What Might 'Numeracy' Mean in Twenty-First Century Canada?

by

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I: Introduction

The son of an eminent Victorian noted in his father's biography that:

My father's letters to Fox show how sorely oppressed he felt by the reading for an examination. His despair over mathematics must have been profound, when he expresses a hope that Fox's silence is due to "your being ten fathoms deep in the Mathematics; and if you are, God help you, for so am I, only with this difference, I stick fast in the mud at the bottom, and there I shall remain." Mr. Herbert says: "He had, I imagine no natural turn for mathematics, and he gave up his mathematical reading before he had mastered the first part of algebra, having had a special quarrel with Surds and the Binomial Theorem."

The father himself wrote on another occasion:

During the three years which I spent at Cambridge my time was wasted, as far as the academical studies were concerned, as completely as at Edinburgh and at school. I attempted mathematics, and even went during the summer of 1828 with a private tutor to Barmouth, but I got on very slowly. The work was repugnant to me chiefly from my not being able to see any meaning in the early steps in algebra. This impatience was very foolish, and in after years I have deeply regretted that I did not proceed far enough at least to understand something of the great leading principles of mathematics, for men thus endowed seem to have an extra sense.

The individual in question, despite his anxieties about being mathematically mired, managed, to put it mildly, to make a significant contribution to nineteenth century society. In fact, it might be suggested that he makes a compelling case for the fact that a good grip on school mathematics is not a necessity for success in later life. This argument might be strengthened if it were revealed that his chosen field of study was science.

But in the seventeen decades that have passed since our anonymous hero lost his struggle with "Surds and the Binomial Theorem", the world has changed dramatically. The thought today of a prospective science student applying for entry to any university, let alone an international powerhouse like Cambridge, without being on the best of terms with the concepts and techniques of intermediate algebra is (to raise a tricky etymological question) 'absurd'.

This brings us, in a rather round-about way, to the theme of our meeting and the topic of this paper, a cluster of innocuous-looking questions about 'numeracy'. What might it mean to be numerate in the early decades of the twenty-first century in a country called Canada? What are the consequences, at both personal and societal levels, of being innumerate in Canada in the twenty-first century? How can we move a citizenry to higher levels of numeracy? How can we modify existing institutions and programs and create new ways of addressing numeracy-related priorities so that we meet the needs of Canadian citizens? This paper is one of two especially commissioned for the Roundtable to help prepare participants for active participation in the meeting. The first paper mainly addresses the "Current Challenges" component of our meeting

theme by examining numeracy in contemporary Canadian society from something of a historical perspective: "Where are we, and how have we gotten here?" This paper is intended to be complementary to the first and to concern itself largely with "Future Possibilities". Its orientation is "Where are we and where do we want to go?"

It will be apparent soon that our attempts to understand more fully the worlds of 'literacy' and 'numeracy' and their interrelations are going to take us into deep and complex waters. A constant and often frustrating reality of academic life, which cuts across all disciplines, is that for all but the most trivial of concepts the question of definition quickly becomes difficult. Some three decades ago, for instance, mathematics educators caught up in the excitement of the so-called 'new math revolution' discovered to their dismay that the bed-rock upon which they had erected their entire curriculum edifice was far from the straight-forward and simple idea they had assumed. The concept of 'set', as in 'set theory', the aforementioned foundation, turns out in fact, to be one of the most richly interconnected ideas in the English language. This is reflected by the fact that of all the words in the Oxford English Dictionary, 'set' is the one which requires the most pages to define. [Curious readers might at this point hazard a guess as to the number of pages required. The answer to this and to the identity of our mystery scientist will be given at the end of this section.] This underacknowledged complexity of a foundational idea may have been one of the contributing factors to the conspicuous and damaging failure of this recent effort to bring learners to mathematical comprehension and competence.

