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CENTRE FOR THE STUDY OF LIVING STANDARDS

# SECTORAL CONTRIBUTIONS TO LABOUR PRODUCTIVITY GROWTH IN CANADA: DOES THE CHOICE OF DECOMPOSITION FORMULA MATTER?

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# Sectoral Contributions to Labour Productivity Growth in Canada: Does the Choice of Decomposition Formula Matter?

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**Abstract**

Using three decomposition formulas (TRAD, CSLS, and GEAD), this article estimates sectoral contributions to business sector labour productivity growth in Canada during the 2000-2010 period. Although at the aggregate economy level there was substantial agreement among the three formulas – with most of business sector labour productivity growth being explained by within-sector productivity improvements –, contribution estimates varied widely at the sectoral level. In particular, there were significant differences in the estimated contributions of construction, manufacturing, and mining and oil and gas extraction. Ultimately, these differences reflect the fact that traditional decomposition formulas (TRAD and CSLS) and the GEAD formula measure distinct economic phenomena. Instead of seeing estimates constructed by the GEAD and traditional formulas as "competing" narratives, the article concludes it is more useful to see them as providing complementing stories about the role of different sectors in driving aggregate labour productivity growth.

## 1 Introduction

An important part of productivity analysis is the estimation of sectoral contributions to aggregate labour productivity growth. Several decomposition formulas have been developed for this purpose. Unfortunately, different decomposition formulas frequently yield significantly different results. The objective of this article is to compare estimates produced by three such formulas in the case of labour productivity growth in the Canadian business sector during the 2000-2010 period.

In the past 20 years, statistical agencies around the world have increasingly favoured chained indexes in place of fixed-base indexes when calculating real output. The advantages of chained indexes over their fixed-base counterparts are well established in the economics literature (see, for

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instance, Whelan (2002)), but a few points are worth highlighting here. Generally speaking, real output estimates in constant prices – i.e. calculated using fixed-base indexes – use relative prices from an arbitrarily chosen base period as the basis for comparison with all the other periods; on the other hand, real output estimates in chained prices – i.e. calculated using chained indexes – take into account the fact that relative prices are constantly changing. The growth rate of real output in constant prices depends on the choice of the base year, becoming increasingly "unbalanced" as one moves further and further away from it. This does not happen with real output in chained prices, since relative prices are updated every period. Thus, in general, chained indexes produce better quality estimates of real output.

A disadvantage of using chained indexes, however, is that real output estimates cease to be additive. When real output is computed using fixed-base Laspeyres quantity and Paasche price indexes,[1] aggregate real output is exactly equal to the sum of its individual components. This is not true when chained indexes are used. In this case, aggregate real output is equal to the sum of its individual components *only* for the chosen reference year (when real output is also equal to nominal output). The difference between the two increases as one moves away from the reference year.

Even though statistical agencies have updated their methods, productivity analysts still often use techniques that *assume* real output is additive. This is particularly true when it comes to decomposition formulas used to find sectoral or regional contributions to aggregate labour productivity growth. Labour productivity estimates are constructed using real output and labour input estimates. Since official real output estimates are now calculated mainly with chained indexes, labour productivity growth decomposition formulas that assume real output in constant prices generate sectoral or regional contributions that do not sum up *exactly* to aggregate labour productivity growth.

Tang and Wang (2004) addressed this issue with their Generalized Exactly Additive Decomposition (GEAD) formula. By taking into account changes in relative prices, the GEAD formula is able to generate sectoral contribution estimates that are perfectly additive irrespective of how real output is calculated. Despite this clear advantage over other decomposition formulas, the GEAD formula is still not widely used by productivity analysts, possibly due to some of its results being perceived as counterintuitive.

Dumagan (2012) compared the GEAD formula to a traditional decomposition formula (TRAD) that assumed real output in constant prices. He concluded that the GEAD formula produced superior estimates from both an empirical and an analytical point of view. The purpose of this

[1] A Laspeyres formula uses base period prices (or quantities, in the case of a price index) as weights, whereas a Paasche formula uses current period prices (or quantities, in the case of a price index) as weights. The Fisher formula is a geometric average of the Laspeyres and Paasche formulas. For a discussion on these and other index number formulas, as well as on many topics central to index number theory, see Diewert (1993).

article is to expand Dumagan's investigation on how estimates produced by the GEAD formula compare to those produced by traditional labour productivity growth decomposition formulas. In order to do so, a third decomposition formula is added to the mix and sectoral contribution estimates for all three formulas are analysed in the case of labour productivity growth in the Canadian business sector during the 2000-2010 period.

This article is organized as follows. Section two presents the TRAD decomposition formula, a variation of that formula developed by the Centre for the Study of Living Standards (CSLS), and the GEAD formula. Section three compares the sectoral contributions calculated by the three formulas for the Canadian business sector during the 2000-2010 period. Section four concludes.

## 2 Decomposing Aggregate Labour Productivity Growth

This section describes three formulas commonly used to calculate sectoral contributions to aggregate labour productivity growth. The first decomposition formula is the TRAD formula, which, according to Dumagan (2012), can be traced back to Denison (1962). It assumes that real output is measured in constant prices – more specifically, using fixed-base Laspeyres quantity and Paasche price indexes –, so that aggregate real output corresponds to the sum of sectoral real output. A second decomposition formula, referred to here as the CSLS decomposition, was developed by the Centre for the Study of Living Standards (CSLS) and is used in several of its articles and reports, including Sharpe (2008), Sharpe (2010) and Sharpe and Thomson (2010). The CSLS formula is essentially a variation of the TRAD formula. It also assumes real output in constant prices, but differs significantly from the TRAD formula in the way it accounts for the contribution of each sector to aggregate productivity growth. Finally, the last formula was developed by Tang and Wang (2004) (see also Diewert (2008) for an alternative formulation) and is perfectly additive regardless of how real output is measured. Following Dumagan (2012), the Tang and Wang decomposition is referred to here as the Generalized Exactly Additive Decomposition (GEAD) formula.[2]

This section is divided into four parts. The first three parts present the TRAD, CSLS, and GEAD formulas, respectively. The fourth part compares the three formulas, noting how different specifications can lead to very different sectoral contributions to aggregate labour productivity growth. This section draws from Dumagan (2012), especially regarding the choice of notation and the derivations of the TRAD and GEAD formulas.

[2]The reader should bear in mind that there are many other labour productivity decompositions that are not discussed or used in this article. See, for instance, Nordhaus (2002), Reinsdorf et al. (2002), Diewert (2008), and Reinsdorf and Yuskavage (2010).

## 2.1 TRAD Decomposition

The TRAD decomposition formula is still widely used to measure the contribution of different sectors to aggregate productivity growth (see, for example, Dekle and Vandenbroucke (2006), IMF (2006), and Usui (2011)). The underlying assumption of this formula is that real output is calculated in *constant prices* using fixed-base Laspeyres quantity and Paasche price indexes at both the aggregate and sectoral levels. When this happens, the sum of sectoral real output (net of intermediate inputs, i.e. value added) $X_t^{*i}$ is equal to the economy's real output $X_t^*$, i.e. $X_t^* = \sum_i X_t^{*i}$, where the superscript $i = 1, 2, ..., N$ denotes the sector and the subscript $t = 1, 2, ..., T$ denotes the time period. Defining labour productivity as output per unit of labour input, aggregate labour productivity $Z_t^* = \frac{X_t^*}{L_t}$ and sectoral productivity $Z_t^{*i} = \frac{X_t^{*i}}{L_t^i}$ where $L_t$ and $L_t^i$ represent labour input used in the aggregate economy and in sector $i$ (respectively) such that $L_t = \sum_i L_t^i$. In this case, since real output is additive:

$$Z_t^* = \frac{X_t^*}{L_t} = \frac{\sum_i X_t^{*i}}{L_t} = \frac{\sum_i Z_t^{*i} L_t^i}{L_t} = \sum_i Z_t^{*i} l_t^i \tag{1}$$

where $l_t^i = \frac{L_t^i}{L_t}$. Thus, aggregate labour productivity $Z_t^*$ is equal to the weighted sum of sectoral labour productivity $Z_t^{*i}$ across all $i$'s, where the weights $l_t^i$ are each sector's labour input shares. Sectoral contributions to aggregate labour productivity growth can be computed by looking at productivity changes between two periods of time:

$$G_t^* = \frac{Z_t^* - Z_{t-1}^*}{Z_{t-1}^*} = \frac{\sum_i (Z_t^{*i} l_t^i - Z_{t-1}^{*i} l_{t-1}^i)}{Z_{t-1}^*} \tag{2}$$

Adding and subtracting $l_t^i Z_{t-1}^{*i}$ to the numerator of equation (2) and collecting terms:

$$G_t^* = \frac{\sum_i [l_t^i (Z_t^{*i} - Z_{t-1}^{*i}) - Z_{t-1}^{*i} (l_t^i - l_{t-1}^i)]}{Z_{t-1}^*} \tag{3}$$

Defining $G_t^{*i} = \frac{Z_t^{*i} - Z_{t-1}^{*i}}{Z_{t-1}^{*i}}$ and noting that $\frac{Z_t^{*i}}{Z_t^*} l_t^i = \frac{Z_t^{*i}}{Z_t^*} \frac{L_t^i}{L_t} = \frac{X_t^{*i}}{X_t^*}$, $\frac{Z_{t-1}^{*i}}{Z_{t-1}^*} l_{t-1}^i G_t^{*i}$ can be added and subtracted to equation (3) so that:

$$\begin{aligned} G_t^* &= \sum_i \frac{Z_{t-1}^{*i}}{Z_{t-1}^*} [l_{t-1}^i G_t^{*i} + (l_t^i - l_{t-1}^i) + (l_t^i - l_{t-1}^i) G_t^{*i}] && (4) \\ &= \sum_i \left[ \frac{X_{t-1}^{*i}}{X_{t-1}^*} G_t^{*i} + \frac{Z_{t-1}^{*i}}{Z_{t-1}^*} \Delta l_t^i + \frac{Z_{t-1}^{*i}}{Z_{t-1}^*} \Delta l_t^i G_t^{*i} \right] && (5) \end{aligned}$$

Equation (5) is the TRAD decomposition formula. According to this formula, sectoral contributions to aggregate productivity growth can be broken down into three effects. The first term of equation (5) represents the **within-sector effect (WSE)**. [3] As the name implies, it measures the contribution to aggregate productivity growth due *solely* to productivity increases experienced by individual sectors. If sectoral labour shares remain unchanged over time ($\Delta l_t^i = 0$), the second and third terms of equation (5) equal zero and the contribution of each sector collapses to the first term, which is the sectoral labour productivity growth weighted by the sector's *real* share in aggregate real output ($\frac{X_t^{*i}}{X_t^*}$).

The other two terms of equation (5) represent two different sectoral reallocation effects. The second term of equation (5) captures the **reallocation level effect (RLE)**. [4] As Denison (1962) realized, aggregate labour productivity can increase even when sectoral labour productivity remains constant, as long as labour moves from sectors with below average labour productivity levels towards sectors with above average labour productivity levels. In the TRAD decomposition, this effect is positive when $\Delta l_t^i > 0$. The ratio between the sector's labour productivity level and the aggregate labour productivity level scales the magnitude of the effect, either increasing it (when $\frac{Z_t^{*i}}{Z_t^*} > 1$) or decreasing it (when $\frac{Z_t^{*i}}{Z_t^*} < 1$).

The third term is the **reallocation growth effect (RGE)**. [5] It captures a phenomenon similar to Baumol's cost disease (see Baumol (1967) and Baumol et al. (1985)) – that is, the propensity of labour to move towards sectors where labour productivity is stagnant or declining ($G_t^{i*} \leq 0$). In the TRAD decomposition, this effect will be positive either when labour has moved towards a sector with positive labour productivity growth ($\Delta l_t^i > 0$ and $G_t^{*i} > 0$) or when labour has moved away from a sector with negative labour productivity growth ($\Delta l_t^i < 0$ and $G_t^{*i} < 0$). The magnitude of the reallocation growth effect depends not only on the $\Delta l_t^i$ and $G_t^{*i}$ but also on the ratio between the sector's labour productivity level and the aggregate labour productivity level ($\frac{Z_t^{*i}}{Z_t^*}$).

## 2.2 CSLS Decomposition

Like the TRAD formula, the CSLS decomposition formula also assumes real output in constant prices calculated using fixed-base Laspeyres quantity and Paasche price indexes, so that sectoral real output sums up to aggregate real output. Starting from equation (1), the *absolute* change in labour productivity between two periods of time is:

[3] Tang and Wang (2004) call this effect the "pure productivity growth effect", while Dumagan (2012) labels it the "within-sector productivity growth effect"

[4] Tang and Wang (2004) name this effect the "relative size change effect", noting that it was an "analog of the Denison effect in Nordhaus (2002)" (p.427) Dumagan (2012), in turn, calls this effect the "static structural reallocation effect".

