denotes significant difference between METHOD2 and METHOD3
Source: Statistics Canada.

We have compared the depreciation estimates produced here by METHOD3 to those that are used by the U.S. Bureau of Economic Analysis (see Appendix D). The U.S. estimates are also generated from age-price profiles—but come from a myriad of unique databases that track the prices of individual assets—and range from studies that were done over 20 years ago to more recent efforts. In contrast, the Canadian estimates reported here are derived from a large scale survey that has been capturing the price of used assets since 1987.

On average, the Canadian depreciation rate is quite similar to the U.S. rate for the machinery and equipment asset classes. The U.S. average is 18%, the Canadian depreciation rate averaged 20%. This difference is not large.

In contrast, there is a considerable difference between the Canadian and U.S. rates for buildings and engineering construction. Here the Bureau of Economic Analysis (BEA) average is 3%, while the Canadian average rate is 8%. But almost all of the Canadian estimates fall in the higher range (Table 4), not so much because the estimated length of life is all that much shorter, but because the estimated DBR's are higher than are assumed in the United States. The price of used assets associated with buildings and engineering assets declines over time at a rate that is faster than is assumed in the United States.

6.2 Ex ante versus ex post estimates of depreciation and length of life

Our comparisons to this point have been based on alternative formulations of our econometric framework. In this section, we compare these econometric results to a geometric profile based on the *ex ante* accounting method described by Equation (4).

Statistics Canada's Investment Survey not only asks for the price of assets upon disposition, but it also asks for the anticipated length of service life when investments are first reported to the agency. As outlined earlier in the study, use of the length of service life, along with a declining-balance constant, provides an alternate way to estimate the average depreciation rate (δ=DBR/L —see Equation [4]).

Estimates of depreciation using expected length of life offer a different way of estimating average depreciation rates. They are *ex ante* measures and they may therefore suffer from inaccurate forecasts. These may either underestimate or overestimate the actual or *ex post* lives. Differences may also occur if service lives have been changing over time, since the *ex post* rates make use of data that precede the post 1987 period from which the *ex ante* estimates are taken.

There are several other reasons that the *ex ante* rates may differ from the *ex post* rates that have to do with the concept of an anticipated length of life, all of which stem from the fact that managers may have in mind a different concept than the expected age of discard. For example, they may have in mind the expected time before disposal, which could be the point at which they sell the asset, rather than the point at which they scrap it. For example, buyers of fleet autos may have in mind the point at which they dispose of the car after the first (three-year) lease. Or managers may have in mind the point at which they expect to lose half of their asset value. In both of these cases, the *ex ante* concept may turn out to be less than the *ex post* estimate.

Another reason for possible discrepancies between *ex ante* and *ex post* rates arises from the heterogeneity of some asset classes. In this case, the composition of the sample of discards may be quite different from the population of investments that is used to calculate the *ex ante* length of life. Assets with shorter lives within any class are more likely to be captured in discards, while longer-lived assets are weighted more heavily in the second. A good example is the class of Shopping Centers, Plazas and Stores. Shopping Centers involve large investments with long service lives and they probably dominate the investment population that supplies the *ex ante* rates. On another hand, stores with shorter lives are likely to be more heavily weighted in the observations on discards. This would produce an *ex ante* estimate that is higher than the *ex post* estimate derived from the pattern of actual discards.

The data source that provides an estimate of the expected *ex ante* length of life offers a much larger number of observations per asset than are available for the *ex post* estimate and this is a distinct advantage. Table 1 contains the number of observations over the period 1985 to 2001 per asset. For our estimates, we use the average over the latter half of the period. There has been a slight decline in the expected service life over the period.

As attractive as this alternate *ex ante* technique is, it still requires the estimation (choice) of the declining-balance rate (DBR). And we have demonstrated, the choice of this DBR in itself involves uncertainty. The DBR can be chosen as 2 as it often is in the accounting world. But this essentially involves an assumption that the associated efficiency or capacity frontier of the asset is constant. If the profile is concave, the DBR will typically be greater than 2 but less than 3.[40]

In order to compare our *ex post* estimates to our *ex ante* estimates, we make use of the DBRs that are yielded by our *ex post* technique and substitute them into the formula δ=DBR/T using an *ex ante* length of life to yield a depreciation rate. Asset-specific estimates of mean service life

40. It should be noted that if the efficiency frontier takes on the profile of a logistic curve (initially concave but then reversing itself to become convex), the DBR may be greater than 3.

(T) are taken from the Capital and Repair Expenditures Survey.[41] The resulting estimates are then compared to the *ex post* rates that are derived from METHOD3 in Tables 7 and 8. Once again, we use only assets that were estimated in the original paper.

The two sets of estimates are quite similar for buildings. The mean for buildings for METHOD3 that estimates a DBR is 9.4%. It is 10.1% for the *ex ante* estimate calculated with the DBRs that result from METHOD3. These differences are not statistically significant.

We also include the estimate of the expected discard age taken from the simultaneous estimate and the *ex ante* expected discard length of life in Table 8 for buildings. The two are quite close—26.7 and 26.5 years, respectively. We conclude that for long-lived assets in the buildings category, the data cannot distinguish between the *ex post* and *ex ante* estimates, the exception being the class of Shopping Centers, Plazas, Malls and stores, which may suffer from the data problem discussed previously. In Table 9, we include the expected length of life for a select set of engineering assets for which adequate data are available to estimate the discard function even if price survival ratio is likely to be deficient (because most assets are discarded at zero price and not sold for positive value). Again the *ex post* estimates are very close to the *ex ante* estimates for the long-lived assets.

These two results suggest that the use of the *ex ante* estimates of length of lives, along with an imputed DBR, for those long-lived assets with infrequent sales where we do not have used-asset prices, promises a reasonable method of filling in our data set of depreciation rates when used-asset prices are not available.

As can be seen from Table 7, there are larger differences between the *ex post* and *ex ante* estimates of depreciation and length of life for machinery and equipment. The econometric average *ex post* estimates of life are higher than the *ex ante* expected service life estimates—some 14.1 years versus 11.3 years.[42] Concomitantly, the average *ex post* depreciation rates are 20.0% versus 24.0% for the *ex ante* estimate.

41. Data on service lives was obtained for the period 1995 to 1997. Mean service lives are investment-weighted.

42. If we estimate the length of life only from those discards with a zero price, then the expected *ex post* life is almost the same as the mean *ex ante* life. The lower *ex post* estimate occurs because it uses a censored data set that ignores the information provided by the asset sales that are made at a positive price. Using the censored data set is akin to trying to estimate the average length of unemployment by examining only the experience of those people who have made a transition from unemployment to employment and ignoring those who remain unemployed.

Table 7 *Ex post* versus *ex ante* depreciation rates for machinery and equipment

Asset category	Depreciation rate		Expected life years	
	Ex post Simultaneous Concave	*Ex ante* DBR[1] from Simultaneous 2<DBR>3	*Ex post* Simultaneous	*Ex ante* Survey
Office furniture, furnishing (e.g., desks, chairs)	0.259	0.281	9.0	8.3
Computers, associated hardware and word processors	0.531	0.496	4.4	4.7
Non-office furniture, furnishings and fixtures (e.g., recreational equipment, etc.)	0.233	0.248	10.0	9.4
Scientific, professional and medical devices (including measuring, controlling, laboratory equipment)	0.222	0.236	9.5	8.9
Heating, electrical, plumbing, air conditioning and refrigeration equipment	0.172	0.211	15.3	12.5
Pollution abatement and control equipment	0.123	0.127	17.2	16.7
Safety and security equipment (including firearms)	0.211	0.215	11.0	10.8
Motors, generators, transformers, turbines, compressors and pumps of all types	0.129	0.178	21.2	15.3
Heavy construction equipment (e.g., loading, hauling mixing, paving, grating)	0.177	0.304	13.6	7.9
Tractors of all types and other field equipment (truck tractors - see 6203)	0.183	0.276	14.2	9.4
Capitalized tooling and other tools (hand, power, industrial)	0.242	0.266	8.8	8.0
Drilling and blasting equipment	0.192	0.221	12.8	11.1
Automobiles and major replacement parts	0.303	0.358	7.9	6.7
Buses (all types) and major replacement parts	0.149	0.209	17.6	12.6
Trucks, vans, truck tractors, truck trailers and major replacement parts	0.238	0.348	10.4	7.1
Locomotives, rolling stock, street and subway cars, other rapid transit and parts	0.103	0.112	25.3	23.3
Ships and boats and major replacement parts	0.098	0.145	25.6	17.3
Aircraft, helicopters, aircraft engines and other major replacement parts	0.084	0.135	23.8	14.8
Other transportation equipment	0.201	0.282	12.6	9.0
Computer-assisted process for production process	0.172	0.210	15.5	12.7
Computer-assisted process for communication and related equipment	0.222	0.259	11.1	9.5
Non-computer assisted process for material handling	0.196	0.251	13.6	10.6
Non-computer assisted process for production process	0.155	0.179	16.1	14
Non-computer assisted process for communication and related equipment	0.232	0.240	11.5	11.1
Other machinery and equipment not elsewhere specified	0.172	0.213	13.5	10.9
Mean	0.200	0.240	14.1	11.3
Standard error of mean	0.017	0.016	1.1	0.8

1. Declining-balance rate.

Note: Outlined asset categories are those where there are significant differences between *ex ante* and *ex post* length of life.

Source: Statistics Canada.

Table 8 *Ex post* versus *ex ante* depreciation rates for buildings

Asset category	Depreciation rate		Expected life in years	
	Ex post Simultaneous Concave	*Ex ante* DBR[1] from Simultaneous	*Ex post* Simultaneous	*Ex ante* Survey
Plants for manufacturing, processing and assembling goods	0.095	0.100	29.2	26.6
Warehouses, refrigerated storage, freight terminals	0.074	0.073	32.8	32.2
Maintenance garages, workshops, equipment storage facilities	0.080	0.095	31.3	28.0
Office buildings	0.064	0.060	34.2	33.3
Shopping centers, plazas, malls, stores	0.122	0.076	16.1	30.7
Other industrial and commercial	0.089	0.098	25.1	23.9
Telephone and cablevision lines, underground and marine cables	0.139	0.116	18.3	20.0
Communication towers, antennae, earth stns. including dishes for state, etc.	0.100	0.205	23.3	13.0
Broadcasting and communication buildings	0.086	0.086	30.4	30.6
Mean	0.094	0.101	26.7	26.5
Standard error of mean	0.007	0.013	2.0	2.1

1. Declining-balance rate
Source: Statistics Canada.

Table 9 *Ex post* versus *ex ante* length of life for engineering construction

Asset category	Expected length of life in years	
	Ex post	*Ex ante*
Highways, roads, streets, including: logging road, signs, guardrail, lighting, etc.	20.9	24.8
Rail track and roadbeds including: signals and interlockers	39.6	36.9
Telephone and cablevision lines, underground and marine cables	18.3	20
Communication towers, antennae, earth stns.	23.3	13
Gas mains and services	43.0	38
Bulk storage	23.5	23
Waste disposal facilities	25.4	36.1
Mean	27.7	27.4
Standard error of mean	3.4	3.4

Source: Statistics Canada.

Most of the major differences occur in four categories—heavy construction, tractors, buses and trucks. These are all categories where heavy motive equipment is found. The differences in these categories are compatible with the explanation that some managers in all of these categories have the concept ‘time to disposal’ rather than ‘time to discard’, when answering the question about the *ex ante* expected length of life.

Explanations for differences between the *ex ante* and the *ex post* estimates must also remember that the prices of used assets may only imperfectly reflect the future stream of earnings of the assets for several reasons. The used assets that are sold may have a higher proportion of ‘lemons’ than the capital stock in general, and therefore may not reflect the average value in use. In addition, the price data used in estimating age-price profiles may be subject to more reporting error than the expected length of life data.

In the face of all these potential problems, it is perhaps surprising to find the congruence that exists between the two estimates. Errors that would lead to divergences in opposite directions cancel one another.