Nor is it the case that this phenomenon is confined to scholars dealing with abstract, socially-constructed ideas. Take for example, the neurologist, Frank Wilson, whose near-universal subject is given in the title of his recent book, *The Hand*. Wilson (1999) notes in the middle of page eight, "But what do we mean by 'the hand'? Should we define it on the basis of its visible physical boundaries? From the perspective of classical surface anatomy the hand extends". More than a page later he concludes, "a precise definition of the hand may be beyond us."

It ought not surprise us then when we scratch the surface of two concepts that are rooted in an area as fundamental to human functioning as language, that it is difficult to make short definitive statements. The number of aspects of human culture which impinge on this discussion is both stimulating and daunting.

The statements of our Victorian scientist should serve to remind us that numeracy is always closely connected to the society in which the potentially numerate live. It therefore follows that in a rapidly changing society conceptions of numeracy need to follow suit. From Charles Darwin's observations (for he was our mystery individual; see Darwin, 1958) we should also note those features of poor teaching which impeded his understanding of elementary algebra. And finally, we ought not let go unremarked his very significant insight that what we might call here, truly numerate individuals, those people who understand some of the "leading principles of mathematics" have an "extra sense". Given the direction of social and technological change in the last century and a half, this 'extra sense' is, in fact, at the heart of the question of numeracy. For the set [a term requiring no fewer than twenty-six, definitional pages in the OED] consisting of 19th century Britons, this sense may indeed have been a bonus. For 21st-century Canadians it may well be a necessity.

II: Assumptions

The Anglo-American mathematician and philosopher, Alfred North Whitehead, once observed that he seldom disagreed with the last 90% of a text. It was only at the beginning of an exposition where he claimed that explicitly or implicitly the author outlined her or his assumptions that he might disagree. With that insight in mind it seems worth making a few of the assumptions underlying this paper quite explicit.

Another 20th century philosopher, Karl Popper, once noted that the purpose of a lecture is to provoke. Presumably one can extend this view to include background papers. This document attempts to set our discussions of numeracy in a very broad context and will argue that any serious, long term deliberations about policy and implementation for programs addressing this theme must include consideration of the issues raised in this intellectually-broad, future-oriented perspective.

The period we have to prepare is limited, our time together short. Long and profound books have been, and will continue to be, written about many of the topics mentioned in the following pages. At this stage in our discussion it would be a mistake either to follow any one issue into thickets of detail, or to pretend that it does not exist. We will be seeking -- very much in keeping with the vision proposed in the latter part of the paper -- a balanced, middle way. One possible consequence of our deliberations may be some recommendations about desirable next steps or possible priorities for action. Readers perplexed, angered, intrigued or excited by any of the potentially provocative statements below may find some general directions for further reading on the issue in question in one or more of the long list of titles contained in the references. The ambitious breadth of coverage and the reality of space constraints dictate a somewhat cryptic/telegraphic style.

The next section consists of a list of factors which have influenced numeracy policy and practices in the past or have the potential to influence them in the future. Before moving to these specific factors a general comment about the importance of this discussion to human society in the 21st century is in order.

At the risk of sounding extreme, I want to argue that progress on the issue of numeracy in world culture in the next few decades should be seen not merely as a small factor in increasing a nation's economic competitiveness -- this seems to be the standard rationale -- but rather as what Whitehead called a "requisite for social progress" (1964). To begin to flesh out this claim one might start with a judgment of yet another emigré thinker with British connections, in this case the political philosopher, Isaiah Berlin. Near the end of his life, which spanned most of the last century, Berlin was asked to reflect on the key aspects of 20th-century life (1990, p. 1). He responded as follows:

There are, in my view, two factors that, above all others, have shaped human history in this century. One is the development of the natural sciences and technology, certainly the greatest success story of our time --to this, great and mounting attention has been paid from all quarters. The other, without doubt, consists in the great ideological storms that have altered the lives of virtually all mankind: the Russian Revolution and its aftermath --

totalitarian tyrannies of both right and left and the explosions of nationalism, racism, and, in places, of religious bigotry, which, interestingly enough, not one among the most perceptive social thinkers of the nineteenth century ever predicted.