[5] Tang and Wang (2004) simply label this effect as the "interaction term", while recognizing that it is similar to the Baumol effect in Nordhaus (2002). Dumagan (2012) refers to this effect as the "dynamic structural reallocation effect".

$$\Delta Z^*_t = Z^*_t - Z^*_{t-1} = \sum\nolimits_i (Z^{*i}_t l^i_t - Z^{*i}_{t-1} l^i_{t-1}) \tag{6}$$

Adding and subtracting $Z^{*i}_{t-1} l^i_{t-1}$, $Z^{*i}_{t-1} l^i_t$, and $Z^{*i}_t l^i_{t-1}$ to equation (6) and collecting terms:

$$\begin{aligned}
\Delta Z^*_t &= \sum\nolimits_i [(Z^{*i}_t - Z^{*i}_{t-1}) l^i_{t-1} + Z^{*i}_{t-1}(l^i_t - l^i_{t-1}) + (Z^{*i}_t - Z^{*i}_{t-1})(l^i_t - l^i_{t-1})] && (7)\\
&= \sum\nolimits_i \Delta Z^{*i}_t l^i_{t-1} + \sum\nolimits_i Z^{*i}_{t-1} \Delta l^i_t + \sum\nolimits_i \Delta Z^{*i}_t \Delta l^i_t && (8)
\end{aligned}$$

Subtracting $\sum_i Z *_{t-1} \Delta l^i_t$ and $\sum_i \Delta Z *_t \Delta l^i_t$ from equation (8), it becomes becomes:[6]

$$\Delta Z^*_t = \sum\nolimits_i \Delta Z^{*i}_t l^i_{t-1} + \sum\nolimits_i (Z^{*i}_{t-1} - Z^*_{t-1}) \Delta l^i_t + \sum\nolimits_i (\Delta Z^{*i}_t - \Delta Z^*_t) \Delta l^i_t \tag{9}$$

Equation (9) is the CSLS decomposition formula. Note that, while the TRAD decomposition stated the contribution of individual sectors in terms of percentage points (relative to the per cent growth in aggregate labour productivity), the CSLS formula looks at absolute changes, i.e. increases in constant dollars per unit of labour. The CSLS formula can be easily modified so that sectoral contributions are stated in percentage points; one needs only divide both sides of equation (9) by $Z^*_{t-1}$:

$$G^*_t = \frac{Z^*_t - Z^*_{t-1}}{Z^*_{t-1}} = \frac{\sum_i \Delta Z^{*i}_t l^i_{t-1}}{Z^*_{t-1}} + \frac{\sum_i (Z^{*i}_{t-1} - Z^*_{t-1}) \Delta l^i_t}{Z^*_{t-1}} + \frac{\sum_i (\Delta Z^{*i}_t - \Delta Z^*_t) \Delta l^i_t}{Z^*_{t-1}} \tag{10}$$

Analogous to the TRAD decomposition, the first term of the CSLS decomposition formula accounts for the WSE, while the two other terms represent the RLE and the RGE, respectively. Although the CSLS formula appears to specify the WSE differently – with sectoral labour shares ($l^i_t$) instead of real output shares ($\frac{X^{*i}_t}{X^*_t}$) used as weights –, the two formulas actually have the same WSE. Recalling that $G^{*i}_t = \frac{Z^{*i}_t - Z^{*i}_{t-1}}{Z^{*i}_{t-1}}$, the first term of equation (10) can be rewritten as

$$\sum\nolimits_i \frac{\Delta Z^{*i}_t l^i_{t-1}}{Z^*_{t-1}} = \sum\nolimits_i \frac{Z^{*i}_{t-1} l^i_{t-1}}{Z^*_{t-1}} \frac{\Delta Z^{*i}_t}{Z^{*i}_{t-1}} = \sum\nolimits_i \left( \frac{\frac{X^{*i}_{t-1}}{L^i_{t-1}} \frac{L^i_{t-1}}{L_{t-1}}}{\frac{X^*_{t-1}}{L_{t-1}}} \right) G^{*i}_t = \sum\nolimits_i \left( \frac{X^{i*}_{t-1}}{X^*_{t-1}} \right) G^{*i}_t \tag{11}$$

which corresponds to the TRAD's WSE.

The crucial difference between the two formulas lies in how the RLE and RGE are specified and interpreted. Much like in the TRAD decomposition, the CSLS's RLE captures aggregate labour productivity changes caused by labour input shifts to sectors with above- or below-average labour productivity levels. In the CSLS decomposition, this effect is positive either when a sector with above-average labour productivity level ($Z^{*i}_t > Z^*_t$) experiences an increase in its labour input share

[6] As Sharpe (2010) notes, because $\sum_i \Delta l^i_t = 0$, the terms $Z^*_{t-1} \Delta l^i_t$ and $\Delta Z^*_t \Delta l^i_t$ both sum to zero when aggregated across all sectors.

($l_t^i > l_{t-1}^i$) or when a sector with below-average labour productivity level ($Z_t^{*i} < Z_t^*$) experiences a reduction in its labour input share ($l_t^i < l_{t-1}^i$). In both the TRAD and CSLS formulas, the magnitude of the RLE is a function of $\Delta l_t^i$. In the case of the TRAD formula, however, the effect depends on the *ratio* between the sectoral labour productivity level and the aggregate labour productivity level, while in the CSLS formula it depends on the *absolute difference* between the two.

The third term of the CSLS decomposition formula represents the RGE, which measures the impact of shifts of labour input to sectors with above- or below-average labour productivity growth. This effect is positive either when $\Delta l_t^i > 0$ and $Z_t^{*i} - Z_t^* > 0$ or when $\Delta l_t^i < 0$ and $Z_t^{*i} - Z_t^* < 0$. Both the TRAD's and the CSLS's RGE are a function of $\Delta l_t^i$. The difference between the two is reflected, once again, by the fact that the TRAD's RGE depends on the ratio between a sector's labour productivity level and the aggregate labour productivity level, whereas the CSLS's RGE is a function of the difference between sectoral labour productivity growth and aggregate labour productivity growth. The differences between the TRAD's and the CSLS's reallocation effects are explored in more detail in section 2.4.

## 2.3 GEAD

The TRAD and the CSLS decompositions share a common assumption: that output is measured in constant prices using fixed-base Laspeyres quantity and Paasche price indexes. This guarantees that real aggregate output is equal to the sum of real sectoral output, which leads to equation (1). The Generalized Exactly Additive Decomposition (GEAD) formula has a different starting point. Noting that:

1. Aggregate *nominal* output ($Y_t$) is always additive across sectors ($Y_t = \sum_i Y_t^i$, where $Y_t^i$ represents output of sector i);

2. By definition, $Y_t = P_t X_t$ and $Y_t^i = P_t^i X_t^i$, where $X_t$ and $X_t^i$ are aggregate and sectoral real output (respectively) and $P_t$ and $P_t^i$ are their corresponding price indexes;

then $Y_t = \sum_i Y_t^i = \sum_i P_t^i X_t^i$. Defining relative prices ($p_t^i$) as the ratio between prices in sector $i$ and economy-wide prices so that $p_t^i = \frac{P_t^i}{P_t}$, it can be noted that the relationship between aggregate labour productivity ($Z_t = \frac{X_t}{L_t}$) and sectoral labour productivity ($Z_t^i = \frac{X_t^i}{L_t^i}$) is:

$$Z_t = \frac{X_t}{L_t} = \frac{\frac{Y_t}{P_t}}{L_t} = \frac{\sum_i Y_t^i}{P_t L_t} = \frac{\sum_i P_t^i X_t^i}{P_t L_t} = \sum\nolimits_i \frac{P_t^i}{P_t} \frac{L_t^i}{L_t} \frac{X_t^i}{L_t^i} = \sum\nolimits_i p_t^i l_t^i Z_t^i \tag{12}$$

While equation (1) holds only when output is calculated using fixed-base Laspeyres quantity and Paasche price indexes, equation (13) is true *regardless* of the index number formula used to calculate real output, because $Y_t = P_t X_t$ and $Y_t^i = P_t^i X_t^i$ *always* hold. In particular equation (13) is true when chained index number formulas are used to measure real output, in which case the additivity of real output does not hold ($X_t \neq \sum_i X_t^i$).

The derivation of the GEAD formula follows similar steps to that of the TRAD formula:

$$G_t = \frac{Z_t - Z_{t-1}}{Z_{t-1}} = \frac{\sum_i (p_t^i l_t^i Z_t^i - p_{t-1}^i l_{t-1}^i Z_{t-1}^i)}{Z_{t-1}} \tag{13}$$

Adding and subtracting $p_t^i l_t^i Z_{t-1}^i$ to the numerator of equation (13):

$$G_t = \frac{\sum_i [p_t^i l_t^i (Z_t^i - Z_{t-1}^i) - Z_{t-1}^i (p_t^i l_t^i - p_{t-1}^i l_{t-1}^i)]}{Z_{t-1}} \tag{14}$$

Defining $G_t = \dfrac{Z_t^i - Z_{t-1}^i}{Z_{t-1}}$ and noting that $\dfrac{Z_t^i}{Z_t} p_t^i l_t^i = \dfrac{Z_t^i}{Z_t} \dfrac{P_t^i}{P_t} \dfrac{L_t^i}{L_t} = \dfrac{Y_t^i}{Y_t}$, $\dfrac{Z_{t-1}^i}{Z_{t-1}} p_{t-1}^i l_{t-1}^i G_t^i$ can be added and subtracted to equation (14) so that

$$\begin{aligned} G_t &= \sum\nolimits_i \frac{Z_{t-1}^i}{Z_{t-1}} [p_{t-1}^i l_{t-1}^i G_t^i + (p_t^i l_t^i - p_{t-1}^i l_{t-1}^i) + (p_t^i l_t^i - p_{t-1}^i l_{t-1}^i) G_t^i] && (15) \\ &= \sum\nolimits_i \left[ \frac{Y_{t-1}^i}{Y_{t-1}} G_t^i + \frac{Z_{t-1}^i}{Z_{t-1}} (p_t^i l_t^i - p_{t-1}^i l_{t-1}^i) + \frac{Z_{t-1}^i}{Z_{t-1}} (p_t^i l_t^i - p_{t-1}^i l_{t-1}^i) G_t^i \right] && (16) \end{aligned}$$

Equation (16) is the GEAD formula. The similarities – as well as the differences – between equation (16) and equation (5) are immediately noticeable. Much like the TRAD and CSLS decompositions, the GEAD breaks down sectoral contribution into three effects: WSE, RLE, and RGE. The first term of the GEAD represents the WSE. Notice that, while in the TRAD formula the WSE was defined as sectoral labour productivity growth weighted by the sector's *real* output share ($\dfrac{X_t^i}{X_t}$), in the GEAD sectoral growth is weighted by the sector's *nominal* output share ($\dfrac{Y_t^i}{Y_t}$).

Regarding the reallocation effects, the GEAD's RLE and RGE do not depend only on how sectoral labour input shares changed over time (as was the case in the TRAD decomposition), but also on relative price movements. Thus, a decline in a sector's labour input share can be offset by an increase in the sector's relative prices. In the GEAD formula, a sector will have a positive RLE when $p_t^i l_t^i > p_{t-1}^i l_{t-1}^i$. The RGE will be positive either when $p_t^i l_t^i > p_{t-1}^i l_{t-1}^i$ and $G_t^i > 0$ or when $p_t^i l_t^i < p_{t-1}^i l_{t-1}^i$ and $G_t^i < 0$. If there are no movements in relative prices ($p_t^i = 1.0$, for all $t$), then the GEAD formula becomes equal to the TRAD formula.[7]

[7] Diewert (2008) notes the difficulty in interpreting the GEAD's RLE and RGE as actual reallocation effects, since the effects of changes in labour shares are mixed with the effects of changes in relative prices. Thus, he proposes an alternative formulation of the GEAD formula, where the two reallocation effects are replaced by a labour input effect and a price effect. Although Diewert's formulation of the GEAD breaks down sectoral contributions into terms that have a more straightforward interpretation, the overall sectoral contributions estimated by his formula will be exactly the same as those estimated by the original GEAD formula.

### 2.4 Comparison between Different Decomposition Formulas

Table 1 compares the different specifications of the WSE, RLE, and RGE found in the TRAD, CSLS, and GEAD decomposition formulas. As mentioned previously, despite the apparent dissimilarity between the TRAD and CSLS within-sector effect, the two effects are actually the same. The differences between the TRAD and CSLS decompositions arise in their treatment of the RLE and RGE. These differences can be better understood with a simple example.