7. Capital stock

7.1 The effect of alternate depreciation rates on capital stock

In the previous section, we have presented several estimates of depreciation rates that come from alternate estimation techniques. On average, the alternate techniques do not differ a great deal. But at the individual asset level, estimates differ on average by 3 percentage points between METHOD2 and METHOD3. On a mean of 20%, this means that the difference in estimates is around 15% of the average.

For some purposes, these ranges may be too large. For those interested in arguing that a specific depreciation rate should be used for a particular asset, more precision may be required. But that is not the purpose of this exercise. In this study, and its predecessor, we are interested in developing a *set* of depreciation rates that are to be used as part of the productivity program. The productivity program needs estimates of the rate of growth of capital stock or capital services. Ultimately, we are interested in whether the alternate estimates of depreciation affect these growth rates. We examine this question here.

In this section, we generate estimates of capital stock based on the three econometric estimates for those categories where *ex post* depreciation estimates exist.

Estimates of capital stock are based on the perpetual inventory model

$$K(t) = I(t) + (1-\delta)K(t-1) \tag{54}$$

where δ represents a (constant) geometric rate of depreciation.

To produce econometric estimates of δ, we estimate the depreciation rates for all assets for whom we have sufficient observations, and then use the estimated depreciation rates from this model to construct aggregate summary rates of depreciation for 29 different asset groups—14 asset groups for structures and engineering, and 15 asset groups for machinery and equipment. These 29 asset groups were constructed using historical information on service life—in effect, by combining individual assets with comparable estimates of service life from survey data. To build aggregate statistics, we compute the weighted mean of *E(t)* and *E(y)* using the relative stock contribution of each asset[43] as a weight. The ratio of the two means of *E(t)* and *E(y)* provides the aggregate DBR. We report depreciation rates for these 29 asset groups in Table 10. It should be noted that for some of those, it was impossible to build an *ex post* estimator. In those cases, we used an *ex ante* estimate, whose computation will be reported in the next section.

43. The stock contribution of an individual asset to the group is provided by the product of its historical investment over the period 1985 to 1999 (in constant dollars) by its expected life.

Table 10 Depreciation rates by aggregate asset classes

Group	Asset category	METHOD1	METHOD2	METHOD3	METHOD final
4001	Industrial building construction	0.0946	0.0854	0.0903	0.0878
4002	Commercial building	0.0710	0.0643	0.0640	0.0650
4003	Institutional building	0.0713	0.0597	0.0597	0.0605
5001	Marine engineering	0.0802	0.0654	0.0660	0.0664
5002	Transportation engineering	0.0755	0.0616	0.0621	0.0625
5003	Waterworks engineering construction	0.0875	0.0714	0.0720	0.0725
5005	Electric power engineering construction	0.0681	0.0556	0.0560	0.0564
5006	Communication engineering construction	0.1208	0.1203	0.1203	0.1205
5007	Oil and gas engineering construction	0.0805	0.0658	0.0664	0.0808
5008	Mining engineering construction	0.1938	0.1581	0.1594	0.1604
5089	Other engineering construction	0.1040	0.0862	0.0869	0.0874
7001	Other transportation equipment	0.0950	0.0968	0.1004	0.0979
7002	Industrial machinery	0.1638	0.1638	0.1636	0.1637
7003	Telecommunication equipment	0.2207	0.2207	0.2207	0.2207
7004	Furniture	0.2277	0.2309	0.2277	0.2277
7005	Software	0.5500	0.5500	0.5500	0.5500
7007	Trucks	0.2333	0.2333	0.2333	0.2333
7008	Automobiles and major replacement parts	0.2437	0.2800	0.2800	0.2800
7009	Agricultural machinery	0.1354	0.1709	0.1709	0.1709
7010	Computers and related machinery and equipment	0.4670	0.4255	0.4670	0.4670
7089	Other machinery and equipment	0.1786	0.1719	0.1786	0.1786

Source: Statistics Canada.

The rates of growth of capital stock are presented in Table 11. The first two columns compare the results using METHOD1 for the original and the new extended database. The rate of growth of machinery and equipment stock over the period 1961 to 2000 decreases from 5.5% to 5.2%, an insignificant amount as a result of extending the database. There are also only small differences for two subperiods: 1961 to 1980 and 1980 to 2000. Differences for the rate of growth of structures is also small—going from 3.28% to 3.38%.

If we compare the three methods using the same extended database (METHOD1, METHOD2, and METHOD3), we find relatively small differences. For the period 1961 to 2000, the three estimates for machinery and equipment are 5.19%, 5.48%, and 5.32%. For buildings, the estimates are 3.38%, 3.45%, and 3.52%.

The productivity program makes use not of capital stock but of capital services. The growth in capital services is just the weighted average of the growth in the capital stock of different assets. The weights chosen are the user cost of capital, which depends essentially on the rate of return of the asset and its rate of depreciation (see Harchaoui and Tarkhani, 2003). This weighting procedure takes into account potential differences in the marginal product of different assets.

We therefore calculate the rate of growth of capital services making use of the estimates of the user cost of capital outlined in Harchaoui and Tarkhani (2003). The results are reported in Table 11 for the machinery and equipment and building structure categories for which we can estimate *ex post* depreciation estimates. Here, the differences also are small.

We conclude that extending the database, improving the imputation methods and experimenting with additional estimation techniques has a minimal impact on the estimates of the growth in capital stock that were presented previously.

Table 11 Comparative growth rates in capital stock for select estimated assets under alternative depreciation regimes

	METHOD1	METHOD1	METHOD2	METHOD3
	Original data	New data		
Growth rate 1961 to 2000				
Machinery and equipment	5.50	5.19	5.48	5.32
Structures	3.28	3.38	3.45	3.52
Growth rate 1961 to 1980				
Machinery and equipment	5.61	5.46	5.61	5.55
Structures	4.62	4.85	4.90	4.93
Growth rate 1980 to 2000				
Machinery and equipment	5.41	4.93	5.36	5.12
Structures	2.02	1.99	2.07	2.18

Source: Statistics Canada.

Table 12 Comparative growth rates in capital services for select estimated assets under alternative depreciation regimes

	METHOD1	METHOD1	METHOD2	METHOD3
	Original data	New data		
Growth rate 1961 to 2000				
Machinery and equipment	7.24	7.25	7.29	7.31
Structures	3.37	3.59	3.64	3.72
Growth rate 1961 to 1980				
Machinery and equipment	6.58	6.62	6.62	6.72
Structures	4.80	5.08	5.11	5.18
Growth rate 1980 to 2000				
Machinery and equipment	7.83	7.89	7.92	7.87
Structures	2.02	2.16	2.24	2.33

Source: Statistics Canada.

7.2 Estimation of capital stock for the productivity program

As emphasized in the previous section, the estimation of depreciation rates is needed for the construction of capital stock series. Our analysis here has provided us with several alternatives, none of which is obviously superior—either on theoretical grounds, on practical grounds, or on empirical grounds.

We have investigated the results for three methods—what we have called the original (METHOD1), and two modifications—METHOD2, and METHOD3. All three methods draw a line through some price-age profiles to obtain an average depreciation rate. METHOD1 takes the proportion of the remaining value over time and estimates a mean depreciation rate. It takes the prices of assets that are sold at a positive price or discarded at a zero prices and averages the two and then estimates the age-price profile. METHOD2 predicts the distribution of discards at zero prices using the actual discards and then uses this prediction for each age group to average down the positive prices that were observed. It proceeds in two separate steps. METHOD3 performs the estimation of the discard function and the price-time profile simultaneously. The three methods also differ slightly in terms of the functional forms imposed on the estimation process. METHOD1 uses an exponential for the price-age profile. METHOD2 uses a Weibull for the discard function and then a Weibull for the price-age profile. METHOD3 uses a Weibull for the discard function and a different functional form than a simple Weibull for the price-age profile. This different form is compatible with a Weibull discard function for the zero price observations and a general type of concave efficiency frontier for the stream of revenue yielded by the assets.

METHOD2 and METHOD3 are both based on a system of two-equations. The two-step procedure treats the discard process and the price-survival curve as independent processes, while the simultaneous approach recognizes the interdependence between the two. Estimating the two simultaneously improves the accuracy of the estimates of both the depreciation rate and the length of life. For example, information from the survival process is useful when it comes to estimating the discard process. Information that an asset is sold for a positive price at time t is useful when estimating the discard process—because it tells us that that asset has yet to be discarded and the higher its price, the longer will be the period before it is discarded. Similarly, information on the nature of the discard process serves to inform the estimation of the depreciation rate (via the declining-balance rage [DBR]) since the discard pattern tells us whether the data on prices are censored or bunched in unusual patterns.

A simultaneous framework also allows the estimators to respect the consistencies between the two processes generating t and y given that those two processes are correlated. In this paper, we show how constraints can be imposed during the simultaneous estimation procedure that takes into account the commonality of certain coefficients in each process. The background theory presented demonstrates how a formal representation of the efficiency process along with the discard processes can be brought together to yield estimating equations for each that have common parameters. The representations of the efficiency and the discard process that are used are quite general—a general hyperbolic function for the efficiency process and a Weibull for the discard process. Both are representations that have many adherents. But they may not be correct in all situations. Nevertheless, our Monte Carlo simulations still show that when they are not appropriate, it is still better to impose consistency across the discard and the survival process for the purpose of estimating the *average* depreciation rate.

In order to evaluate the three estimation techniques, we have employed a commonly used technique—that of a Monte Carlo simulation experiment. And in doing so, we paid particular attention to the desirability of the different estimation techniques in the face of non-random sampling. The survey that produced the price data was designed to provide a random sample of investment data—not used-asset price data by age class. The latter was a by-product of the former. Since our purpose is to estimate the average depreciation rate, having a sample of used-asset prices that is not representative of the entire spectrum of possible prices can distort estimates of the average depreciation rate.

Our simulation results show that the survival measure (METHOD1) was the least desirable, even when the price data were drawn in a random way. We also asked whether failure to have ideal data would create a larger problem using METHOD1 or METHOD2—and found that METHOD2 would be more desirable. In situations where the one-hoss-shay efficiency profile[44] is a good working assumption and price information[45] is generated by a random positive-price sampling process, the two-step approach turns out to be very efficient.

We also investigate the properties of METHOD3. We found that it too was superior to METHOD1, but that it frequently (concave efficiency profiles or non-random positive-prices sampling) was also superior to METHOD2—especially when the positive-price data is generated in a non-random fashion.

Our reweighting addresses the problem with non-random data and, after the reweighting is applied, differences across *ex post* estimates produced by METHOD2 and METHOD3 generally are not statistically significant. Therefore, for the purposes of proceeding to choose an *ex post* estimate of the average depreciation rate, we chose to average the two.

When it comes to the choice of an *ex ante* estimator as opposed to an *ex post* estimator, a different set of data issues arise. While the *ex ante* estimator may involve bias from inherent management optimism, it benefits from a larger sample that is more likely to represent a random sample of the universe of interest—because the survey is aimed at catching investments, not discards. And the *ex ante* approach does not suffer a different data problem that besets the *ex post* approach—the large number of asset classes in engineering construction for which adequate data on asset prices do not exist because there are few used-asset markets for these assets.

But the approach that uses *ex ante* information only for estimating expected length of life ignores price data and thus is inherently less efficient in its use of information since it cannot provide a DBR. The *ex post* approach provides estimates of both actual length of life and the rate of depreciation.

Fortunately, the estimates of the *ex post* and the *ex ante* length of life are very similar and the *ex ante* estimate has the advantage that it is relatively easy to obtain. It can be acquired through a relatively straightforward survey question and does not require the type of econometric estimation techniques that have been used here—that depend upon the collection of many years of data. Therefore, we proceed by combining the information that we have obtained from both the *ex ante* and the *ex post* estimates.

44. The one-hoss-shay profile corresponds to a DBR of 2.

45. Random information on price of assets could be built by evaluating a sample of assets in production. However, when this information is provided by the market on transactions, there is no way to control whether the assets that are transacted are representative of the population of assets in production.

After considering these issues, we have adopted the following approach.