And so we can think of teachers of mathematics, science and technological studies straddling these twin peaks of 20th-century history, scientific achievement and social strife. The great challenge, of course, is to teach these scientific subjects in such a way as to make learners less susceptible to arguments promoting continuing social discord.

Considering these issues from the vantage point of another of the great contemporary 'problematiques', the environmental crisis, leads us to very much the same conclusions. We are confronted with individuals who have difficulty relating to and understanding the natural world and their part in it, particularly with respect to the concept of growth. Here again the fundamental issues are those of balance, waste, fairness and justice. While it may be far from obvious, these are, in fact, all concepts which are closely related to a comprehensive vision of 21st-century numeracy.

III: The Multiple Contexts of Numeracy in Twenty-First Century Canada

Our discussions of 'numeracy' are, by necessity, located in a multidimensional contemporary world. Among the characteristics of that world pertinent to this discussion are:

- a. **Flux:** We live in a period of rapid and dramatic social and economic change (Castells, 1996). The effects of these changes are often disorienting and stressful for the individuals who are caught up in them (Gleick, 1999).
- b. **Computerization:** The driving force behind many of these changes is directly connected to the various devices of new information technology. (Borgmann, 1999; Kurzweil, 1999; Mitchell, 1999; Robertson, 1998).
- c. **Technology, Science and Mathematics:** These technological devices are, very often, based on scientific research which is, in turn, most often, highly mathematical (Bailey, 1996; Dawkins, 1998; Dyson, 1999; Feynman, 1985). For better, or for worse, the contemporary world is increasingly a mathematical one (Davis and Hersh, 1983; Hobart and Schiffman, 1998).
- d. Literacy, Numeracy and Symbols: Both literacy and numeracy can be interpreted narrowly or broadly. Much of the rest of this paper is an argument for understanding numeracy in a broad sense. Literacy narrowly defined is little more than being able to read at a rudimentary level. Literacy more broadly defined has to do with the ability to use symbol systems effectively. Narrowly defined numeracy fits entirely within a broad definition of literacy. More generally, numeracy is to mathematics as literacy is to language (Steen, 1990). Given how central language, including symbol systems like mathematics, is to the essence of being human (Cassirer, 1965), it should not be surprising that the connections between language and mathematics are subtle, difficult and deep (Pimm, 1987).

- e. **The Mathematical Worldview:** The Cartesian view of mathematics, science and epistemology has been extremely influential and highly effective from a materialistic viewpoint (Diamond, 1999; Everdell, 1997). With its emphasis on abstraction and the separation of mind and body it has been much more problematic from more broadly-based perspectives (Damasio, 1995; Davis and Hersh, 1986, Eglash, 1999; Joseph, 1991; Lakoff and Johnson, 1999; Postman, 1992; Toulmin, 1990).
- f. **Numerate Nations:** The nations which will thrive in the twenty-first century [even if one takes only a narrow economic perspective on what that might mean] will be those who have a citizenry characterized by comfort, competence and caring about the discipline of mathematics, i.e., a highly numerate population (Devlin, 1998; Rutherfod and Ahlgren, 1990).
- g. **International Tests of Achievement:** In the last decade much of the discussion about numeracy has been generated by a set of international studies of mathematics and science achievement. The 'horse race' mentality shown in the media representation of achievement in studies such as TIMSS (The Third International Mathematics and Science Study) is often used more for political than educational purposes. Perhaps the most revealing aspect of these studies are those components that look at national teaching styles. The true fact of the matter may well be that even the highest scoring nations are not serving their students well (Stigler and Hiebert, 1999).
- h. **Paradoxes -- Humans and Mathematics:** The relationships between the discipline of mathematics and various human subgroups are exceptionally diverse.
- i. **Mathematics is Rooted in Human Action:** Several leading mathematical thinkers and historians of mathematics have come to see the roots of mathematics in human actions such as collecting, comparing, rearranging, shaping, selecting and arguing (Mac Lane, 1986). The historian Grattan-Guiness begins his recent encyclopedia with a consideration of what he calls the "eight ancient roots of mathematics" which include "spacing and distancing, which led to geometry", and "balancing and weighing, which led to statics" (1998, p. 19).
- j. Humans are Natural Mathematicians: Studies are emerging (Butterworth, 1999; Dehaene, 1997) which would seem to indicate that humans are, in some quite fundamental ways, 'wired' to do mathematics. Most of the abilities discovered to this point are, in some way, numerical. It seems likely that there may, as well, be some inherent geometric predispositions connected to a sophisticated ability to perceive pattern (King, 1993). Such abilities would support some of the work done by the anthropologist, Ellen Dissanayake (1995) who sees humans as "homo aestheticus". This view argues that art is central to human evolutionary adaptation and that the aesthetic faculty is a basic psychological component of every human being (Barrow, 1996).
- k. **Some People Love Mathematics:** There are individuals, a very small segment of the total population, whose passion for mathematics parallels the sorts of feelings other individuals have for the arts. These individuals are often identified at an early age (Albers