Table 1: Comparison between the TRAD, CSLS, and GEAD Decomposition Formulas

| | Within-Sector Effect | Reallocation Level Effect | Reallocation Growth Effect |
|---|---|---|---|
| TRAD | $\sum_i \frac{X^{*i}_{t-1}}{X^*_{t-1}} G^{*i}_t$ | $\sum_i \frac{Z^{*i}_{t-1}}{Z^*_{t-1}} \Delta l^i_t$ | $\sum_i \frac{Z^{*i}_{t-1}}{Z^*_{t-1}} \Delta l^i_t G^{*i}_t$ |
| CSLS | $\sum_i \frac{\Delta Z^{*i}_t l^i_{t-1}}{Z^*_{t-1}}$ | $\sum_i \frac{(Z^{*i}_{t-1} - Z^*_{t-1})\Delta l^i_t}{Z^*_{t-1}}$ | $\sum_i \frac{(\Delta Z^{*i}_t - \Delta Z^*_t)\Delta l^i_t}{Z^*_{t-1}}$ |
| GEAD | $\sum_i \frac{Y^i_{t-1}}{Y_{t-1}} G^i_t$ | $\sum_i \frac{Z^i_{t-1}}{Z_{t-1}} (p^i_t l^i_t - p^i_{t-1} l^i_{t-1})$ | $\sum_i \frac{Z^i_{t-1}}{Z_{t-1}} (p^i_t l^i_t - p^i_{t-1} l^i_{t-1}) G^i_t$ |

Table 2: Two-Sector Economy Example: Nominal Output, Real Output in Constant Prices, Prices, Labour Input, and Labour Productivity

Period 0

| | $Y^i_0$ | $\frac{Y^i_0}{Y_0}$ | $X^{*i}_0$ | $\frac{X^{*i}_0}{X^*_0}$ | $P^i_0$ | $p^i_0$ | $L^i_0$ | $l^i_0$ | $Z^{*i}_0$ |
|---|---|---|---|---|---|---|---|---|---|
| *Total Economy* | *215* | *1.00* | *200* | *1.00* | *108* | *1.00* | *50* | *1.00* | *4.0* |
| Sector A | 165 | 0.77 | 150 | 0.75 | 110 | 1.02 | 30 | 0.60 | 5.0 |
| Sector B | 50 | 0.23 | 50 | 0.25 | 100 | 0.93 | 20 | 0.40 | 2.5 |

Period 1

| | $Y^i_1$ | $\frac{Y^i_1}{Y_1}$ | $X^{*i}_1$ | $\frac{X^{*i}_1}{X^*_1}$ | $P^i_1$ | $p^i_1$ | $L^i_1$ | $l^i_1$ | $Z^{*i}_1$ |
|---|---|---|---|---|---|---|---|---|---|
| *Total Economy* | *380* [+77%] | *1.00* .. | *230* [+15%] | *1.00* .. | *165* [+54%] | *1.00* .. | *50* [+0%] | *1.00* .. | *4.6* [+15%] |
| Sector A | 300 [+82%] | 0.79 .. | 150 [+0%] | 0.65 .. | 200 [+82%] | 1.21 .. | 20 [-33%] | 0.40 .. | 7.5 [+50%] |
| Sector B | 80 [+60%] | 0.21 .. | 80 [+60%] | 0.35 .. | 100 [+0%] | 0.61 .. | 30 [+50%] | 0.60 .. | 2.7 [+7%] |

Notes:
1) All variables as defined previously;
2) Numbers in square brackets indicate per cent growth experienced between period 0 and period 1.

Assume a two sector economy, where sector A is a high productivity sector responsible for three-fourths of the economy's real output (net of intermediate goods) in period 0 and sector B is a low productivity sector that accounts for the remainder of that economy's production. In period

1, sector A experiences substantial productivity gains, prompting firms to reduce the total amount of labour input used in the sector while maintaining the same level of production. Sector B also experiences an increase in labour productivity and, at the same time, soaks up the excess labour from sector A. Table 2 summarizes these developments. Note that it is initially assumed that real output is calculated using fixed-base Laspeyres quantity and Paasche price indexes so that it is additive, i.e. the economy's real output is the sum of real output in sectors A and B.

Since labour productivity in both sectors increased, it can be expected that both sectors will have positive WSE. On the other hand, the fact that labour moved from a sector that had above-average labour productivity level and growth (sector A) to a sector that had below-average labour productivity level and growth (sector B) implies that, on aggregate, the contributions of the RLE and RGE to aggregate labour productivity growth will be negative. Lastly, it is also known that the overall magnitude of the WSE will be greater than that of the joint reallocation effects because the economy experienced positive labour productivity growth during the period.

Table 3: Two-Sector Economy Example: TRAD, CSLS, and GEAD Decompositions of Aggregate Labour Productivity Growth Using Real Output in Constant Prices

A) TRAD decomposition

| | Total Contribution | Within-Sector Effect | Reallocation Level Effect | Reallocation Growth Effect |
|---|---|---|---|---|
| | (percentage point contribution to aggregate labour productivity growth) | | | |
| *Total Economy* | *15.0* | *39.2* | *-12.5* | *-11.7* |
| Sector A | 0.0 | 37.5 | -25.0 | -12.5 |
| Sector B | 15.0 | 1.7 | 12.5 | 0.8 |

B) CSLS decomposition

| | Total Contribution | Within-Sector Effect | Reallocation Level Effect | Reallocation Growth Effect |
|---|---|---|---|---|
| | (percentage point contribution to aggregate labour productivity growth) | | | |
| *Total Economy* | *15.0* | *39.2* | *-12.5* | *-11.7* |
| Sector A | 23.0 | 37.5 | -5.0 | -9.5 |
| Sector B | -8.0 | 1.7 | -7.5 | -2.2 |

C) GEAD decomposition

| | Total Contribution | Within-Sector Effect | Reallocation Level Effect | Reallocation Growth Effect |
|---|---|---|---|---|
| | (percentage point contribution to aggregate labour productivity growth) | | | |
| *Total Economy* | *15.0* | *39.9* | *-16.8* | *-8.1* |
| Sector A | 14.0 | 38.4 | -16.2 | -8.1 |
| Sector B | 1.0 | 1.6 | -0.6 | 0.0 |

Table 3 confirms our intuition by presenting the contributions to aggregate labour productivity growth for sectors A and B according to the TRAD and CSLS formulas. As expected, sector A and sector B had positive WSE, with identical contributions to aggregate labour productivity growth in both decompositions. The *aggregate* contributions of the RLE and RGE were also identical in both formulas. However, *sectoral* RLEs and RGEs in the CSLS and TRAD formulas were very different. Sector B had positive RLE and RGE in the TRAD decomposition, but negative effects in the CSLS decomposition. Sector A had negative contributions in both decomposition formulas, but the magnitude of both the RLE and RGE were substantially greater in the TRAD decomposition.

To understand the differences between the reallocation effects in the two formulas, their different specifications must be analysed. As noted earlier, the reallocation effects in the TRAD formula are a function of the *ratio* between each sector's labour productivity level and aggregate labour productivity level. If $\Delta l_t^i > 0$ and $G_t^{*i} > 0$, then the sector's reallocation effects are *never* negative, even if the sector had below-average labour productivity level and growth. In case the sector had below-average labour productivity level and growth however, this positive contribution will *always* be offset by the negative contribution of the sector with above-average labour productivity level and growth where $\Delta l_t^i < 0$. This is exactly what happens in this example, where the positive reallocation effects of sector B were completely offset by the negative reallocation effects in sector A.

The reallocation effects in the CSLS formula work in a different way. What matters here is not the ratio, but the *difference* between a sector's labour productivity level (growth) and aggregate labour productivity level (growth). Thus, if labour moves from a sector with above-average labour productivity level (growth) to a sector with below-average labour productivity level (growth), the reallocation effects of *both* sectors are negative. In this example, the TRAD formula penalizes only the sector with above-average labour productivity level (growth) that suffered a reduction in its labour share, whereas the CSLS formula penalizes both the sector that had above-average labour productivity level (growth) and the sector that had below-average labour productivity level (growth).

Thus, while the TRAD and CSLS formulas yield the exact same results at the aggregate economy level – i.e., the sum of the WSE, RLE, and RGE *across all sectors* is always the same –, the total estimated contribution of each sector to aggregate labour productivity growth – i.e. the sum of *each* sector's WSE, RLE, and RGE – can be very different. In this example, for instance, the TRAD decomposition shows that sector B was responsible for the entirety of labour productivity growth experienced by the economy during the period, because the positive WSE of sector A was completely offset by the sector's negative reallocation effects. The story told by the CSLS formula is almost the exact opposite, with sector A explaining all of aggregate labour productivity growth. In fact, the CSLS formula says something more: aggregate labour productivity growth could have been even higher, if not for the increase in sector B's labour share.

Table 2 also presents nominal output and price deflator estimates for sectors A and B, which allows sectoral contributions to aggregate labour productivity growth to be calculated using the GEAD formula. To fully exemplify the differences between the GEAD formula and the TRAD and CSLS formulas, it is assumed that prices were set to 1.0 in a period prior to period 0, so that some relative price movements can already be observed in period 0.[8] Furthermore, from period 0 to period 1, sector A prices almost double while sector B prices remain constant. How do these movements in relative prices affect the estimated contributions to aggregate labour productivity growth?

First, note that the GEAD's WSE is determined by nominal output shares in period $t-1$ (in this case period 0) instead of real output shares. Sector A's nominal output share is slightly larger than its real output share because the price increase experienced in period 0 was more than enough to offset the lack of real output growth in the sector. Since sector A is also the sector which experienced above-average labour productivity growth, the larger weight of its nominal output share implies a stronger WSE. Not only that, this stronger contribution more than makes up for the smaller contribution of sector B's WSE, leading to an overall larger contribution of the WSE to aggregate labour productivity than the one observed in the TRAD and CSLS decompositions.

Second, a significant increase in sector A's relative price in period 1 dampens the magnitude of the sector's RLE and RGE – although (in this example) it is not enough to change the negative sign of both reallocation effects. Recall that in the TRAD decomposition, sector A's WSE was completely offset by its negative reallocation effects. This does not happen in the GEAD decomposition, where the WSE completely dominates the two reallocation effects. The GEAD decomposition also changes sector B's reallocation effects. In the TRAD decomposition, both reallocation effects were positive for sector B. In the GEAD, since sector B's prices remained constant in the two periods while sector A's prices almost doubled, sector B's relative price fell substantially, causing the sector's RLE to be slightly negative and its RGE to be zero. In the end, the overall reallocation effects in the GEAD formula had the same negative sign as in the TRAD and CSLS formulas, but with a larger magnitude, which was caused by a stronger reallocation level effect.

In all three cases discussed above, it is clear that the sum of sectoral contributions was exactly equal to aggregate labour productivity growth. In the cases of the TRAD and CSLS formulas, this also happens, but only because the real output measure used to calculate labour productivity was in constant prices. In the case of the GEAD formula, the contributions will always sum up to aggregate labour productivity growth, regardless of how real output was calculated. Table 4 shows the same economy described in Table 2, but this time real output is in chained prices, calculated using chained Fisher quantity and price indexes.

Note that real output remains the same at the sectoral level (it is assumed that this is the lowest possible level of aggregation), but not at the total economy level. In particular, the additivity of

[8] If prices were set to 1.0 in period 0, then nominal output shares would be equal to real output shares in that period, causing the contribution of the within-sector effect to be the same in all three formulas.

real output no longer holds, so that total economy real output can be above or below the sum of sectoral real output. As a consequence, real output shares no longer sum up to 1.0. Obviously, this change in total economy real output also affects implicit price deflators and aggregate labour productivity.

Table 4: Two-Sector Economy Example: Nominal Output, Real Output in Chained Prices, Prices, Labour Input, and Labour Productivity

Period 0

| | $Y_0^i$ | $\frac{Y_0^i}{Y_0}$ | $X_0^i$ | $\frac{X_0^i}{X_0}$ | $P_0^i$ | $p_0^i$ | $L_0^i$ | $l_0^i$ | $Z_0^i$ |
|---|---|---|---|---|---|---|---|---|---|
| *Total Economy* | *215* | *1.00* | *190* | *1.00* | *113* | *1.00* | *50* | *1.00* | *4.0* |
| Sector A | 165 | 0.77 | 150 | 0.79 | 110 | 0.97 | 30 | 0.60 | 5.0 |
| Sector B | 50 | 0.23 | 50 | 0.23 | 100 | 0.88 | 20 | 0.40 | 2.5 |

Period 1

| | $Y_1^i$ | $\frac{Y_1^i}{Y_1}$ | $X_1^i$ | $\frac{X_1^i}{X_1}$ | $P_1^i$ | $p_1^i$ | $L_1^i$ | $l_1^i$ | $Z_1^i$ |
|---|---|---|---|---|---|---|---|---|---|
| *Total Economy* | *380* [+77%] | *1.00* .. | *222* [+17%] | *1.00* .. | *171* [+51%] | *1.00* .. | *50* [+0%] | *1.00* .. | *4.4* [+17%] |
| Sector A | 300 [+82%] | 0.79 .. | 150 [+0%] | 0.67 .. | 200 [+82%] | 1.17 .. | 20 [-33%] | 0.40 .. | 7.5 [+50%] |
| Sector B | 80 [+60%] | 0.21 .. | 80 [+60%] | 0.36 .. | 100 [+0%] | 0.59 .. | 30 [+50%] | 0.60 .. | 2.7 [+7%] |

Notes:
1) All variables as defined previously;
2) Numbers in square brackets indicate per cent growth experienced between period 0 and period 1.

Table 5 provides estimates for the sectoral contributions to aggregate labour productivity growth according to the TRAD, CSLS, and GEAD formulas when real output is calculated in chained prices. Although the estimates change slightly from those in Table 3, the main stories are the same. In all three formulas, the positive WSE dominates the negative (joint) reallocation effects. In the TRAD formula, sector B continues being the sole contributor to aggregate labour productivity growth, while in the CSLS formula it actually hinders productivity growth. In the GEAD formula, sector A still accounted for most of the aggregate labour productivity growth, although sector B also had a positive (albeit small) contribution.

The main difference between these estimates and the ones presented in Table 3 is that the sectoral contributions generated by the TRAD and CSLS formulas no longer sum up to aggregate labour productivity growth. Because real output was calculated using chained indexes, sectoral contributions sum up to aggregate labour productivity growth *only* in the case of the GEAD formula.