1. For those assets where we can employ used-asset prices for *ex post* estimates of depreciation, we take the average of the depreciation rate derived from the two-step (METHOD2) and the simultaneous approach (METHOD3).[46]
2. For these estimates, we calculate an implicit DBR from Equation (4) using the *ex post* depreciation rate and the *ex ante* length of life.
3. For those machinery and equipment and building assets where heterogeneity or data availability prevent us from estimating a relevant *ex post* depreciation rate, we apply an imputed DBR to the *ex ante* service life. The imputed DBR for a given asset is derived from its corresponding average DBR from the 22 group levels when available, otherwise, from the general class of the asset.
4. For the engineering asset estimates, we have few *ex post* estimates as guides. Therefore, we make use of the *ex ante* estimates of length of life. But the DBR used is derived from combining available *ex post* estimates for all assets in Building and Engineering Construction.
5. For software, we used *ex ante* service life with a DBR of 1.65.
6. In the interests of simplification, we average the DBRs across all machinery and equipment assets, all buildings, and all engineering construction giving estimates of 2.3, 2.1, and 2.3, respectively and use these with the *ex ante* expected length of lives.[47] The average DBRs show that the rate of decline is slightly above the double-declining-balance rate of 2, and that machinery and equipment tend to lose their value at a slightly faster pace than do the buildings and engineering construction categories.

The resulting estimates are reproduced in Table 13.

With these estimates in hand, we now calculate the growth rate in the entire capital stock over the period from 1960 to 2000. Subperiods from 1960 to 1980 and 1980 to 2000 are also provided (Table 13).

The growth rates that are provided by the new estimates are also compared in Table 14 to the growth rates that were generated in our previous paper (Gellatly, Tanguay and Yan, 2002). The rate of growth of machinery and equipment falls from 5.56% over the period to 5.41%. Over the period from 1961 to 2000, the rate of growth of buildings increases from 3.43% to 4.10%. The rate of growth of engineering construction falls from 3.35% to 3.02%.

46. In this step and all subsequent ones, professional judgement is exercised when one or the other of the two estimates is unreasonable—using our estimates of length of life as reference.
47. With the exception of the four assets where the *ex ante* length of life was significantly smaller than the *ex post* rate. In these cases, the *ex post* rate was used.

Table 13 List of depreciation rates of all assets used in the capital stock estimates

Major group	Asset group	Asset	Definition	Estimated depreciation rate	Surveyed lives 1985 to 2001
Buildings	Commercial and institutional buildings	1004	Laboratories, research and development centers	0.066	32.4
		1012	Automotive dealerships	0.087	24.5
		1013	Office buildings	0.060	33.3
		1014	Hotels, motels and convention centers	0.059	36.0
		1015	Restaurants, fast food outlets, bars and nightclubs	0.087	23.0
		1016	Shopping centers, plazas and stores	0.070	30.7
		1018	Theatre, performing arts and cultural centers	0.067	31.8
		1019	Indoor recreational buildings	0.069	31.2
		1201	Educational buildings	0.062	34.7
		1202	Student residences	0.055	39.1
		1203	Religious buildings	0.047	45.6
		1204	Hospitals and other health centres	0.061	35.1
		1205	Nursing homes	0.060	35.6
		1206	Day care centers	0.076	27.9
		1207	Libraries	0.059	35.9
		1208	Historical sites	0.094	23.3
		1209	Penitentiaries, detention centers and courthouses	0.060	35.4
		1210	Museums, science centers and public archives	0.046	46.2
		1211	Fire stations	0.081	26.4
		1212	Post offices	0.118	18.2
		1214	Armouries, barracks, drill halls and other military type structures	0.096	22.3
		1299	Other institutional/government buildings	0.075	28.6
		1999	Other building constructions	0.071	30.0
		2201	Passenger terminals (such as air, boat, bus and rail)	0.065	32.9
		3001	Broadcasting and communication buildings	0.086	30.6
	Industrial buildings	1001	Manufacturing plants	0.089	26.6
		1006	Warehouses, refrigerated storage and freight terminals	0.068	32.2
		1007	Grain elevator and terminals	0.071	30.0
		1008	Maintenance garages, workshops and equipment storage facilities	0.084	28.0
		1009	Railway shops and engine houses	0.080	32.1
		1010	Aircraft hangars	0.096	26.7
		1011	Service stations	0.123	17.4
		1021	Farm buildings	0.095	27.0
		1022	Bunkhouses, dormitories, camp cookeries and camps	0.161	13.3
		1099	Other industrial and commercial buildings	0.085	23.9
		3401	Mine buildings	0.180	12.2
		3402	Mine buildings for beneficiation treatment of minerals (excluding smelters and refineries)	0.168	13.1
		5999	Other construction (1999/other buildings)	0.150	21.0

Table 13 List of depreciation rates of all assets used in the capital stock estimates (continued)

Major group	Asset group	Asset	Definition	Estimated depreciation rate	Surveyed lives 1985 to 2001
Machinery and equipment	Computers	6002	Computers and associated hardware	0.467	4.7
	Computerized Equipment	6401	Computerized material handling equipment	0.191	13.4
		6402	Computerized production equipment for manufacturing	0.174	12.7
		6403	Computerized communication equipment	0.225	9.5
		6410	Computerized production process - crushers and grinders	0.204	12.6
		6413	Computerized production process - other	0.176	14.6
		6499	Other computerized machinery and equipment	0.314	8.2
	Furniture equipment	6001	Office furniture and furnishing	0.235	8.3
		6003	Non-office furniture, furnishings and fixtures	0.214	9.4
	Heavy machinery	6009	Motors, generators, transformers, turbines, compressors and pumps	0.130	15.3
		6010	Heavy construction equipment*	0.172	13.9
		6011	Tractors of all types and other field equipment*	0.171	14.5
		6013	Drilling and blasting equipment	0.192	11.1
		6028	Underground load, haulage and dump equipment (such as slusher and muck cars)	0.208	10.2
	Equipment attached to building	6005	Heating, electrical, plumbing, air conditioning and refrigeration equipment	0.167	12.5
		6006	Pollution abatement and control equipment	0.151	16.7
		6007	Safety and security equipment	0.200	10.8
		6008	Sanitation equipment	0.218	10.7
	Non-computerized equipment	6601	Non-computerized material handling equipment	0.182	10.6
		6602	Non-computerized production equipment for manufacturing	0.154	14.0
		6603	Non-computerized communication equipment	0.214	11.1
		6610	Non-computerized production process - crushers and grinders	0.171	15.0
		6613	Non-computerized production process - other	0.201	12.8
	Other transport equipment	6205	Locomotives, rolling stock, street/subway cars, other rapid transit and major parts*	0.103	25.3
		6206	Ships and boats*	0.104	26.5
		6207	Aircraft, helicopter and aircraft engines*	0.082	27.9
		6299	Other transportation equipment*	0.201	12.6
	Road transport equipment	6201	Automobiles and major replacement parts*	0.280	8.1
		6202	Buses and major replacement parts*	0.149	17.4
		6203	Trucks, vans, truck tractors, truck trailers and major replacement parts*	0.227	10.6
		6204	All - terrain vehicles and major replacement parts*	0.190	11.6
	Scientific equipment	6004	Scientific, professional and medical devices	0.229	8.9
	Tooling equipment	6012	Capitalized tooling and other tools*	0.233	8.0
	Software	6021	Software, own-account	0.330	5.0
		6022	Software, pre-package	0.550	3.0
		6023	Software, custom-design	0.330	5.0
	Other machinery and equipment	6014	Salvage equipment	0.151	15.4
		6015	Industrial containers (transportable types)*	0.160	12.9
		6016	Navigational aids and weather measurement equipment	0.225	11.1
		8999	Other machinery and equipment (not specified elsewhere)	0.166	10.9

Table 13 List of depreciation rates of all assets used in the capital stock estimates (continued)

Major group	Asset group	Asset	Definition	Estimated depreciation rate	Surveyed lives 1985 to 2001
Machinery and equipment	Machinery and equipment related to electricity production	9001	Gas generators and turbines	0.130	22.9
		9002	Steam and vapour turbines	0.130	26.4
		9010	Electric motors and generators	0.130	23.9
		9011	Electric transformers, static converters and inductors	0.130	30.3
		9012	Electric switchgear and switching apparatus	0.130	28.0
		9013	Electric control and protective equipment	0.229	15.0
		9015	Measuring, checking or automatically controlling instruments and apparatus	0.233	23.0
		9091	Electricity meters	0.233	23.9
		9092	Electric water heaters	0.167	13.4
		9093	Nuclear reactor parts, fuel elements and heavy water	0.130	20.1
		9094	Hydraulic turbines	0.130	37.3
		9095	Boilers	0.166	26.2
		9099	Other machinery and equipment	0.166	16.9
	Machinery and equipment specific to mining and oil and gas production	6027	Raise borers and raise climbers	0.286	9.0
		6029	Mine hoists, cages, ropes and skips	0.286	9.0
		6411	Computerized production process – flotation and cyanidation	0.286	9.0
		6412	Computerized production process – gravitational concentration devices	0.286	9.0
		6611	Non-computerized production process – flotation and cyanidation	0.286	9.0
		6612	Non-computerized production process – gravitational concentration devices	0.286	9.0
Engineering	Engineering	1002	Oil refineries	0.118	22.6
		1003	Natural gas processing plants	0.106	25.1
		1005	Pollution, abatement and controls	0.095	23.1
		1017	Parking lots and parking garages	0.085	25.9
		1020	Outdoor recreational (such as parks, open stadiums, golf courses and ski resorts)	0.099	22.2
		1213	Waste disposal facilities	0.087	25.4
		2001	Docks, wharves, piers and terminals	0.078	28.1
		2002	Dredging and pile driving	0.104	21.2
		2003	Breakwaters	0.211	10.4
		2004	Canals and waterways	0.046	47.7
		2005	Irrigation and land reclamation projects	0.049	44.9
		2099	Other marine construction	0.071	31.0
.		2202	Highways, roads and streets (including logging roads)	0.089	24.8
		2203	Runways (including lighting)	0.073	30.0
		2204	Rail track and roadbeds	0.060	36.9
		2205	Bridges, trestles and overpasses	0.062	35.6
		2206	Tunnels	0.039	56.6
		2299	Other transportation engineering	0.073	30.0
		2401	Reservoirs (including dams)	0.056	39.0
		2402	Trunk and distribution mains for waterworks	0.077	28.4
		2412	Water pumping stations and filtrations plants	0.062	35.6
		2413	Water storage tanks	0.207	10.6
		2499	Other waterworks construction	0.092	23.9
		2601	Sewage treatment and disposal plants (including pumping stations)	0.099	22.2
		2602	Sanitary and storm sewers, trunk and collection lines and open storm ditches	0.076	28.8

Table 13 List of depreciation rates of all assets used in the capital stock estimates (concluded)

Major group	Asset group	Asset	Definition	Estimated depreciation rate	Surveyed lives 1985 to 2001
Engineering	Engineering	2603	Lagoons	0.081	27.0
		2699	Other sewage system construction	0.100	22.0
		2801	Electric power construction	0.096	23.0
		2811	Production plant - steam	0.055	40.0
		2812	Production plant - nuclear	0.051	43.0
		2813	Production plant - hydraulic	0.048	46.0
	Electrical lines	2814	Electrical transmission lines - overhead	0.051	43.0
		2815	Electrical transmission lines - underground	0.049	45.0
		2816	Electrical distribution lines - overhead	0.067	33.0
		2817	Electrical distribution lines - underground	0.063	35.0
	Engineering	2899	Other construction (not specified elsewhere)	0.063	35.0
	Communication engineering	3002	Telephone and cablevision lines	0.122	20.0
		3003	Communication towers and antennas	0.107	13.0
	Engineering	3099	Other communication engineering	0.146	16.0
		3201	Gas mains and services	0.070	38.0
		3202	Pumping stations, oil	0.296	9.0
		3203	Pumping stations. gas	0.083	32.0
		3204	Bulk storage	0.113	23.0
		3205	Oil pipelines	0.116	23.0
		3206	Gas pipelines	0.081	33.0
		3216	Exploration drilling	0.167	16.0
		3217	Development drilling	0.167	16.0
		3218	Production facilities in oil and gas engineering	0.167	16.0
		3219	Enhanced recovery projects	0.167	16.0
		3220	Drilling expenditures, pre-mining, research and other	0.167	16.0
		3221	Geological and geophysical expenditures	0.167	16.0
		3299	Other oil and gas facilities	0.074	36.0
		3403	Mining engineering - below surface (shafts, drifts, daises)	0.147	15.0
		3404	Tailing disposal systems and settling ponds	0.157	14.0
		3411	Mine site exploration	0.137	16.0
		3412	Mine site development	0.137	16.0
		3413	Exploration and deposit appraisal - off mine sites	0.137	16.0
		4999	Other engineering construction	0.122	18.0

Note: Asterisk* and bold format for asset labels indicate that we detected a problem in the anticipated *ex ante* life and replaced its estimate with *ex post* mean service life.