- and Alexanderson, 1985). Their influence on the world has been particularly dramatic for the past 400 years (from Descartes to Gates through Newton and Turing); (Schechter, 2000: Winner, 1996).
- 1. **Most People are Indifferent to or Dislike Mathematics:** If different subgroups of the Canadian population were to be classified according to their feelings about the subject of mathematics on a scale from 'loathe' to 'love' with 'dislike', 'neutral' and 'like' being the intermediate positions, the majority of most populations would fall into the negative and neutral categories. There would, for some age groups, be distinct gender-related trends as well with females tending to be more negative about the subject than males [note, for instance, the statistics emerging from the testing of Ontario schoolchildren in grades 3 and 6 by the Educational Quality Assurance Organization where there is a sharp drop in "liking mathematics" by both boys and girls over the three-year period; see http://www.eqao.com], (Tobias, 1993).
- m. The Unsatisfactory History of Mathematics Teaching: The statements in the preceding section are, on the surface, paradoxical. On the one hand humans are natural mathematicians; on the other, most of them dislike the subject and are not very competent in it. How might one account for this? One obvious possibility is that our ways of teaching mathematics have, for most learners, not been effective (Davis, 1996; Higginson, 1999).
- n. **The Potential of the Devices of New Information Technology for Teaching and Learning:** In the discussions about ways in which the 21st century -- the information age -- might be significantly different from previous eras, the field of education is often seen as an area which is specially ripe for change (Papert, 1993; Tapscott, 1998). Realizing the potential of technology in education has, however, been a long and frustrating saga (Cuban, 1986). It remains to be seen if systems such as the World Wide Web can become exceptions to the rule -- followed with almost identical rhetoric in the last century by radio, typewriters, film and television -- of excessive hype followed by embarrassing failure (Stoll, 1995).
- o. Learning in Context: One major criticism of traditional mathematics instruction is that it becomes abstract and symbolic too quickly for many learners. Studies (Lave, 1988; Lave and Wenger, 1991) have revealed that it is not uncommon to have learners function quite well in a social context with precisely the same mathematical concepts they struggle with in school. The centrality of a socially meaningful context for literacy is fundamental to the theories and practices of Paulo Freire and his followers (Freire, 1996). This principle is of major importance in the numeracy field. The work of Walkerdine (1990) and other scholars has pointed to the power of parent child interactions with young children for the formation of both attitudes and comptetence. The record of programs such as Head Start in the US and the recommendations of the Mustard Commission in Canada need to be carefully considered for any full implementation of a numeracy strategy.
- p. **The Importance of Diversity in Learning:** Educational institutions have often been characterized as being exceptionally conservative. A time traveller returning to earth, it is