Table 5: Two-Sector Economy Example: TRAD, CSLS, and GEAD Decompositions of Aggregate Labour Productivity Growth Using Real Output in Chained Prices

A) TRAD decomposition

| | Total Contribution | Within-Sector Effect | Reallocation Level Effect | Reallocation Growth Effect |
|---|---|---|---|---|
| | (percentage point contribution to aggregate labour productivity growth) | | | |
| *Total Economy* | *15.8* | *41.2* | *-13.2* | *-12.3* |
| Sector A | 0.0 | 39.5 | -26.3 | -13.2 |
| Sector B | 15.8 | 1.8 | 13.2 | 0.9 |

B) CSLS decomposition

| | Total Contribution | Within-Sector Effect | Reallocation Level Effect | Reallocation Growth Effect |
|---|---|---|---|---|
| | (percentage point contribution to aggregate labour productivity growth) | | | |
| *Total Economy* | *15.8* | *41.2* | *-13.2* | *-12.3* |
| Sector A | 23.4 | 39.5 | -6.3 | -9.7 |
| Sector B | -7.6 | 1.8 | -6.8 | -2.5 |

C) GEAD decomposition

| | Total Contribution | Within-Sector Effect | Reallocation Level Effect | Reallocation Growth Effect |
|---|---|---|---|---|
| | (percentage point contribution to aggregate labour productivity growth) | | | |
| *Total Economy* | *17.1* | *39.9* | *-15.3* | *-7.6* |
| Sector A | 15.7 | 38.4 | -15.1 | -7.6 |
| Sector B | 1.4 | 1.6 | -0.2 | 0.0 |

# 3 Sectoral Contributions to Business Sector Labour Productivity Growth in Canada, 2000-2010

This section looks at how different sectors contributed to aggregate labour productivity growth in Canada during the 2000-2010 period according to the TRAD, CSLS, and GEAD decomposition formulas. The section is divided into four parts. The first part provides an overview of the data used in this article, describing adjustments made and possible data limitations. The second part describes nominal output, real output, prices, labour input, and labour productivity trends observed in Canada during the 2000-2010 period for both the business sector as a whole and two-digit NAICS sectors. The third part analyses sectoral contribution estimates to aggregate labour productivity growth in the Canadian business sector during the 2000-2010 period. Six sets of estimates are presented, two for each of the three decomposition formulas. The fourth part discusses which of the three formulas produces “better” estimates.

### 3.1 Data

Statistics Canada constructs productivity estimates for the Canadian business sector, two-digit NAICS sectors, and three-digit NAICS subsectors. These estimates span a variable period of time, depending on which program they belong to – estimates from the multifactor productivity program currently go all the way back to 1961 and up to 2011 (CANSIM Tables 383-0021/22), while estimates from the labour productivity program span the 1997-2010 period (CANSIM Table 383-0011). Both sets of productivity estimates use real GDP in chained 2002 dollars.

Since Statistics Canada constructs real GDP estimates in both constant 2002 dollars and chained 2002 dollars, an option would have been to use Statistics Canada's official productivity numbers and calculate a new set of productivity estimates using real GDP in constant 2002 dollars. Doing so, however, could create additional problems. In particular, there is no guarantee that all the data adjustments made by Statistics Canada would have been replicable, causing our labour productivity estimates in constant prices to not be perfectly comparable to the official Statistics Canada estimates in chained prices. In order to circumvent this problem, two sets of labour productivity estimates – one in chained 2002 dollars and the other in constant 2002 dollars – were constructed by the author using Statistics Canada data on nominal GDP, real GDP, and hours worked. These estimates span the 2000-2010 period and refer to the Canadian business sector as a whole and two-digit NAICS sectors. Below, details on how these estimates were constructed are provided:

1. Statistics Canada provides nominal GDP estimates for the aggregate business sector and two-digit NAICS sectors (excluding non-business sector activities) in Canada up to 2008.[9] Two main adjustments were made to Statistics Canada numbers. First, nominal GDP from imputed rent of owner-occupied dwellings was, per usual practice, excluded from total GDP of the finance, insurance, real estate, rental and leasing (FIRE) sector. Second, the business sector components of educational services, health care and social assistance, and other services (except public administration) were combined under a single aggregate called other private services. After these adjustments were made, our nominal GDP estimates matched Statistics Canada's estimates almost perfectly.

2. Statistics Canada calculates real GDP in both constant 2002 dollars and chained 2002 dollars (CANSIM Table 379-0027). At the two-digit NAICS level, however, these estimates refer to total economic activity, i.e. they include both business sector and non-business sector activities. Thus, they are not consistent with our nominal GDP estimates, which include

[9] Nominal GDP series were expanded to 2010 by using provincial sectoral shares (CANSIM Table 379-0028) and provincial nominal GDP at basic prices (CANSIM Table 384-0001), both of which go all the way up to 2010, to calculate national estimates. Combining these two data sets provides us with nominal GDP estimates for two-digit NAICS sectors (total economic activity) and total economy. To obtain business sector estimates, we assumed that business sector shares of two-digit NAICS sectors remained at their 2008 levels.

only business sector activities. To construct real GDP estimates consistent with our nominal GDP estimates, it was assumed that business sector implicit price deflators were equal to total economy price deflators. This allowed real GDP estimates in both constant 2002 dollars and chained 2002 dollars to be constructed.[10] For most sectors, this assumption is not particularly problematic – almost the entirety of economic activity in agriculture, forestry, fishing and hunting, for instance, is considered a business sector activity. The only sectors where this can be considered a strong assumption are educational services; health care and social assistance; arts, entertainment and recreation; and other services (except public administration).

3. The labour input measure used to construct our estimates was actual hours worked, taken directly from Statistics Canada's labour productivity program. Labour productivity was calculated as real GDP per hour worked.

Overall, our labour productivity estimates were reasonably close to the official numbers. While Statistics Canada's official estimates show business sector labour productivity growing at an average annual rate of 0.76 per cent (chained 2002 dollars) during the 2000-2010 period, our estimates show an average annual growth of 0.91 per cent (chained 2002 dollars) and 0.86 per cent (constant 2002 dollars). The difference between our estimates and the official ones is caused, as mentioned above, by the use of different implicit price deflators.

### 3.2 Output, Prices, Labour Input, and Labour Productivity Trends in Canada, 2000-2010

Nominal GDP in the Canadian business sector increased at an average annual rate of 4.10 per cent during the 2000-2010 period (Table 6).[11] Nominal GDP growth was particularly strong in mining and oil and gas extraction (8.88 per cent per year) and construction (8.22 per cent per year). Due to this above-average growth, the nominal GDP of the two sectors as a share of business sector GDP increased substantially – in the case of mining and oil and gas extraction from 7.9 per cent

[10] Two specific adjustments were made to our real GDP estimates: 1) Imputed rent for owner-occupied dwellings was subtracted from total real GDP in the FIRE sector, for both the constant 2002 dollar and chained 2002 dollar estimates; 2) Real GDP for other private services (the aggregation of educational services, health care and social assistance, and other services (except public administration)) was calculated using a chained Fisher quantity index for the estimates in chained 2002 dollars and a fixed-base Laspeyres quantity index for the estimates in constant 2002 dollars.

[11] Instead of using only starting and end points of a particular series, growth rates presented in this article were calculated as the period average of annual growth rates, e.g. for the 2000-2010 period, reported growth rates are the average of the annual growth experienced in 2001, 2002, ..., 2010. This ensures that growth rates are consistent with our estimates of sectoral contributions to aggregate labour productivity growth, which were calculated on a year-to-year basis. Although sectoral contribution estimates could have been calculated for the entire 2000-2010 period, since contributions are crucially dependent of output and labour shares in the *initial period*, large swings in these shares in subsequent periods could bias estimated contributions significantly.

in 2000 to 9.7 per cent in 2010, while in construction from 6.5 per cent to 9.4 per cent. At the same time, nominal GDP in the manufacturing sector fell 1.22 per cent per year, causing a marked decline in the sector's share of business sector nominal GDP, from 24.4 per cent in 2000 to 14.4 per cent in 2010.

Table 6: Nominal GDP in Canada, Business Sector and Two-Digit NAICS Sectors, 2000-2010

| | $Y^i_{2000}$ ($ millions) | $Y^i_{2010}$ ($ millions) | Ave. Annual Growth (%) | $\frac{Y^i_{2000}}{Y_{2000}}$ (%) | $\frac{Y^i_{2010}}{Y_{2010}}$ (%) |
|---|---|---|---|---|---|
| *Business sector industries* | *769,682* | *1,141,075* | *4.10* | *100.0* | *100.0* |
| Agriculture, forestry, fishing and hunting | 22,137 | 24,996 | 1.60 | 2.9 | 2.2 |
| Mining and oil and gas extraction | 60,906 | 110,904 | 8.88 | 7.9 | 9.7 |
| Utilities | 26,242 | 33,811 | 2.63 | 3.4 | 3.0 |
| Construction | 49,648 | 107,125 | 8.22 | 6.5 | 9.4 |
| Manufacturing | 187,462 | 164,007 | -1.22 | 24.4 | 14.4 |
| Wholesale trade | 50,931 | 80,680 | 4.78 | 6.6 | 7.1 |
| Retail trade | 51,311 | 86,503 | 5.39 | 6.7 | 7.6 |
| Transportation and warehousing | 43,896 | 67,064 | 4.38 | 5.7 | 5.9 |
| Information and cultural industries | 32,150 | 53,228 | 5.18 | 4.2 | 4.7 |
| FIRE | 108,272 | 180,127 | 5.23 | 14.1 | 15.8 |
| Professional, scientific and technical services | 43,566 | 76,907 | 5.87 | 5.7 | 6.7 |
| ASWMRS | 20,367 | 37,712 | 6.44 | 2.6 | 3.3 |
| Arts, entertainment and recreation | 7,009 | 10,858 | 4.53 | 0.9 | 1.0 |
| Accommodation and food services | 23,263 | 33,621 | 3.78 | 3.0 | 2.9 |
| Other Private Services | 42,522 | 73,532 | 5.63 | 5.5 | 6.4 |

Notes:
1) All variables as defined previously;
2) FIRE - Finance, insurance, real estate, rental and leasing; ASWMRS - Administration and support, waste management and remediation services;
3) Growth rates calculated as the arithmetic average of annual growth rates.

Table 7 presents two sets of price deflators, the first one calculated using a fixed-base Paasche price index and the second one calculated using a chained Fisher price index. It is clear from the numbers that, for the time period in question, the differences between the two sets of deflators are minor. The only two sectors where there is a (potentially) significant difference between price deflators are manufacturing (which saw prices rising 0.70 per cent per year according to the constant price deflator and 0.44 per cent per year according to the chained price deflator) and wholesale trade (1.52 per cent per year vs. 1.93 per cent per year).

Almost 60 per cent of the total increase in business sector nominal GDP was caused by rising prices. At the business sector level, prices rose at an average annual rate of 2.38 per cent according to the constant price deflator and 2.33 per cent according to the chained price deflator. The sectors that saw the most significant price increases were mining and oil and gas extraction (7.77-7.79 per cent per year) and construction (4.33-4.44 per cent per year). Although prices in manufacturing increased at a much slower rate than overall business sector prices, the fact that they did increase implies that real GDP growth in the sector was even lower than nominal GDP growth.

In fact, this is exactly what Table 8 shows, with real GDP in the manufacturing sector declining 1.85 per cent per year according to the constant price deflator or 1.64 per cent per year according to the chained price deflator. It is interesting to note, also, that most of the nominal GDP growth experienced by the mining and oil and gas extraction sector was due to price increases. Real GDP growth in the sector was only 0.60-0.71 per cent per year during the 2000-2010 period. The construction sector, on the other hand, saw not only rapid price increases but also fast real GDP growth. The sector had the second highest real growth rate among all two-digit NAICS sectors (3.56-3.67 per cent per year), only behind retail trade (3.72-3.92 per cent per year). Other sectors that experienced robust real GDP growth were FIRE (3.36-3.37 per cent per year) and administration and support, waste management and remediation services (ASWMRS) (3.40-3.42 per cent per year). At the business sector level, real GDP grew at an average annual rate of 1.65-1.70 per cent during the 2000-2010 period.

While nominal GDP in mining and oil and gas extraction as a share of business sector GDP increased substantially from 2000 to 2010, the sector's real GDP share actually fell during the period, from 6.6 per cent in 2000 to 5.9-6.0 per cent (depending on which deflator is used). Manufacturing also saw a marked decline in its real GDP share, from 24.1-24.3 per cent to 16.9-17.1 per cent, although the magnitude of this decline was not as marked as it was in nominal terms. The construction sector's real GDP share, on the other hand, increased in the period, from 6.6-6.7 per cent to 7.9 per cent. Another interesting development was the increase in the FIRE sector's real GDP share, from 14.1 per cent to 16.6-16.7 per cent.

Hours worked in the Canadian business sector increased at an average annual rate of 0.78 per cent during the 2000-2010 period (Table 9). Hours worked saw particularly fast growth in mining and oil and gas extraction (3.91 per cent per year), construction (3.66 per cent per year), and ASWMRS (3.37 per cent per year). Conversely, agriculture, forestry, fishing and hunting (-2.91 per cent per year) and manufacturing (-2.50 per cent per year) experienced a decline in total hours worked.

The changes in sectoral hours worked as a share of total hours worked in the business sector during the 2000s were not as drastic as those seen in terms of nominal GDP or real GDP shares. The two most significant changes were in construction, where the hours worked share increased from 8.3 per cent in 2000 to 11.0 per cent in 2010, and manufacturing, where the share decreased from 18.3 per cent to 13.1 per cent.