Source: Statistics Canada.

Table 14 Estimates of old and new growth rates in capital stock

	1961 to 2000	1961 to 1980	1981 to 2000
Total capital stock			
Old	…	…	…
New	4.08	5.12	3.09
Machinery and equipment			
Old	5.56	5.75	5.40
New	5.37	5.56	5.19
Building			
Old	3.43	4.67	2.31
New	4.05	5.54	2.65
Engineering			
Old	3.35	4.68	2.15
New	3.02	4.37	1.74

… not applicable
Source: Statistics Canada.

The primary difference between the two sets of estimates does not come from our new econometric estimates. As the previous section has shown, the new estimates are quite similar to those previously estimated—for those assets where the existence of used-asset prices permitted estimates to be derived. These assets consist primarily of machinery and equipment and a few of the building classes where there are used-asset markets.

The main difference between the capital stock growth estimates presented here and the previous set comes from revised depreciation estimates for buildings and engineering assets and for which there are few transactions. In these classes, previous estimates came from a formula that used an estimate of the *ex ante* length of life but which did not use any price information to help ascertain the DBR. And in the previous estimates, the DBR was arbitrarily chosen as having a value of 0.9—a value that implicitly presumes a shape that is quite different from the value produced by our *ex post* estimates.

These differences in the growth rate of capital stock need to be set in context. Rates of growth of capital stock in the productivity program are inserted into a formula for multifactor productivity growth that weights the rate of growth of capital stock by the share of GDP (gross domestic product) going to capital. This share is around one-third. Thus the change in the rate of growth of capital stock, weighted by the share of capital, would lead to small changes in the multifactor productivity measure.

We also present differences in the rates of growth of capital services between the old and the new estimates (Table 15). The differences between the old and the new estimates are small here as well.

Table 15 Estimates of old and new growth rates in capital services

	1961 to 2000	1961 to 1980	1981 to 2000
Total capital stock			
Old	…	…	…
New	5.30	5.85	4.77
Machinery and equipment			
Old	7.30	6.76	7.79
New	7.31	6.67	7.92
Building			
Old	3.65	5.05	2.39
New	3.93	5.38	2.55
Engineering			
Old	3.82	5.27	2.52
New	3.56	5.01	2.18

… not applicable
Source: Statistics Canada.

Table 16 Average declining balance rate for calculation of capital stock

Asset category	DBR[1]
Building construction	2.1
Engineering construction	2.3
Machinery and equipment (excluding software)	2.3

1. Declining balance rate.
Source: Statistics Canada.

8. Summary

The productivity program at Statistics Canada requires estimates of both outputs and inputs to the production process. Inputs are classified as intermediate materials, labour and capital. Intermediate materials are products that are essentially completely consumed over the course of one year in the production process. Capital, on the other hand, is provided by assets whose life extends beyond one period and whose use therefore extends over several years.

Measures of capital that are applied to the production process in any particular year require information on investments that have been made over a period of time, *and* some method of weighting investments of different vintages. Estimates of depreciation are used for the latter task. For example, the net value of capital today from an investment made last year is just the gross investment made last year minus the value by which it has declined because of use—the amount it has depreciated. Net values of investments from different years are then summed to provide an aggregate value of capital that is employed in the production process today.

Estimates of depreciation are therefore central to attempts to provide summary measures of the amount of capital that is being applied to the production process. But obtaining estimates of the rate of depreciation (the amount of depreciation in a particular year divided by its initial value) provides numerous difficulties. While depreciation is a concept that is applied directly to the accounts of companies and is used in the calculation of taxes owed to the government, the commonly used concepts are not always perceived as being those required by the productivity program. This can occur for a number of reasons—not the least of which is that depreciation allowances used for taxation purposes may differ from the 'real' rate, either because the tax system lags changes in the world or because the tax system may deliberately choose a rate that is different because it is attempting to stimulate or decelerate investment.

The statistical community has, therefore, long wrestled with alternate methods of estimating depreciation rates. Originally, estimates derived from tax codes were generally chosen in North America. These rates were then arbitrarily adjusted in order to try to accommodate what were widely perceived to be outdated estimates in the tax code. More recently, the United States made use of the prices of used assets to estimate depreciation. And the Canadian productivity accounts made use of *ex ante* estimates of the length of life derived from a survey of what was expected in the way of life upon initial investment and several arbitrary assumptions about the rate of decline of an asset (what has been referred to here as the DBR or declining-balance rate).

In 2003, the Productivity Accounts at Statistics Canada moved to make use of used-asset prices in estimating the rate of depreciation for calculating the growth of capital stock and capital services (see Harchaoui and Tarkhani, 2003). A background paper (Gellatly, Tanguay and Yan, 2002) describes how depreciation rates for a range of assets were estimated by employing used-asset prices. It also compared differences that arose from the *ex post* estimates and an alternate method that used *ex ante* estimates of expected length of life—finding that the differences between the two were not large across most asset classes.

Gellatly, Tanguay and Yan (2002) use Weibull survival models to estimate patterns of economic depreciation based on rich samples of used-asset prices and discards. Two variants of the estimation framework were proposed: a simple linear model estimated via average prices, and models that generate depreciation estimates directly from the entire sample of micro-data. The second used a maximum likelihood formulation of the price survival function that adjusted for patterns of digit preference.

The depreciation profiles generated by the econometric techniques were, on balance, accelerated, producing convex age-price curves. Declines in value early in life were apparent for many assets in the machinery and equipment class, as well as for certain structures. Evidence that rates of depreciation are constant over the service life was, on balance, mixed.

This paper extends the earlier work. It does so in several ways. First, it enlarges the database on used-asset prices and makes use of additional editing techniques on that database. This enlarges the number of observations to around 30,000. The size of this database is unique.

Second, it revisits the issue of the choice of the estimation technique. In our original version (Gellatly, Tanguay and Yan, 2002), we compared a very simple ordinary least squares model to what was referred to as "a maximum likelihood survival model" and chose the latter. In this paper, we extend our econometric techniques. We explore several other econometric techniques than were used in the original study and we investigate the differences between the different estimates. We ask if there are clear advantages of one technique over the latter, both in terms of making use of background theory, and in terms of their ability to handle different datasets. In the latter case, we use Monte Carlo simulation techniques to examine the ability of each to provide accurate estimates in the face of both misspecification of functional forms and imperfect data.

We discover that differences in the econometric estimates stemmed not so much from differences in techniques but rather from the nature of the sample that was being used. The data are not generated by a process that is necessarily random and this may have an influence on the different econometric techniques. The paper examines the nature of the data and finds evidence of non-randomness in the distributions of the used-asset prices that are generated by the survey source and corrects for this.

Differences in the econometric formulations can give rise to discordant impressions about how rapidly asset values erode over the course of service life. Therefore, we briefly discuss the advantages and disadvantages of each technique—based on theoretical and practical considerations. Some of the techniques require less precision in specifying underlying functional forms; others are more consistent but require knowledge about the functional forms. Since these considerations do not yield strong preferences for one technique over another, we also use Monte Carlo simulation techniques to discriminate among the various estimates. The simulation results suggest a slight preference for a technique that simultaneously estimates both the discard process and the price-age profile.

But when we compare the differences in the depreciation rates produced by the three different techniques, we find that, after we account for the non-random nature of the process that generated the data and reweight our sample, the differences in the estimated mean depreciation rates across the three methods are small. And the estimates of individual assets were generally not significantly different from one another. More importantly, the rates of growth of capital services associated with each estimate of the depreciation rate are quite similar. Since our purpose is to estimate the growth in capital stock and capital services as part of the process by which productivity estimates are produced, we conclude that for our purposes there is little to choose between our estimates—at least for those assets where we have an adequate number of observations.

We also compared the estimates derived from our econometric *ex post* approach to *ex ante* methods using estimates of the expected length of life of assets. We do so for two reasons. First, it is inherently interesting to know whether the two estimates yield approximately the same results. Do managers predict the length of life of their assets correctly? If accounting records are based on these *ex ante* predictions, we would like to know how accurate they are. Second, knowing whether *ex post* and *ex ante* estimates are approximately the same is important if we are to produce estimates of depreciation of those assets where we cannot do so via the *ex post* technique but can do so via the *ex ante* approach. There are a large number of fixed assets that fall in the building and engineering construction categories where we have an *ex ante* prediction of the length of life but where we do not have enough used-asset transactions to employ the *ex post* technique.

We find that the *ex ante* and *ex post* approaches are approximately the same for those assets where we have enough observations to provide estimates of both. The *ex ante* approach suffers a number of problems. Managers have to correctly forecast length of life in a changing world.

They need to have in mind an optimal maintenance schedule when they provide expectations on length of life. The *ex post* approach in turn suffers from other difficulties. Discard data can suffer from a number of imperfections—not the least of which is accuracy of recall of the original purchase price, all relevant upgrades, and the asset's age. Despite these problems, the two techniques provide remarkably similar results.

We therefore combine information from both approaches to generate depreciation rates across our asset classes. We propose a set of depreciation rates that make use of both the *ex ante* and *ex post* approaches. The *ex ante* information that is provided in Statistics Canada's surveys only pertains to the expected length of life of the asset. Derivation of a (geometric) depreciation rate from the expected life of the asset also requires a shape parameter of the rate—what is referred to as the DBR (the declining-balance rate). It is this parameter that determines how much of total life-time depreciation occurs early in life. And here we make use of information on similar assets where we have been able to estimate the *ex post* approach to infer what the DBR is likely to be.

Despite the progress that has been made in updating the database, and modifying the estimation techniques, the new growth rates in capital stock and capital services are not very different than those previously used.

Finally, it must be stressed that the adequacy of any set of depreciation estimates depends on the use to which they are being put. Statistics Canada's standard for quality is that its estimates pass a "fitness for use" test. The motivation for this paper is to produce depreciation estimates that have a degree of accuracy that is appropriate for the productivity program. Throughout this exercise, we have asked how robust our estimates of productivity growth are to the various econometric techniques employed once the data are reweighted to take into account non-randomness. In the end, we find little difference across the alternate estimates of depreciation derived from the different estimation techniques. But that is for the construction of the productivity accounts. We are not necessarily advocating their use for determining capital consumption allowances asset by asset in the tax code. For that purpose, we believe the estimates reported here might provide useful starting information—but they would need to be bolstered by case studies and other information.