sometimes claimed, would be baffled by procedures in most hospitals and business settings but might well feel quite at home in many elementary and secondary school classrooms. This be as it may, it is the case that many Canadian teachers are much more open to a range of pedagogic possibilities than was the case even twenty years ago. There are at least two significant reasons for this change. One is social. Demographic changes have made many teachers aware, both directly and indirectly, of different styles of learning often linked to cultural preferences. A second influence has been academic in the form of the theory of Howard Gardner, professor of education at Harvard, about "multiple intelligences" (1999, 1993). Claiming that schools have traditionally overemphasized logical/mathematical and linguistic forms of intelligence, Gardner made a compelling argument for the recognition of musical, bodily-kinesthetic, spatial, interpersonal and intrapersonal forms of intelligence. Many educators have found Gardner's ideas both stimulating and liberating. When combined with the cultural factor this theory has legitimized in the minds of large numbers of teachers a broader range of techniques and approaches to curriculum.

- q. Ways of Seeing Mathematics -- Tool, Language and Art: Teachers open to a broader pedagogic repertoire are often more likely to entertain a wider set of possibilities for conceptions of a discipline. In this regard mathematics has been one of the area most resistance to change. One initiative which has had considerable success in this regard is the "Tomorrow's Mathematics Classroom" (Higginson and Flewelling, 1997) Vision Project. A team of educators met at Queen's University in the summer of 1996 and generated a common vision statement for education in mathematics in Canada in the 21st century. At the core of this initiative is a tripartite conception of the discipline, as a tool, as a language and as an art. [It is important to stress that this is not an 'either-or' system; at different times and in different contexts one wants to see mathematics as a tool, as a language or as an art; most learners get an excessive exposure to mathematics as a tool and very little of mathematics as an art; for many mathematicians on the other hand, it is the art face which is critical.] Roles for the teacher corresponding to the three conceptions of the discipline are: informer, facilitator and artist. The learner categories are: complier, cognizer and creator. [The four 'Vision Statements' are available electronically at: http://tilp.educ.queensu.ca/faculty/tmc/].
- r. **Recent Developments in Mathematics:** There was a saying in computer education circles in the 1980's, "Don't ask how the computer fits into your discipline, ask rather what your discipline is now that computers exist." Some fifteen years later it would appear that at least for the discipline of mathematics there was some validity in that recommendation because the subject is shifting in significant ways because of the influence of technology (Bailey, 1996). The trends have been in the direction of the visual -- there has been something of a renaissance in geometry and the computationally demanding (Casti, 1995; Cohen and Stewart, 1995; Coveney and Highfield, 1995; Flake, 1998; Gleick, 1987; Hall, 1992; Holland, 1998; McNeill and Freiberger, 1993; Penrose, 1994; Pickover, 1990; Prusinkiewicz and Lindenmayer, 1990; Schroeder, 1991; Singh, 1999).

s. **Recent Developments in Mathematical Publishing:** There has been a dramatic increase in recent years in the publication rate of quite high-level books on mathematical topics aimed at the intelligent layperson. Quite distinct from the long-standing 'recreational puzzles' literature which has quite a variable mathematics component -- in the hands of an author like Martin Gardner there can be some guite sophisticated ideas very deftly handled -- these titles (Berlinski, 1995; Berlinski, 2000; Bunch, 1999; Casti, 2000; Cole, 1997; Gazale, 1999; Gazale, 2000; Guedj, 1997; Ifrah, 1999; Kaplan, 1999; Lang, 1994; Maor, 1994; Nahin, 1998; Peterson, 1998; Seife, 2000) make few concessions to the reader and argue forcefully for the satisfactions that come from understanding profound ideas. Two of the texts above -- Kaplan and Seife -- deal, for instance, with the concept of zero in almost poetic terms. At the same time there has continued to be a number of more standard treatments of mathematics from both pure and applied directions (Brookhart, 1998; Changeux and Connes, 1995; Devlin, 1997; Willis, 1995). It is regrettable that with one significant exception (Enzensberger, 1998) these publications are not directed at the younger reader. It remains to be seen whether this is the beginning of a long-term trend or simply a marketing response to the coincidence of two outstandingly successful titles, Simon Singh's (1997) fine treatment of the solution of the 350-year old Fermat conjecture, first as a prize-winning video and then as a book, and James Gleick's (1987) superb introduction to non-linear dynamics.