Using our two sets of real GDP estimates and hours worked, labour productivity estimates were constructed for the business sector and two-digit NAICS sectors in Canada from 2000 to 2010 (Table 10). During the period, labour productivity in the Canadian business sector grew at an average annual rate of 0.86 per cent according to the constant 2002 dollar estimates – from \$35.57 per hour in 2000 to \$ 38.74 per hour in 2010 – and 0.91 per cent according to the chained 2002

Table 7: Implicit Price Deflators for Canada, Business Sector and Two-Digit NAICS Sectors, 2000-2010

A) Constant 2002 Dollars

| | $P^i_{2000}$ (2002 = 100) | $P^i_{2010}$ (2002 = 100) | Ave. Annual Growth (%) | $p^i_{2000}$ (%) | $p^i_{2010}$ (%) |
|---|---|---|---|---|---|
| *Business sector industries* | *98.0* | *123.7* | *2.38* | *100.0* | *100.0* |
| Agriculture, forestry, fishing and hunting | 84.1 | 87.1 | 0.63 | 85.9 | 70.4 |
| Mining and oil and gas extraction | 116.7 | 201.8 | 7.79 | 119.1 | 163.2 |
| Utilities | 97.5 | 111.2 | 1.39 | 99.5 | 89.9 |
| Construction | 96.1 | 147.6 | 4.44 | 98.1 | 119.3 |
| Manufacturing | 98.3 | 105.3 | 0.70 | 100.4 | 85.1 |
| Wholesale trade | 99.4 | 115.4 | 1.52 | 101.4 | 93.3 |
| Retail trade | 99.1 | 114.0 | 1.42 | 101.2 | 92.2 |
| Transportation and warehousing | 94.1 | 122.6 | 2.69 | 96.0 | 99.1 |
| Information and cultural industries | 100.1 | 123.4 | 2.12 | 102.1 | 99.7 |
| FIRE | 97.6 | 116.8 | 1.81 | 99.6 | 94.5 |
| Professional, scientific and technical services | 95.3 | 128.3 | 3.03 | 97.2 | 103.8 |
| ASWMRS | 93.9 | 124.8 | 2.90 | 95.8 | 100.9 |
| Arts, entertainment and recreation | 91.8 | 122.5 | 2.95 | 93.7 | 99.0 |
| Accommodation and food services | 95.3 | 127.8 | 2.98 | 97.3 | 103.3 |
| Other Private Services | 93.2 | 128.9 | 3.30 | 95.1 | 104.2 |

B) Chained 2002 Dollars

| | $P^i_{2000}$ (2002 = 100) | $P^i_{2010}$ (2002 = 100) | Ave. Annual Growth (%) | $p^i_{2000}$ (%) | $p^i_{2010}$ (%) |
|---|---|---|---|---|---|
| *Business sector industries* | *98.0* | *123.2* | *2.33* | *100.0* | *100.0* |
| Agriculture, forestry, fishing and hunting | 86.6 | 90.6 | 0.70 | 88.4 | 73.5 |
| Mining and oil and gas extraction | 118.2 | 201.8 | 7.77 | 120.6 | 163.8 |
| Utilities | 95.2 | 110.5 | 1.58 | 97.1 | 89.7 |
| Construction | 95.9 | 145.8 | 4.33 | 97.8 | 118.3 |
| Manufacturing | 99.2 | 103.6 | 0.44 | 101.2 | 84.1 |
| Wholesale trade | 97.0 | 117.3 | 1.93 | 98.9 | 95.2 |
| Retail trade | 97.7 | 114.5 | 1.61 | 99.6 | 92.9 |
| Transportation and warehousing | 94.5 | 122.7 | 2.66 | 96.3 | 99.6 |
| Information and cultural industries | 99.7 | 123.5 | 2.18 | 101.7 | 100.3 |
| FIRE | 98.0 | 117.1 | 1.80 | 99.9 | 95.1 |
| Professional, scientific and technical services | 94.6 | 129.0 | 3.16 | 96.5 | 104.7 |
| ASWMRS | 93.6 | 124.7 | 2.92 | 95.5 | 101.2 |
| Arts, entertainment and recreation | 91.4 | 122.5 | 2.99 | 93.3 | 99.5 |
| Accommodation and food services | 95.5 | 127.4 | 2.93 | 97.4 | 103.4 |
| Other Private Services | 93.2 | 129.8 | 3.37 | 95.1 | 105.3 |

Notes:
1) All variables as defined previously;
2) FIRE - Finance, insurance, real estate, rental and leasing; ASWMRS - Administration and support, waste management and remediation services;
3) Growth rates calculated as the arithmetic average of annual growth rates.

Table 8: Real GDP in Canada, Business Sector and Two-Digit NAICS Sectors, 2000-2010

A) Constant 2002 Dollars

| | $X^{*i}_{2000}$ ($ millions) | $X^{*i}_{2010}$ ($ millions) | Ave. Annual Growth (%) | $\frac{X^{*i}_{2000}}{X^{*}_{2000}}$ (%) | $\frac{X^{*i}_{2010}}{X^{*}_{2010}}$ (%) |
|---|---|---|---|---|---|
| *Business sector industries* | *785,491* | *922,567* | *1.65* | *100.0* | *100.0* |
| Agriculture, forestry, fishing and hunting | 26,312 | 28,711 | 1.11 | 3.3 | 3.1 |
| Mining and oil and gas extraction | 52,183 | 54,958 | 0.60 | 6.6 | 6.0 |
| Utilities | 26,914 | 30,403 | 1.28 | 3.4 | 3.3 |
| Construction | 51,655 | 72,576 | 3.56 | 6.6 | 7.9 |
| Manufacturing | 190,617 | 155,727 | -1.85 | 24.3 | 16.9 |
| Wholesale trade | 51,256 | 69,891 | 3.22 | 6.5 | 7.6 |
| Retail trade | 51,766 | 75,893 | 3.92 | 6.6 | 8.2 |
| Transportation and warehousing | 46,670 | 54,719 | 1.65 | 5.9 | 5.9 |
| Information and cultural industries | 32,126 | 43,152 | 3.03 | 4.1 | 4.7 |
| FIRE | 110,890 | 154,182 | 3.36 | 14.1 | 16.7 |
| Professional, scientific and technical services | 45,731 | 59,929 | 2.76 | 5.8 | 6.5 |
| ASWMRS | 21,697 | 30,208 | 3.42 | 2.8 | 3.3 |
| Arts, entertainment and recreation | 7,634 | 8,864 | 1.53 | 1.0 | 1.0 |
| Accommodation and food services | 24,406 | 26,313 | 0.77 | 3.1 | 2.9 |
| Other Private Services | 45,633 | 57,042 | 2.26 | 5.8 | 6.2 |

B) Chained 2002 Dollars

| | $X^{i}_{2000}$ ($ millions) | $X^{i}_{2010}$ ($ millions) | Ave. Annual Growth (%) | $\frac{X^{i}_{2000}}{X_{2000}}$ (%) | $\frac{X^{i}_{2010}}{X_{2010}}$ (%) |
|---|---|---|---|---|---|
| *Business sector industries* | *785,057* | *926,103* | *1.70* | *100.0* | *100.0* |
| Agriculture, forestry, fishing and hunting | 25,549 | 27,593 | 0.95 | 3.3 | 3.0 |
| Mining and oil and gas extraction | 51,519 | 54,967 | 0.71 | 6.6 | 5.9 |
| Utilities | 27,560 | 30,602 | 1.12 | 3.5 | 3.3 |
| Construction | 51,757 | 73,467 | 3.67 | 6.6 | 7.9 |
| Manufacturing | 188,914 | 158,307 | -1.64 | 24.1 | 17.1 |
| Wholesale trade | 52,511 | 68,802 | 2.80 | 6.7 | 7.4 |
| Retail trade | 52,536 | 75,564 | 3.72 | 6.7 | 8.2 |
| Transportation and warehousing | 46,472 | 54,647 | 1.67 | 5.9 | 5.9 |
| Information and cultural industries | 32,242 | 43,084 | 2.97 | 4.1 | 4.7 |
| FIRE | 110,515 | 153,801 | 3.37 | 14.1 | 16.6 |
| Professional, scientific and technical services | 46,068 | 59,612 | 2.63 | 5.9 | 6.4 |
| ASWMRS | 21,750 | 30,248 | 3.40 | 2.8 | 3.3 |
| Arts, entertainment and recreation | 7,665 | 8,861 | 1.49 | 1.0 | 1.0 |
| Accommodation and food services | 24,362 | 26,395 | 0.82 | 3.1 | 2.9 |
| Other Private Services | 45,622 | 56,654 | 2.19 | 5.8 | 6.1 |

Notes:
1) All variables as defined previously;
2) FIRE - Finance, insurance, real estate, rental and leasing; ASWMRS - Administration and support, waste management and remediation services;
3) Growth rates calculated as the arithmetic average of annual growth rates.

Table 9: Hours Worked in Canada, Business Sector and Two-Digit NAICS Sectors, 2000-2010

| | $L^i_{2000}$ (millions) | $L^i_{2010}$ (millions) | Ave. Annual Growth (%) | $l^i_{2000}$ (%) | $l^i_{2010}$ (%) |
|---|---|---|---|---|---|
| *Business sector industries* | *22,083* | *23,812* | *0.78* | *100.0* | *100.0* |
| Agriculture, forestry, fishing and hunting | 997 | 737 | -2.91 | 4.5 | 3.1 |
| Mining and oil and gas extraction | 324 | 461 | 3.91 | 1.5 | 1.9 |
| Utilities | 167 | 202 | 2.02 | 0.8 | 0.8 |
| Construction | 1,833 | 2,610 | 3.66 | 8.3 | 11.0 |
| Manufacturing | 4,037 | 3,114 | -2.50 | 18.3 | 13.1 |
| Wholesale trade | 1,597 | 1,584 | -0.04 | 7.2 | 6.7 |
| Retail trade | 2,754 | 3,145 | 1.36 | 12.5 | 13.2 |
| Transportation and warehousing | 1,412 | 1,479 | 0.51 | 6.4 | 6.2 |
| Information and cultural industries | 631 | 691 | 0.95 | 2.9 | 2.9 |
| FIRE | 1,644 | 1,937 | 1.66 | 7.4 | 8.1 |
| Professional, scientific and technical services | 1,566 | 1,926 | 2.11 | 7.1 | 8.1 |
| ASWMRS | 1,014 | 1,406 | 3.37 | 4.6 | 5.9 |
| Arts, entertainment and recreation | 370 | 459 | 2.28 | 1.7 | 1.9 |
| Accommodation and food services | 1,660 | 1,713 | 0.34 | 7.5 | 7.2 |
| Other Private Services | 2,076 | 2,348 | 1.25 | 9.4 | 9.9 |

Notes:
1) All variables as defined previously;
2) FIRE - Finance, insurance, real estate, rental and leasing; ASWMRS - Administration and support, waste management and remediation services;
3) Growth rates calculated as the arithmetic average of annual growth rates.

dollar estimates – from \$35.55 per hour to \$38.89 per hour. This difference is so small, however, that it seems safe to assume it is not particularly relevant.

Since labour input is the same for both sets of labour productivity estimates, the sole source of differences between the two sets of estimates is real output. Thus, exactly as seen when price deflator and real output trends were discussed, there were only two sectors where a substantial difference in labour productivity growth rates could be observed. First, wholesale trade, where labour productivity grew 3.27 per cent per year according to the constant dollar estimates, but only 2.86 per cent per year according to the chained dollar estimates; second, manufacturing, where labour productivity growth was 0.61 per cent per year according to the constant dollar estimate and 0.85 per cent per year according to the chained dollar estimate. For all the other sectors, the differences in labour productivity growth between the two sets of estimates were minor.

During the 2000-2010 period, the two-digit NAICS sector that experienced the fastest labour productivity growth was agriculture, forestry, fishing and hunting (4.12-4.25 per cent per year), followed by wholesale trade (2.86-3.27 per cent per year) and retail trade (2.35-2.54 per cent per year). Conversely, the sectors that saw the worst labour productivity performances were mining and oil and gas extraction (-2.80/-2.63 per cent per year), arts, entertainment and recreation (-0.65/-0.61 per cent per year), and utilities (-0.66/-0.51 per cent per year).