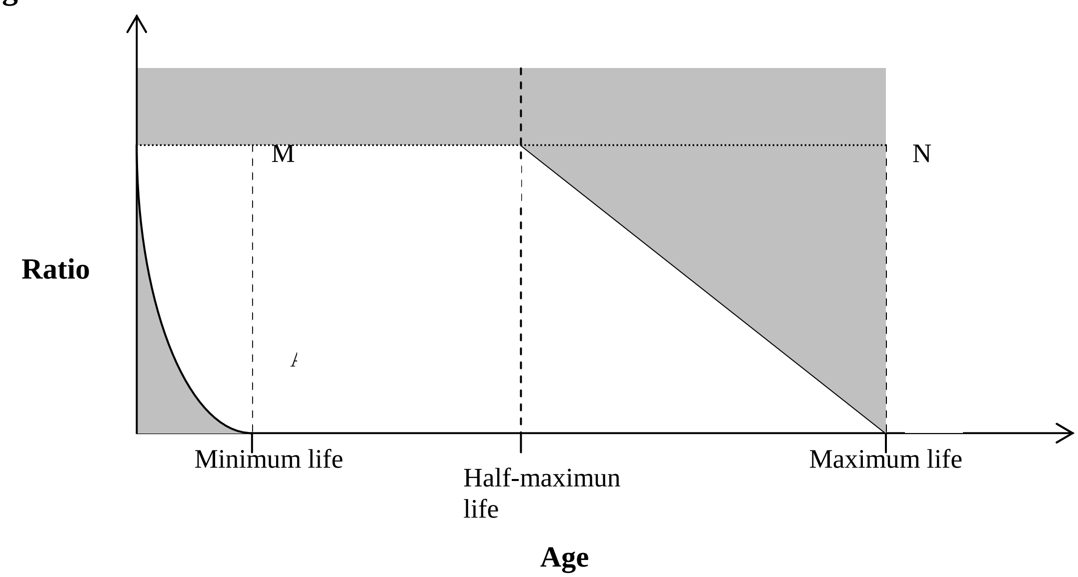
Appendix A. Edit strategy

1. Stage—Generating depreciation profiles for individual assets

In preparing the asset samples for estimation, we identified subsets of records that, relative to the majority of observations in their asset categories, exhibited either highly undervalued resale prices in early stages of service life, or highly overvalued resale prices at late stages. We removed these outlier observations from the asset samples. In principle, we could identify outliers on an asset-by-asset basis via visual examinations of age-survival plots. However, this entails a high degree of subjective judgement, and may give rise to inconsistencies in the treatment of certain types of observations across asset categories. Consequently, we start by developing a set of systematic rules. These are described below.

First, we calculate minimum and maximum survival times for a given asset using information on discards—observations with a selling price of zero, but with information on gross book value and age. We begin by assuming that the retirement age of an asset (expressed in log form) follows normal distribution. We represent this graphically in Figure A1. The lower and upper bounds correspond to the youngest and oldest retirement ages at the 10% confidence level. Minimal survival time is defined at the lower bound, and maximum survival time is defined at the upper bound weighted by an adjustment factor of 1.2.[48]

Figure A1 Outlier identification



Source: Statistics Canada.

All observations in areas A, B and C are removed from sample. Area A includes observations which have "unreasonably low" survival rates at an early age. This area is bounded by a quadratic frontier connecting point U (the "start" point[49]) and the minimum age M (i.e., the lower

48. This weighting adjustment was made in order to define roughly symmetrical rejection areas on both sides of the distribution.
49. That is, the point corresponding to a zero age and a survival rate of unity.

boundary below which zero sale prices are rejected). Area B includes observations which have "unreasonably high" survival rates well into their service life. This area is bounded by a linear frontier connecting point V (corresponding to a survival rate that equals one-half of maximum life) and point N (maximum life). Area C identifies all observations with survival rates greater than one (i.e., assets that appreciate in constant dollars).

In addition to this generic editing procedure, a number of specific edits were made on particular assets. These procedures 1) eliminated those observations on asset discards which exhibit large gross book values; the identification process in this instance is carried out on the data as a whole—rather than on an age-cohort specific basis, and results in 352 observations being dropped; 2) eliminated 56 observations which involved "abnormally low price ratios for relatively young buildings"; 3) removed 732 observations which had age values in excess of 3 times the average expected surveyed service life; 4) eliminated an additional 1,355 observations that had an excessively large gross book value (which had 'made it through' the previous filters); and 5) data from the financial and rental industries for automobiles. In total, around 8% of the original database was dropped as a result of these five specific filtering processes.

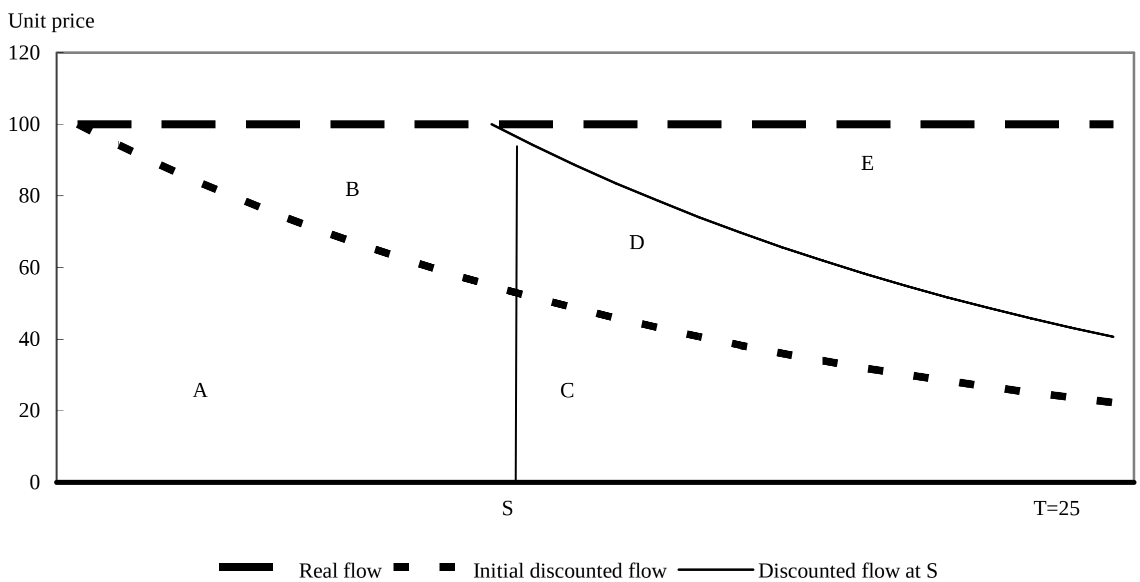
Appendix B. A note on the impact of discounting on the estimation of depreciation rates using information on the price of used assets

When making decisions as to how to evaluate the price of assets that yield a future stream of earnings, firms must decide how to discount those earnings. Market information on the price of used assets then depends on the discount rates that are used by market participants.

We will start with the simplest case as illustrated by Figure B1 where the length of life of an asset is fixed with a value of 25 years, and capital's price at any point in time reflects its future flows of services. We assume that these flows are constant over the life cycle.

If there is no discounting, the loss of value will be spread equally over T, with 1/T being the loss that occurs in each period.

Figure B1 Discounted flow of services under fixed duration



Source: Statistics Canada.

At point S, the undiscounted price ratio would be (T-s)/T or

P(S) = (E+D+C)/(E+D+C+A+B) in Figure B1.

With discounting, the flow of services provided at a distant period is worth less than those in closer periods. When there is a sale at time *s,* with discounting, the price at point S—$S_o(s)$— becomes

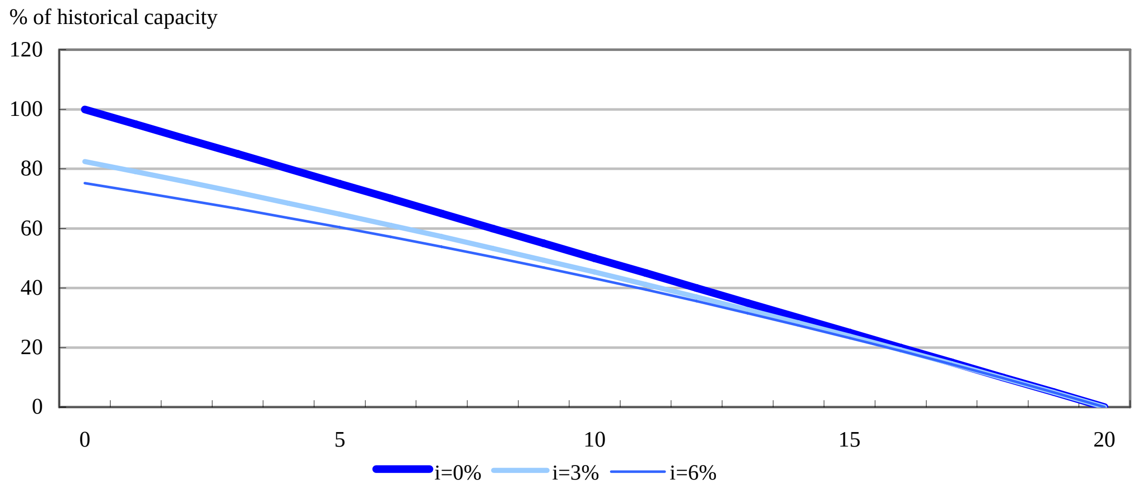
P(S) = (D+C)/(C+A) or

$$S_o(s) = \frac{e^{is}\int_s^T e^{-iy}dy}{\int_0^T e^{-iy}dy} = \frac{1-e^{-i(T-s)}}{1-e^{-iT}} \tag{B1}$$

from which it follows that $S(s) = -ln(1 - S_o(s)(1 - e^{-iT}))/iT$.

Figure B2 shows the impact of various discount rates on an asset's prices when the length of life is 20 years, the flow of services (capacity) is constant and discounting is exponential. The discounted prices converge to nondiscounted prices as remaining life approach zero.

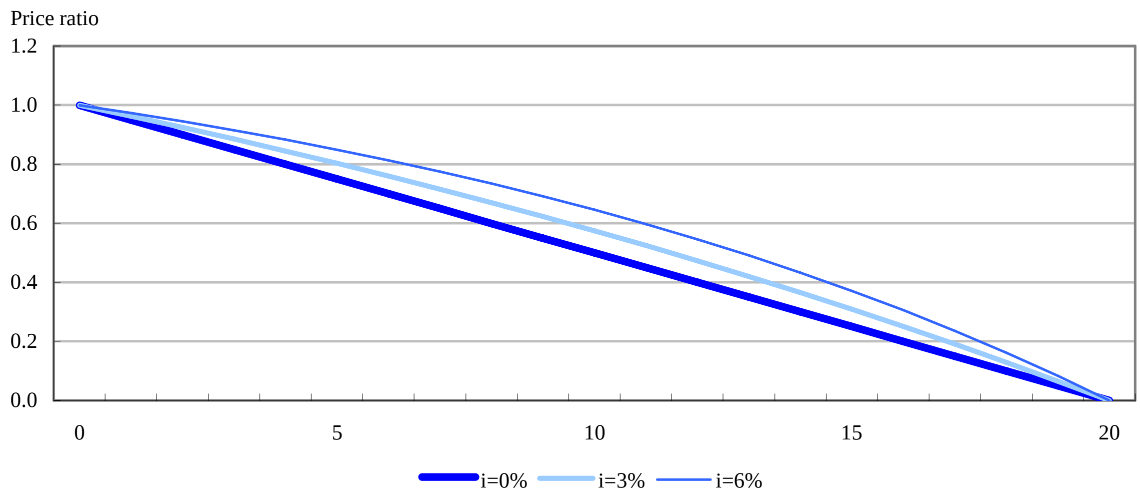
Figure B2 Equilibrium prices for an asset with a 20-year life expectancy with constant capacity and rational discounting



Source: Statistics Canada.

Figure B3 presents the price profiles when prices are rescaled to take on a value of 1 at the beginning of the period.

Figure B3 Observed price ratios under discounting for an asset with a 20-year life expectancy with constant capacity



Source: Statistics Canada.

This example assumes a fixed length of life and certainty about the flows of earnings derived from the assets. The results changes when it is recognized that the service life is random and that there is considerable risk facing a buyer because the future flow of services is not known with certainty.

We shall examine the impact of discounting when t is random. If service lives are random, economic depreciation will reflect the expected loss of value and this expectation will not be constant as in the previous example.

We provide an example for a discrete world. We compute the expected loss of value at each point in time—because this is what determines how much the price of an asset will decline over time. At year one, the expected loss is provided by the full value of assets that are discarded at year one, plus half of the value of those assets that will be discarded after 2 years,[50] plus one-third of the value of the assets discarded at three years, etc. Suppose, as before, that y is the loss of value, $f(t)$ the distribution function[51] of the asset's lives and s the point at which the asset is valued.

Therefore, at year 1, the expected loss is:

$$E(y \mid s=1) = 1 * f(t \mid t=1) + ½ * f(t \mid t=2) + 1/3 * f(t \mid t=2) + \dots$$

The expected loss at year 1 will be higher than would have occurred if the durations were not random. When we arrive at year 2, we have successfully weathered the first period and there is no more risk linked to year 1. Therefore, the term $f(t \mid t=1)$ disappears and the survivors are, on average, more efficient than the starting population since all one-year assets have been discarded.