IV: Four Perspectives on Numeracy

In this section of the paper four different, but related, 'interpretations' of numeracy are outlined based largely on the observations and contextual remarks of the previous sections.

(i) Perspective One - Level Zero: "Numeracy Problems? What Numeracy Problems"

This perspective is included largely for purposes of completeness and to provide something of a 'baseline'. There may, however, be people who would make a case for it. Essentially, this is a position of denial. Numeracy, at least in its 'post-school' form is not an issue from this perspective. Individuals whose school experience in mathematics was not a happy one but who have gone on to success in later life might argue this way. Consider your genetic makeup. If you are an uncoordinated young male, 1.5m tall and weighing in at 70kg then your future is unlikely to include roles as a successful jockey or as a starting centre in the *National Basketball Association*. Similarily, some folks got the math gene and others didn't. Go with the flow; do whatever it is that you can do and don't worry about it.

This perspective also introduces the '900 pound gorilla' (in the sense of a looming problem that one would prefer to ignore), namely the public school system. Currently there is something of a 'gentlemen's agreement' that the 'first shot' at numeracy is given to public schools. Should an individual, for whatever reason, not manage to emerge from that system performing up-to-speed in a numeracy sense, other agencies are free to offer their services at whatever point that individual decides that she or he would like to try to change their numeracy status. This moves us into the direction of our second perspective.

(ii) Perspective Two - Level One: "Numeracy as Remedial Arithmetic"

The most common and comfortable way to think about numeracy programs is to consider them to be nothing other than ways of dealing with the needs of learners who have, for whatever reason, not mastered the techniques of basic arithmetic. Some of the clients for these programs will have encountered fundamental arithmetic operations, and concepts like measurement and percent at an earlier period in their lives in school, probably without much success. Others may never have had that opportunity. Looked at 'locally' such programs often have much to commend them. The focus is tight, the teaching staff can often identify with the students in that their own experiences with some of the curriculum topics may have been less than captivating as well, and there are almost no 'territorial' issues. The schools have done what they can and are, for the most part, happy to have other agencies take up the challenge. This is not to say that the task is an easy one. Almost all of the pedagogic and psychological barriers which existed in school settings still need to be considered. Sometimes the issue of motivation is an exception to this rule. With a clearer picture of how curriculum topics relate to personal aspirations; often in the form of employment, many students are prepared to try much harder than they did in school.

What we have then in this case is an important enterprise, being pursued with intelligence and vigour by many individuals in a broad range of settings, often with considerable success and grateful clients. There remain, however, enough challenges to fill the agendas of many meetings such as this one. While clearly acknowledging the importance of activities of this sort, there is also a danger in allowing this conception to remain as 'the' conception of numeracy. This takes us to our third scenario.

(iii): Perspective Three - Level Two: "Numeracy as Quantitative Literacy and the Comprehension of Systems"

The major weakness of the level-one conception of numeracy is that it is too narrow. It is essentially a perspective that has remained unchanged for many decades and it 'applies' to a restricted set of employment settings and to a limited potential clientele. Basic arithmetic is not a sufficient qualification for full participation in either the social or commercial worlds in the 21st-century. Our new electronic tools permit us to grapple with the very large data sets that accompany the contemporary activities of most medium to large-sized enterprises. To flourish in these environments individuals will need a broader and deeper set of mathematical skills and appreciation. In particular, they will need to have some understanding of the principles underlying the mathematical sub-disciplines of statistics and probability and the theory of systems.