Table 10: Labour Productivity in Canada, Business Sector and Two-Digit NAICS Sectors, 2000-2010

A) Constant 2002 Dollars

| | $Z^{*i}_{2000}$ ($ per hour) | $Z^{*i}_{2010}$ ($ per hour) | Ave. Annual Growth (%) | $\frac{Z^{*i}_{2000}}{Z^{*}_{2000}}$ (%) | $\frac{Z^{*i}_{2010}}{Z^{*}_{2010}}$ (%) |
|---|---|---|---|---|---|
| *Business sector industries* | *35.57* | *38.74* | *0.86* | *100.0* | *100.0* |
| Agriculture, forestry, fishing and hunting | 26.39 | 38.96 | 4.25 | 74.2 | 100.6 |
| Mining and oil and gas extraction | 161.06 | 119.21 | -2.80 | 452.8 | 307.7 |
| Utilities | 161.16 | 150.51 | -0.51 | 453.1 | 388.5 |
| Construction | 28.18 | 27.81 | -0.10 | 79.2 | 71.8 |
| Manufacturing | 47.22 | 50.01 | 0.61 | 132.7 | 129.1 |
| Wholesale trade | 32.10 | 44.12 | 3.27 | 90.2 | 113.9 |
| Retail trade | 18.80 | 24.13 | 2.54 | 52.8 | 62.3 |
| Transportation and warehousing | 33.05 | 37.00 | 1.17 | 92.9 | 95.5 |
| Information and cultural industries | 50.91 | 62.45 | 2.13 | 143.1 | 161.2 |
| FIRE | 67.45 | 79.60 | 1.69 | 189.6 | 205.4 |
| Professional, scientific and technical services | 29.20 | 31.12 | 0.65 | 82.1 | 80.3 |
| ASWMRS | 21.40 | 21.49 | 0.06 | 60.2 | 55.5 |
| Arts, entertainment and recreation | 20.63 | 19.31 | -0.61 | 58.0 | 49.8 |
| Accommodation and food services | 14.70 | 15.36 | 0.45 | 41.3 | 39.6 |
| Other Private Services | 21.98 | 24.29 | 1.02 | 61.8 | 62.7 |

B) Chained 2002 Dollars

| | $Z^{i}_{2000}$ ($ per hour) | $Z^{i}_{2010}$ ($ per hour) | Ave. Annual Growth (%) | $\frac{Z^{i}_{2000}}{Z_{2000}}$ (%) | $\frac{Z^{i}_{2010}}{Z_{2010}}$ (%) |
|---|---|---|---|---|---|
| *Business sector industries* | *35.55* | *38.89* | *0.91* | *100.0* | *100.0* |
| Agriculture, forestry, fishing and hunting | 25.63 | 37.44 | 4.12 | 72.1 | 96.3 |
| Mining and oil and gas extraction | 159.01 | 119.23 | -2.63 | 447.3 | 306.6 |
| Utilities | 165.03 | 151.50 | -0.66 | 464.2 | 389.5 |
| Construction | 28.24 | 28.15 | 0.00 | 79.4 | 72.4 |
| Manufacturing | 46.80 | 50.84 | 0.85 | 131.6 | 130.7 |
| Wholesale trade | 32.88 | 43.44 | 2.86 | 92.5 | 111.7 |
| Retail trade | 19.08 | 24.03 | 2.35 | 53.7 | 61.8 |
| Transportation and warehousing | 32.91 | 36.95 | 1.20 | 92.6 | 95.0 |
| Information and cultural industries | 51.10 | 62.35 | 2.07 | 143.7 | 160.3 |
| FIRE | 67.22 | 79.40 | 1.70 | 189.1 | 204.2 |
| Professional, scientific and technical services | 29.42 | 30.95 | 0.52 | 82.7 | 79.6 |
| ASWMRS | 21.45 | 21.51 | 0.05 | 60.3 | 55.3 |
| Arts, entertainment and recreation | 20.72 | 19.31 | -0.65 | 58.3 | 49.6 |
| Accommodation and food services | 14.68 | 15.41 | 0.50 | 41.3 | 39.6 |
| Other Private Services | 21.98 | 24.13 | 0.95 | 61.8 | 62.0 |

Notes:
1) All variables as defined previously;
2) FIRE - Finance, insurance, real estate, rental and leasing; ASWMRS - Administration and support, waste management and remediation services;
3) Growth rates calculated as the arithmetic average of annual growth rates.

In terms of labour productivity levels, the sectors with the highest levels in 2010 were utilities ($150.51-151.50 per hour), mining and oil and gas extraction ($119.21-119.23 per hour), and FIRE ($79.40-79.60 per hour). The lowest labour productivity levels could be found in accommodation and food services ($15.36-15.41 per hour), arts, entertainment and recreation ($19.31 per hour), and ASWMRS ($21.49-21.51 per hour).

### 3.3 Sectoral Contributions to Business Sector Labour Productivity Growth in Canada, 2000-2010

Using the TRAD, CSLS, and GEAD decomposition formulas, contributions of two-digit NAICS sectors to business sector labour productivity growth in Canada during the 2000-2010 period were calculated. Two sets of estimates were constructed for each of the three formulas – one using real output in constant 2002 dollars (Table 11) and the other using real output in chained 2002 dollars (Table 12). A comparison between Tables 11 and 12 shows that, while the estimated sectoral contributions differ significantly from formula to formula, the differences due to the use of real GDP in constant prices or in chained prices are relatively small and do not alter the overall results of each decomposition formula. The main difference between the two sets of estimates is simply that sectoral contributions calculated using the TRAD and CSLS formulas do not sum up to business sector labour productivity growth when real GDP is in chained 2002 dollars. Given that these are minor differences, this section focuses on the first set of estimates, which uses real GDP in constant 2002 dollars.

At the aggregate economy level, the TRAD and CSLS formulas tell exactly the same story. According to both formulas, the entirety of business sector labour productivity growth in Canada during the 2000-2010 period (0.86 per cent per year) is explained by within-sector effects (WSE). In fact, the WSE contribution was higher than actual labour productivity growth – 0.95 percentage points or 110.6 per cent of business sector labour productivity growth. Aggregate labour productivity growth was dampened by negative reallocation level (RLE) and reallocation growth (RGE) effects, with the RLE reducing growth by 0.03 percentage points (-3.6 per cent) and the RGE accounting for a 0.06 percentage point (-6.6 per cent) reduction in growth.

Table 11: Sectoral Contributions to Business Sector Labour Productivity Growth in Canada, Real GDP in Constant 2002 Dollars, 2000-2010

A) Percentage Point Contributions

| | TRAD | | | | CSLS | | | | GEAD | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TC | WSE | RLE | RGE | TC | WSE | RLE | RGE | TC | WSE | RLE | RGE |
| *Business sector industries* | *0.86* | *0.95* | *-0.03* | *-0.06* | *0.86* | *0.95* | *-0.03* | *-0.06* | *0.86* | *0.82* | *0.23* | *-0.18* |
| Agriculture, forestry, fishing and hunting | 0.00 | 0.13 | -0.11 | -0.01 | 0.15 | 0.13 | 0.03 | -0.01 | -0.05 | 0.10 | -0.14 | -0.01 |
| Mining and oil and gas extraction | -0.01 | -0.18 | 0.20 | -0.03 | -0.06 | -0.18 | 0.15 | -0.03 | 0.26 | -0.25 | 0.65 | -0.14 |
| Utilities | 0.01 | -0.02 | 0.04 | -0.01 | 0.01 | -0.02 | 0.04 | -0.01 | -0.02 | -0.02 | 0.02 | -0.01 |
| Construction | 0.19 | -0.01 | 0.21 | 0.00 | -0.08 | -0.01 | -0.06 | 0.00 | 0.36 | -0.02 | 0.39 | 0.00 |
| Manufacturing | -0.56 | 0.11 | -0.67 | 0.00 | -0.04 | 0.11 | -0.15 | 0.00 | -0.83 | 0.10 | -0.93 | -0.01 |
| Wholesale trade | 0.17 | 0.23 | -0.06 | 0.00 | 0.22 | 0.23 | 0.00 | 0.00 | 0.11 | 0.23 | -0.11 | -0.01 |
| Retail trade | 0.23 | 0.18 | 0.04 | 0.00 | 0.15 | 0.18 | -0.03 | 0.00 | 0.15 | 0.18 | -0.02 | 0.00 |
| Transportation and warehousing | 0.05 | 0.07 | -0.02 | 0.00 | 0.07 | 0.07 | 0.00 | 0.00 | 0.07 | 0.06 | 0.00 | 0.00 |
| Information and cultural industries | 0.10 | 0.09 | 0.01 | 0.00 | 0.09 | 0.09 | 0.00 | 0.00 | 0.09 | 0.09 | 0.00 | 0.00 |
| FIRE | 0.39 | 0.25 | 0.14 | 0.00 | 0.32 | 0.25 | 0.07 | 0.00 | 0.30 | 0.24 | 0.05 | 0.00 |
| Professional, scientific and technical services | 0.12 | 0.04 | 0.08 | 0.00 | 0.02 | 0.04 | -0.02 | 0.00 | 0.16 | 0.03 | 0.13 | 0.00 |
| ASWMRS | 0.08 | 0.00 | 0.08 | 0.00 | -0.05 | 0.00 | -0.05 | 0.00 | 0.09 | 0.00 | 0.09 | 0.00 |
| Arts, entertainment and recreation | 0.01 | -0.01 | 0.01 | 0.00 | -0.02 | -0.01 | -0.01 | 0.00 | 0.01 | -0.01 | 0.02 | 0.00 |
| Accommodation and food services | 0.00 | 0.01 | -0.01 | 0.00 | 0.03 | 0.01 | 0.02 | 0.00 | 0.02 | 0.01 | 0.00 | 0.00 |
| Other Private Services | 0.09 | 0.06 | 0.03 | 0.00 | 0.04 | 0.06 | -0.02 | 0.00 | 0.14 | 0.06 | 0.08 | 0.00 |

B) Per Cent Contributions

| | TRAD | | | | CSLS | | | | GEAD | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TC | WSE | RLE | RGE | TC | WSE | RLE | RGE | TC | WSE | RLE | RGE |
| *Business sector industries* | *100.0* | *110.1* | *-3.6* | *-6.6* | *100.0* | *110.1* | *-3.6* | *-6.6* | *100.0* | *94.9* | *26.5* | *-21.4* |
| Agriculture, forestry, fishing and hunting | 0.3 | 14.5 | -13.1 | -1.1 | 16.9 | 14.5 | 3.4 | -1.0 | -5.5 | 11.7 | -15.7 | -1.5 |
| Mining and oil and gas extraction | -1.6 | -21.1 | 22.8 | -3.3 | -7.1 | -21.1 | 17.3 | -3.4 | 30.1 | -29.4 | 75.2 | -15.7 |
| Utilities | 1.7 | -2.4 | 5.2 | -1.0 | 0.7 | -2.4 | 4.1 | -1.0 | -2.0 | -2.2 | 1.8 | -1.5 |
| Construction | 22.3 | -1.7 | 24.1 | -0.1 | -8.8 | -1.7 | -6.7 | -0.4 | 41.8 | -2.5 | 44.8 | -0.5 |
| Manufacturing | -64.7 | 13.0 | -77.8 | 0.1 | -4.1 | 13.0 | -17.6 | 0.4 | -96.3 | 12.2 | -107.6 | -0.9 |
| Wholesale trade | 19.3 | 26.7 | -6.9 | -0.4 | 26.1 | 26.7 | -0.2 | -0.4 | 12.2 | 26.1 | -13.2 | -0.6 |
| Retail trade | 26.4 | 21.4 | 4.9 | 0.0 | 17.8 | 21.4 | -3.6 | 0.0 | 17.8 | 20.9 | -2.9 | -0.3 |
| Transportation and warehousing | 5.8 | 7.8 | -1.9 | -0.2 | 7.9 | 7.8 | 0.2 | -0.2 | 7.8 | 7.5 | 0.4 | -0.1 |
| Information and cultural industries | 11.3 | 10.6 | 1.1 | -0.3 | 10.9 | 10.6 | 0.6 | -0.3 | 10.1 | 10.6 | -0.1 | -0.4 |
| FIRE | 45.3 | 29.3 | 15.9 | 0.1 | 37.2 | 29.3 | 7.9 | 0.0 | 34.7 | 28.3 | 6.1 | 0.3 |
| Professional, scientific and technical services | 13.9 | 4.2 | 9.7 | 0.0 | 2.3 | 4.2 | -1.8 | 0.0 | 18.5 | 4.0 | 14.5 | 0.0 |
| ASWMRS | 9.1 | 0.2 | 9.0 | -0.1 | -6.3 | 0.2 | -6.2 | -0.2 | 10.8 | 0.1 | 10.7 | -0.1 |
| Arts, entertainment and recreation | 0.8 | -0.7 | 1.7 | -0.1 | -2.0 | -0.7 | -1.2 | -0.1 | 1.4 | -0.7 | 2.3 | -0.2 |
| Accommodation and food services | 0.0 | 1.5 | -1.5 | 0.0 | 3.8 | 1.5 | 2.3 | 0.0 | 2.1 | 1.5 | 0.5 | 0.0 |
| Other Private Services | 10.2 | 6.9 | 3.3 | 0.0 | 5.0 | 6.9 | -2.0 | 0.1 | 16.6 | 6.7 | 9.7 | 0.2 |

Notes:
1) TC - Total Contribution; WSE - Within-Sector Effect; RLE - Reallocation Level Effect; RGE - Reallocation Growth Effect.
2) FIRE - Finance, insurance, real estate, rental and leasing; ASWMRS - Administration and support, waste management and remediation services;

Table 12: Sectoral Contributions to Business Sector Labour Productivity Growth in Canada, Real GDP in Chained 2002 Dollars, 2000-2010

A) Percentage Point Contributions

| | TRAD | | | | CSLS | | | | GEAD | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TC | WSE | RLE | RGE | TC | WSE | RLE | RGE | TC | WSE | RLE | RGE |
| *Business sector industries* | *0.87* | *0.96* | *-0.03* | *-0.06* | *0.87* | *0.96* | *-0.03* | *-0.06* | *0.91* | *0.84* | *0.27* | *-0.20* |
| Agriculture, forestry, fishing and hunting | 0.00 | 0.12 | -0.11 | -0.01 | 0.14 | 0.12 | 0.03 | -0.01 | -0.05 | 0.10 | -0.13 | -0.01 |
| Mining and oil and gas extraction | -0.01 | -0.17 | 0.20 | -0.03 | -0.05 | -0.17 | 0.15 | -0.03 | 0.27 | -0.23 | 0.65 | -0.15 |
| Utilities | 0.01 | -0.03 | 0.04 | -0.01 | 0.00 | -0.03 | 0.04 | -0.01 | -0.02 | -0.02 | 0.02 | -0.01 |
| Construction | 0.20 | -0.01 | 0.21 | 0.00 | -0.07 | -0.01 | -0.06 | 0.00 | 0.36 | -0.01 | 0.38 | 0.00 |
| Manufacturing | -0.51 | 0.17 | -0.67 | 0.00 | 0.02 | 0.17 | -0.15 | 0.00 | -0.82 | 0.15 | -0.97 | -0.01 |
| Wholesale trade | 0.14 | 0.20 | -0.06 | 0.00 | 0.20 | 0.20 | 0.00 | 0.00 | 0.11 | 0.20 | -0.08 | -0.01 |
| Retail trade | 0.21 | 0.17 | 0.04 | 0.00 | 0.14 | 0.17 | -0.03 | 0.00 | 0.16 | 0.17 | -0.01 | 0.00 |
| Transportation and warehousing | 0.05 | 0.07 | -0.02 | 0.00 | 0.07 | 0.07 | 0.00 | 0.00 | 0.07 | 0.07 | 0.00 | 0.00 |
| Information and cultural industries | 0.10 | 0.09 | 0.01 | 0.00 | 0.09 | 0.09 | 0.00 | 0.00 | 0.09 | 0.09 | 0.00 | 0.00 |
| FIRE | 0.39 | 0.25 | 0.14 | 0.00 | 0.32 | 0.25 | 0.07 | 0.00 | 0.30 | 0.25 | 0.06 | 0.00 |
| Professional, scientific and technical services | 0.11 | 0.03 | 0.08 | 0.00 | 0.01 | 0.03 | -0.02 | 0.00 | 0.16 | 0.03 | 0.14 | 0.00 |
| ASWMRS | 0.08 | 0.00 | 0.08 | 0.00 | -0.05 | 0.00 | -0.05 | 0.00 | 0.09 | 0.00 | 0.09 | 0.00 |
| Arts, entertainment and recreation | 0.01 | -0.01 | 0.01 | 0.00 | -0.02 | -0.01 | -0.01 | 0.00 | 0.01 | -0.01 | 0.02 | 0.00 |
| Accommodation and food services | 0.00 | 0.01 | -0.01 | 0.00 | 0.03 | 0.01 | 0.02 | 0.00 | 0.02 | 0.01 | 0.00 | 0.00 |
| Other Private Services | 0.08 | 0.06 | 0.03 | 0.00 | 0.04 | 0.06 | -0.02 | 0.00 | 0.15 | 0.05 | 0.09 | 0.00 |