---

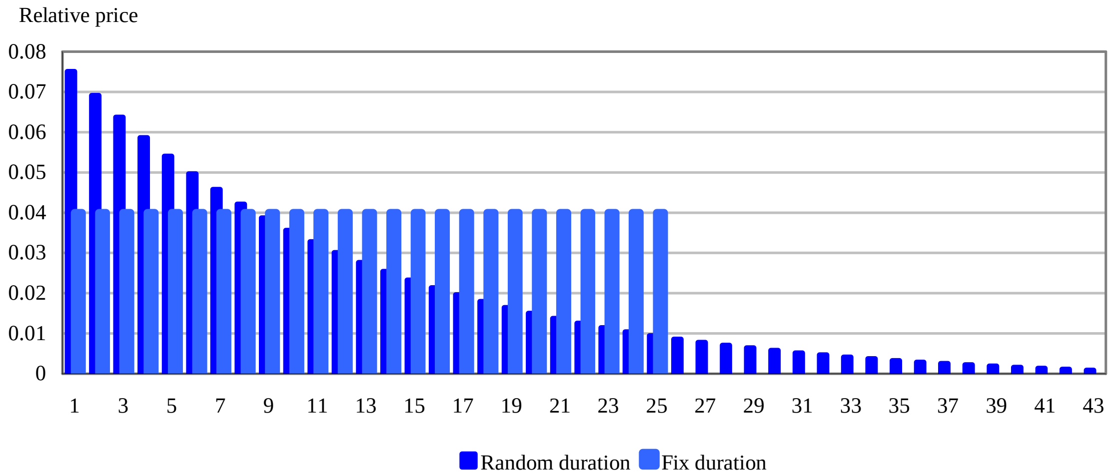
50. We assume that the value of each asset is spread equally across each year that it is alive.
51. In case of continuous process, $f(t)$ would be the density.

$$E(y \mid s = 2) = ½ * f(t \mid t = 2) + 1/3 * f(t \mid t = 2) + ...$$

Therefore, in a random world, the risk, and consequently the cost, is more concentrated at the starting period.

Figure B4 presents the difference between two assets with the same mean length of life—one that is certain and one that has a random duration. The density of the second one is a Gamma of parameter 2 and its expectation corresponds to T, which, in this case, is 25 years.

Figure B4 Expected loss of value with and without risk on duration when individuals capacity are constant

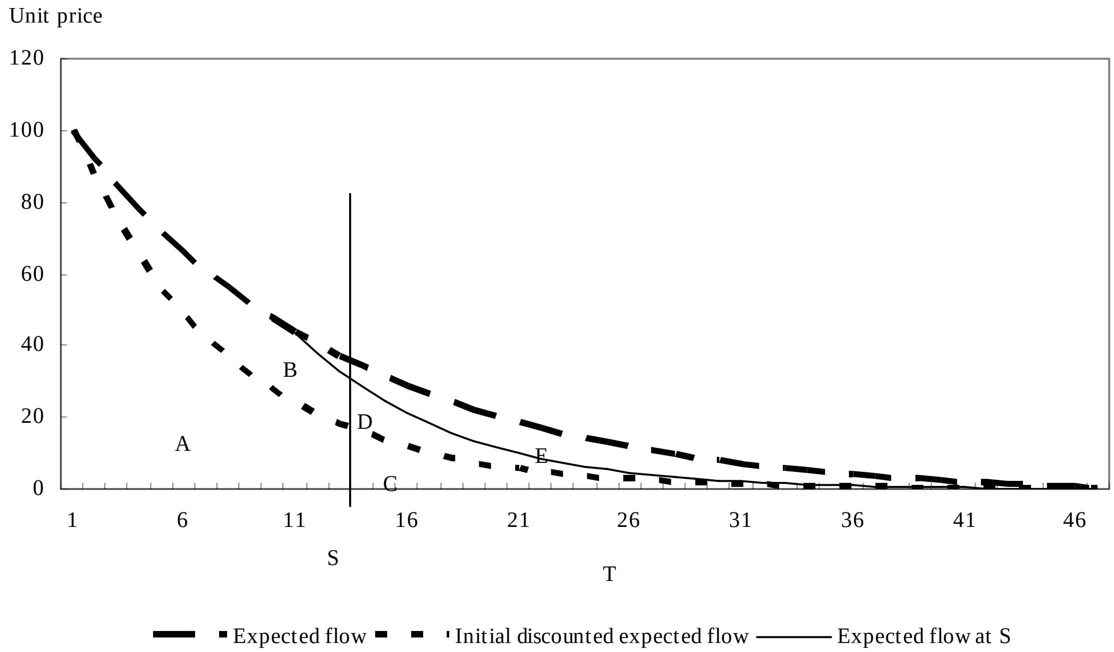


Source: Statistics Canada.

Discounting will have less of an impact in a world of uncertainty—when t is random—since the expected loss decreases dramatically over time. As Figure B4 demonstrates, the value of the flow in year 25 that is to be discounted back to year 1 is much smaller in a world of uncertainty than in the world of certainty.

Figure B5 reproduces the results for a world of uncertainty previously described in Figure B1 for a world of certainty.

Figure B5 Discounted flow of services under random durations



Sources: Statistics Canada.

As before, the price ratio without discounting is (E+D+C)/(E+D+C+A+B) while the observed ratio with discounting is (D+C)/(C+A) .

The ratios are now dominated by the value of A, making them closer, one to another, than they were in a world of certainty.

Let $f_d(y)$ denote the discounted loss of value at any point y on the scale of time. If individual capacity profiles are constant, we have:

$$f_d(y) = e^{-iy} f(y) \qquad \text{(B2)}$$

where $f(y)$, the undiscounted instantaneous loss of value is:

$$f(y) = \int_y^{+\infty} \frac{f(t)}{t} dt \qquad \text{(B3)}$$

with $f(t)$ being the density function that described the discard process.

The observed price ratio $S_o(y)$ at point s is therefore:

$$S_o(s) = \frac{e^{is}\int_s^{\infty} e^{-iy} f(y)dy}{\int_0^{\infty} e^{-iy} f(y)dy} . \qquad \text{(B4)}$$

It should be noted that if *f(y)* is exponential, discounting will have no impact on the price ratio. Indeed, we will have:

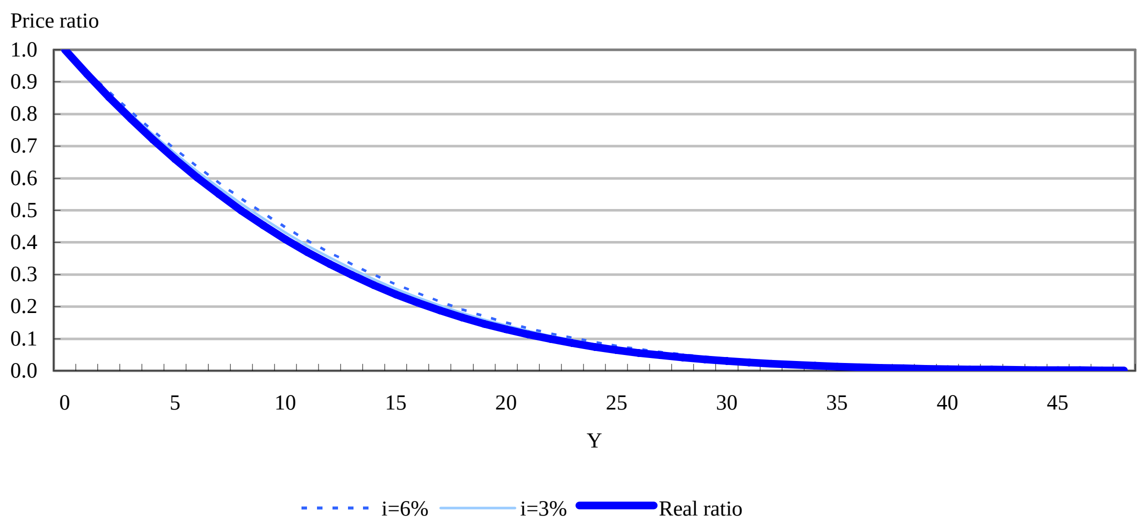
$$
\begin{aligned}
S_o(s) &= \frac{e^{is}\int_s^\infty \lambda\, e^{-iy}e^{-\lambda y}dy}{\int_0^\infty \lambda\, e^{-iy}e^{-\lambda y}dy} = \frac{e^{is}\int_s^\infty \lambda\, e^{-y(\lambda+i)}dy}{\int_0^\infty \lambda\, e^{-y(\lambda+i)}dy} \\
&= \frac{e^{is}\int_s^\infty (\lambda+i)e^{-y(\lambda+i)}dy}{\int_0^\infty (\lambda+i)e^{-y(\lambda+i)}dy} = e^{-\lambda s - is + is} = e^{-\lambda s} = S(s).
\end{aligned}
\tag{B5}
$$

If *f(y)* has any other shape than exponential, the gap between undiscounted and discounted price profiles increases with *i*, *E(t)* and the magnitude of the departure of *f(y)* from an exponential probability density function (p.d.f.)

Figure B6 presents the impact of discounting when *t* is Weibull and capacity profiles are constant. We see that the main impact of discounting has all but disappeared.

Since introducing the discount factor has a small impact on the result, we have ignored the real discount factor embodied in price formation in the estimates produced in the paper.

Figure B6 Observed price ratios under discounting for assets with an expected life of 20 years with constant individual capacities



Source: Statistics Canada.

Appendix C. Ex post weighting of price data for estimating depreciation rates[52]

1. Background

Various econometric models are used to estimate economic depreciation. A database containing information on assets discarded by companies is used for this purpose. Acquisition and resale prices are known, as well as the assets' useful life. The aim is to infer results for the overall population of assets used by companies. The representativeness of the database used must therefore be ascertained. Two problems arise:

- Survey respondents are a sub-category of companies having made investments. There is an initial selection bias, insofar as we have no information on the value of assets belonging to non-investor companies. Absent further information, the impact of this initial bias cannot be assessed and is not dealt with in this note.
- Assets that were subject to a transaction are the only ones whose price is observed. We are unaware of the extent to which their observed decline in value is representative of assets in production, whether or not subject to a transaction. We propose to alleviate, at least in part, this second source of error.

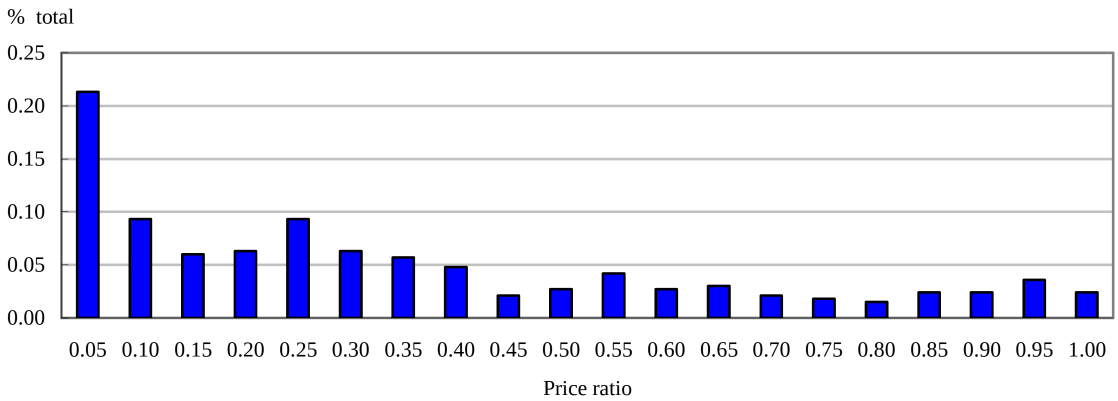
2. Issue

We are attempting to describe the relationship between prices and asset age. Once prices have been expressed in real dollar terms, their ratio[53] is deemed strictly to decline in relation to the time axis. Initially, we have no knowledge of the process causing the decline in value and no specification regarding the function describing the decline, other than the fact that it is strictly diminishing. We may, however, examine the price ratio distribution between 0 and 1.