The article by Elizabeth Church (1999) published in the Globe and Mail's 'Report on Business' just over a year ago captures much of this argument from the perspective of the banking industry. The headline reads, "Bankers grapple with math gap" and the summary states, "Mathematics has become a stumbling block to advancement in the ever-changing banking industry, as employees are being required to do more than just add and subtract. But one firm hopes its training packages will upgrade worker's skills." Thus we enter the exciting world of 'life-long learning' and find ourselves on margins of the heavily-hyped, and very topical environment of 'on-line education' (training?).

In this perspective the clear and separate relation between numeracy programs and main-line educational institutions begins to unravel. The corporate sector is starting to have a larger voice in influencing school curricula generally and they, understandably, have a particular interest in literacy and numeracy. We have the example from England, for instance, where the terminology of 'numeracy' has not only entered, but now dominates the discussions of mathematics in school. From the outside, this latest 'Thatcherite' initiative would appear to be exceptionally conservative with classroom teachers being stripped of much of their autonomy. The long-term effects of this remain to be seen. The United States experience remains more similar to the Canadian one with the high-profile 'Standards' of the National Council of Teachers of Mathematics maintaining a commitment to a language-rich, problem-solving emphasis in spite of very heavy criticism. The work of Steen and his colleagues (1999, 1997, 1990, 1989; Galbraith et al, 1992) reflects a particularly well-developed rationale for a level-two approach to numeracy.

(iv): Perspective Four - Level Three: "Numeracy as Mathesis/ Patterns of Organized Knowledge"

A shift from level-one to level-two thinking about numeracy might meet the concerns of some individuals. However, even if we were to be so fortunate as to be able to implement this shift successfully, problems would remain. Just replacing 'remedial' arithmetic with what is, effectively, 'modern' arithmetic, is not sufficient as a numeracy 'ambition'. There are at least two reasons for this. First, the arithmetic perspective, while undoubtedly important, is only part of the mathematical spectrum. Perhaps equally important, but virtually ignored in most school curricula and numeracy programs, is a geometric or visual perspective. Second, the exclusively arithmetic orientation can be interpreted so that it omits any consideration of fundamental human and social concerns. The geometric, broader and more embracing, 'mathetic' [from the Greek word for organized knowledge] perspective addresses more easily some fundamental human, philosophical, aesthetic and historical questions.

Humans are, by their nature, perceptive about patterns and the symbol systems which evolve from them. This way of looking at the world from a mathematical/aesthetic perspective is potentially very powerful. One corollary, for instance, is the provision of a possible alternative to the current and very destructive materialist worldview in the form of an extended variation on the life of the mind. Over two centuries ago the German poet and philosopher, Friedrich von Schiller wrote of the "aesthetic state". The ultimate numeracy program, in the form of this level-three conception of the cultivation in learners of a set of mathematical/aesthetic abilities, might, surprisingly, be a significant step in the direction of that utopian dream.

V: Questions

To speculate so broadly is to beg questions at many levels. Our immediate queries need to be rooted in the here and now. What are our present and near-future circumstances? What needs to be done in the next few months/years? But to ask only those questions is to do a moral disservice to our children. The world we inhabit is, with all its wonders and weaknesses, to an extent unappreciated by most of its inhabitants, the consequence of some quite fundamental decisions about the sort of mathematics we have chosen to privilege. A not insignificant number of problems facing humans at the turn of the millennium might be addressed by a recasting of our mathematics/numeracy conceptions. This -- perhaps Darwin might see it as the search for the "extra sense" that he saw mathematics providing -- is the real challenge which faces us as educators and as citizens. What better time to start than in the remaining months of the UNESCO-designated World Mathematical Year?

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