B) Per Cent Contributions

| | TRAD | | | | CSLS | | | | GEAD | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TC | WSE | RLE | RGE | TC | WSE | RLE | RGE | TC | WSE | RLE | RGE |
| *Business sector industries* | *100.0* | *110.5* | *-3.2* | *-7.3* | *100.0* | *110.5* | *-3.2* | *-7.3* | *100.0* | *92.3* | *30.0* | *-22.3* |
| Agriculture, forestry, fishing and hunting | -0.1 | 13.8 | -12.7 | -1.1 | 16.5 | 13.8 | 3.7 | -1.0 | -5.1 | 10.9 | -14.8 | -1.3 |
| Mining and oil and gas extraction | -0.6 | -19.7 | 22.7 | -3.6 | -6.2 | -19.7 | 17.3 | -3.7 | 29.4 | -25.6 | 72.0 | -17.1 |
| Utilities | 1.0 | -3.1 | 5.1 | -1.0 | -0.1 | -3.1 | 4.1 | -1.0 | -1.7 | -2.7 | 2.6 | -1.6 |
| Construction | 23.2 | -0.9 | 24.2 | -0.1 | -7.8 | -0.9 | -6.5 | -0.4 | 40.1 | -1.5 | 42.1 | -0.5 |
| Manufacturing | -58.5 | 19.3 | -77.5 | -0.4 | 1.9 | 19.3 | -17.5 | 0.0 | -90.8 | 16.8 | -106.7 | -0.9 |
| Wholesale trade | 15.9 | 23.2 | -6.8 | -0.5 | 22.7 | 23.2 | -0.1 | -0.4 | 11.9 | 21.8 | -9.3 | -0.6 |
| Retail trade | 24.6 | 19.7 | 4.9 | 0.0 | 16.1 | 19.7 | -3.6 | 0.0 | 17.3 | 18.4 | -1.0 | -0.2 |
| Transportation and warehousing | 5.9 | 8.0 | -1.9 | -0.2 | 8.1 | 8.0 | 0.3 | -0.2 | 7.7 | 7.3 | 0.4 | -0.1 |
| Information and cultural industries | 11.0 | 10.3 | 1.0 | -0.3 | 10.5 | 10.3 | 0.5 | -0.3 | 9.8 | 9.9 | 0.3 | -0.3 |
| FIRE | 45.0 | 29.2 | 15.7 | 0.1 | 37.0 | 29.2 | 7.8 | 0.0 | 33.6 | 27.1 | 6.3 | 0.2 |
| Professional, scientific and technical services | 12.9 | 3.3 | 9.6 | 0.0 | 1.4 | 3.3 | -1.9 | -0.1 | 17.9 | 3.0 | 14.9 | 0.0 |
| ASWMRS | 9.0 | 0.1 | 8.9 | -0.1 | -6.3 | 0.1 | -6.2 | -0.2 | 10.4 | 0.1 | 10.4 | -0.1 |
| Arts, entertainment and recreation | 0.8 | -0.8 | 1.7 | -0.1 | -2.1 | -0.8 | -1.2 | -0.1 | 1.4 | -0.7 | 2.3 | -0.2 |
| Accommodation and food services | 0.2 | 1.7 | -1.5 | 0.0 | 3.9 | 1.7 | 2.3 | 0.0 | 2.1 | 1.6 | 0.5 | 0.0 |
| Other Private Services | 9.7 | 6.4 | 3.3 | 0.0 | 4.4 | 6.4 | -2.0 | 0.1 | 16.0 | 6.0 | 9.9 | 0.2 |

Notes:
1) TC - Total Contribution; WSE - Within-Sector Effect; RLE - Reallocation Level Effect; RGE - Reallocation Growth Effect.
2) FIRE - Finance, insurance, real estate, rental and leasing; ASWMRS - Administration and support, waste management and remediation services;

The differences between the two formulas emerge when the role of each two-digit NAICS sector in explaining business sector labour productivity growth is analysed separately. In the TRAD formula, the FIRE sector was responsible for almost half of aggregate labour productivity growth – 0.39 percentage points or 45.3 per cent of overall growth. Retail trade (0.23 percentage points or 26.4 per cent of total growth), construction (0.19 percentage points or 22.3 per cent), and wholesale trade (0.17 percentage points or 19.3 per cent) were also fundamental in driving aggregate labour productivity growth in the period. The economy's labour productivity performance was hindered, however, by the massive negative contribution of the manufacturing sector (-0.56 percentage points or -64.7 per cent of total growth), which was caused by a very strong (and negative) RLE. Other than manufacturing, the only sector which had a negative (albeit very small) contribution to business sector labour productivity growth was mining and oil and gas extraction.

In the CSLS formula, FIRE is still the sector with the highest contribution to business sector labour productivity growth (0.32 percentage points or 37.2 per cent of total growth), although its contribution is smaller than it was in the TRAD formula. Other sectors that played an important role in driving aggregate labour productivity growth according to the CSLS formula were wholesale trade (0.22 percentage points or 26.1 per cent of total growth), retail trade (0.15 percentage points or 17.8 per cent), and agriculture, forestry, fishing and hunting (0.15 percentage points or 16.9 per cent). Note that, while construction had an important positive contribution in the TRAD formula, it had a *negative* contribution in the CSLS formula (-0.08 percentage points or -8.8 per cent of total growth). Furthermore, the contribution of manufacturing in the CSLS formula, while still negative, was not nearly as strong as it was in the TRAD formula (-0.04 percentage poitns or -4.1 per cent of total growth). It is interesting to note, also, that five out of the 15 two-digit NAICS sectors had a negative contribution to aggregate labour productivity growth according to the CSLS formula (vs. two in the TRAD formula).

Recalling our discussion about each decomposition formula in the previous section, it is clear that these differences are caused solely by how the RLE and RGE are calculated in the TRAD and CSLS formulas. Take the case of the construction sector, for instance, which had a positive RLE in the TRAD formula and a negative one in the CSLS formula. Since there was an increase in the sector's labour share (from 8.3 per cent in 2000 to 11.0 per cent in 2010) during the period, the TRAD formula sees the sector's RLE as positive. The CSLS formula, on the other hand, attributes to the sector a negative RLE because of its below-average labour productivity level.

What explains, however, the huge differences in the manufacturing sector's RLE? In the TRAD formula, the sector's RLE contributed -0.67 percentage points to aggregate labour productivity growth, while in the CSLS formula it contributed only -0.15 percentage points. This difference is explained by the fact that the TRAD formula depends on the ratio between the labour productivity level in manufacturing and in the business sector, whereas in the CSLS formula it depends on the difference between the two. Overall, as shown by Table 11, the TRAD formula produces RLE estimates with much higher magnitudes than the CSLS formula.

The GEAD formula provides a very different perspective on sectoral contributions to business sector labour productivity growth. First, at the aggregate economy level, the magnitude of the WSE in the GEAD formula is noticeably smaller than that of the TRAD and CSLS formulas (0.82 percentage points vs. 0.95 percentage points). This difference is driven by the fact that, in the GEAD formula, the within-sector effect is determined by nominal shares instead of real shares.

While the WSE still accounts for almost the entirety of business sector labour productivity growth in the period (94.9 per cent), the joint reallocation effects (RLE + RGE) now have a small positive contribution (5.1 per cent). More precisely, the RLE now has a positive sign, while the RGE retains its negative sign. What is more, the magnitude of both reallocation effects under the GEAD formula is significantly higher than in the other two formulas. In the GEAD formula, the RLE explained 0.23 percentage points of total labour productivity growth versus -0.03 percentage points in the TRAD and CSLS formulas. The same applies for the RGE, which explained -0.18 percentage points of aggregate productivity growth in the GEAD formula versus -0.06 percentage points in the TRAD and CSLS formulas.

According to the GEAD formula, the three sectors that contributed the most to business sector labour productivity growth were: construction (0.36 percentage points or 41.8 per cent of total growth), FIRE (0.30 percentage points or 34.7 per cent), and mining and oil and gas extraction (0.26 percentage points or 30.1 per cent).[12] In general, all sectors had positive contributions, with the exception of utilities; agriculture, forestry, fishing and hunting; and manufacturing. Of these three sectors, manufacturing was the only one where the magnitude of the contribution actually mattered, reducing aggregate labour productivity growth by -0.83 percentage points (or 96.3 per cent of total growth). Much like the TRAD formula, this was caused by a massive RLE – in fact, the RLE in the GEAD formula was even stronger than that of the TRAD formula.

### 3.4 Three Decomposition Formulas, Three Stories

The comparison between the estimates produced by the TRAD, CSLS, and GEAD formulas highlights an important problem: despite some similarities, all three decomposition formulas paint very different pictures of which sectors drove labour productivity growth in the Canadian business sector during the 2000-2010 period. Why are estimated contributions so different? Which set of estimates provides a more accurate picture of economic reality?

[12]It is interesting to note that, for the 2000-2008 period, the contribution of mining and oil and gas extraction to business sector labour productivity growth was much higher. Almon and Tang (2011) show that the sector accounted for 0.64 percentage points of the average annual labour productivity growth of 0.79 per cent experienced by the Canadian economy in the period, slightly over 80 per cent of total growth. This difference was caused by the spike in oil prices in 2008. For details on Almon and Tang's estimates (and on how they compare to our estimates), see Appendix Table.

The differences between the TRAD and CSLS formulas are caused solely by the way each formula assigns sectoral contributions due to reallocation effects. In the TRAD formula, for instance, a sector's RLE will be positive as long as its labour share increases, regardless of whether the sector had above-average or below-average labour productivity level. In the CSLS formula, however, a sector with below-average labour productivity level will have a negative RLE if its labour share increases.

In our opinion, the way the TRAD formula deals with reallocation effects is quite problematic. At the aggregate level, if labour moved to sectors with below-average labour productivity level (growth), the sign of the overall RLE (RGE) would be negative. Sectors with below-average labour productivity levels that experienced an increase in their labour share would, however, have a *positive* RLE (which would be offset by the negative RLE of the sectors with above-average labour productivity that experienced a reduction in their labour share). The CSLS formula prevents this from happening by making reallocation effects a function of the difference between a sector's labour productivity level (growth) and the aggregate labour productivity level (growth).

Despite this advantage, some of the results of the CSLS formula can also be considered counter-intuitive. When labour moves from a sector with below-average labour productivity level towards a sector with above-average labour productivity level, for instance, *both* sectors will have a positive RLE. The positive RLE of the low productivity sector, which seems unwarranted, happens exactly because people are leaving the sector. Although this dynamic represents a limitation of the CSLS formula, we still believe that the CSLS formula produces better reallocation effect estimates than the TRAD formula. Because of how they are specified, the CSLS's reallocation effects tend to be well distributed among sectors, which minimizes the impact of strange reallocation dynamics, while the TRAD's reallocation effects tend to be concentrated in specific sectors, magnifying the dynamic discussed in the previous paragraph. In the end, however, an argument can be made for the impossibility of assigning reallocation effects across sectors in a satisfactory way.

The TRAD and CSLS formulas – which represent the "traditional" formulas analysed in this article – differ from the GEAD formula because they do not incorporate *price effects* into sectoral contributions. Sectoral contributions to aggregate labour productivity growth are calculated based solely on sectoral labour productivity and labour shares. In this sense, the TRAD and CSLS formulas capture only *quantity effects*. Sectoral contributions in both formulas represent how the increase in the *volume* of goods and services produced per hour in a particular sector (along with labour movements) affects the increase in the *volume* of goods and services produced per hour at the aggregate level. These formulas can therefore be seen as measuring the impact of (sectoral) *real* variables on aggregate labour productivity growth.

In the case of the GEAD formula, estimated contributions have a different meaning. A sector's contribution to aggregate productivity growth, while obviously dependent on the sector's labour productivity growth (and labour movements), is also dependent on how the economy values that sector's output, which is determined by relative prices. By incorporating price effects into contributions, the GEAD formula captures the overall *economic significance* of different sectors to aggregate labour productivity growth.