Following is an example[54] based on manufacturing plant data.

52. See also Tanguay and Lavallée, 2006.
53. The ratio is Ps/Pi, where Pi is the initial value of the investment and Ps is the resale price at the s point in time.
54. It should be noted that two-thirds of the sample were excluded (their price was zero) and each estimation procedure takes this component into account in a particular way.

Figure C1 Distribution of observations based on price ratio – manufacturing plants



Source: Statistics Canada.

Given the aim is to infer, on the basis of available data, statistics on the population of assets in production, it is desirable that data properties match those of a random sample of the population. It should be recalled, however, that this is not the case, inasmuch as only prices subject to a transaction are available. The form of the distribution described above, having been based on a random sample, is open to speculation. We believe it should converge toward a uniform distribution. We shall, therefore, attempt to achieve a weighting that will assist us in recreating a uniform price ratio distribution. This weighting will assist us in making up for the lack of uniformity in the distribution of observations, which is liable to affect statistical analyses such as linear regression.

3. Procedure

We shall proceed from the hypothesis that price ratios may be considered empirical realizations of a survival function, the form of which is unknown. Within duration models, the survival function expresses the likelihood that an entity with a limited lifespan will survive beyond a certain point on the time axis. It thus provides the same information as the distribution function,[55] being y, a random variable describing the lifespan of a unit of value within a given asset. The value diminishes with time, so long as the asset is in use. The price ratio may accordingly be interpreted as the surviving fraction which declines progressively. This fraction is taken to be $S(y)$, resulting in:

$$S(y) = 1 - F(y)$$

where $F(y)$ is the distribution function, namely, the likelihood that a unit of value will be lost prior to point y being attained.

55. This refers to the Cumulative Density Function or CDF.

The inverse function[56] of $F(y)$ may be described based on the theorems of fundamental transformation of probability distribution.

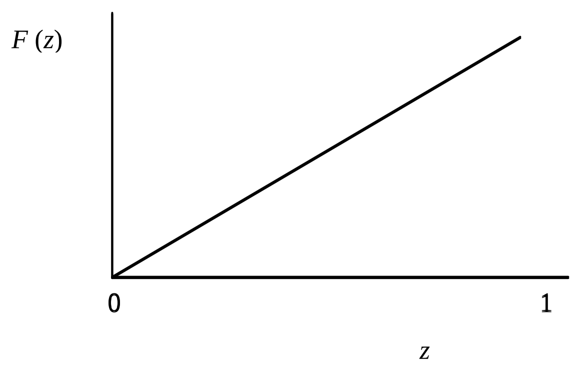
Where $z = F(y)$, this implies that $y = F^{-1}(z)$. This shows a direct match between the space of y, bounded by 0 though right-infinite, and that of F, bounded by 0 and 1.

Given the transformation is monotonic, for any value α contained between 0 and 1, the probability that z is less than α is:

$$\text{Prob}(z < \alpha) = \text{Prob}(F(y) < \alpha) = \text{Prob}(y < F^{-1}(\alpha)) = F(F^{-1}(\alpha)) = \alpha .$$

Consequently, the distribution function of z is $F(F-1(z)) = z$.

The result is a projection of a broken straight line on the first bisector between 0 and 1.



The law that generates this result is a uniform distribution between 0 and 1. The result itself is the basis of data generation processes such as the Monte Carlo simulations.[57] It was also used in generalized residual approaches, for example, the construction of specification tests.[58] Hence, any random sample constructed on the basis of empirical realizations of survival proportion data must converge towards a uniform distribution.

Regarding price data, intuition dictates that, between the time of investment and that of disposal, the entire relative price range must perforce be covered by an asset in production. During the initial period, value diminishes at a more rapid rate, resulting in a larger amount of observations of short duration. However, this is compensated by the fact that the corresponding reference on

56. See Greene, W.H. 1993. *Econometric Analysis.* Second edition. Englewood Cliffs, NJ.: Prentice Hall.
57. In fact, when generating a random sample, use is made initially of uniform distribution, to which is then applied an inverse function. See: Davidson, R. and J.G. MacKinnon. 1993. *Estimation and Inference in Econometrics*, N.Y.: Oxford University Press.
58. See Lancaster, T. 1985. "Generalized Residuals and Heterogeneous Duration Model: With Applications to the Weibull Model." *Journal of Econometrics.* 28, 1: 155–169.

the time scale is also shorter in length. For example, a drop in the initial value from 100% to 90% occurs more quickly than a decrease from 15% to 5%.

These results may easily be ascertained numerically using simulated data and will not be discussed at greater length. In fact, this would be tantamount to introducing circular reasoning. Generating random data based on any law always calls for a uniform distribution, even though this step may be not apparent, as is the case with off-the-shelf software. This type of software combines uniform distribution and inverse functions to generate random numbers. Constructing the resulting distribution functions is analogous to returning to square one.

We shall discuss, rather, how this result may be reintroduced into the database in order to restore, at least in part, properties similar to those of a random draw.

This merely requires *ex post* application to the price distribution of a weighting structure designed so that empirical data distribution is uniform within the price space. Empirical price distribution is shown as

$$\hat{F}(y) = \frac{\sum_{i=1}^{n} I_i(y)}{n}$$

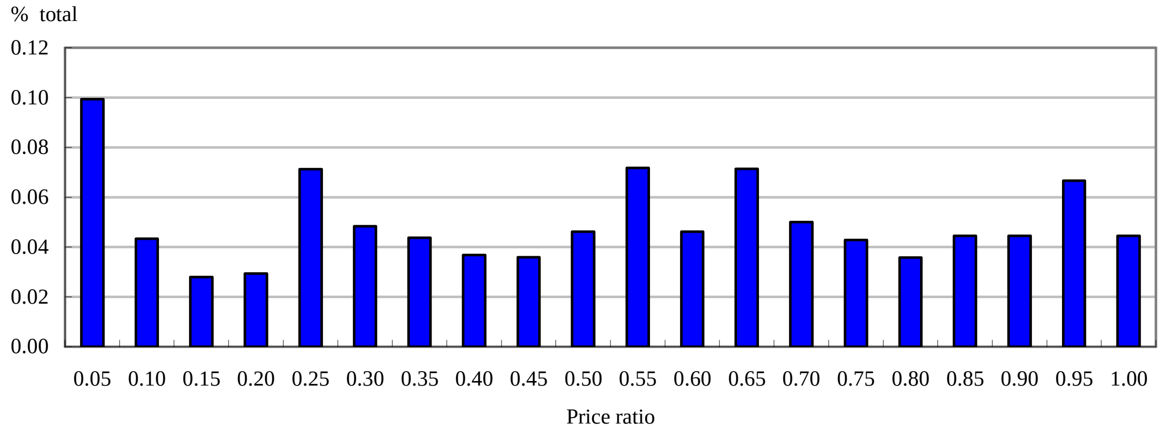
where $I_i(y) = 1$ if the measured value of observation i is less than y, otherwise it is 0, and n represents the total number of observations.

We shall simply distribute the sample over a given number k of fixed-width intervals on the time scale and assign the same probability P to each interval. Weighting w_k will then be computed within each k interval using the ratio P/P_k, ,where P_k is the empirical probability specific to the interval. This is shown as $w_i = w_k = P/P_k$, where $i \in k$. Given these weightings, the empirical weighted price distribution is represented by

$$\hat{F}_w(y) = \frac{\sum_{i=1}^{n} w_i I_i(y)}{\sum_{i=1}^{n} w_i}.$$

For example, referring once again to the histogram set out above and assuming that the sample is divided into five intervals of 0.2 in width with a P value of 20%, the following histogram, to which *ex post* weighting was applied, will be obtained.

Figure C2 Weighted distribution of observations based on price ratio – manufacturing plants' weight *ex post*



Source: Statistics Canada.

Monte Carlo simulations have demonstrated that estimations based on a non-random sample may be improved by this approach. Its main benefits are:

- its simplicity;
- the fact that it may be introduced *ex ante*, prior to the introduction of the econometric model per se. Thus, it does not require robust working hypotheses.

We shall describe the process based on an example taken from the Kelley Blue Book, a widely used source of information for estimating the depreciation of automobiles. Table C1 sets out the prices of two models of automobile of various ages between 1 and 18 years. Prices are expressed in terms of relative value compared to a new model. Further, ratio adjustment is required to take into account the probability of survival at each age. The final ratio used is therefore constructed on the basis of the product of the price ratio multiplied by the probability of survival.

Our concern is with the average depreciation rate, which could be estimated based on price regression (or a function of price) compared to age (or a function of age). However, if it is assumed that the rate is constant and geometric in shape, an average rate may be estimated for each cell using

$$1 - R^{\frac{1}{Age}}$$

where R is the relative price per age.

A statistic is then derived based on cell average.

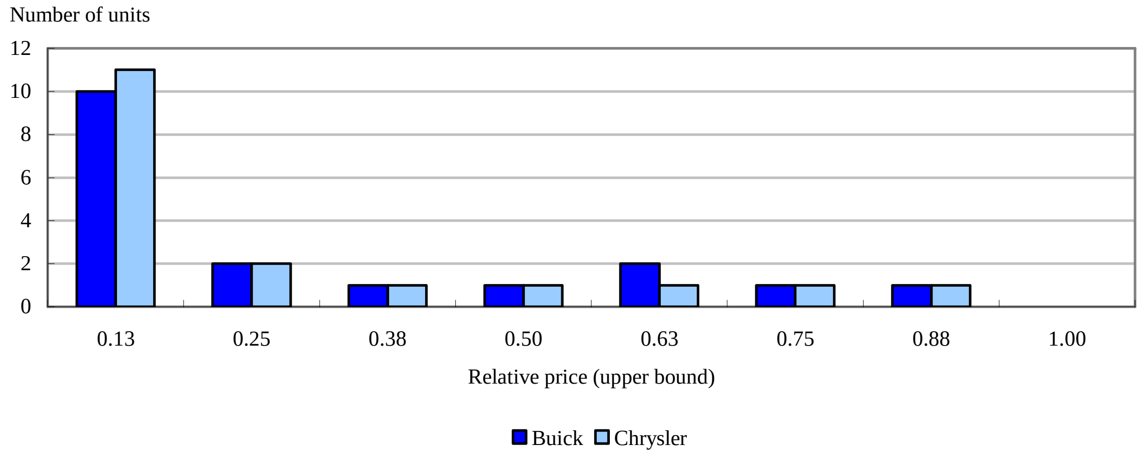
Table C1 Relative prices of two models of automobile based on the Kelley Blue Book and the average depreciation rate prior to reweighting

Years	P(t>S)*	Relative prices				Average depreciation rate	
		Excluding discards		Including discards		Including discards	
		Buick	Chrysler	Buick	Chrysler	Buick	Chrysler
1	0.9988	0.8633	0.8257	0.8622	0.8246	0.1367	0.1743
2	0.9901	0.7435	0.6801	0.7361	0.6734	0.1377	0.1753
3	0.9666	0.6410	0.5608	0.6195	0.5420	0.1378	0.1754
4	0.9220	0.5523	0.4621	0.5092	0.4261	0.1379	0.1755
5	0.8526	0.4740	0.3794	0.4042	0.3234	0.1387	0.1762
6	0.7582	0.4034	0.3087	0.3058	0.2341	0.1404	0.1779
7	0.6433	0.3391	0.2482	0.2181	0.1597	0.1432	0.1805
8	0.5164	0.2790	0.1953	0.1441	0.1009	0.1475	0.1846
9	0.3892	0.2227	0.1491	0.0867	0.0580	0.1537	0.1906
10	0.2731	0.1639	0.1050	0.0448	0.0287	0.1654	0.2018
11	0.1770	0.1261	0.0772	0.0223	0.0137	0.1716	0.2077
12	0.1051	0.0892	0.0523	0.0094	0.0055	0.1824	0.2180
13	0.0567	0.0614	0.0344	0.0035	0.0019	0.1932	0.2284
14	0.0276	0.0441	0.0236	0.0012	0.0007	0.1999	0.2347
15	0.0120	0.0320	0.0164	0.0004	0.0002	0.2050	0.2396
16	0.0046	0.0190	0.0093	0.0001	0.0000	0.2194	0.2534
17	0.0016	0.0088	0.0041	0.0000	0.0000	0.2432	0.2761
18	0.0005	0.0051	0.0023	0.0000	0.0000	0.2542	0.2867
Average	…	…	…	…	…	0.1727	0.2087

… not applicable
* Survival probability according to Micro-economic Analysis Division estimates
Source: Statistics Canada.