The case of the Canadian mining and oil and gas extraction sector clearly illustrates the difference between the GEAD and the other two decomposition formulas analysed in this article. In the CSLS and TRAD formulas, mining and oil and gas extraction had a *small, negative* contribution to business sector labour productivity growth during the 2000-2010 period. According to the GEAD formula, however, the sector had a *large, positive* contribution.

Looking only at real variables, mining and oil and gas did indeed contribute very little to overall business sector labour productivity growth in Canada during the 2000-2010 period. Real GDP in the sector increased only 0.60 per cent per year (vs. 1.65 per cent per year at the business sector level); the sector's labour share did increase, from 1.5 per cent in 2000 to 1.9 per cent in 2010, but remained quite small; and labour productivity in the sector experienced a sharp drop of 2.80 per cent per year (vs. an increase of 0.86 per cent per year at the business sector level). These developments are captured by the TRAD and CSLS formulas and result in a small, negative contribution of mining and oil and gas to business sector labour productivity growth.

When relative prices are included in the equation, the story looks very different. Mining and oil and gas prices more than doubled during the period, increasing 7.79 per cent per year, while business sector prices rose only 2.38 per cent per year. The GEAD formula captures this shift in relative prices, which was more than enough to offset the lacklustre productivity performance of the mining and oil and gas extraction sector, causing it to have a large, positive contribution to business sector labour productivity growth. Thus, the GEAD formula captures the fact that mining and oil and gas extraction played a fundamental role in driving economic growth in Canada during this past decade.

The above discussion underlines that sectoral contributions calculated by the TRAD and CSLS formulas are not strictly comparable with those calculated by the GEAD formula. In the first two formulas, sectoral contributions are a function only of real variables, whereas in the GEAD formula they are also a function of changes in relative prices. From this perspective, it is impossible to say which set of estimates provides a more accurate picture of economic reality because the GEAD formula is, ultimately, measuring something very different from the TRAD and CSLS formulas.

# 4 Conclusion

This article analysed sectoral contributions to business sector labour productivity growth in Canada during the 2000-2010 period using three different decomposition formulas – TRAD, CSLS, and GEAD. The first two formulas assume real output in constant prices, while the GEAD formula does not make any particular assumption regarding how real output estimates were constructed.

All three formulas break down sectoral contributions into within-sector, reallocation level, and reallocation growth effects. The TRAD and CSLS formulas produce the exact same estimates of within-sector effects, but allocate sectoral contributions due to the reallocation effects in different ways. The GEAD formula differs from the first two formulas in that it takes into account how movements in relative prices impact the three effects.

During the 2000-2010 period, business sector labour productivity increased at an average annual rate of 0.86 per cent (or 0.95 per cent, depending on how real GDP is calculated). At the aggregate economy level, the TRAD and CSLS formulas tell the same story, with the within-sector effect accounting for over 100 per cent of labour productivity growth in the period and the negative reallocation effects actually hindering growth. In terms of sectoral contributions, there were also important similarities between the estimates produced by the two formulas. In both formulas, FIRE was the sector that had the highest positive contribution to business sector labour productivity growth. Other sectors that played an important role in both formulas were wholesale and retail trade.

There were also, however, important differences in how the TRAD and CSLS formulas allocated sectoral contributions to business sector labour productivity growth. In the TRAD formula, construction had a strong, positive contribution to aggregate productivity growth, whereas in the CSLS formula the sector actually dampened productivity growth. Conversely, the CSLS formula attributes a somewhat important role to agriculture, forestry, fishing and hunting, whereas in the TRAD formula the sector's contribution is zero. The biggest difference between the two formulas can be seen in the role of manufacturing. In the TRAD formula, the reduction of the manufacturing sector's labour share caused a massive, negative reallocation level effect, which was more than enough to overcome the sector's positive within-sector effect. In the CSLS formula, the sector also experienced a negative reallocation effect, but its magnitude was much lower – less than one fourth of the magnitude of TRAD's RLE –, with the total contribution of the manufacturing sector to aggregate labour productivity growth being only slightly negative.

The story told by the GEAD formula, in turn, is quite different from that of the other two formulas. At the aggregate economy level, the within-sector effect is still the main driving force of business sector labour productivity growth, but the magnitude of the effect is lower than it was in the TRAD and CSLS formulas. Furthermore, the joint reallocation effects are actually positive. The construction, FIRE, and mining and oil and gas sectors had the highest contributions to aggregate labour productivity growth, while manufacturing was the "villain" of the story, with a huge negative contribution to productivity growth.

The three decomposition formulas provide alternative narratives as to which sectors drove aggregate productivity growth in Canada over the past decade. Some parts of these three narratives point in the same direction. In particular, at the aggregate economy level, all three formulas show that most (if not all) of labour productivity growth was caused by within-sector productivity improvements, with sectoral reallocation either hindering productivity growth or improving it by a very small margin. When it comes to assessing the role of each individual sector in aggregate labour productivity growth the room for agreement is much smaller. It is true that there is some agreement regarding the importance of FIRE. Nevertheless, the differences appear to be much more significant. What is the actual role of construction, manufacturing, or mining and oil and gas extraction in explaining business sector labour productivity growth? Which formula produces more accurate estimates?

Although estimates produced by the CSLS and TRAD formulas can be compared – and, in our opinion, the CSLS formula assigns reallocation effects in a more logical way than the TRAD formula –, they are not strictly comparable to estimates calculated using the GEAD formula. In the TRAD and CSLS formulas, sectoral contributions reflect only the impact of *real* variables on aggregate labour productivity growth, whereas in the GEAD formula they also incorporate the effect of changes in *relative prices* to capture the overall economic significance of different sectors in the economy. This explains why, in the GEAD formula, mining and oil and gas had such a strong contribution to business sector labour productivity growth, despite the sector's lacklustre productivity performance.

From this perspective, it is impossible to say which set of estimates provides a more accurate picture of economic reality because the GEAD formula is, ultimately, measuring something very different from the TRAD and CSLS formulas. Instead of seeing estimates constructed by the GEAD and traditional formulas as "competing" narratives, it is more useful to see them as providing complementing stories about the role of different sectors in driving aggregate labour productivity growth.

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Appendix Table: Sectoral Contributions to Business Sector Labour Productivity Growth in Canada, Chained 2002 Dollars, 2000-2008

A) Percentage Point Contributions

| | TRAD | | | CSLS | | | GEAD | | | Almon and Tang (GEAD) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TC | WSE | RE | TC | WSE | RE | TC | WSE | RE | TC | WSE | RE |
| *Business sector industries* | *0.84* | *0.90* | *-0.05* | *0.84* | *0.90* | *-0.05* | *0.95* | *0.69* | *0.27* | *0.80* | *0.42* | *0.38* |
| Agriculture, forestry, fishing and hunting | 0.01 | 0.15 | -0.14 | 0.18 | 0.15 | 0.03 | -0.03 | 0.12 | -0.16 | -0.07 | 0.07 | -0.14 |
| Mining and oil and gas extraction | -0.01 | -0.27 | 0.26 | -0.08 | -0.27 | 0.19 | 0.77 | -0.42 | 1.19 | 0.65 | -0.48 | 1.13 |
| Utilities | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.02 | 0.00 | -0.02 | -0.02 | -0.02 | 0.00 |
| Construction | 0.24 | 0.00 | 0.24 | -0.07 | 0.00 | -0.07 | 0.43 | -0.01 | 0.43 | 0.30 | -0.04 | 0.34 |
| Manufacturing | -0.51 | 0.16 | -0.68 | 0.01 | 0.16 | -0.15 | -0.98 | 0.15 | -1.12 | -0.91 | 0.06 | -0.97 |
| Wholesale trade | 0.16 | 0.19 | -0.03 | 0.19 | 0.19 | 0.00 | 0.08 | 0.19 | -0.11 | 0.10 | 0.21 | -0.11 |
| Retail trade | 0.21 | 0.19 | 0.02 | 0.17 | 0.19 | -0.02 | 0.14 | 0.18 | -0.05 | 0.09 | 0.21 | -0.12 |
| Transportation and warehousing | 0.05 | 0.05 | 0.00 | 0.05 | 0.05 | 0.00 | 0.02 | 0.05 | -0.03 | 0.01 | 0.00 | 0.01 |
| Information and cultural industries | 0.10 | 0.11 | -0.01 | 0.11 | 0.11 | -0.01 | 0.06 | 0.11 | -0.06 | 0.05 | 0.12 | -0.07 |
| FIRE | 0.32 | 0.21 | 0.11 | 0.26 | 0.21 | 0.05 | 0.15 | 0.21 | -0.05 | 0.17 | 0.14 | 0.03 |
| Professional, scientific and technical services | 0.12 | 0.05 | 0.07 | 0.04 | 0.05 | -0.01 | 0.14 | 0.05 | 0.08 | 0.11 | 0.02 | 0.09 |
| ASWMRS | 0.10 | 0.00 | 0.09 | -0.06 | 0.00 | -0.07 | 0.10 | 0.00 | 0.10 | 0.10 | 0.01 | 0.09 |
| Arts, entertainment and recreation | 0.00 | -0.01 | 0.01 | -0.02 | -0.01 | -0.01 | 0.01 | -0.01 | 0.02 | 0.01 | -0.01 | 0.02 |
| Accommodation and food services | 0.00 | 0.00 | -0.01 | 0.02 | 0.00 | 0.01 | 0.00 | 0.00 | 0.00 | -0.01 | 0.03 | -0.04 |
| Other Private Services | 0.05 | 0.05 | 0.00 | 0.05 | 0.05 | 0.00 | 0.09 | 0.05 | 0.03 | 0.22 | 0.10 | 0.12 |

B) Per Cent Contributions

| | TRAD | | | CSLS | | | GEAD | | | Almon and Tang (GEAD) | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TC | WSE | RE | TC | WSE | RE | TC | WSE | RE | TC | WSE | RE |
| *Business sector industries* | *100.0* | *106.5* | *-6.5* | *100.0* | *106.5* | *-6.5* | *100.0* | *72.2* | *27.8* | *100.0* | *52.5* | *47.5* |
| Agriculture, forestry, fishing and hunting | 1.2 | 17.6 | -16.4 | 21.1 | 17.6 | 3.5 | -3.6 | 13.0 | -16.5 | -8.8 | 8.8 | -17.5 |
| Mining and oil and gas extraction | -0.7 | -31.9 | 31.2 | -9.9 | -31.9 | 22.0 | 80.9 | -44.0 | 124.9 | 81.3 | -60.0 | 141.3 |
| Utilities | 0.6 | 0.0 | 0.6 | 0.4 | 0.0 | 0.4 | -2.4 | 0.0 | -2.4 | -2.5 | -2.5 | 0.0 |
| Construction | 28.9 | 0.2 | 28.7 | -7.9 | 0.2 | -8.1 | 44.8 | -0.5 | 45.3 | 37.5 | -5.0 | 42.5 |
| Manufacturing | -60.9 | 19.3 | -80.2 | 1.3 | 19.3 | -18.1 | -102.2 | 15.6 | -117.8 | -113.8 | 7.5 | -121.3 |
| Wholesale trade | 18.8 | 22.5 | -3.7 | 22.4 | 22.6 | -0.2 | 8.7 | 19.8 | -11.1 | 12.5 | 26.3 | -13.8 |
| Retail trade | 24.7 | 22.1 | 2.7 | 19.7 | 22.1 | -2.4 | 14.3 | 19.2 | -4.9 | 11.3 | 26.3 | -15.0 |
| Transportation and warehousing | 5.6 | 5.8 | -0.2 | 5.8 | 5.8 | 0.0 | 1.8 | 4.9 | -3.1 | 1.3 | 0.0 | 1.3 |
| Information and cultural industries | 11.8 | 13.5 | -1.7 | 12.8 | 13.5 | -0.7 | 5.8 | 12.0 | -6.2 | 6.3 | 15.0 | -8.8 |
| FIRE | 38.0 | 24.8 | 13.2 | 31.2 | 24.8 | 6.4 | 16.2 | 21.6 | -5.4 | 21.3 | 17.5 | 3.8 |
| Professional, scientific and technical services | 14.5 | 6.2 | 8.2 | 4.6 | 6.2 | -1.6 | 14.2 | 5.4 | 8.8 | 13.8 | 2.5 | 11.3 |
| ASWMRS | 11.4 | 0.5 | 10.8 | -7.3 | 0.5 | -7.8 | 10.9 | 0.5 | 10.4 | 12.5 | 1.3 | 11.3 |
| Arts, entertainment and recreation | 0.4 | -1.2 | 1.6 | -2.5 | -1.2 | -1.3 | 1.1 | -1.0 | 2.1 | 1.3 | -1.3 | 2.5 |
| Accommodation and food services | -0.6 | 0.4 | -0.9 | 1.8 | 0.4 | 1.4 | 0.5 | 0.3 | 0.2 | -1.3 | 3.8 | -5.0 |
| Other Private Services | 6.2 | 6.4 | -0.3 | 6.5 | 6.4 | 0.1 | 9.0 | 5.6 | 3.5 | 27.5 | 12.5 | 15.0 |

Notes:

1) TC - Total Contribution; WSE - Within-Sector Effect; RE - Joint reallocation effects (reallocation level effect + reallocation growth effect).

2) FIRE - Finance, insurance, real estate, rental and leasing; ASWMRS - Administration and support, waste management and remediation services;

3) Almon and Tang (GEAD) results from Almon and Tang (2011).