In the above example, it may be noted that depreciation rates vary according to age range and that they tend to increase with age. However, merely using cell average is tantamount to assigning implicitly the same weight to each age. Obviously, this would not be the distribution derived from a random sampling of automobiles in service. The Figure C3 shows the distribution of price cells for ratios contained between 0 and 1.

Figure C3 Distribution of cells used for estimating the average depreciation rate based on data from the Kelley Blue Book prior to reweighting



Source: Statistics Canada.

The reweighting technique merely involves assigning an equal weight to each relative price range. In the example shown, 18 cells are divided into 7 classes,[59] resulting in the assignment to each of a weight of 18/7. Individual weights for each year are constructed by dividing the weight of each class by the number of observations included, except for empty cells, the weight of which is zero. Table C2 shows results and impact of reweighting on derived statistics.

59. In fact, the cell structure was configured for eight classes, though the last is consistently empty.

Table C2 Relative prices of two models of automobile according to the Kelley Blue Book and average depreciation rate after reweighting

	Relative prices		Average depreciation rate		Weight *ex post*	
	Including discards		Including discards			
Years	**Buick**	**Chrysler**	**Buick**	**Chrysler**	**Buick**	**Chrysler**
1	0.8622	0.8246	0.1367	0.1743	2.5714	2.5714
2	0.7361	0.6734	0.1377	0.1753	2.5714	2.5714
3	0.6195	0.5420	0.1378	0.1754	1.2857	2.5714
4	0.5092	0.4261	0.1379	0.1755	1.2857	2.5714
5	0.4042	0.3234	0.1387	0.1762	2.5714	2.5714
6	0.3058	0.2341	0.1404	0.1779	2.5714	1.2857
7	0.2181	0.1597	0.1432	0.1805	1.2857	1.2857
8	0.1441	0.1009	0.1475	0.1846	1.2857	0.2338
9	0.0867	0.0580	0.1537	0.1906	0.2571	0.2338
10	0.0448	0.0287	0.1654	0.2018	0.2571	0.2338
11	0.0223	0.0137	0.1716	0.2077	0.2571	0.2338
12	0.0094	0.0055	0.1824	0.2180	0.2571	0.2338
13	0.0035	0.0019	0.1932	0.2284	0.2571	0.2338
14	0.0012	0.0007	0.1999	0.2347	0.2571	0.2338
15	0.0004	0.0002	0.2050	0.2396	0.2571	0.2338
16	0.0001	0.0000	0.2194	0.2534	0.2571	0.2338
17	0.0000	0.0000	0.2432	0.2761	0.2571	0.2338
18	0.0000	0.0000	0.2542	0.2867	0.2571	0.2338
Weighted average	…	…	…	…	0.1479	0.1836

… not applicable
Source: Statistics Canada.

This example clearly demonstrates the aggregate bias problems arising from regression estimates based on economic aggregates, where real distribution of units at the macro level is not taken into account. In this respect, it is quite apparent that 17- and 18-year-old units should not have the same regression weighting as those of 1-year-old units, inasmuch as the risk of loss at 1 year applies to practically all automobiles put into circulation, whereas very few will be likely to decrease in value at an advanced age. As a result, the unweighted estimate used in the example introduces an overestimation of the depreciation rate in the order of 15%.

Appendix D. Comparison of Canadian and U.S. depreciation rates

This appendix compares the depreciation rates used by the Bureau of Economic Analysis and those derived for use by Statistics Canada's productivity program.

The Bureau of the Economic Analysis (BEA) in the United States has chosen to use estimates of depreciation derived from used-asset price data—and the Bureau of Labor Statistics (BLS) has adopted much the same rates. These estimates are derived from a pioneering set of studies by Hulten and Wykoff (1981). Since the original studies that were done in the 1980s, the estimates have been extended to new assets and modified by a number of special studies—some done by academics, others by policy analysts in government (Fraumeni, 1997; Gravel, 2005). All of the studies make use of data sets that have been collected from disparate sources that yield the price of used assets in second-hand markets. And almost all of the studies have suffered from a lack of data on discard patterns and therefore have had to assume a discard pattern and arbitrarily adjust downward the positive prices observed in market transactions for the assets that were discarded at zero prices that are not observed in used-asset markets.

In contrast, the Canadian data that are used in this study have the advantage that they are derived from a similar source—a large comprehensive survey of investments done by Canadian companies—and from collecting data on dispositions that were sold at a positive price and discarded at a zero price. The data are developed from recent surveys, which collected prices of assets disposed of between 1987 and 2001.

The estimation techniques that were used in the two countries are relatively similar. The U.S. estimates basically use a two-step procedure—similar to that used in METHOD2 in this paper—except that the first stage is arbitrarily imposed using assumptions about the length of life of an asset and the distribution of discards around that life. The estimates outlined in this paper combine in a simultaneous framework both the discard function and the age-price profile.

In this note, we ask how the two sets of estimates compare. To do so, we start by estimating the implicit BEA rate from its capital stock and investment data. Since capital stock is built up from investment and capital stock data from the formula, $K_t = I_{t-1} + (1-\delta)K_{t-1}$ where K is capital, I is investment and is δ the rate of depreciation. The rate of depreciation can be deduced from BEA capital stock and investment data for the period 1987 to 2003. These rates are presented in Table D1.

In order to compare the BEA asset categories to the Statistics Canada asset classes, we have constructed a concordance between the two asset classes (Table D2). These concordances were used to construct BEA rates[60] that are then compared to the estimate of depreciation derived from the simultaneous estimation method (METHOD3) in Table D3.

60. Simple averages were used to combine BEA categories during this exercise.

Table D1 List of assets and depreciation rates for the Bureau of Economic Analysis

BEA[1] assets	BEA[1] asset names	BEA[1] rates
1	Computers and peripheral equipment	0.50
2	Software	0.49
3	Communications	0.14
4	Medical equipment and instruments	0.17
5	Non-medical instruments	0.15
6	Photocopy and related equipment	0.21
7	Office and accounting equipment	0.37
8	Fabricated metal products	0.12
9	Steam engines	0.05
10	Internal Combustion engines	0.23
11	Metalworking machinery	0.12
12	Special industrial machinery	0.11
13	General industrial equipment	0.10
14	Electric transmission and distribution	0.05
15	Light trucks (including utility vehicles)	0.22
16	Other trucks, buses and truck trailers	0.21
17	Autos	0.22
18	Aircraft	0.08
19	Ships and boats	0.06
20	Railroad equipment	0.06
21	Household furniture	0.15
22	Other furniture	0.13
23	Agricultural machinery	0.12
24	Farm tractors	0.16
25	Construction machinery	0.17
26	Construction tractors	0.18
27	Mining and oilfield machinery	0.16
28	Service industry machinery	0.18
29	Household appliances	0.18
30	Other electrical	0.20
31	Other	0.16
32	Office, including medical buildings	0.03
33	Commercial	0.03
34	Hospitals and special care	0.02
35	Manufacturing	0.03
36	Electric	0.02
37	Other power	0.02
38	Communication	0.02
39	Petroleum and natural gas	0.07
40	Mining	0.05
41	Religious	0.02
42	Educational	0.02
43	Other buildings	0.03
44	Railroads	0.02
45	Farm	0.02
46	Other	0.02

1. Bureau of Economic Analysis.
Source: Statistics Canada.

Table D2 Concordance between Canadian productivity accounts and Bureau of Economic Analysis asset categories

Canadian assets	Canadian asset names	BEA[1] asset categories
1	Office furniture, furnishing (e.g., desks, chairs)	6,7
2	Non-office furniture, furnishings and fixtures (e.g., recreational equipment, etc.)	21,22
3	Motors, generators, and transformers	9,10
4	Computer-assisted process	4,5
5	Non-computer-assisted process	4,5
6	Communication equipment	3
7	Tractors and heavy construction equipment	23 to 27
8	Computers, associated hardware and word processors	1
9	Trucks, vans, truck tractors, truck trailers and major replacement parts	15 to 17
10	Automobiles and major replacement parts	17
11	Other machinery and equipment	27 to 31
12	Electrical equipment and scientific devices	4, 5
13	Other transportation equipment	18 to 20
14	Pollution abatement and control equipment	18 to 20
15	Software	2
16	Plants for manufacturing	35
17	Farm building, maintenance garages, and warehouses	43
18	Office buildings	35
19	Shopping centers and accommodations	33
20	Passenger terminals, warehouses	33
21	Other buildings	43
22	Institutional building construction	41, 42
23	Transportation engineering construction	44
24	Electric power engineering construction	36, 37
25	Communication engineering construction	38
26	Downstream oil and gas engineering facilities	39
27	Upstream oil and gas engineering facilities	39
28	Other engineering construction	36 to 38

1. Bureau of Economic Analysis.
Source: Statistics Canada.

Table D3 Comparison of Canadian and U.S. depreciation rates

	Asset class	METHOD3	BEA[1] rates
1	Office furniture, furnishing (e.g., desks, chairs)	0.24	0.29
2	Non-office furniture, furnishings and fixtures (e.g., recreational equipment, etc.)	0.23	0.14
3	Motors, generators, and transformers	0.12	0.14
4	Computer-assisted process	0.17	0.16
5	Non-computer-assisted process	0.14	0.16
6	Communication equipment	0.23	0.14
7	Tractors and heavy construction equipment	0.16	0.16
8	Computers, associated hardware and word processors	0.45	0.50
9	Trucks, vans, truck tractors, truck trailers and major replacement parts	0.21	0.22
10	Automobiles and major replacement parts	0.27	0.22
11	Other machinery and equipment	0.17	0.18
12	Electrical equipment and scientific devices	0.18	0.16
13	Other transportation equipment	0.10	0.07
14	Pollution abatement and control equipment	0.15	0.07
15	Software	0.50	0.49
16	Plants for manufacturing	0.09	0.03
17	Farm buildings, maintenance garages, and warehouses	0.08	0.03
18	Office buildings	0.07	0.03
19	Shopping centers and accommodations	0.10	0.03
20	Passenger terminals, warehouses	0.07	0.03
21	Other buildings	0.07	0.03
22	Institutional building construction	0.07	0.02
23	Transportation engineering construction	0.05	0.02
24	Electric power engineering construction	0.09	0.02
25	Communication engineering construction	0.12	0.02
26	Downstream oil and gas engineering facilities	0.06	0.07
27	Upstream oil and gas engineering facilities	0.08	0.07
28	Other engineering construction	0.13	0.02

1. Bureau of Economic Analysis.
Sources: Statistics Canada.

On average, the Canadian depreciation rate is quite similar for the machinery and equipment asset classes. The U.S. average is 18%, the Canadian depreciation rate averaged 20%. These differences are not large.

In contrast, there is a considerable difference between the Canadian and U.S. rates for buildings and engineering construction. Here the BEA average is 3%, while the Canadian rate averaged 8%. These differences occur mainly because of the very low declining-balance rates (DBRs) that are used in the U.S. estimates—that are based on a very small number of studies. A recent U.S. study of the depreciation rate of office buildings suggest a higher rate of depreciation than is used by the BEA (Deloitte and Touche, 2002). This study suggests a length of life around 21 years, and using a DBR of around 2 would yield a depreciation rate about equal to the rates reported here for Canada.

While we do not have a large number of Canadian building and engineering categories with a sufficient number of observations on used-asset prices for estimation purposes (at least compared to the number of asset classes used for the depreciation estimates for machinery and equipment), they cover a substantial part of total investment in building and engineering and have enough observations to permit us to develop meaningful estimates of depreciation rates and DBRs (Table D3). The results show that the DBR for these long-lived assets is much higher than that derived from the historical U.S. studies and that consequently so too is the depreciation rate.

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